MINIMIZING INTERFERENCE POTENTIAL AMONG MOVING ENTITIES

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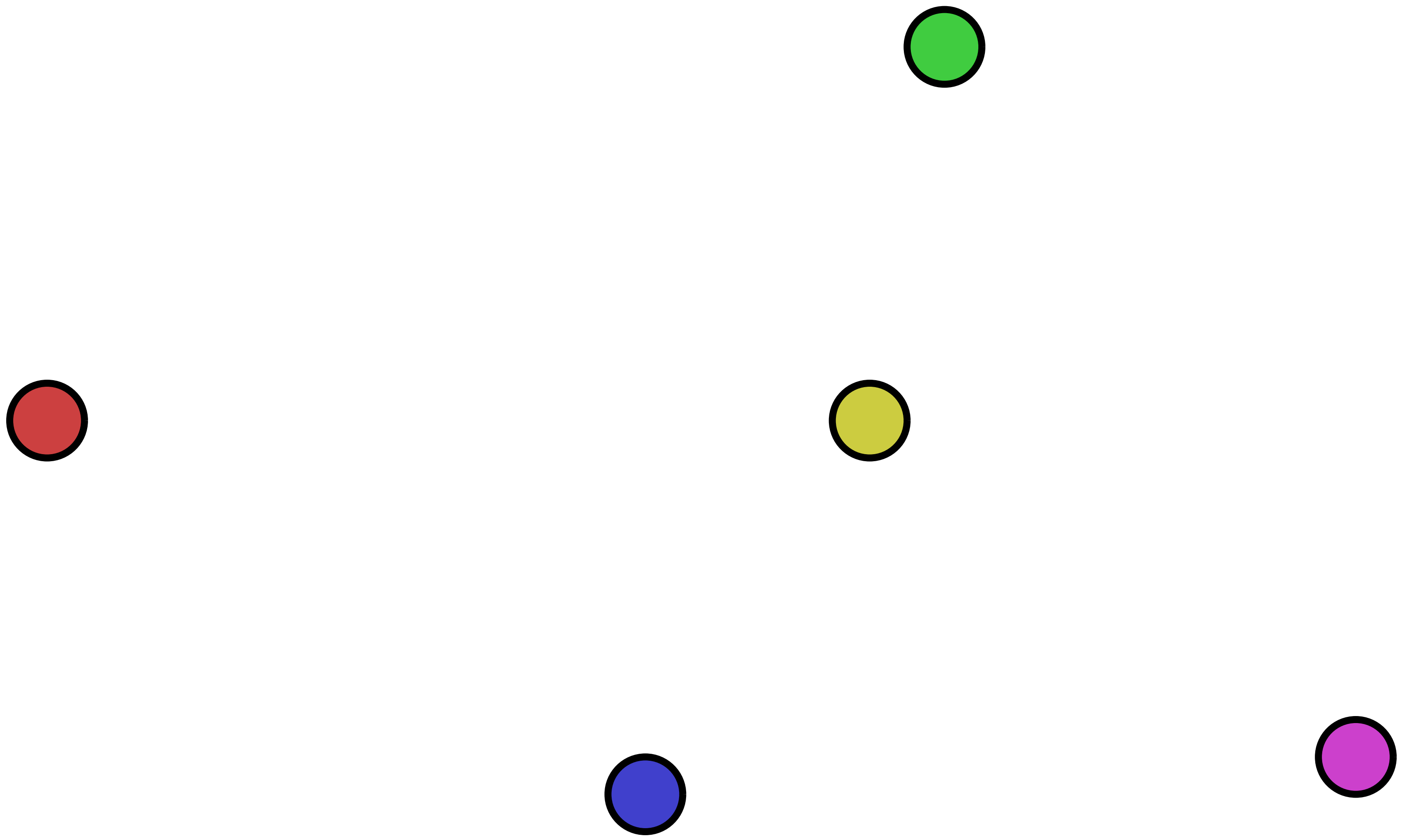
Moving entities

- Airplanes
- Smart phones
- Mobile transmitters

Equi-sized balls in $\mathbb{R}^d$ that move with bounded speed.

Suppose we know their positions now.



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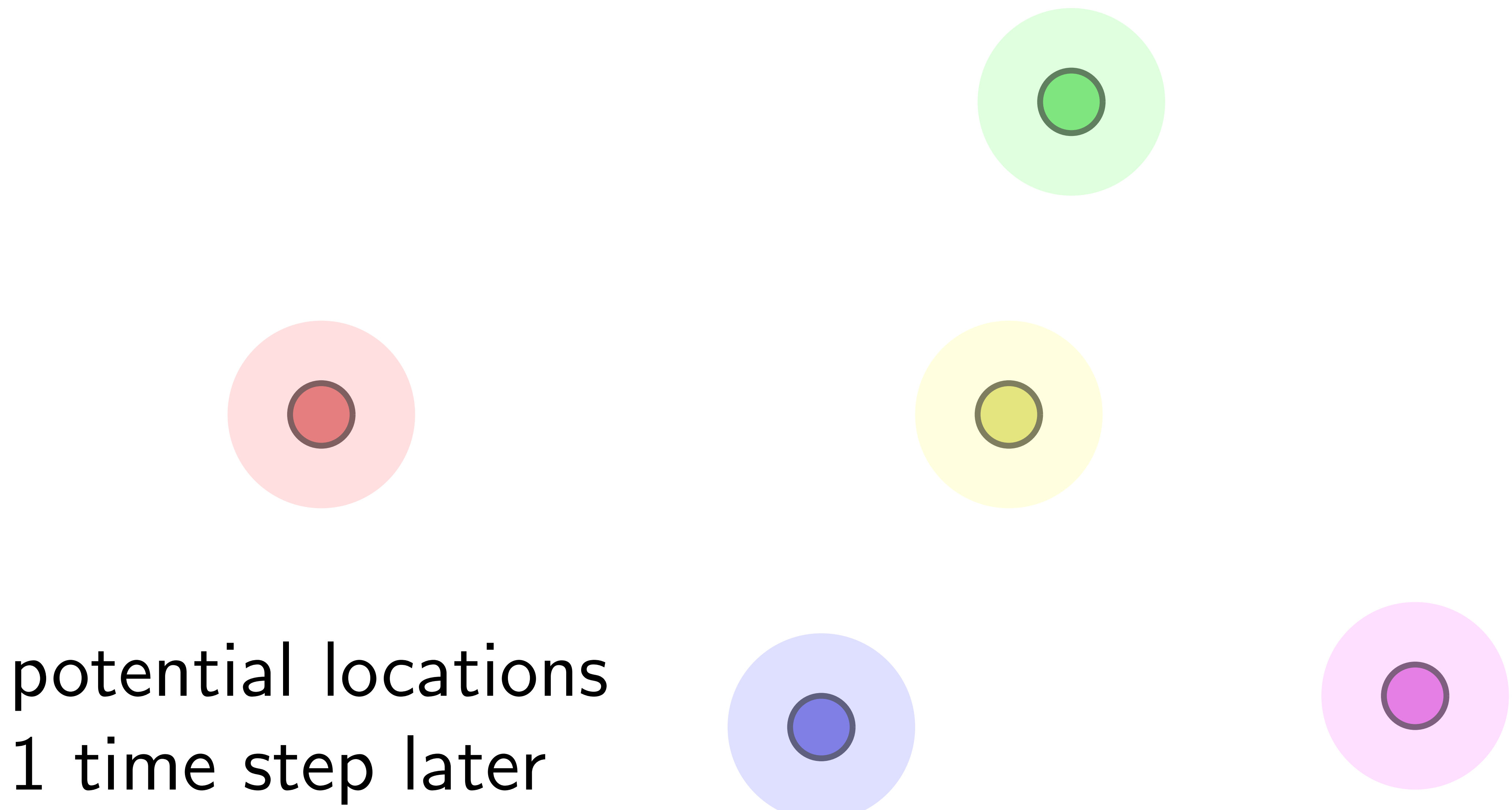

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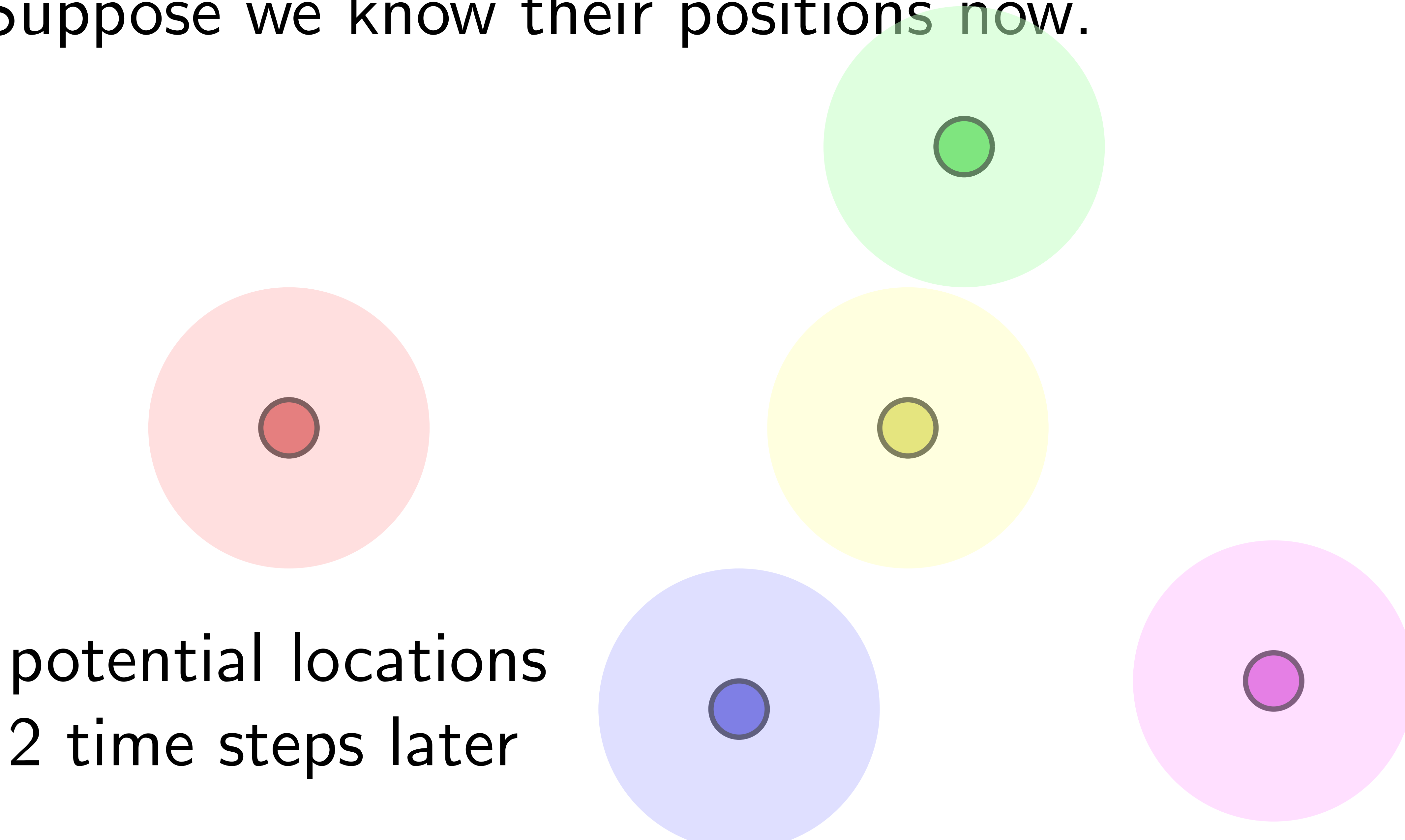

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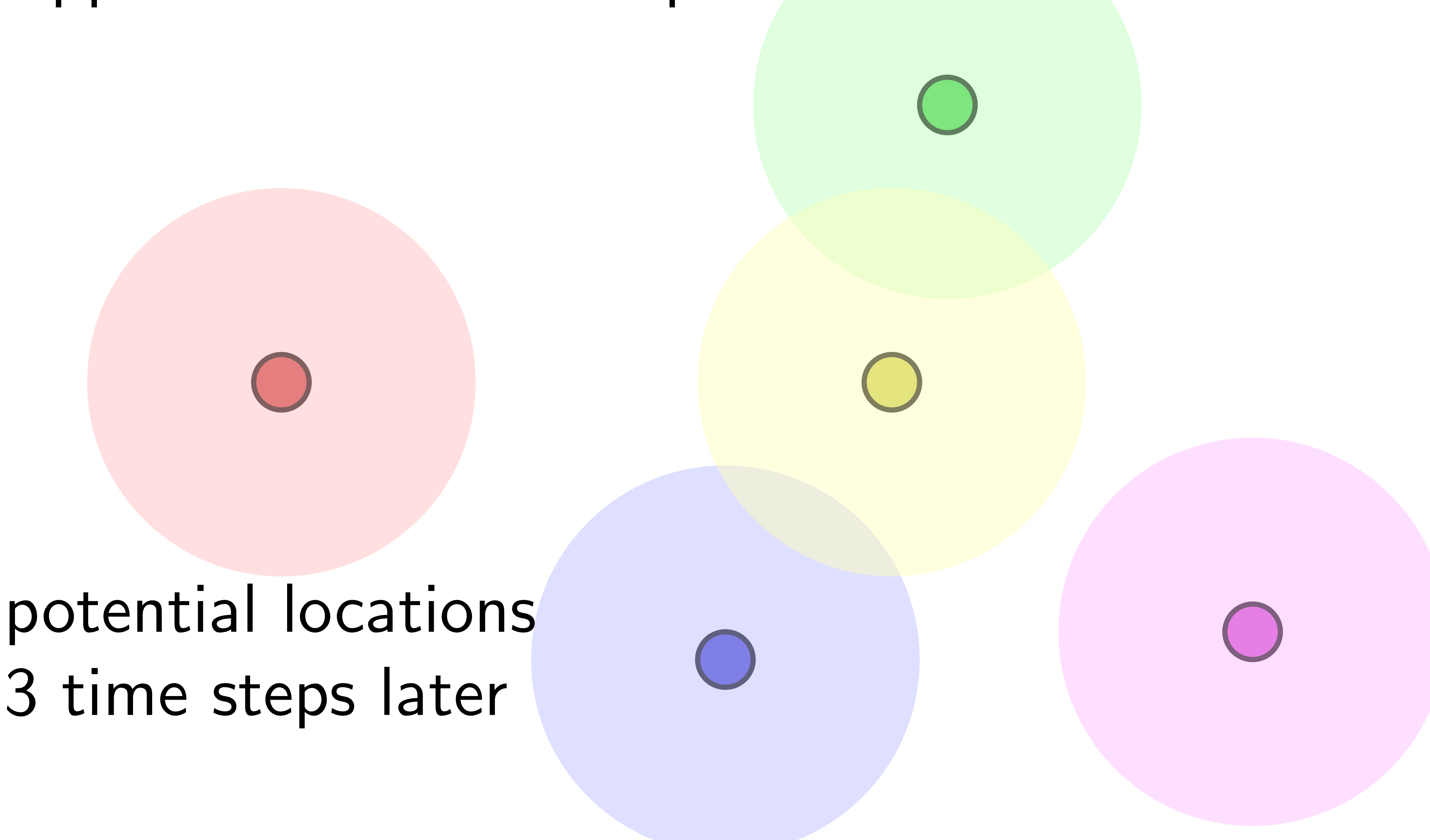

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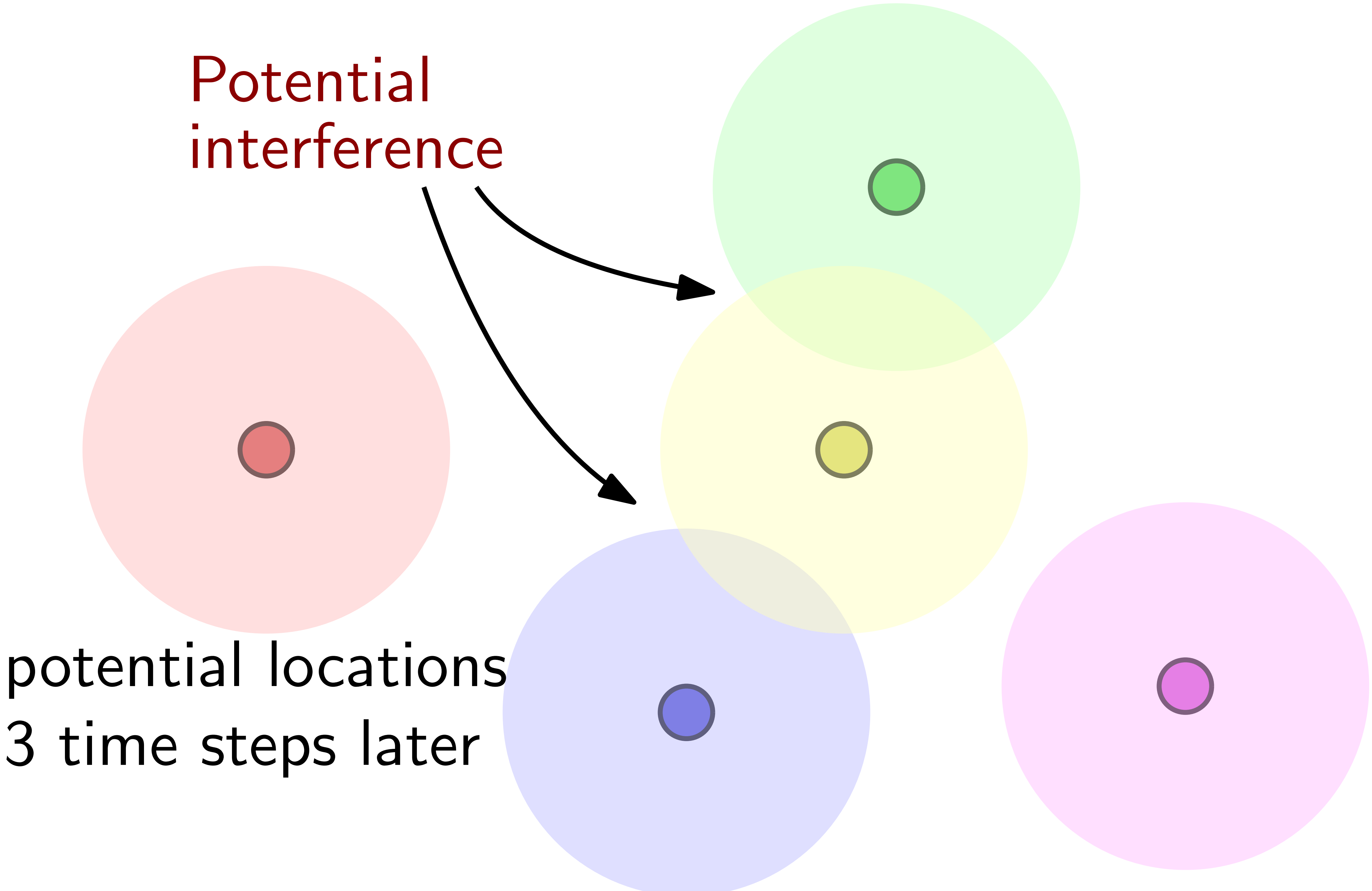

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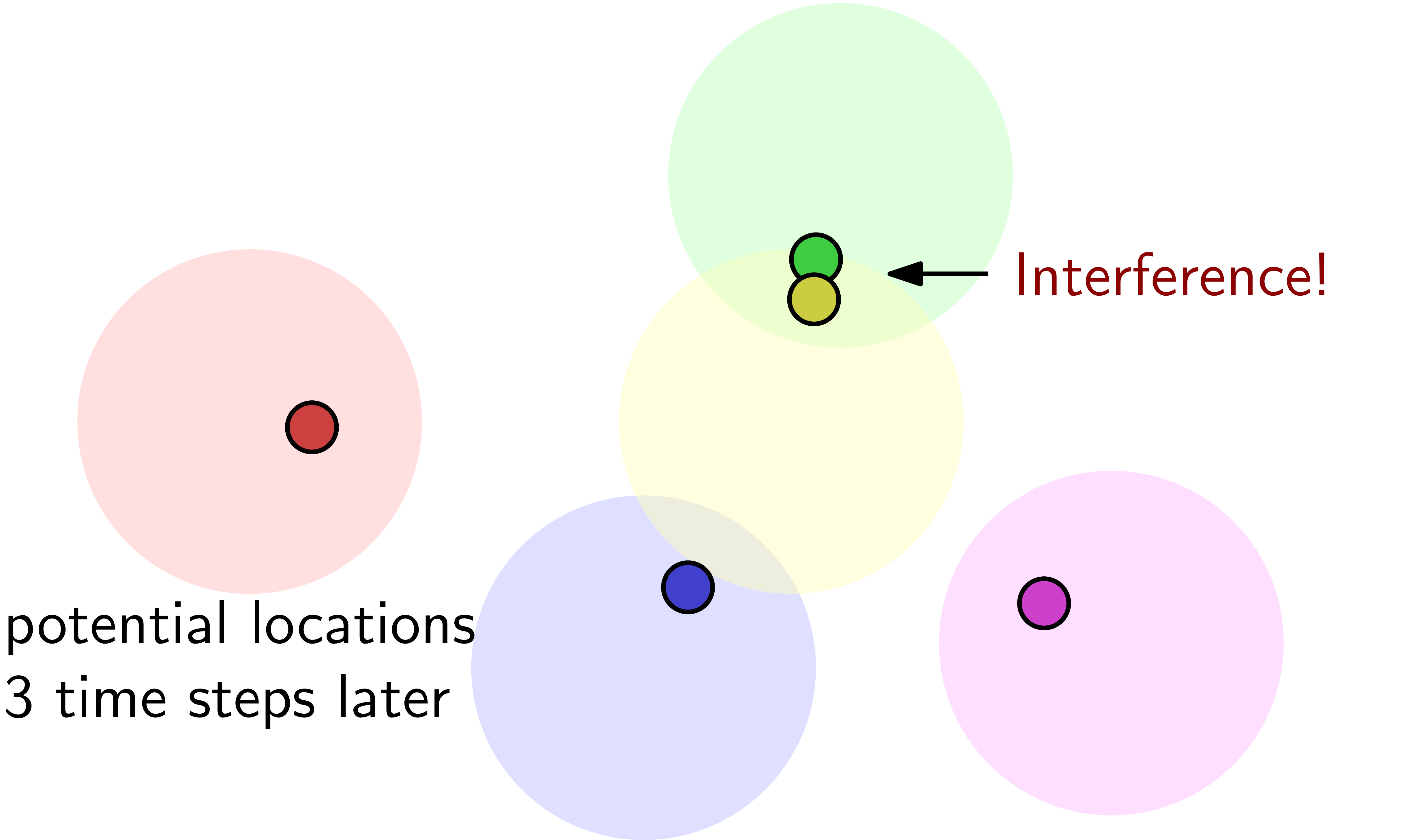

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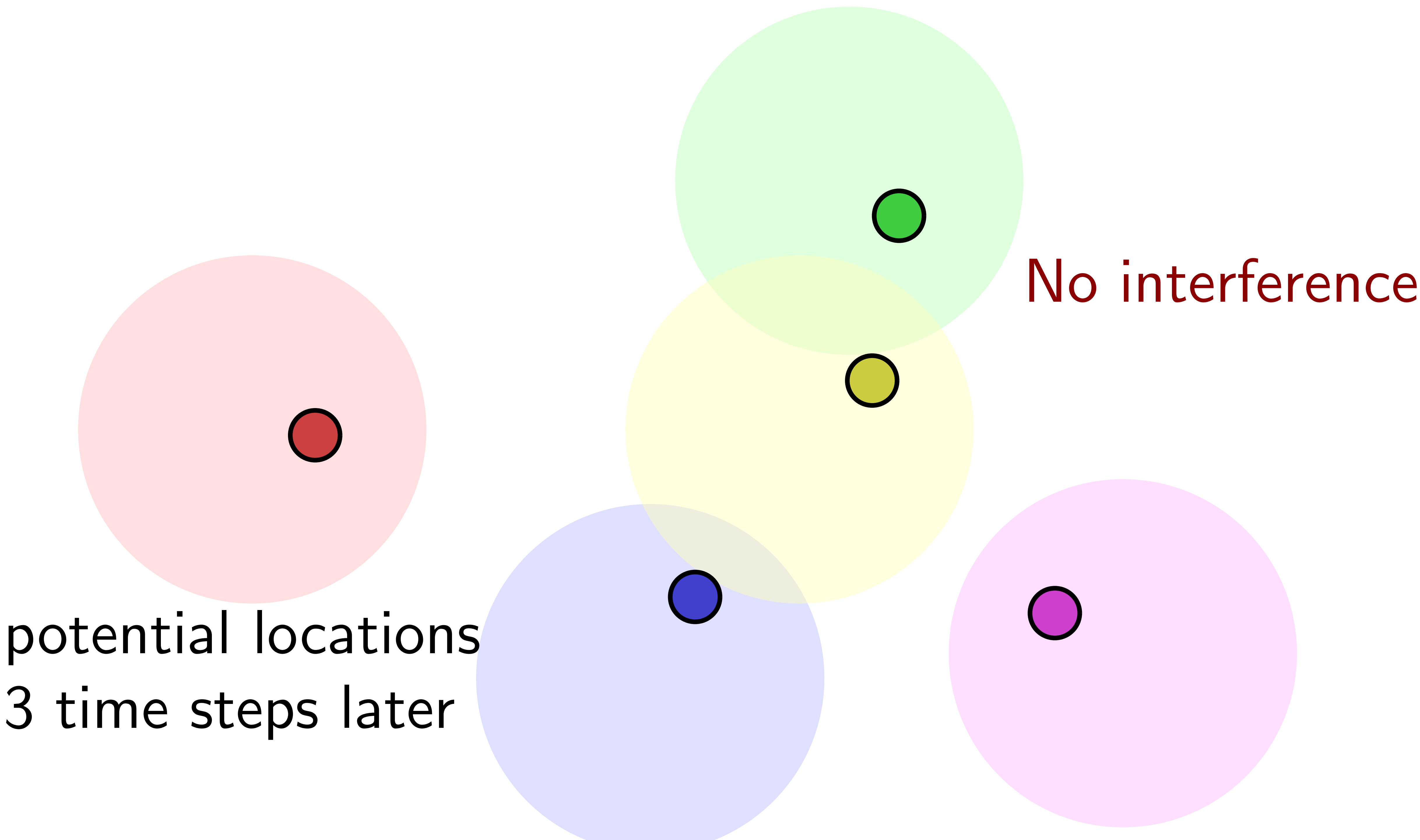
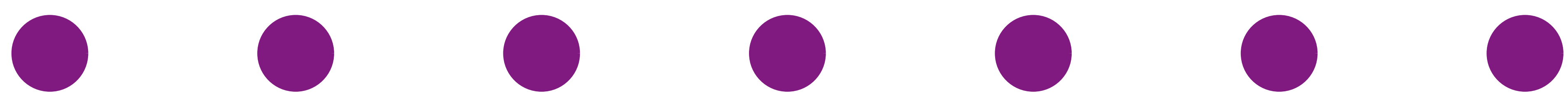


Goal: Minimize potential interference

How: Query one entity per time step for its position



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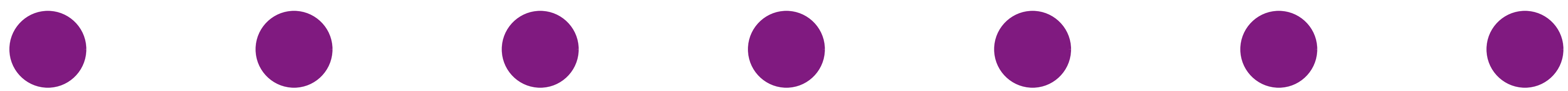
$t = 0$



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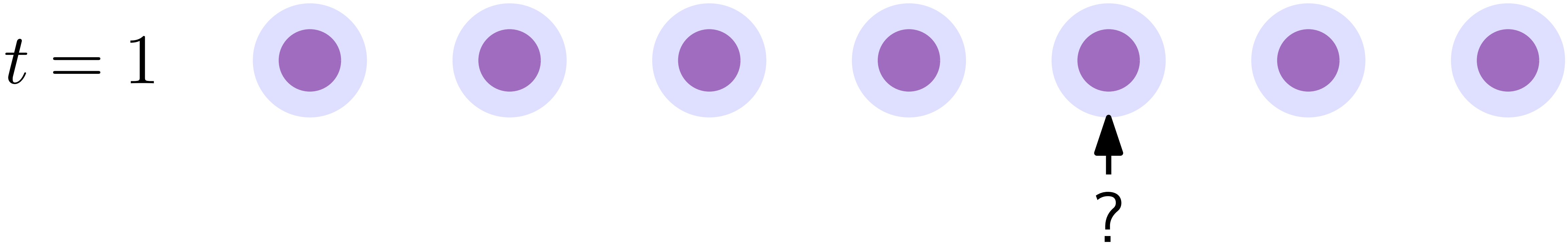

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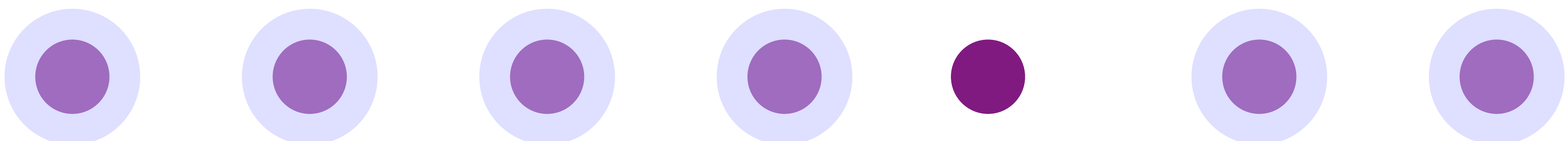
$t = 1$



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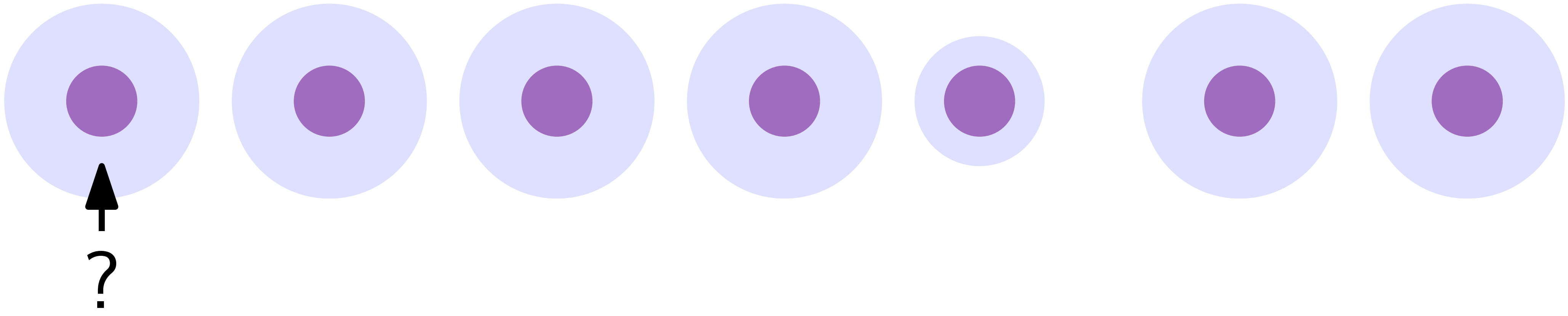
$t = 2$



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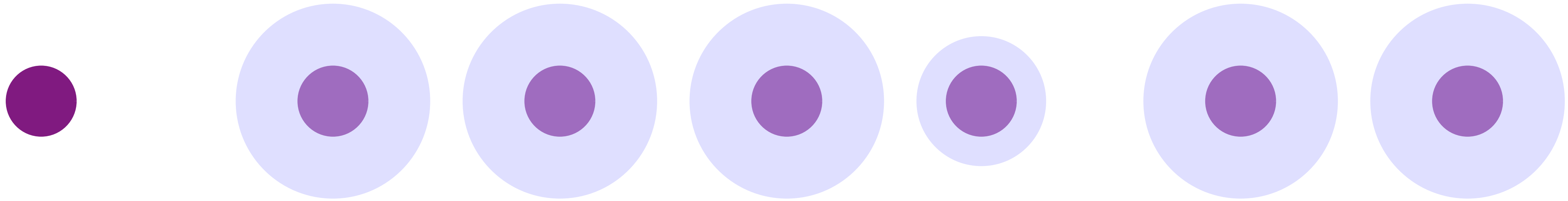

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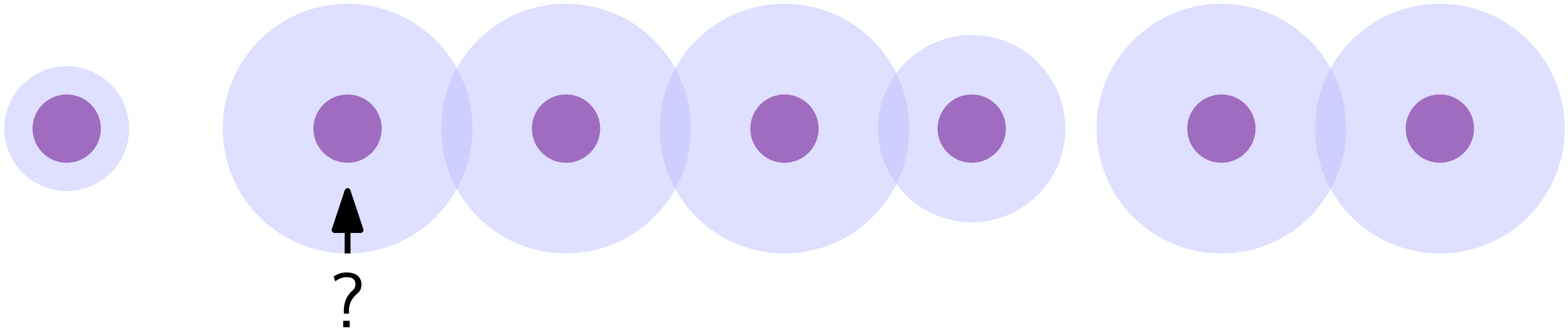
$t = 3$



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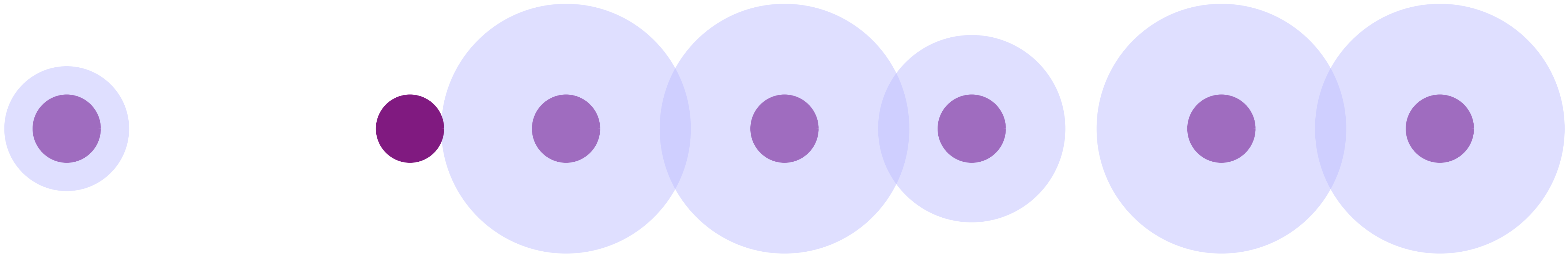

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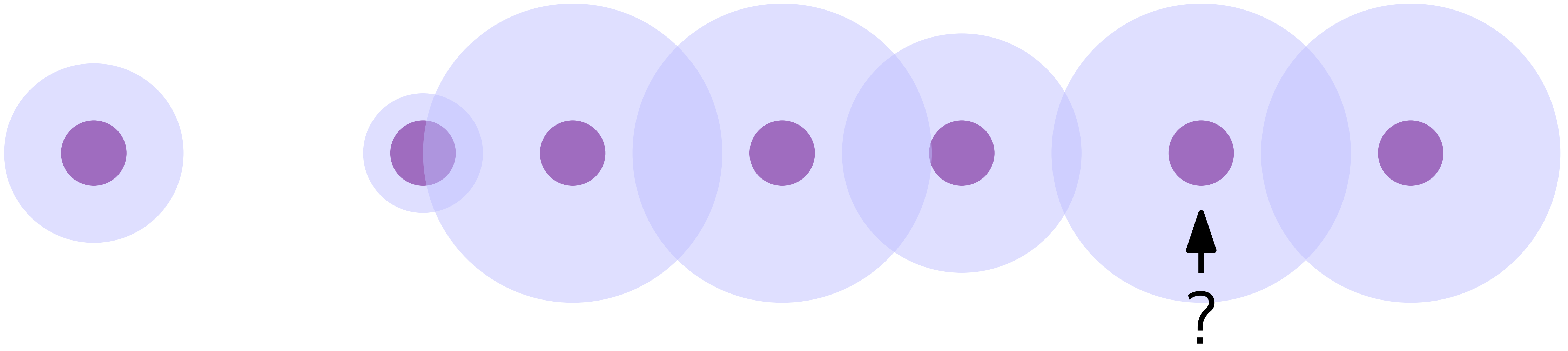
$t = 4$



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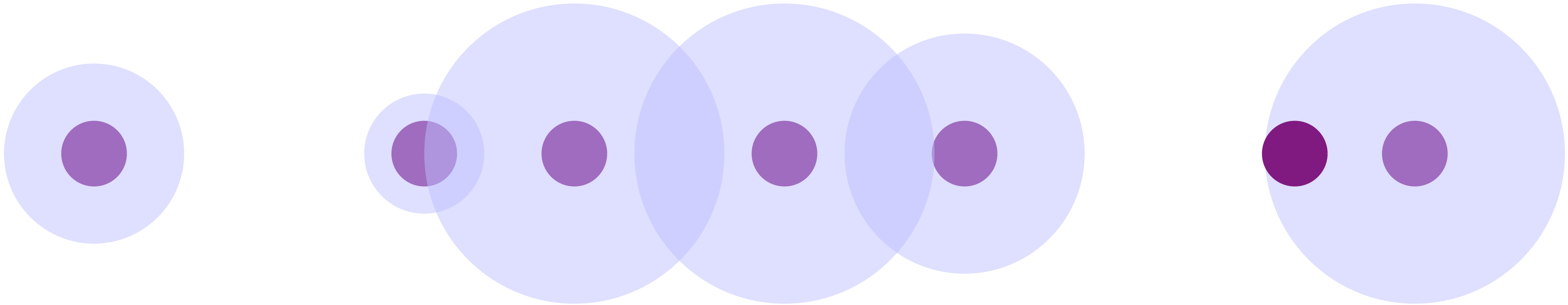

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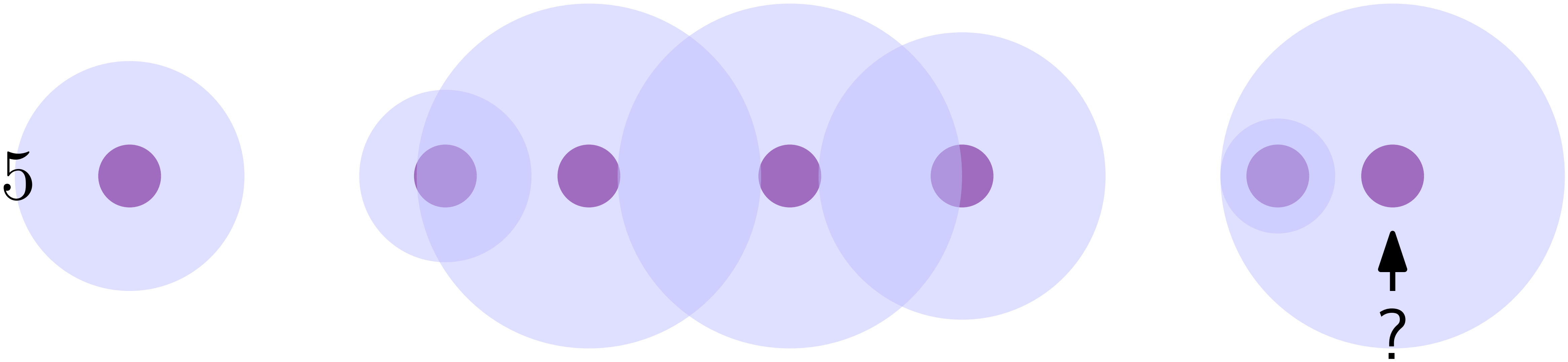
$t = 5$



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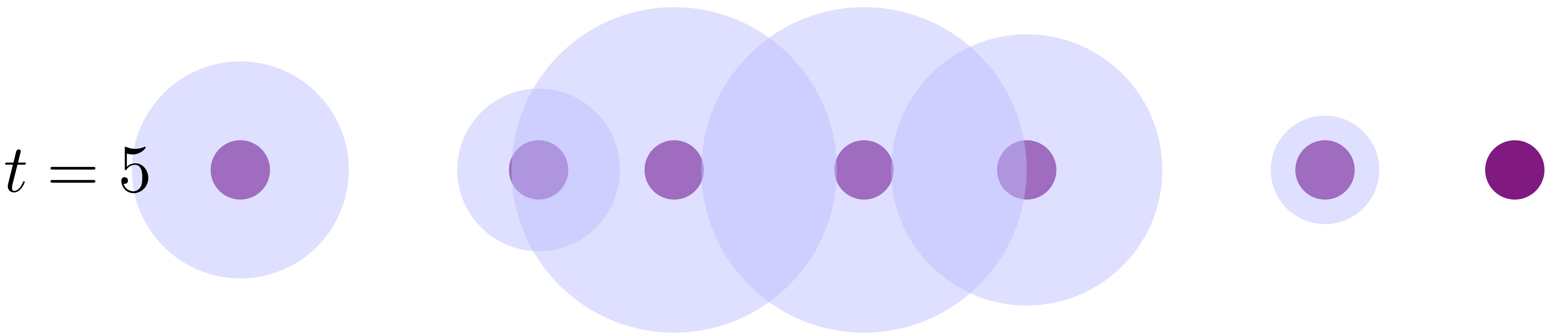

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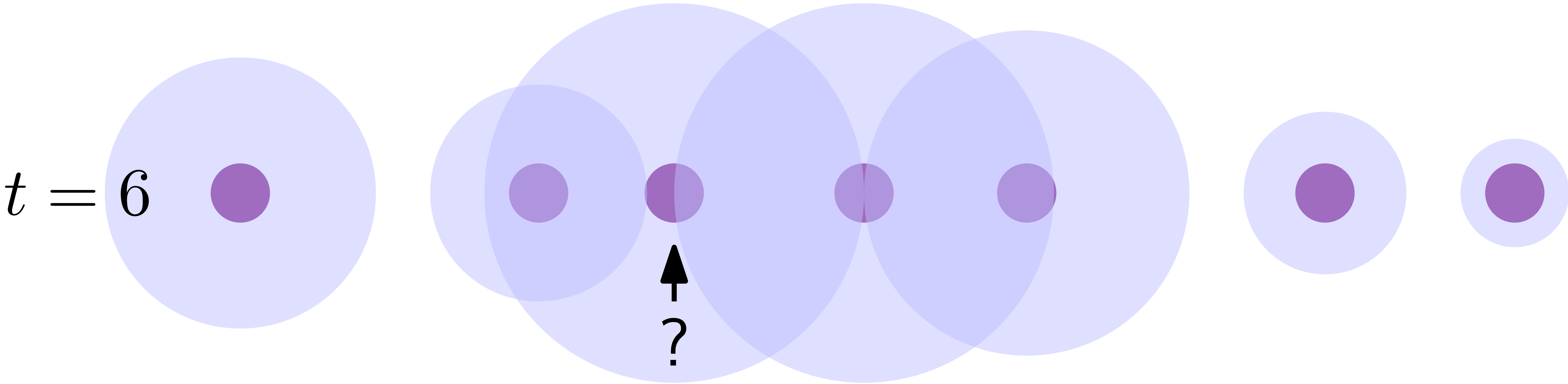

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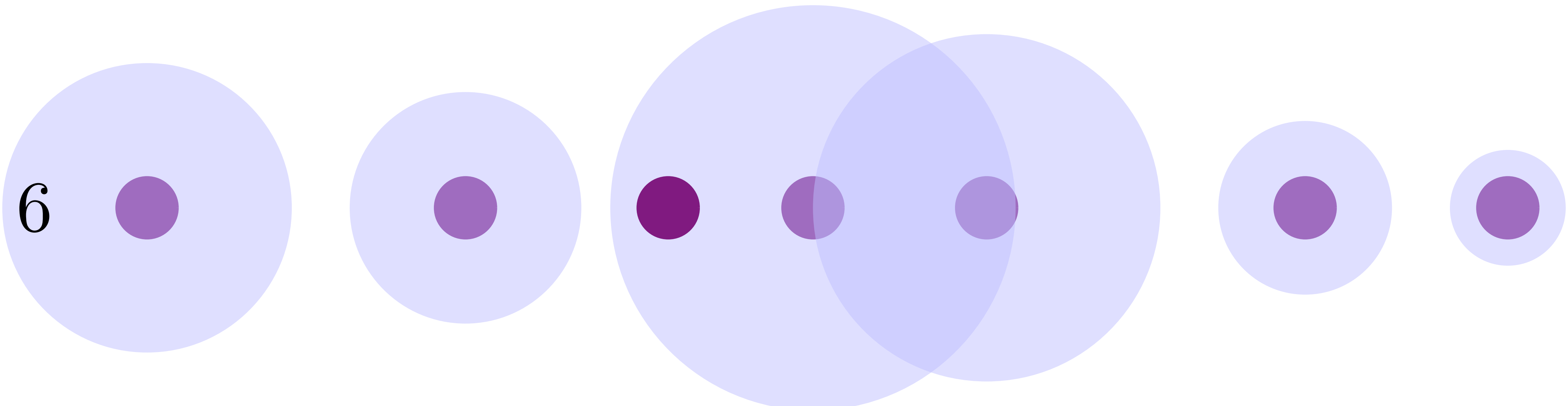
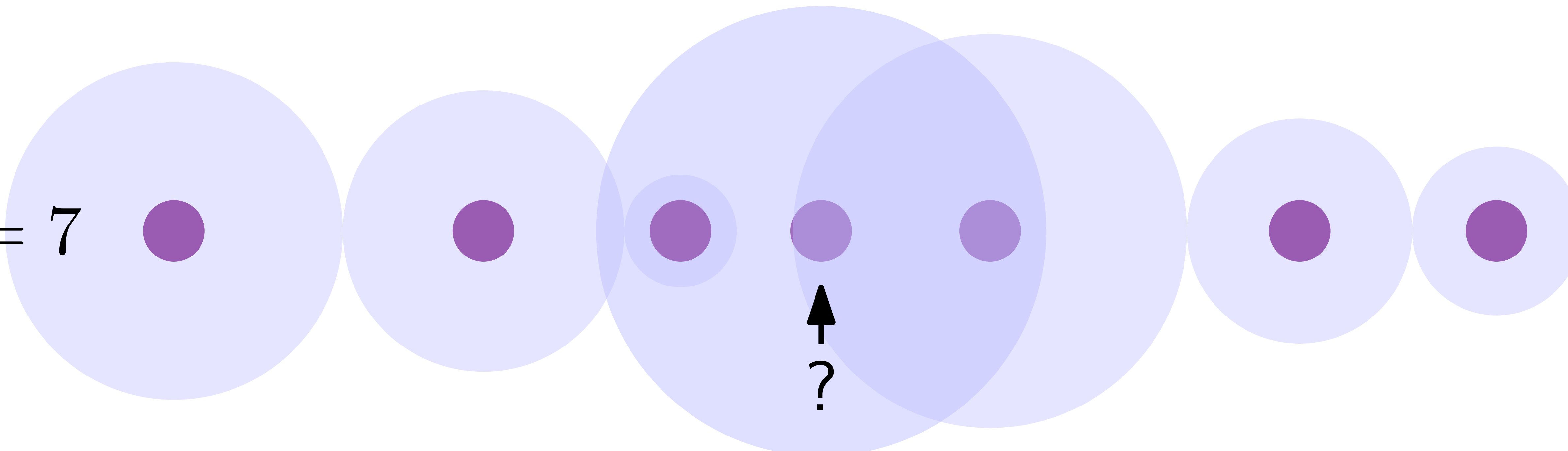
$t = 6$



Goal: Minimize potential interference

How: Query one entity per time step for its position

$t = 7$



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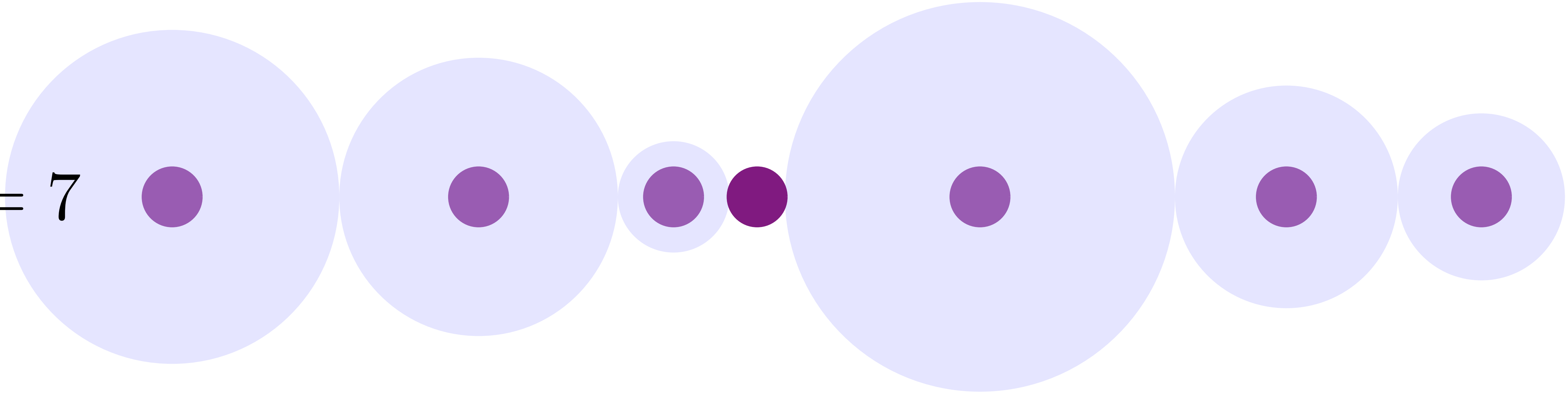

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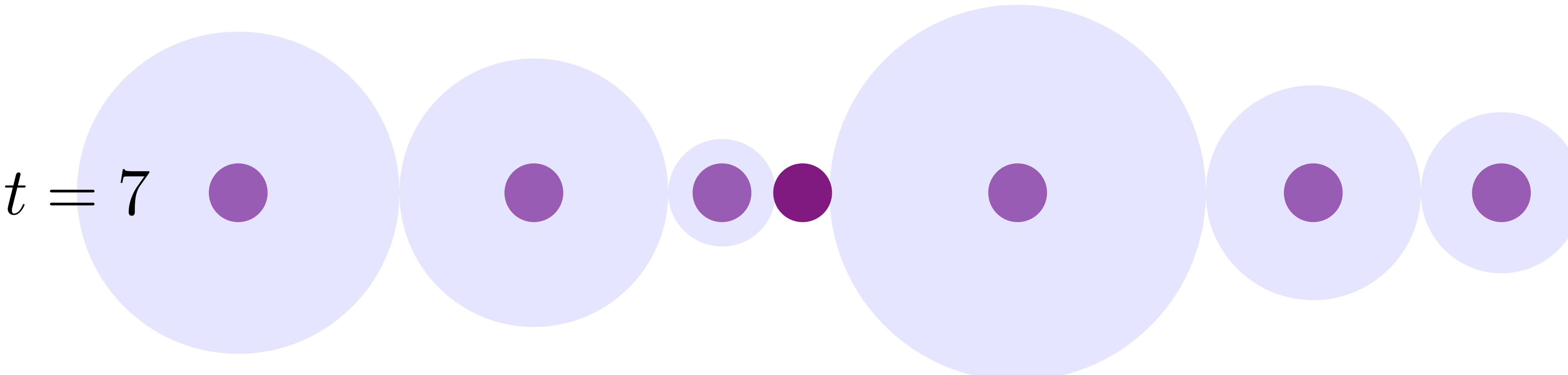

Related Goals:

Minimize queries to calculate some function of entities

Kahan '91, Erlebach&Hoffmann '15 (survey)

Query reveals partial information

Kirkpatrick '09 (hyperbolic dovetailing)

Measures of potential interference

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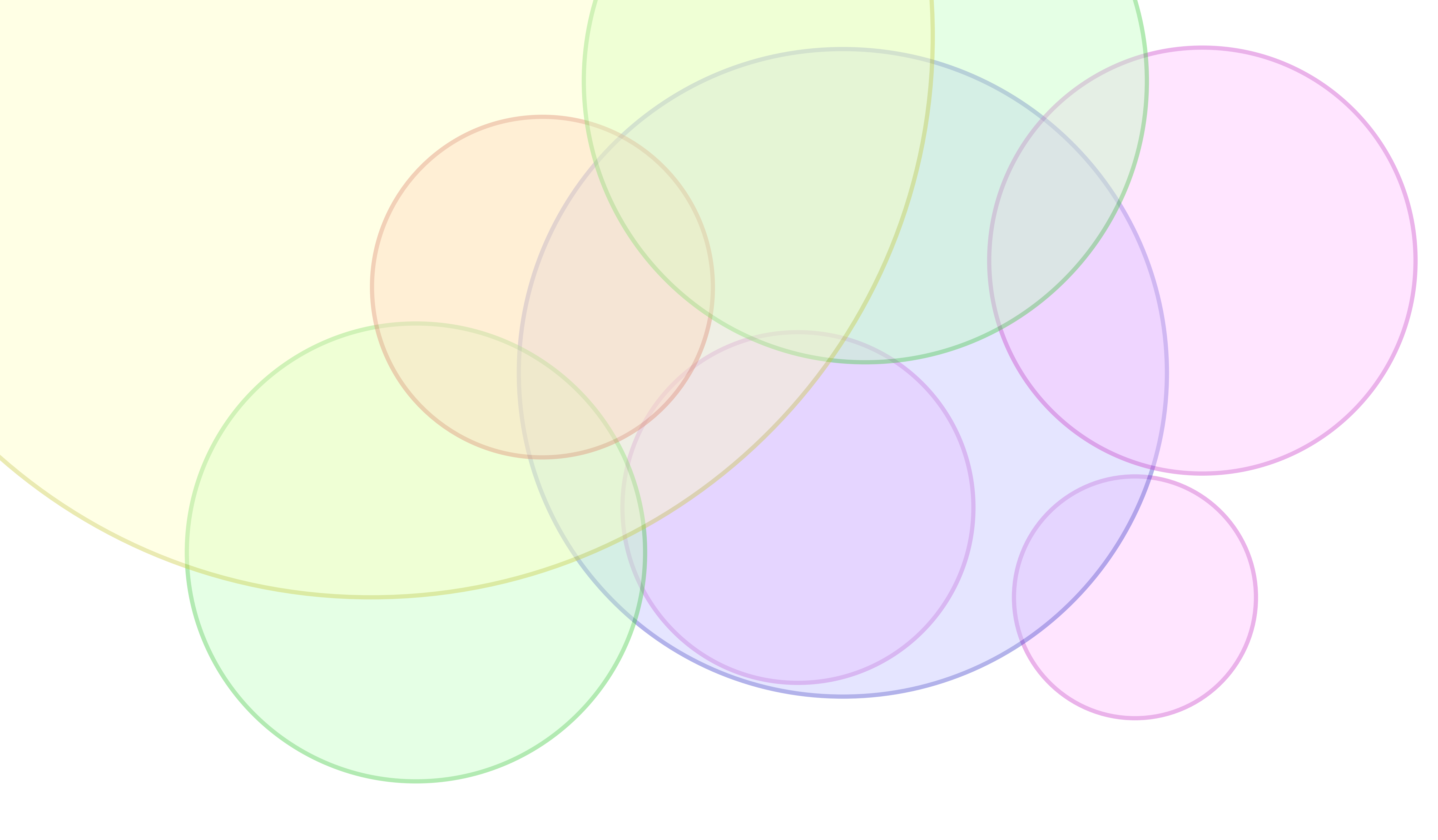
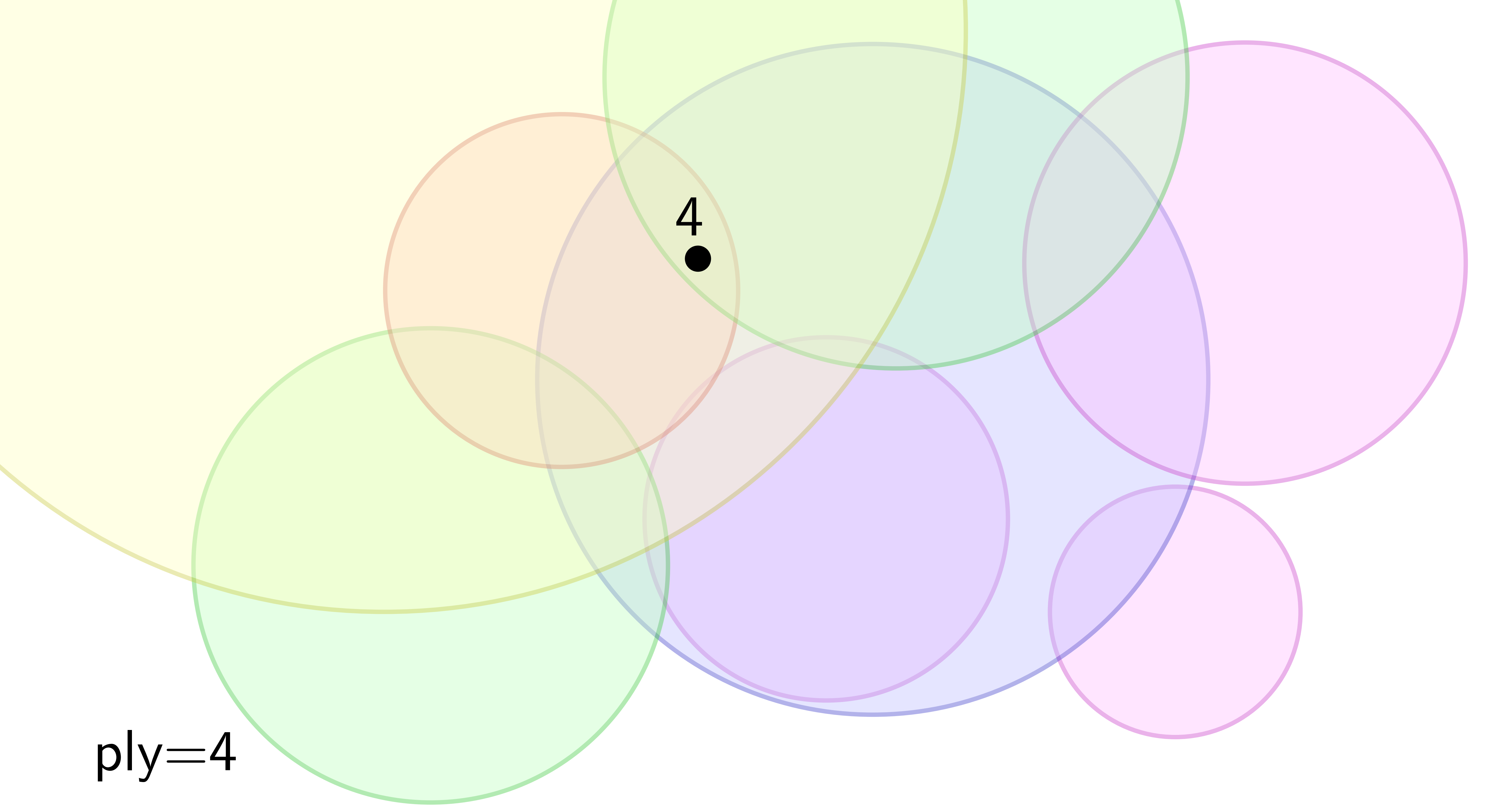
Ply: max number of regions covering any point



Measures of potential interference

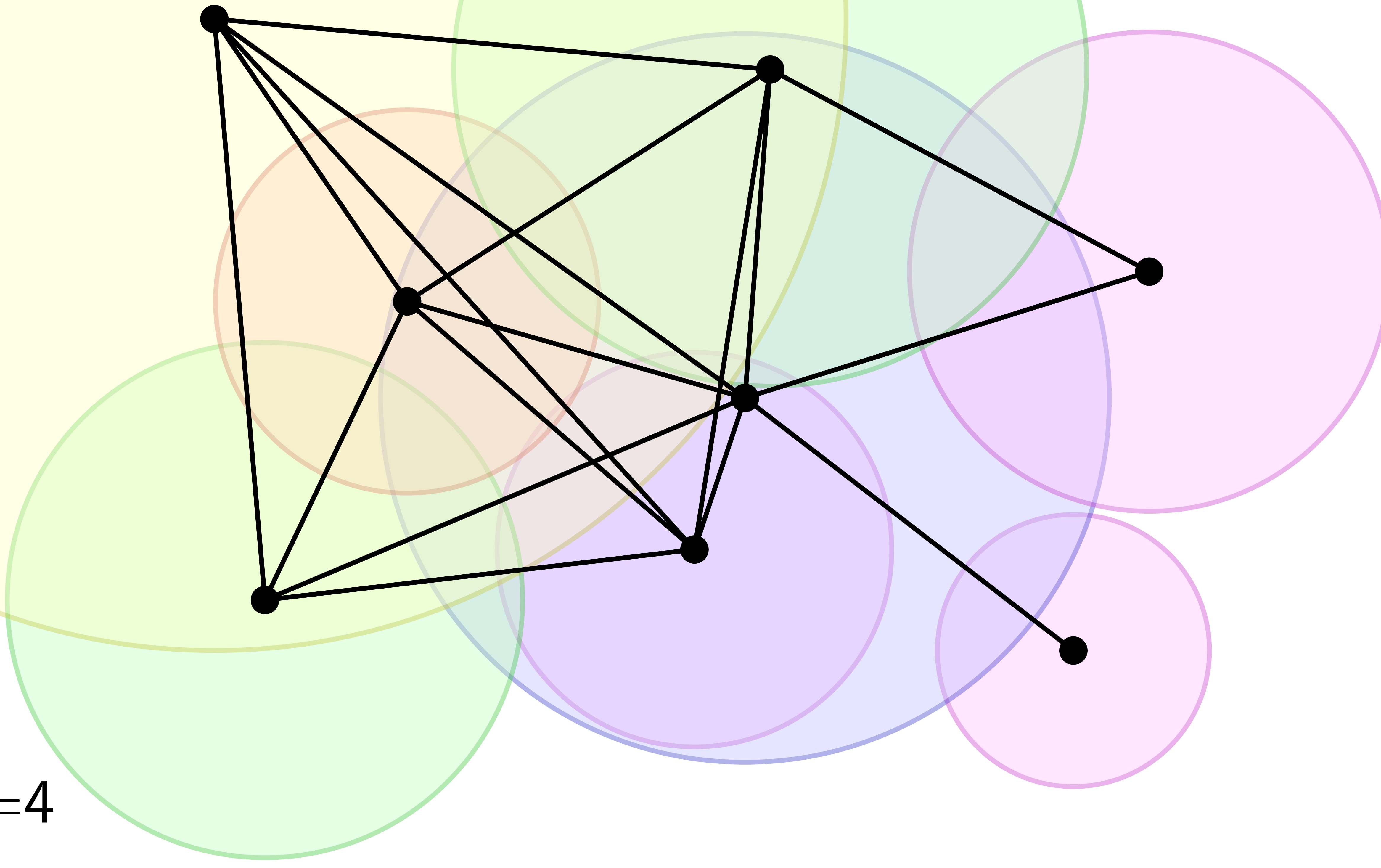
Ply: max number of regions covering any point $= 4$



ply$=$4

Measures of potential interference

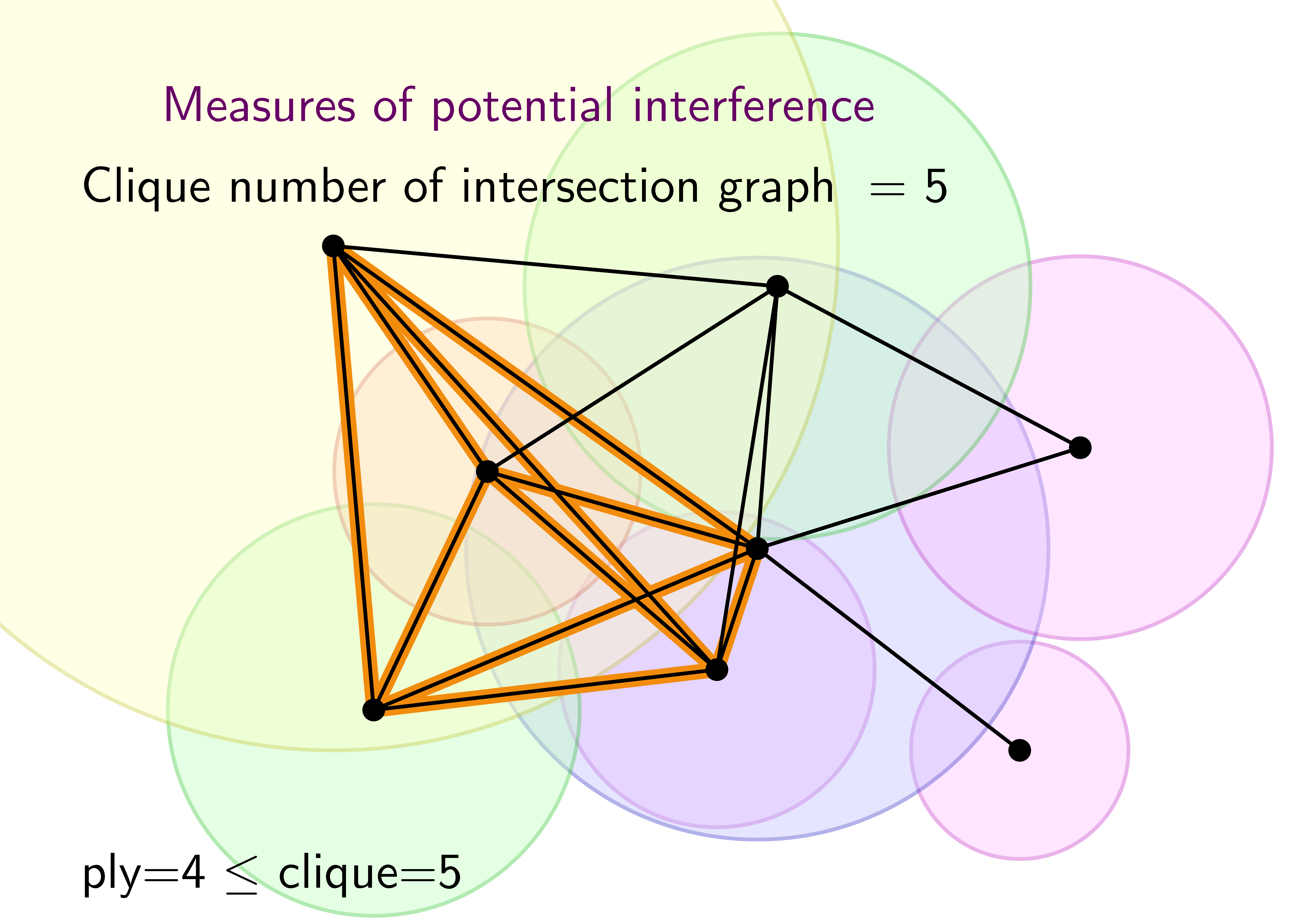
Clique number of intersection graph



ply=4

Measures of potential interference

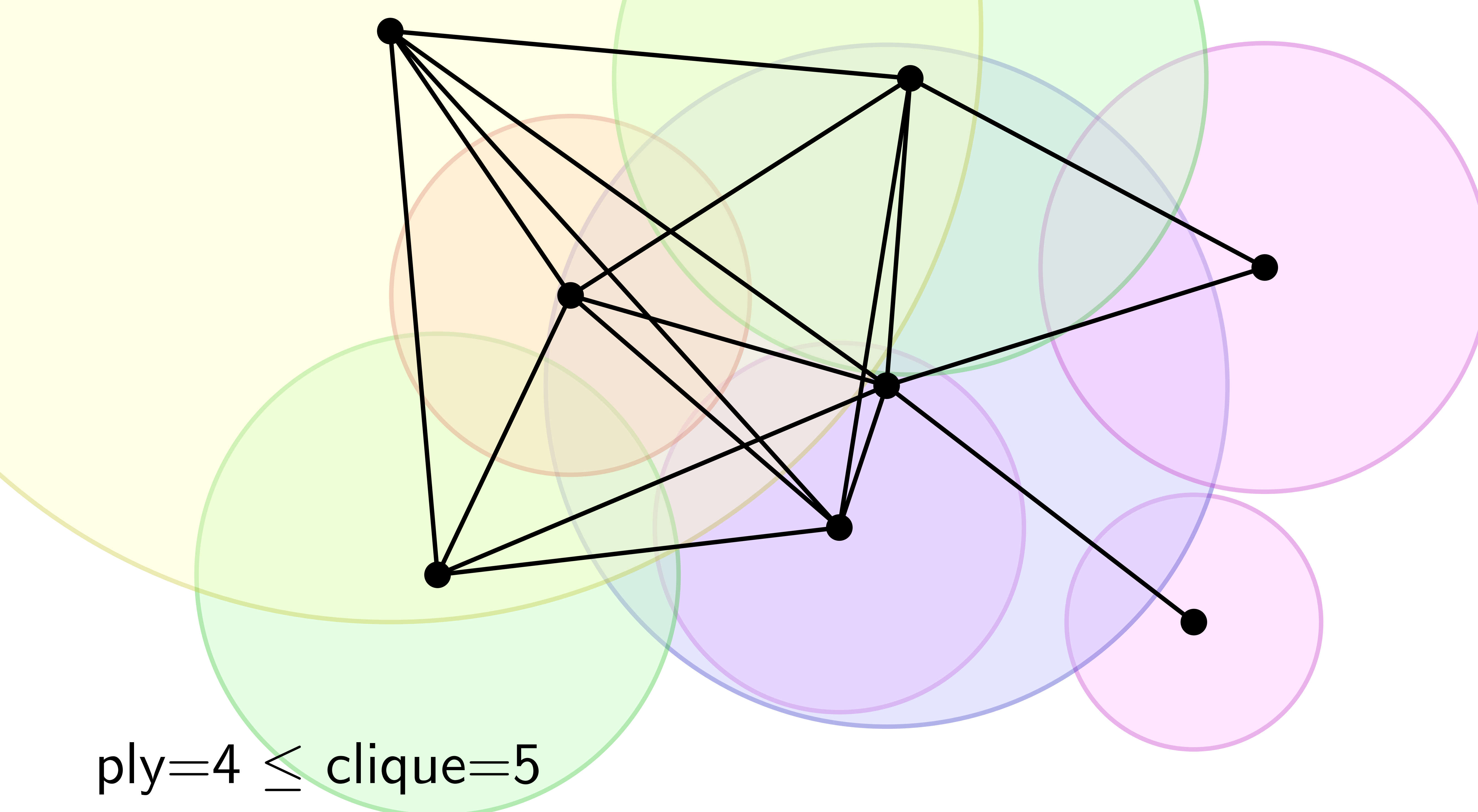
Clique number of intersection graph $= 5$



ply$=4 \leq$ clique$=5$

Measures of potential interference

Chromatic number of intersection graph



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Measures of potential interference

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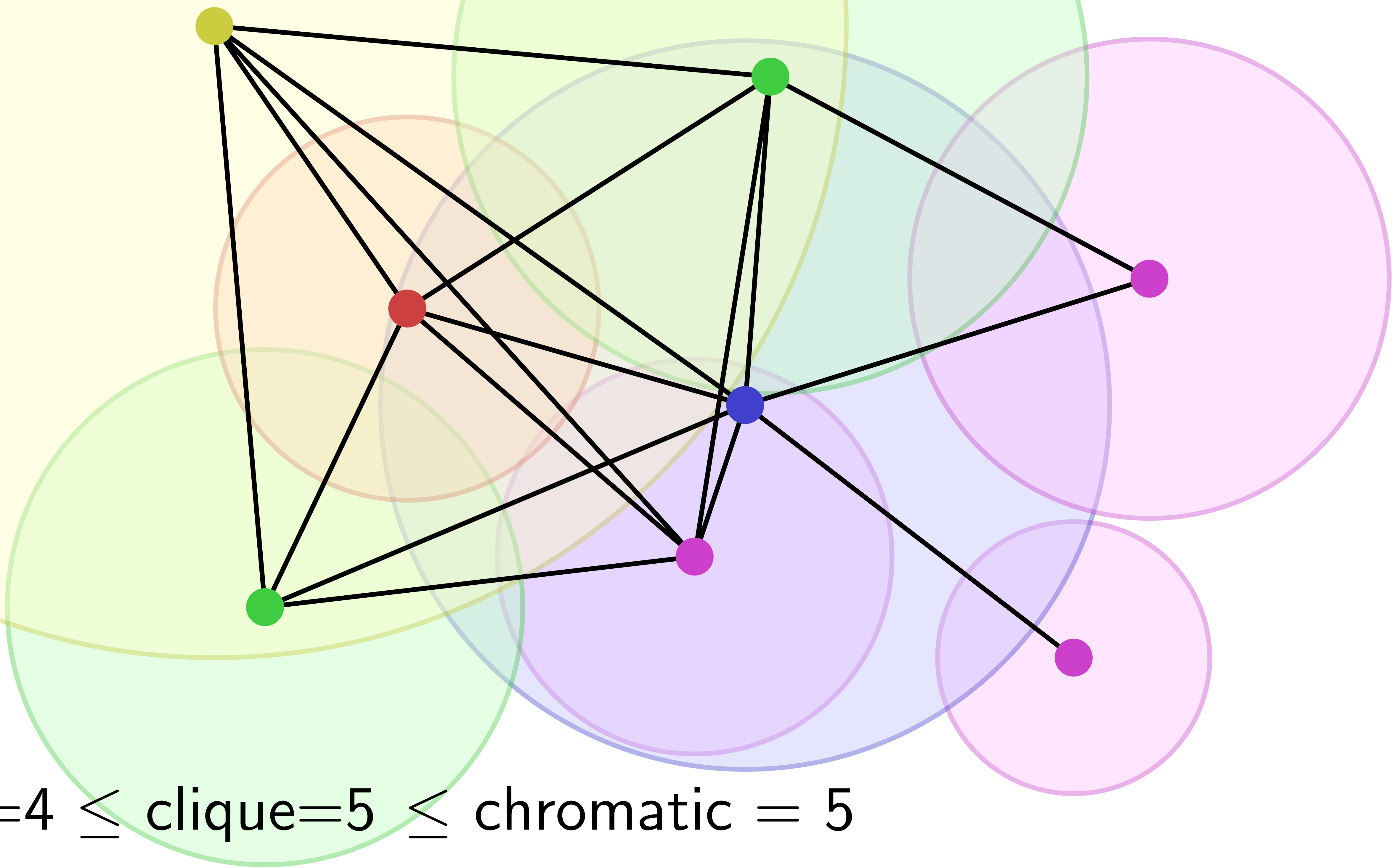

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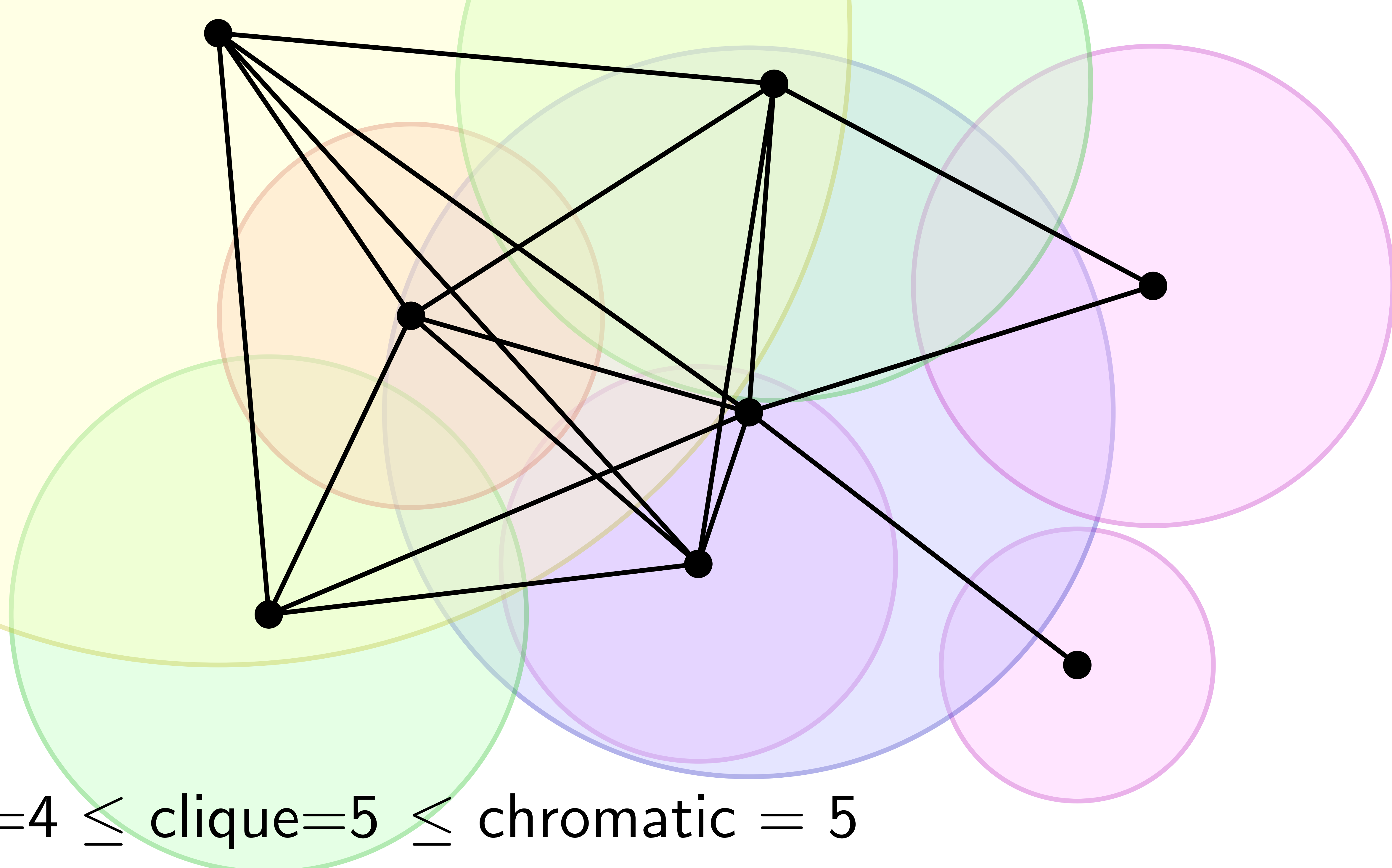
Measures of potential interference

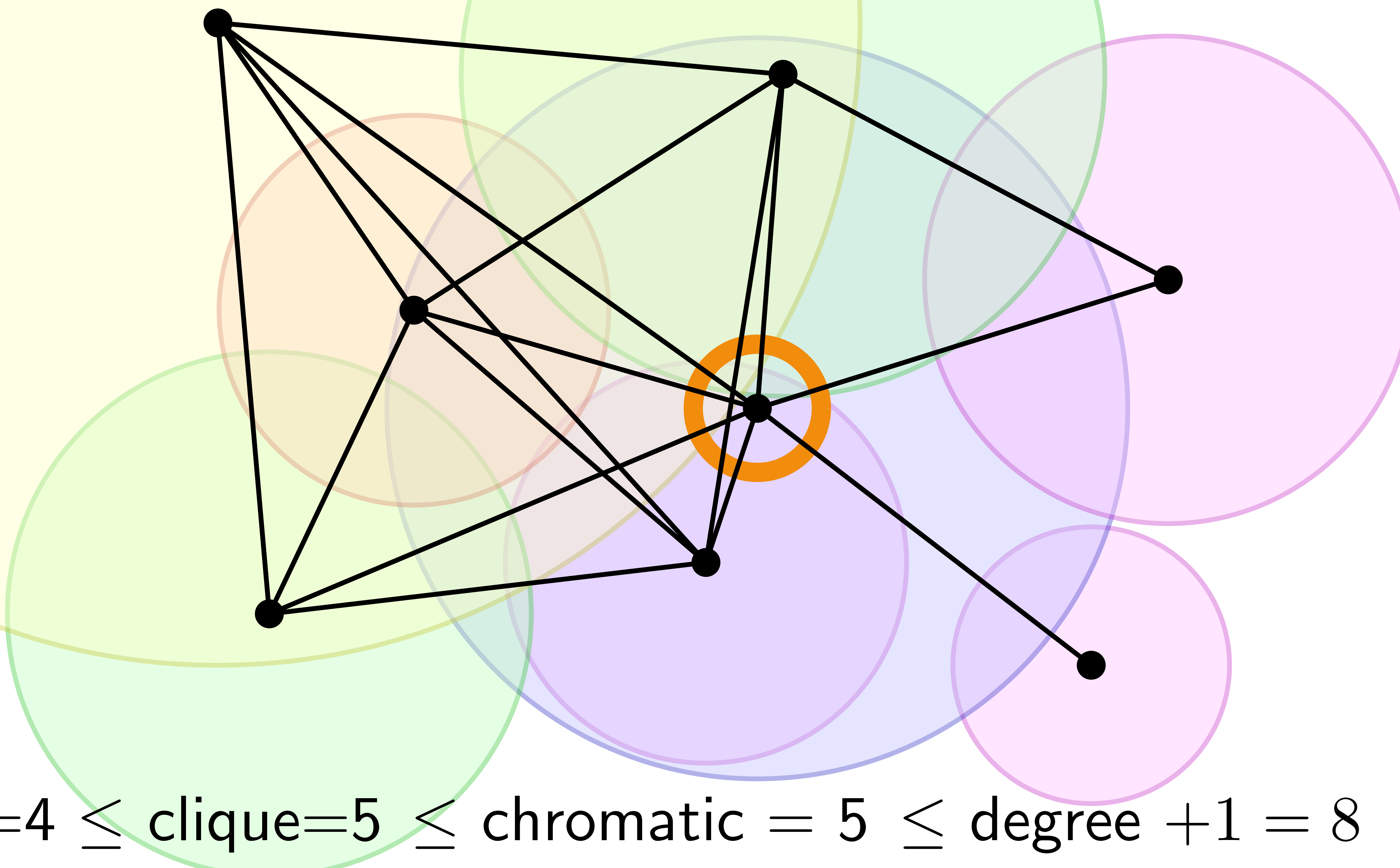
Max degree of intersection graph



ply=4 $\leq$ clique=5 $\leq$ chromatic = 5

Measures of potential interference

Max degree of intersection graph $= 7$



ply=4 $\leq$ clique=5 $\leq$ chromatic $= 5 \leq$ degree $+1 = 8$

Our main result

Theorem. *For* n *entities, the Adaptive Bucket Strategy keeps* **degree** $O(x^*)$ *during any time interval* $T = [a, b]$ *where* x^* *is the maximum* **ply** *observed by the best strategy over the interval* $[a - |T|, b]$, *provided* $|T| > c_d n$.

Query to reduce interference potential

at a target time [E,K,Löffler,Staals '16]

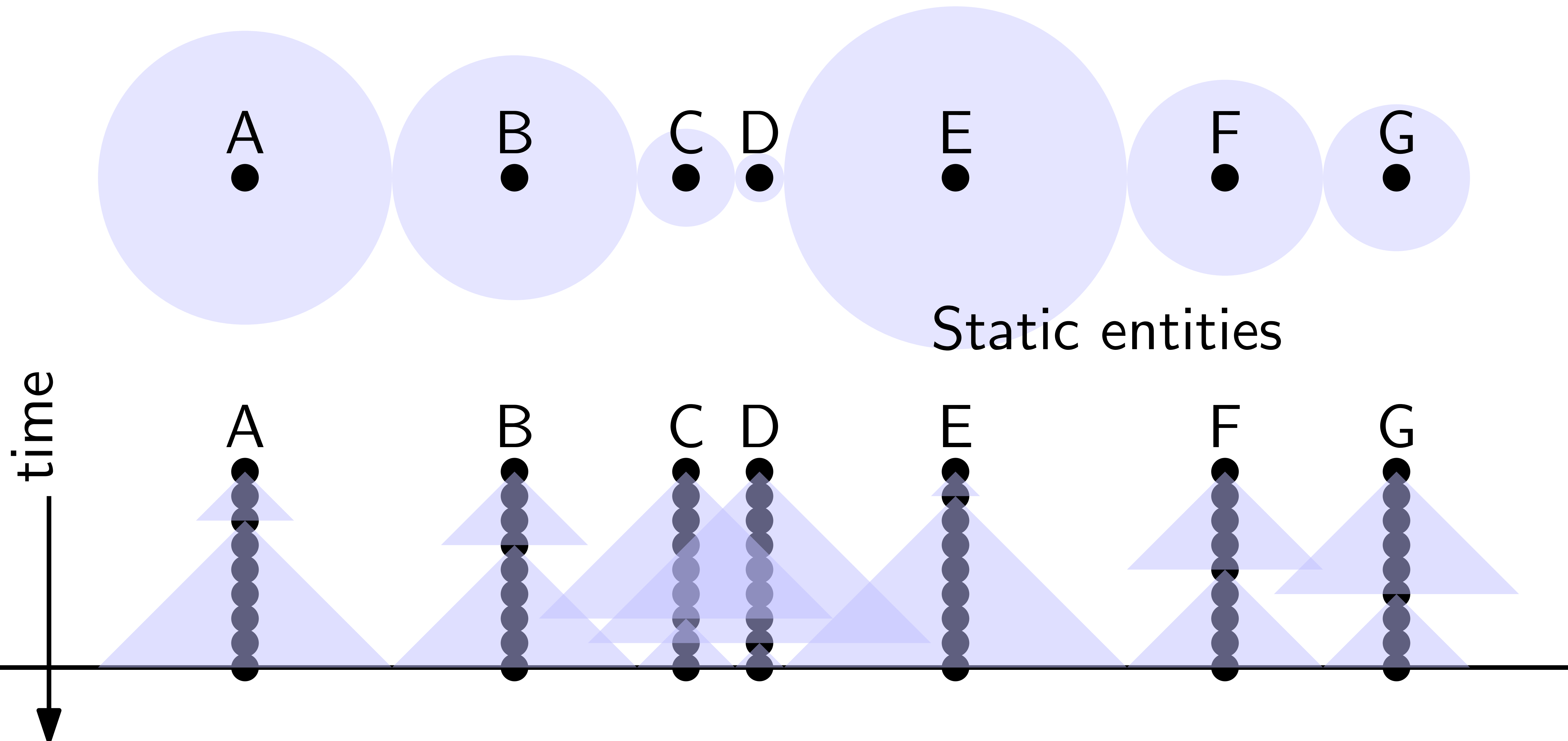
Static entities

Query to reduce interference potential

at a target time [E,K,Löffler,Staals '16]

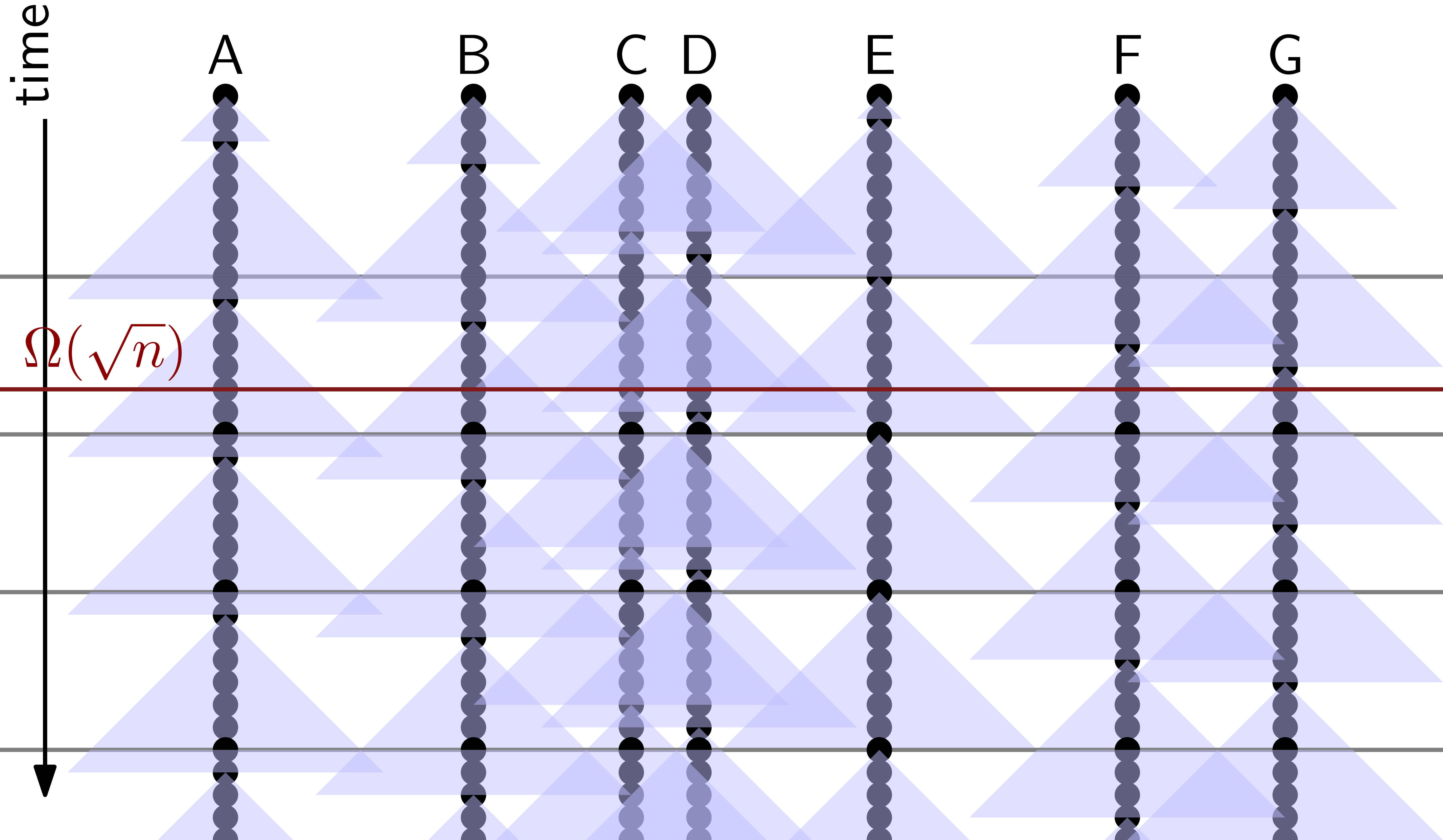
Optimal: Query EABFGCD

NP-complete even for static entities



Query to reduce interference potential

Repeat at a target time [E,K,Löffler,Staals '16]



Query to reduce interference potential

at **all** times [this paper]

Let's start with static point entities $e_1, \ldots, e_n$.

Query to reduce interference potential

at **all** times [this paper]

Let's start with static point entities $e_1, \ldots, e_n$.

	e_1	e_2	e_3	e_4	e_5	e_6	e_7
$r_i(1)$	11	7	3	3	8	7	7

x-**radius** $r_i(x)$ = distance from e_i to its x^{th}-nearest entity

To keep **degree** $< x$, must query e_i every $r_i(x)$ steps.

If $\underbrace{\sum_{i=1}^{n} \frac{1}{r_i(x)}}_{x\text{-density}} > 1$ this is impossible.

Observation

Unavoidable interference

The **congestion** of a set $\mathcal{E}$ of entities is

$$x_{\mathcal{E}} = \max\{x | \sum_{e_i \in \mathcal{E}} \frac{1}{r_i(x)} > 1\}.$$

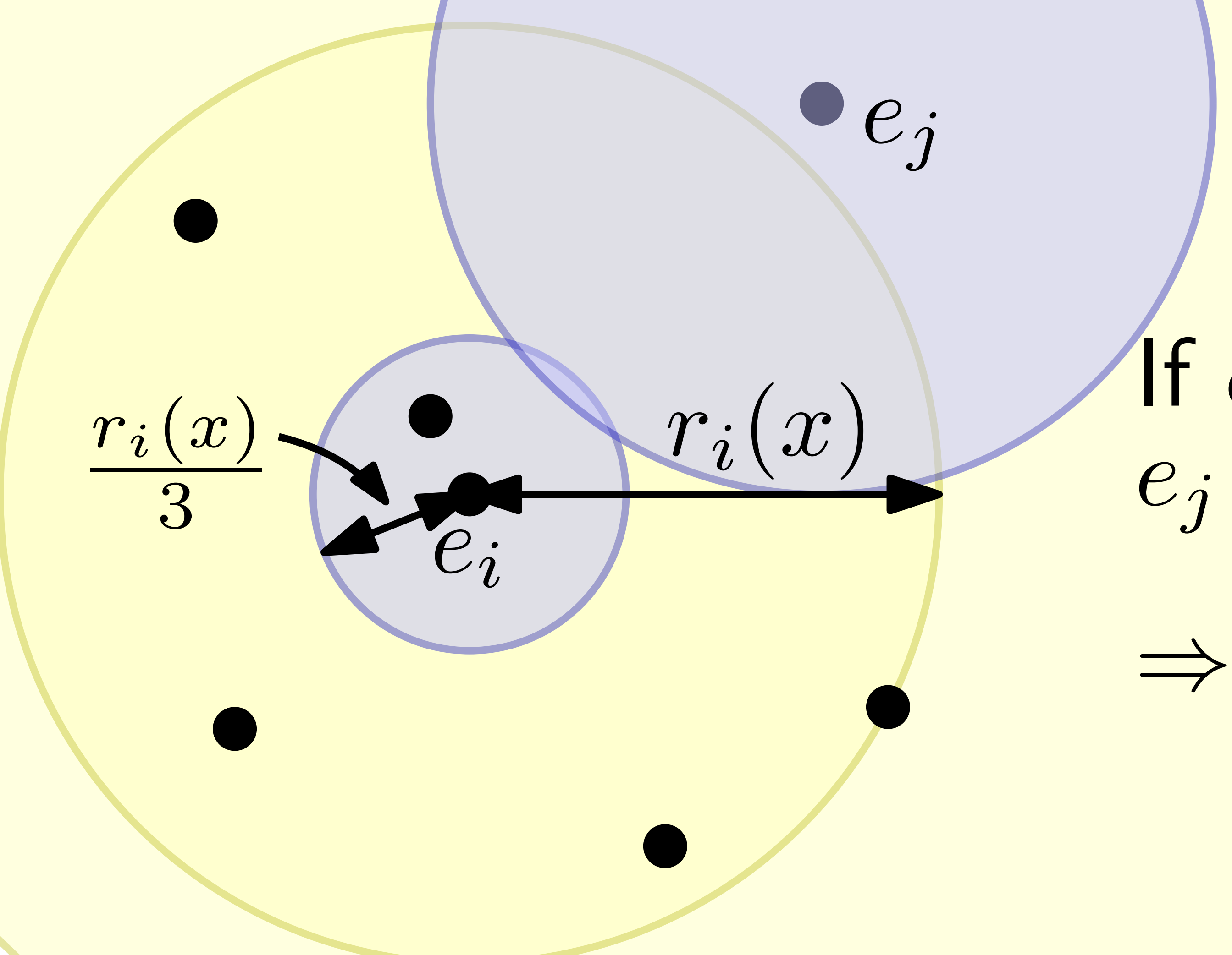
Theorem. *Any query strategy for n static entities $\mathcal{E}$ suffers **ply** $\Omega(x_{\mathcal{E}})$ at some time in any n consecutive time steps.*

Algorithm to avoid interference

Weighted Round Robin

Idea: Query entity e_i so its uncertainty region has radius $< r_i(k(x_{\mathcal{E}}+1))/3$.

Let $x = k(x_{\mathcal{E}}+1)$



If e_j's region intersects e_i's then e_j is nbr$_x$ of e_i.

$\Rightarrow$ max **degree** $\leq x$

Algorithm to avoid interference

Theorem. *Any query strategy for n static entities $\mathcal{E}$ suffers* **ply** $\Omega(x_{\mathcal{E}})$ *at some time in any n consecutive time steps.*

Theorem. *For static entities, Weighted Round Robin maintains* **degree** $O(x_{\mathcal{E}})$ *at all times.*

Algorithm to avoid interference

Theorem. *Any query strategy for n static entities $\mathcal{E}$ suffers* **ply** $\Omega(x_{\mathcal{E}})$ *at some time in any n consecutive time steps.*

Theorem. *For static entities, Weighted Round Robin maintains* **degree** $O(x_{\mathcal{E}})$ *at all times.*

Theorem. *For n static point entities $\mathcal{E}$, if $\omega_{\mathcal{E}}$ is the minimum ply achievable at some target time, $x_{\mathcal{E}} \in O(\omega_{\mathcal{E}} \log^d(n/\omega_{\mathcal{E}}))$ and for some such $\mathcal{E}$ this is tight.*

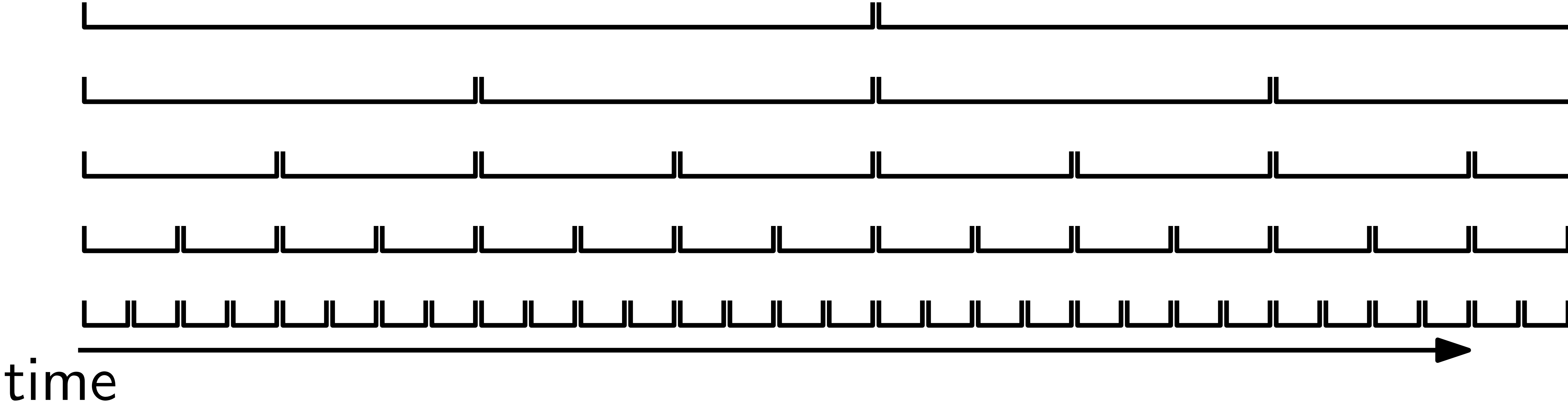
Dynamic entities

Complications due to movement:

1. Strategy knows **perceived** (last queried) location rather than true (current) location
2. Congestion $x_{\mathcal{E}}$ varies over time

Bucket Strategy for fixed tolerance x

Idea: Schedule e_i for a future time interval (**bucket**) based on perceived x-radius $\widetilde{r}_i(x,t)$ when queried.



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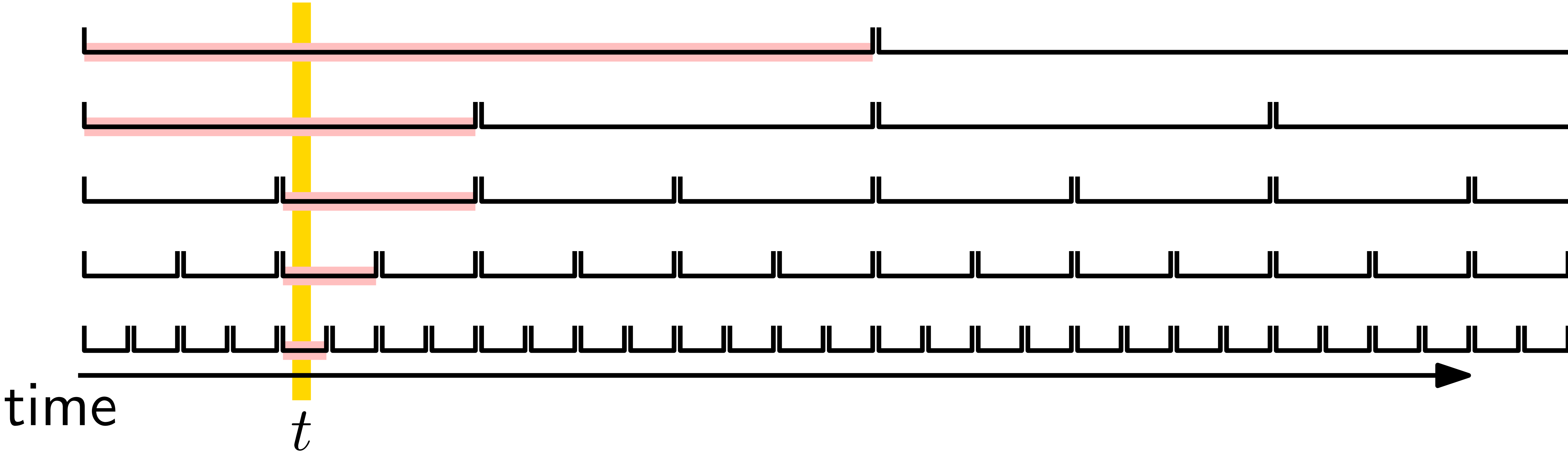
Repeat for all time t
  Query entity e_i from shortest active bucket
  Move e_i to next bucket of length 2^b
    where $b = \lfloor \lg(\widetilde{r}_i(x,t)/Q) \rfloor$



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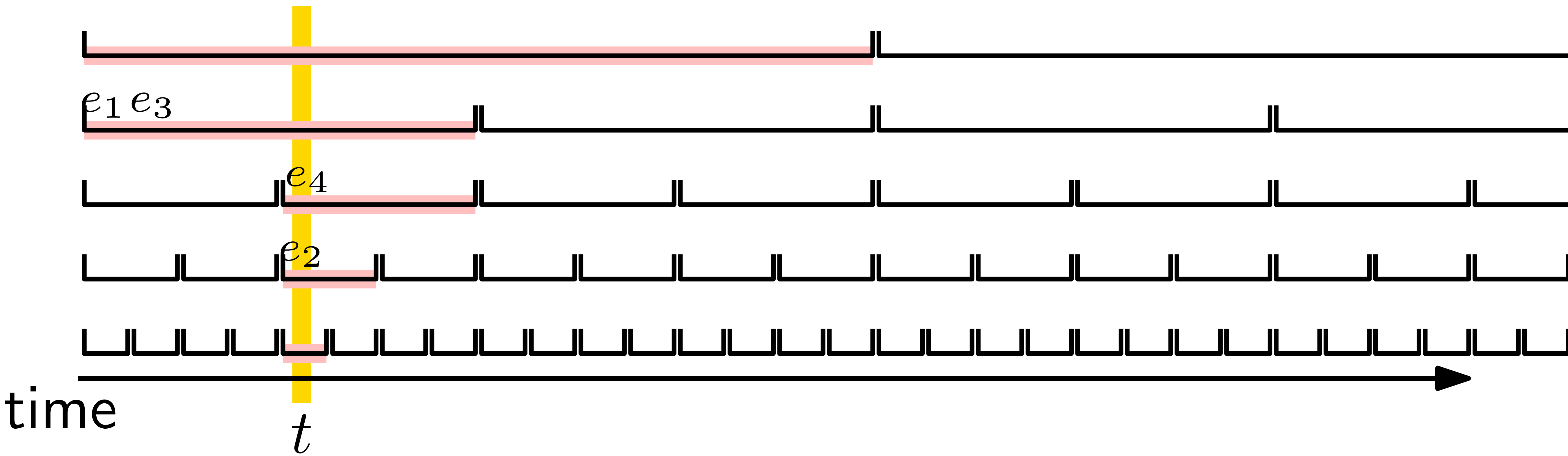

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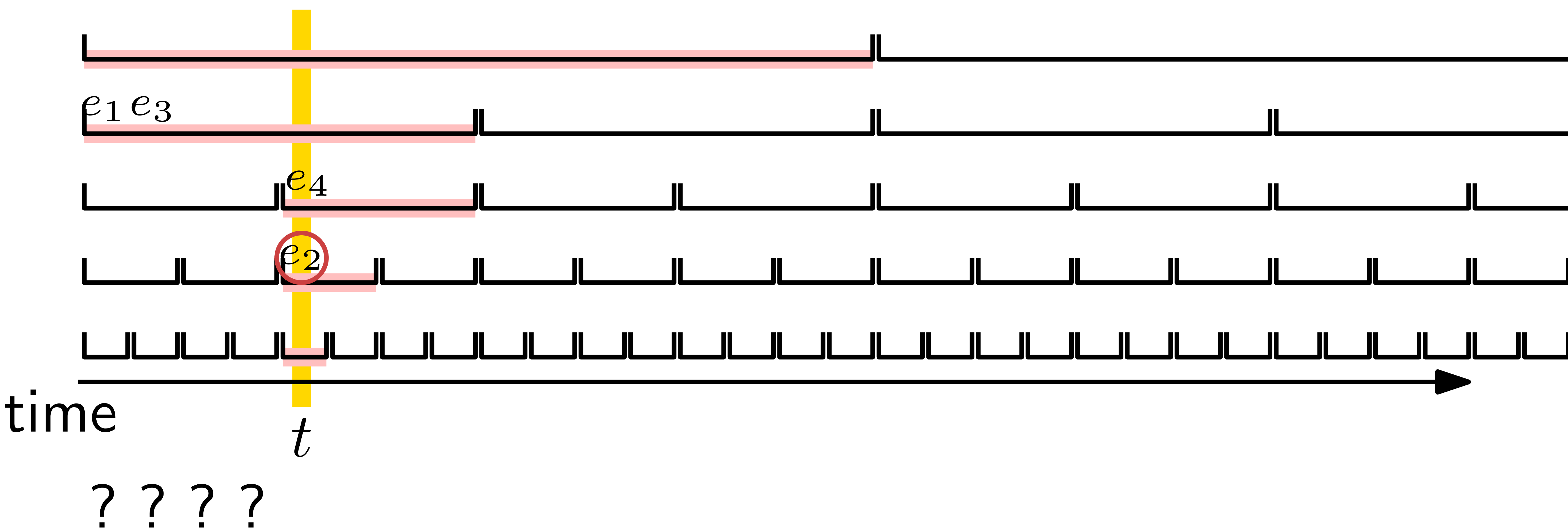

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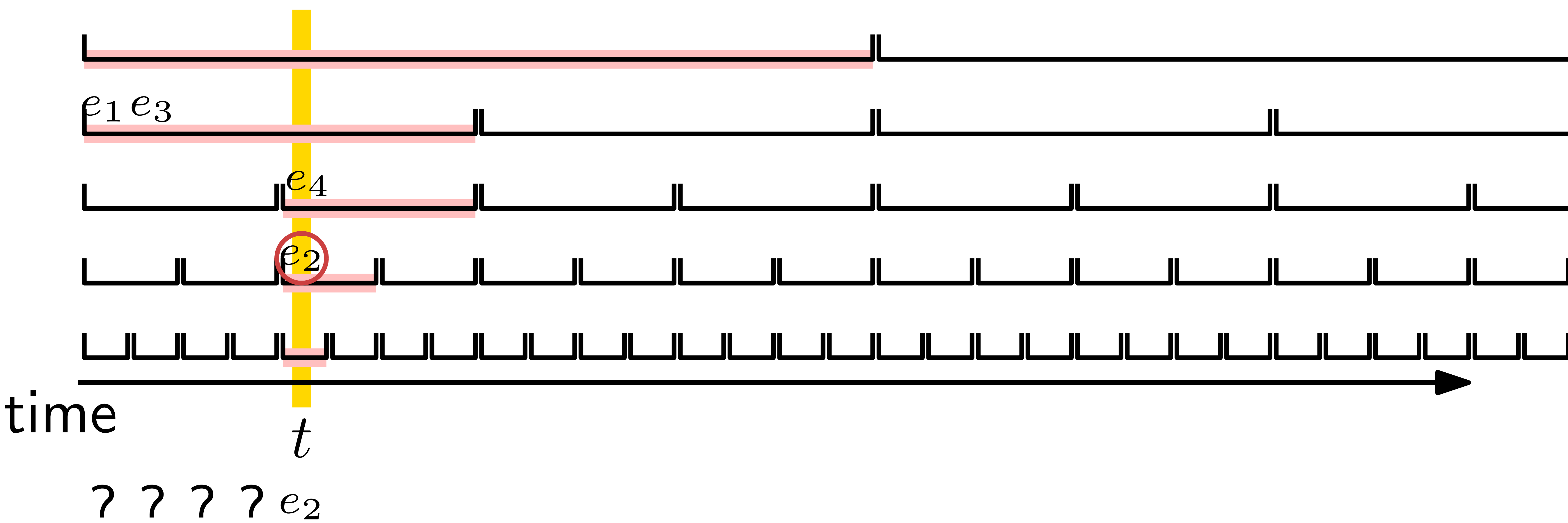

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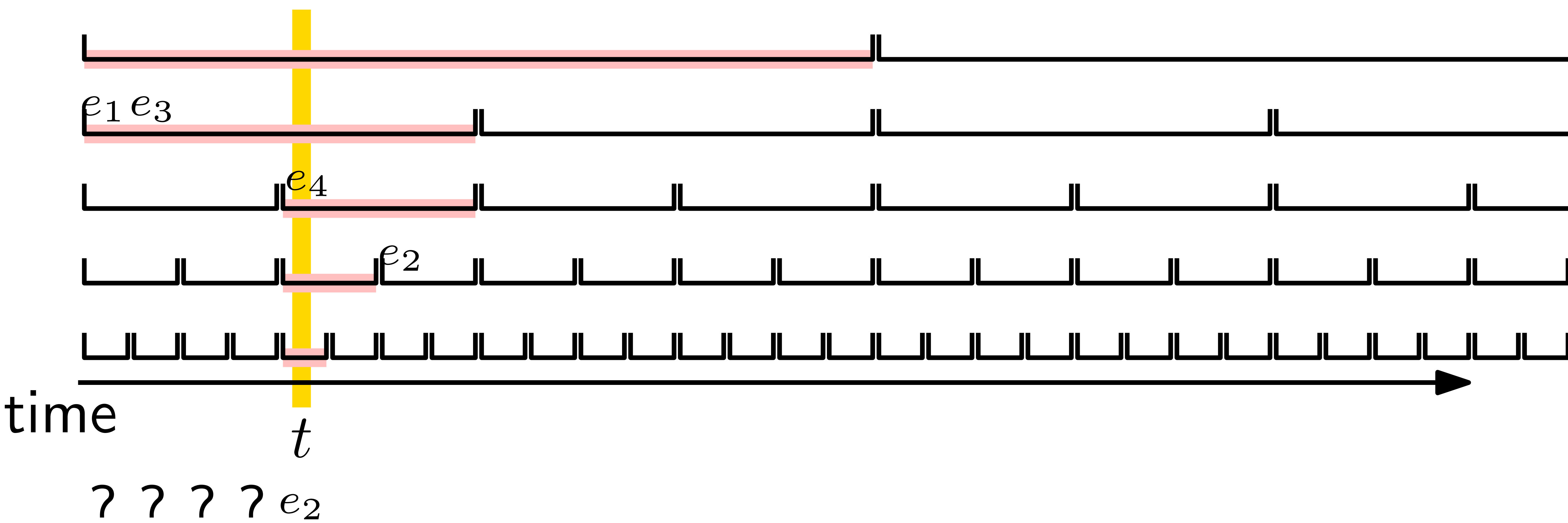

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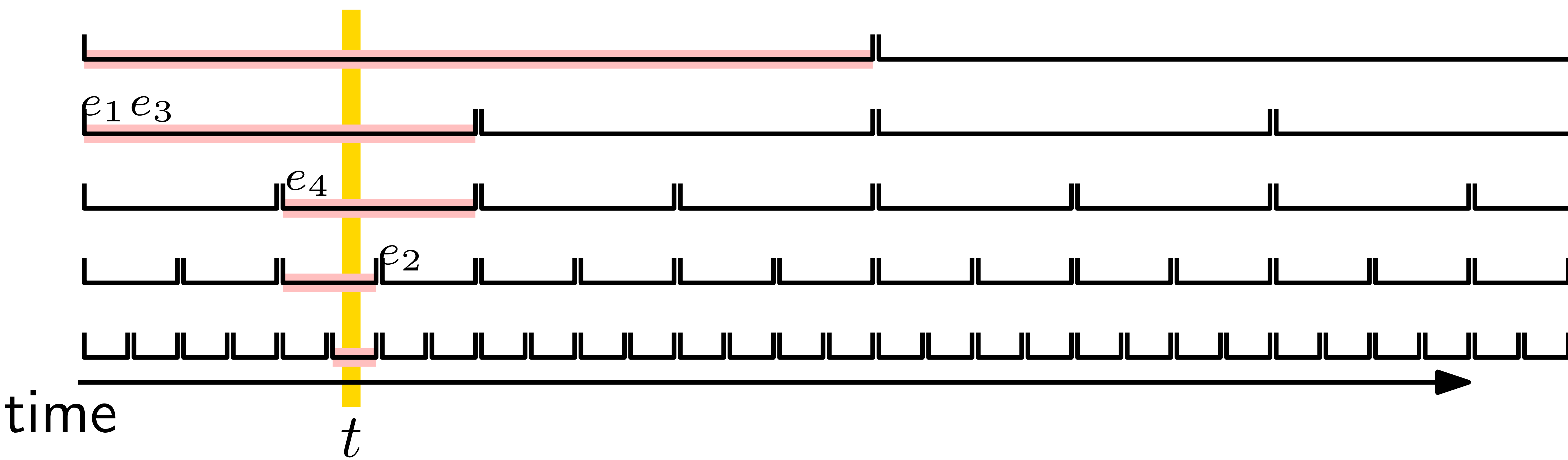

? ? ? ? e_2

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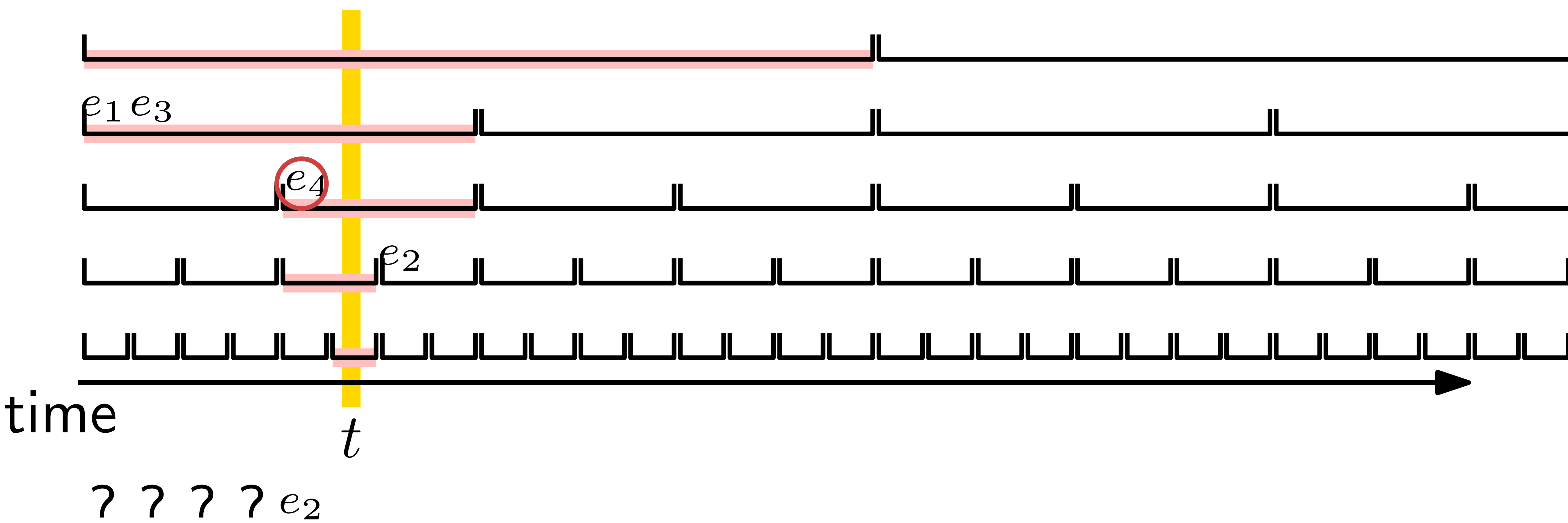

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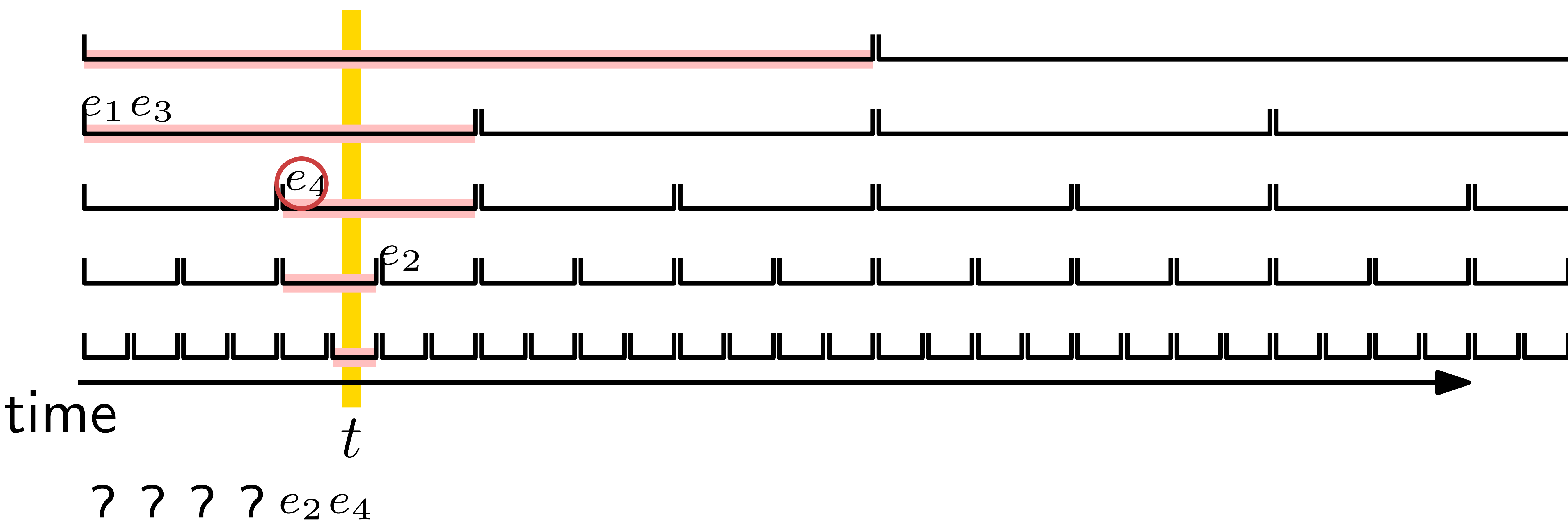

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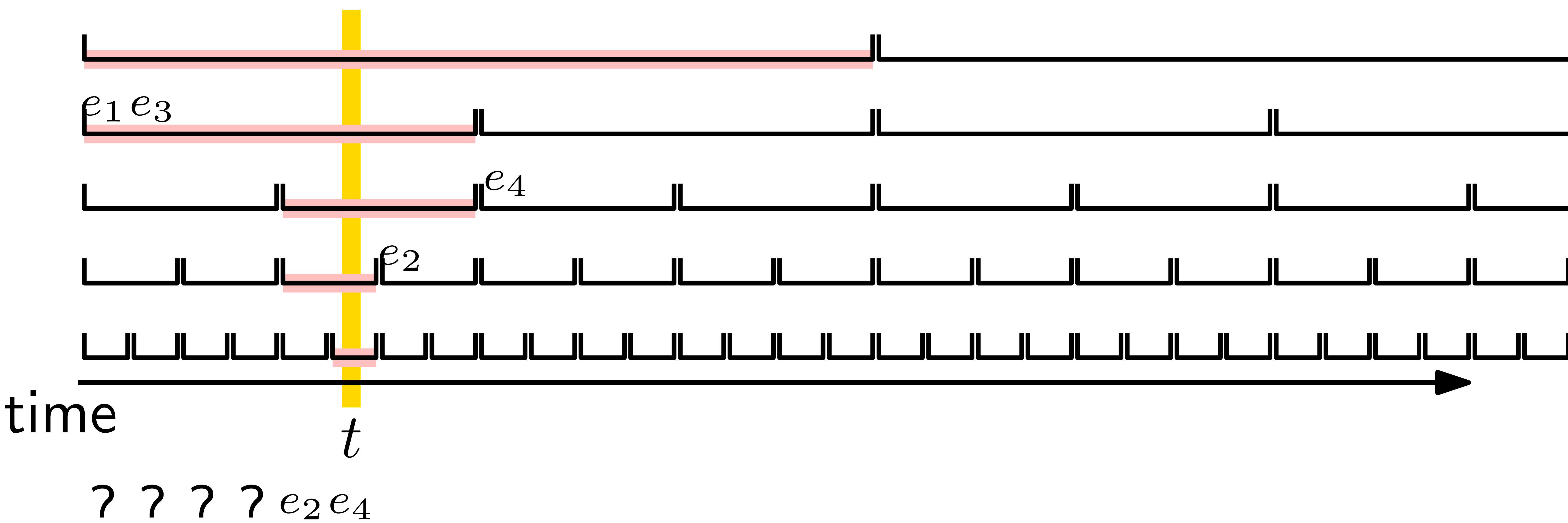

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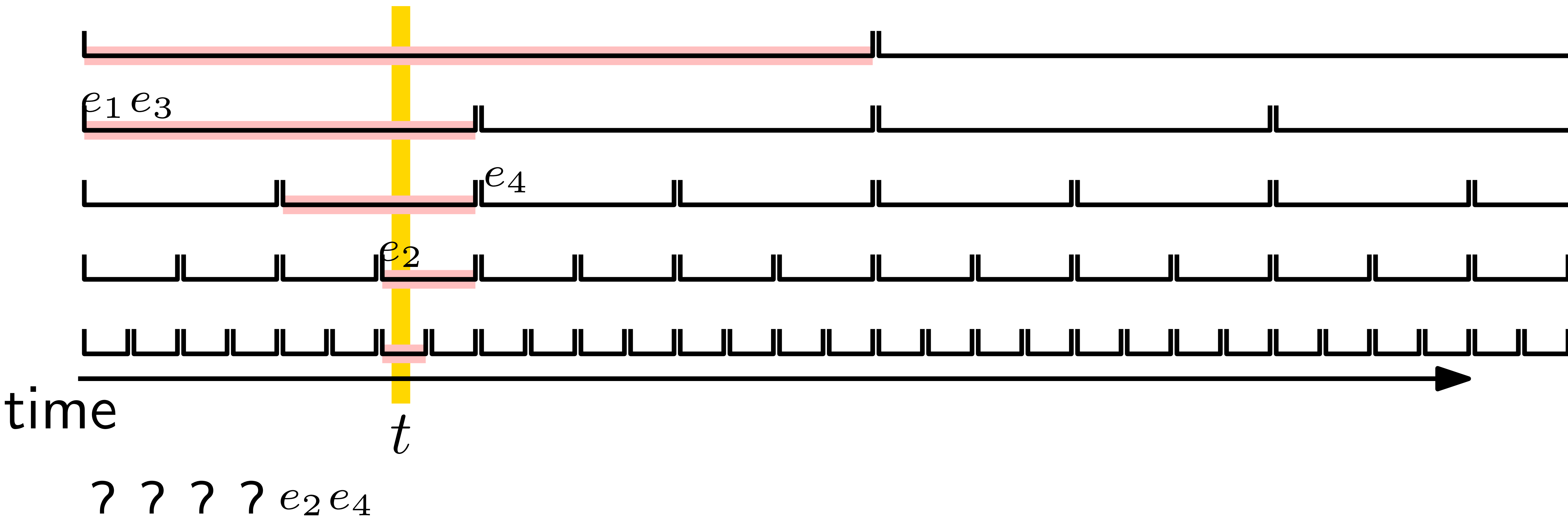

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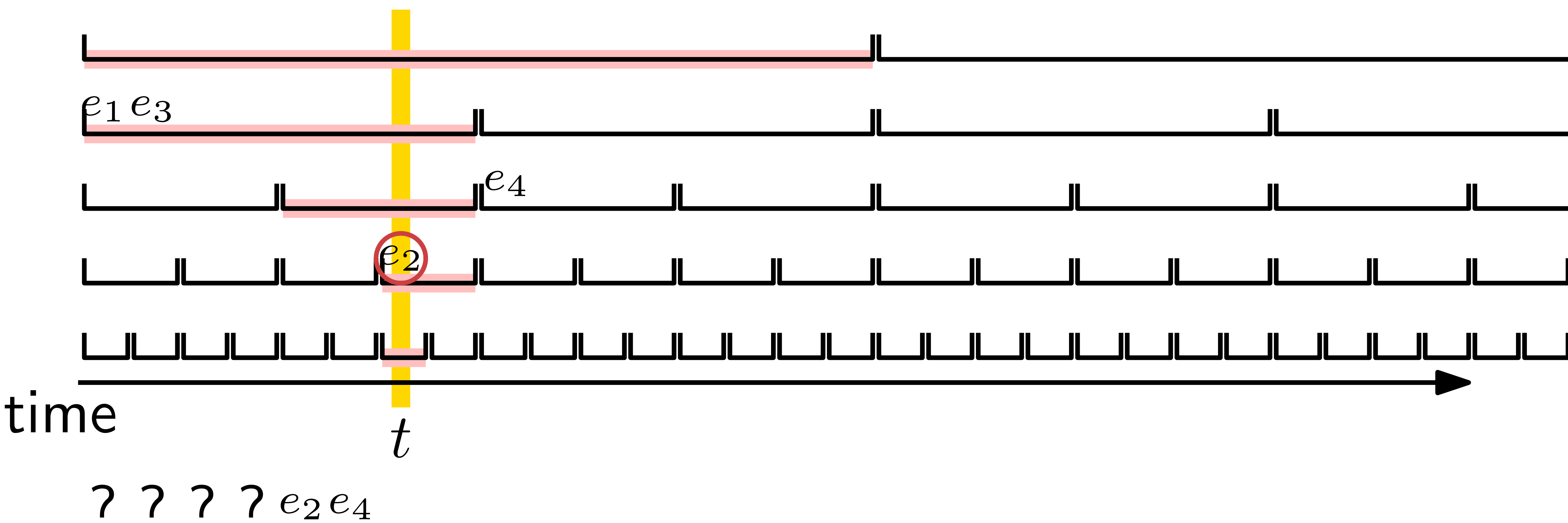

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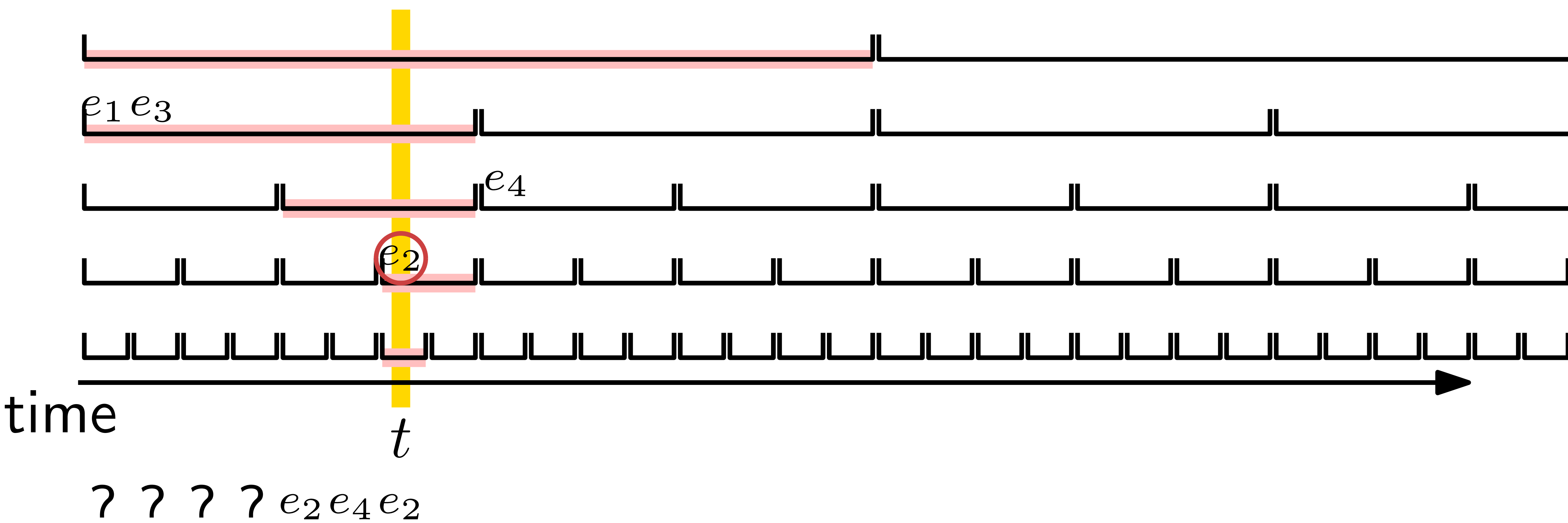

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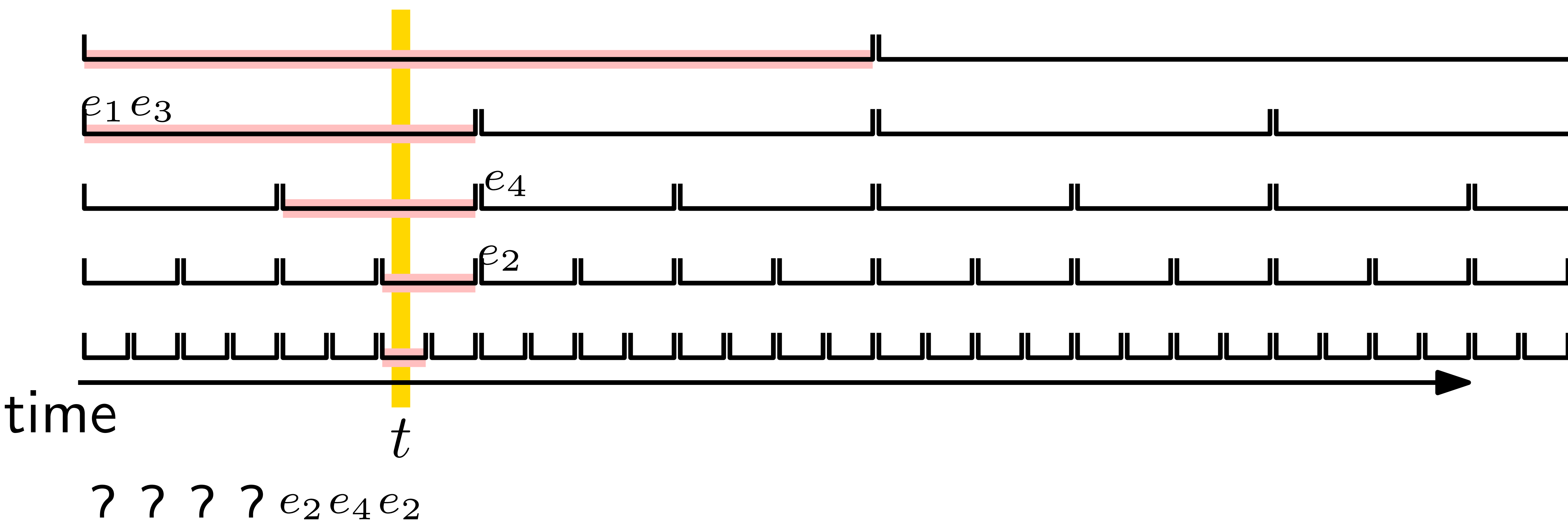

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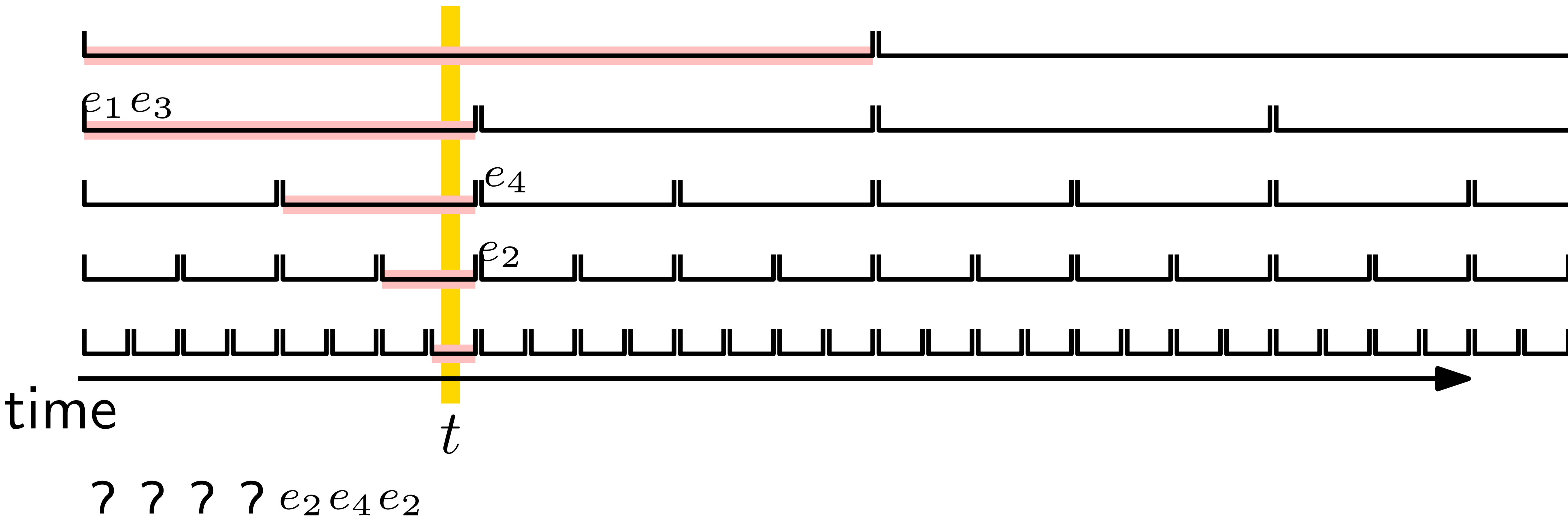

Bucket Strategy for fixed tolerance x

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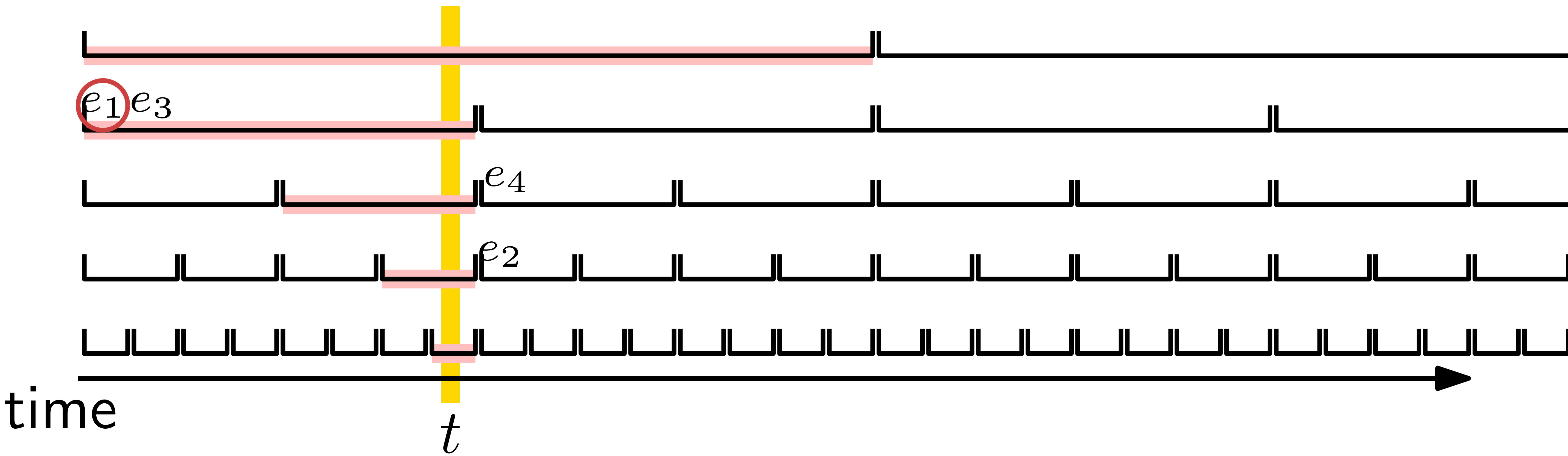

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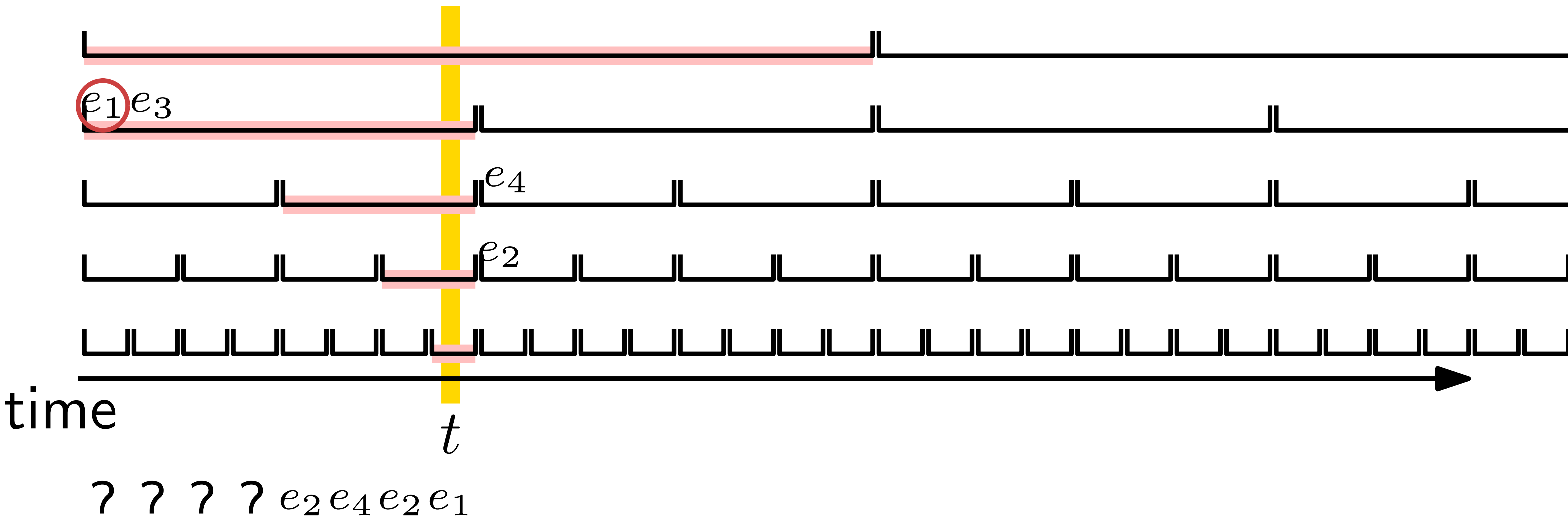

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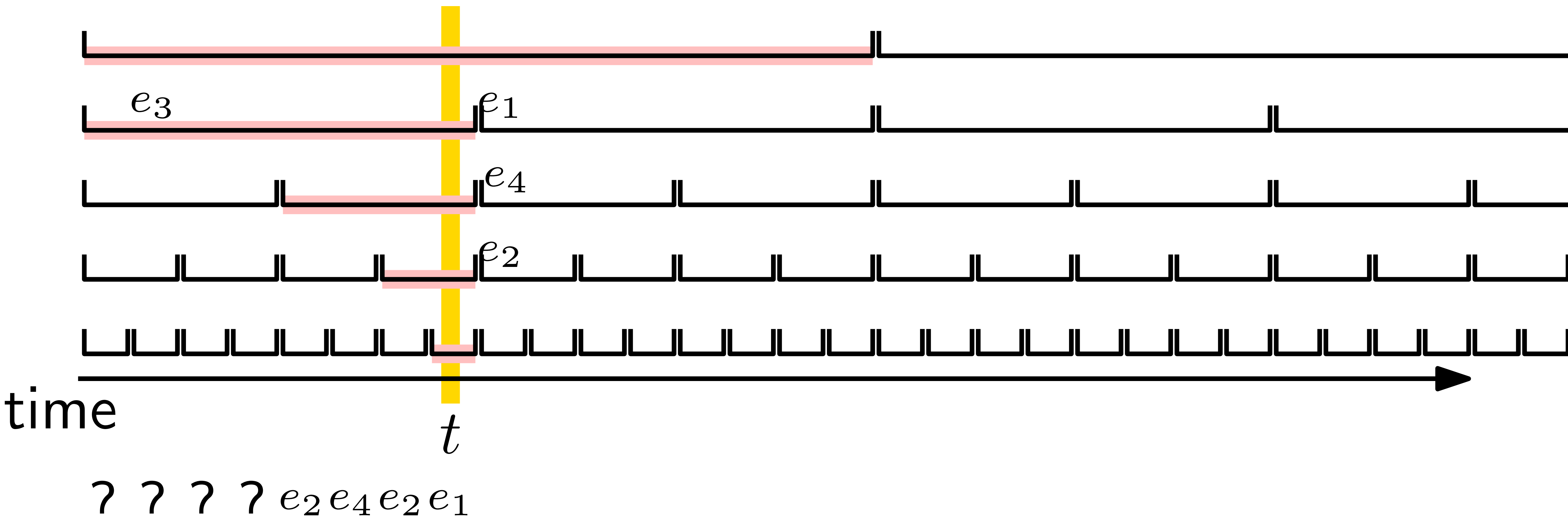

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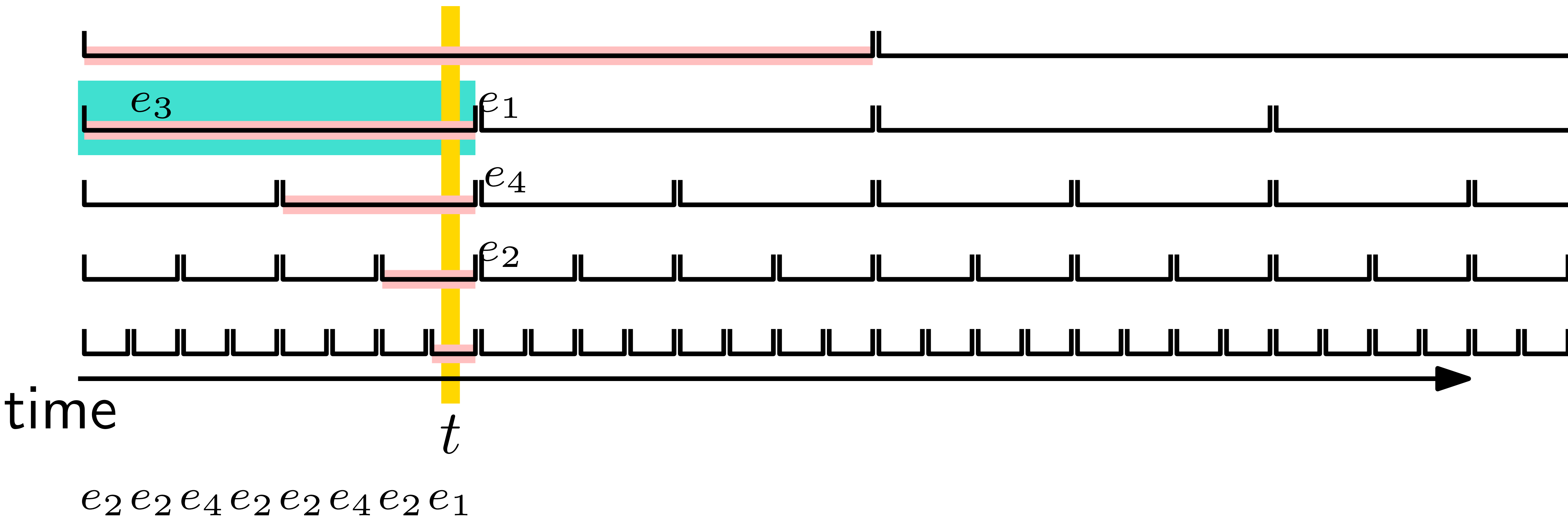
  If any ending bucket is uncleared, fail



Unavoidable interference - moving entities

Sustained x-density implies high ply

Lemma. *For any time interval T with $|T| \geq |E|$, if $\sum_{t \in T} \sum_{e_i \in \mathcal{E}} \frac{1}{r_i(x,t)} \geq c_d |T|$ then any query strategy suffers* **ply** $\Omega(x)$ *at some time in T.*

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   perform speculative queries in parallel

Future Work

A simpler optimal strategy

A distributed optimal strategy

Other measures of interference potential

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A simpler optimal strategy

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Other measures of interference potential

Thank you

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kissing in dim d

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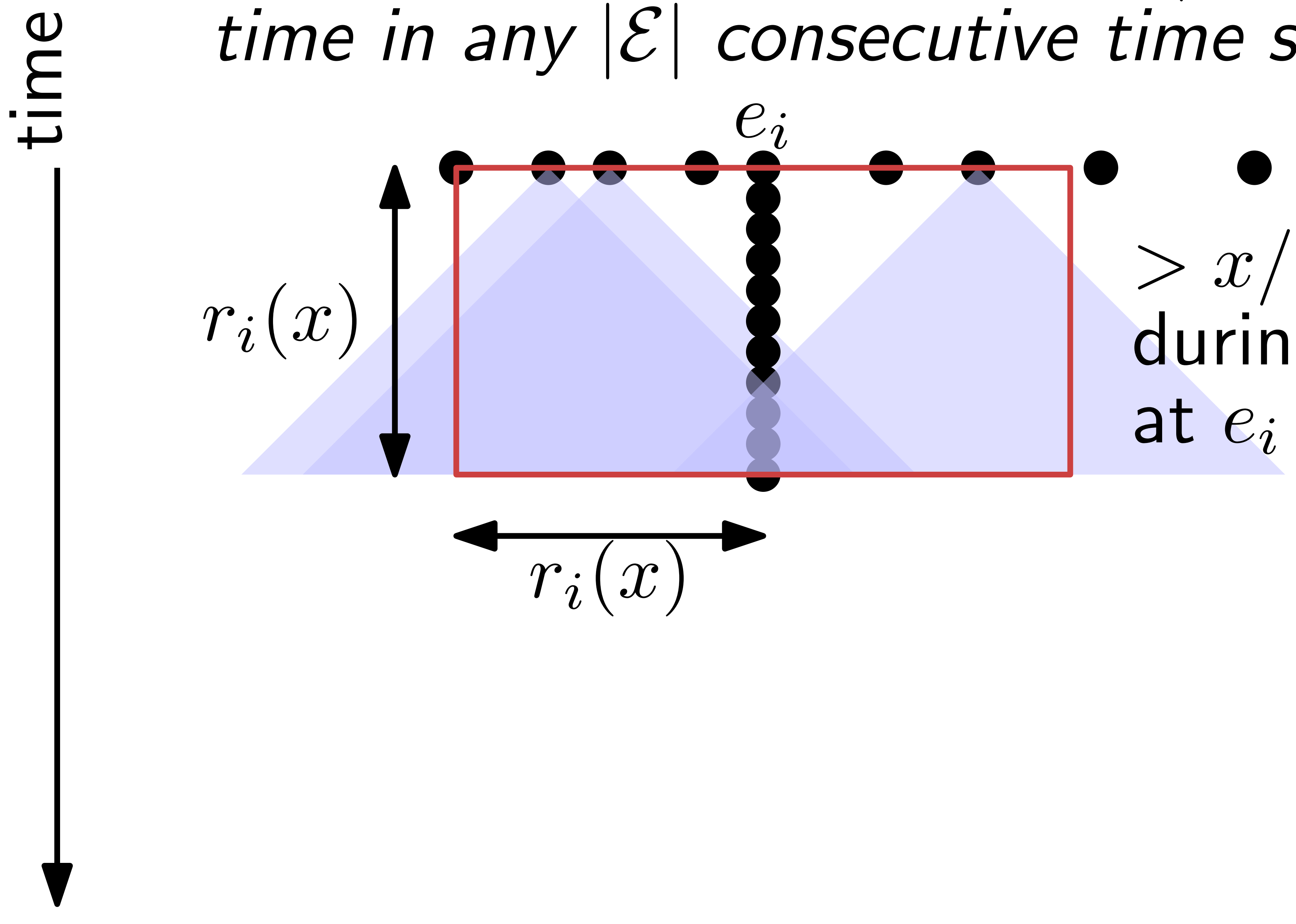

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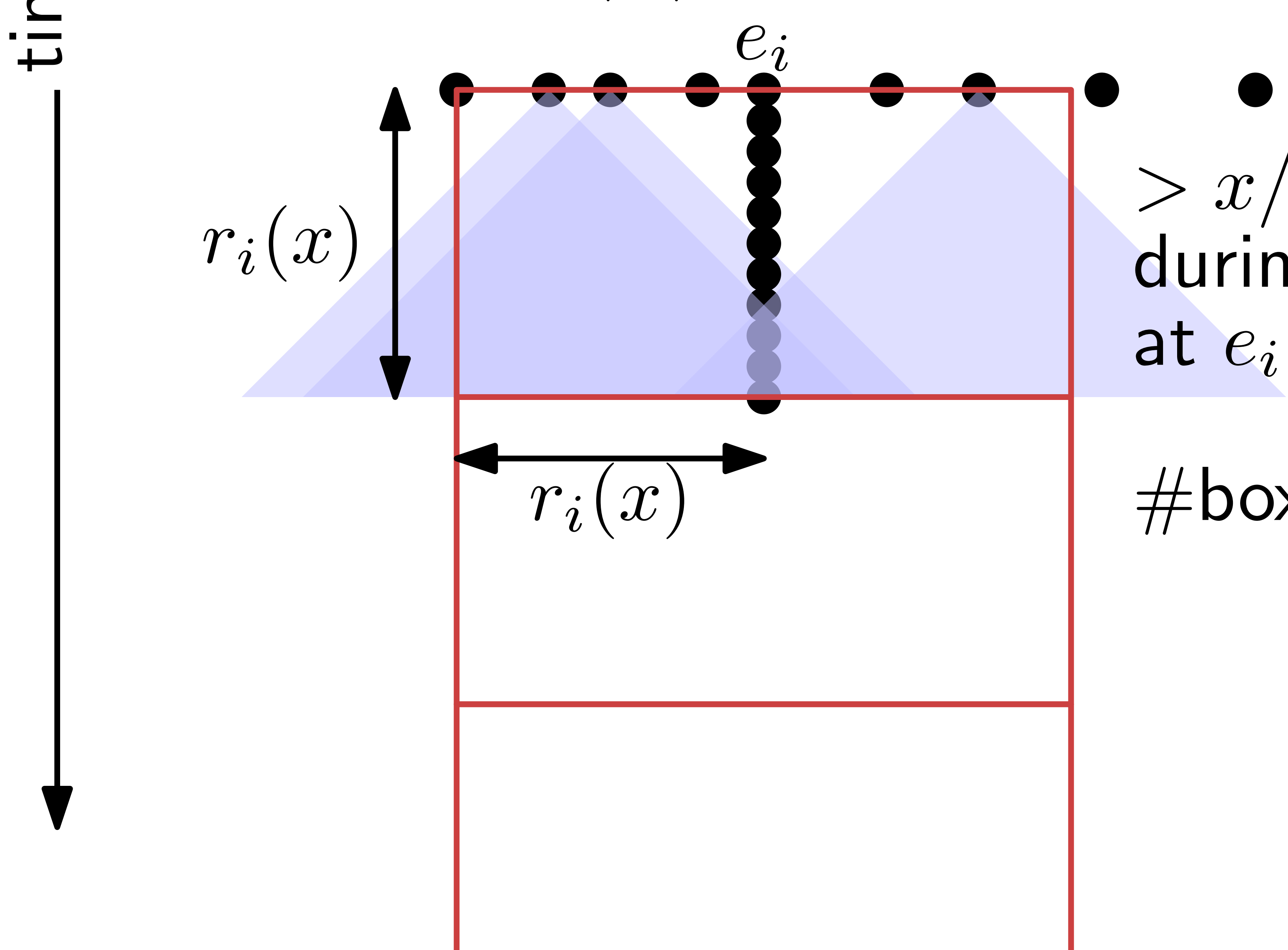

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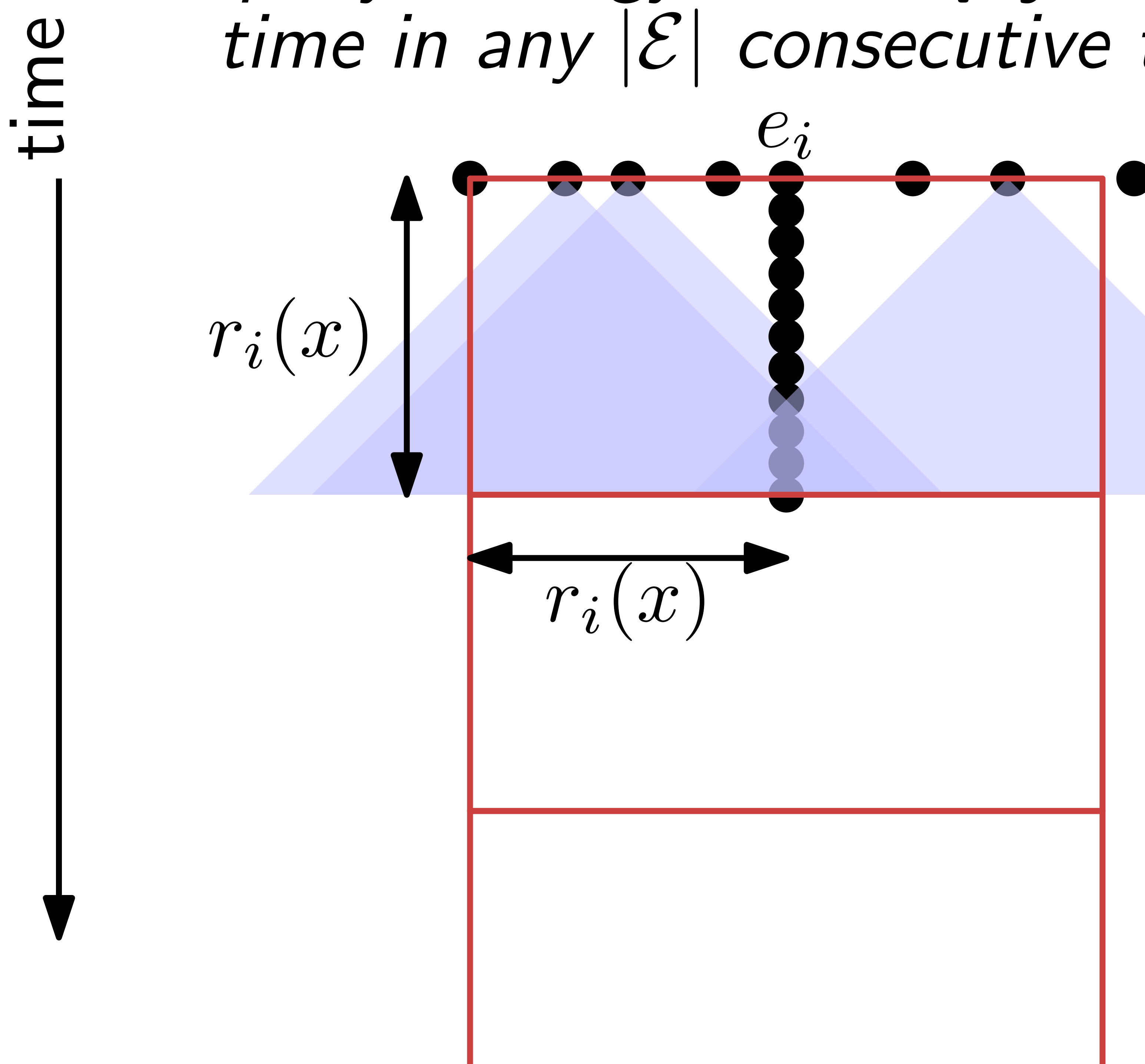

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Each entity is nbr$_x$ of at most $x\kappa_d$ entities e_i.

geometric lemma

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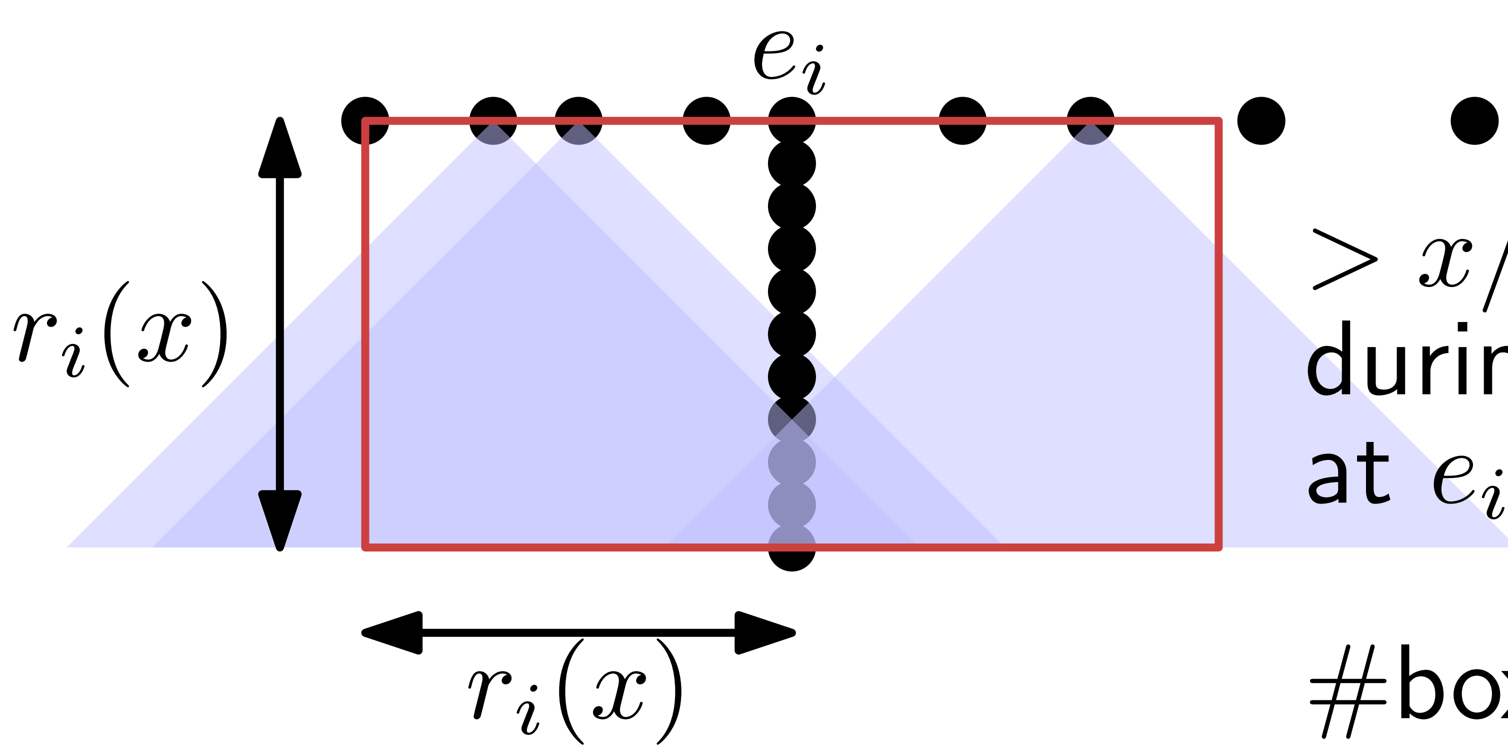
time



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$\#\text{boxes} \geq \sum_{e_i \in \mathcal{E}} \left\lfloor \frac{|\mathcal{E}|}{r_i(x)} \right\rfloor$

$|\mathcal{E}| = \#\text{queries} \geq \frac{1}{x\kappa_d} \frac{x}{2} \#\text{boxes}$

$> \frac{|\mathcal{E}|}{4\kappa_d} \sum_{e_i \in \mathcal{E}} \frac{1}{r_i(x)} \geq |\mathcal{E}|$

$\Rightarrow\Leftarrow$

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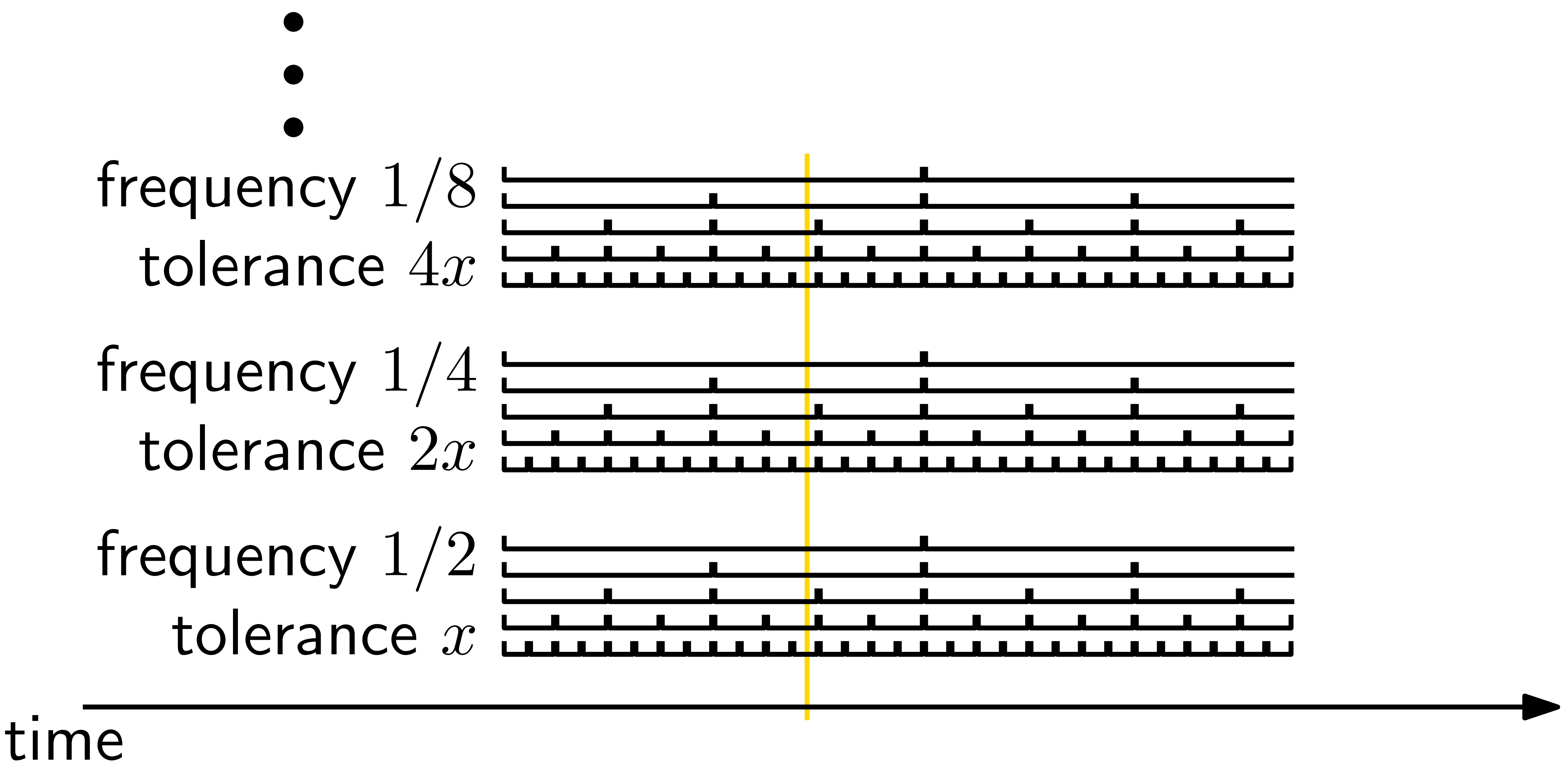
geometric lemma

Adapt to larger tolerance

Rough idea: Run multiple Bucket Strategies in parallel



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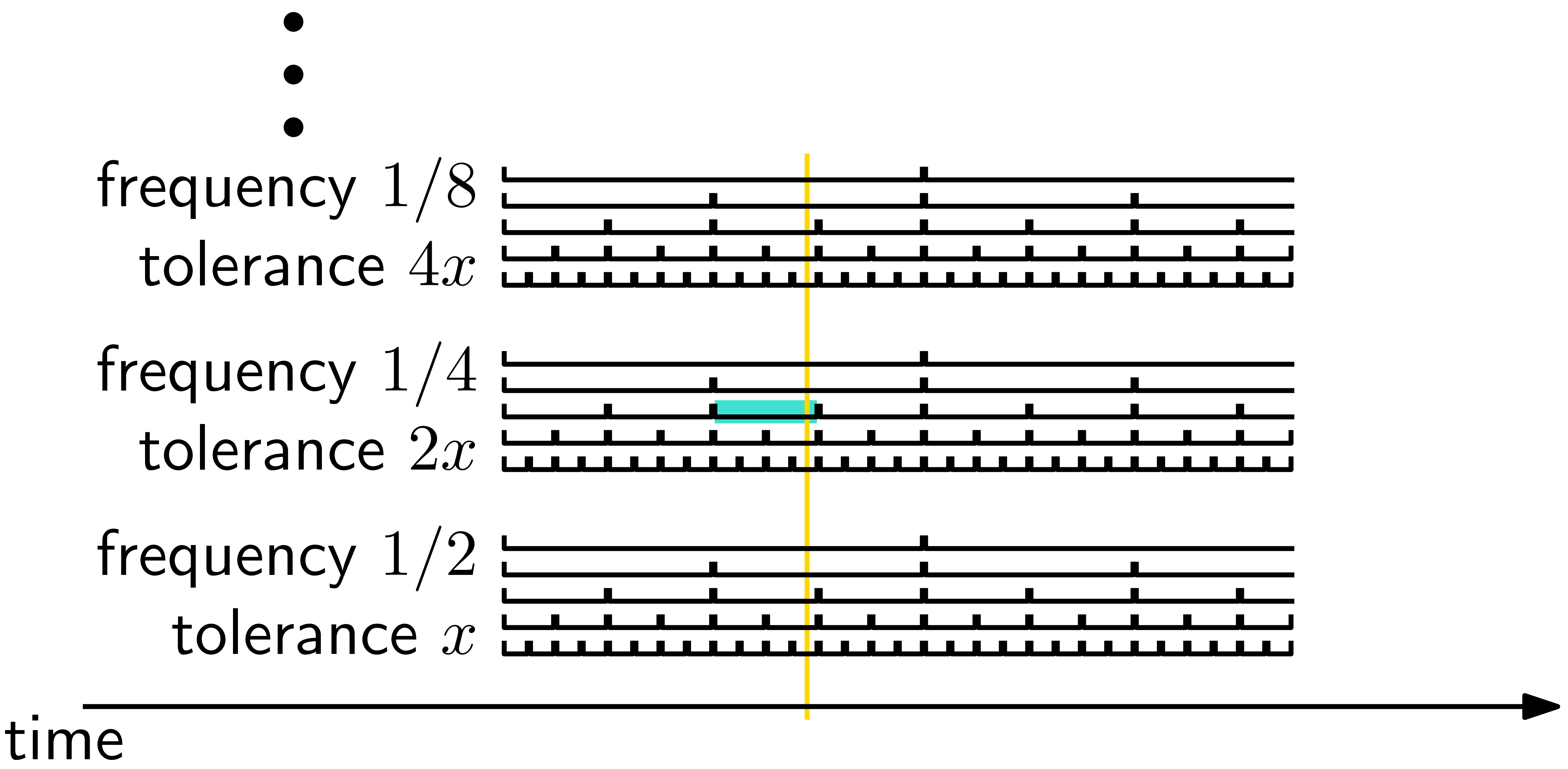
Failure of any implies ply $\Omega(x)$ unavoidable [scaling lem]



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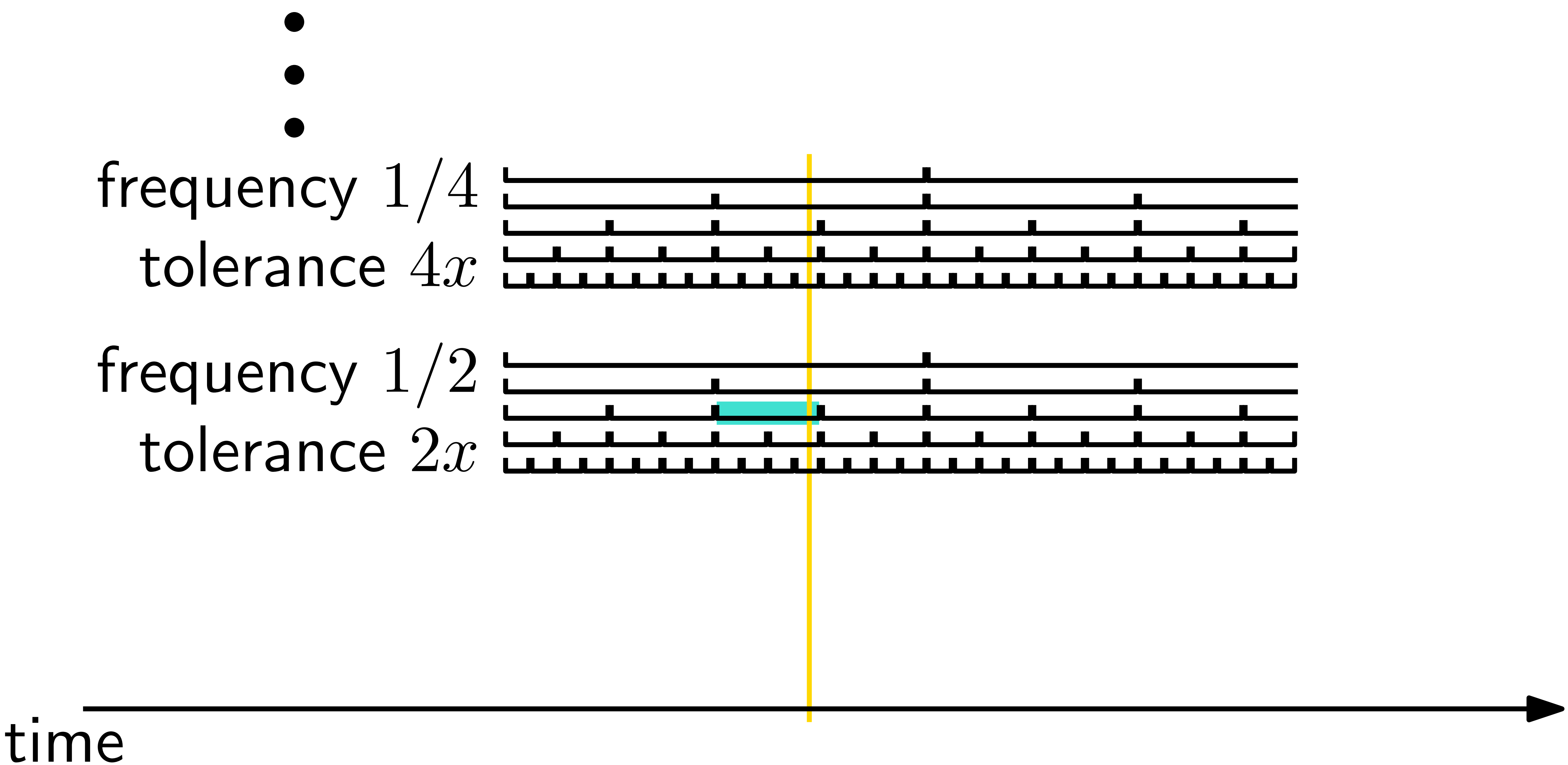

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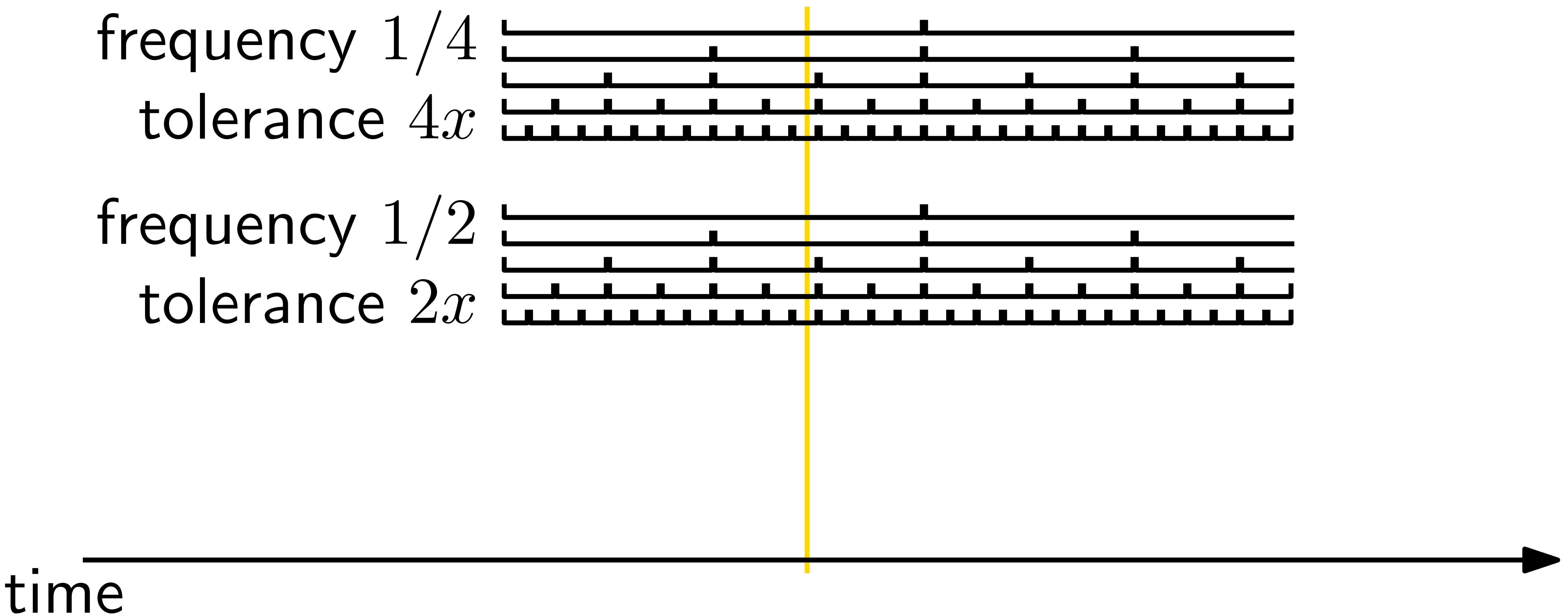
- Failure of any implies ply $\Omega(x)$ unavoidable [scaling lem]
- Stop strategy for x and divvy up its frequency
  - Empty all active buckets to special queues

$\vdots$



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Round-robin query all entities.

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If $|E_0| > n/2$ then restart
If $|E_0| = 0$ then add Bucket Strategy $x/2$

region cannot contain more than $x/2$ entities in the next n steps

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for $k = 1$ to $\lg(n)$
Round-robin query E_{k-1} for $n/2^k$ steps
Let E_k be those that are **not** $(x/2)$-safe for $n/2^k$ steps
If $|E_k| > n/2^k$ then restart
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