Reading:


You may discuss problems with other people in the class, but you must write up your own solutions. If you do discuss a problem with someone else or you use an outside resource, you must acknowledge them. Do not copy solutions from anyone. A star (⋆) means that the problem may be somewhat difficult.

1. Prove the the Schnyder wood for a triangulated (i.e., maximal) planar graph has only three bi-directed edges. What edges are these?

2. Prove that Tutte’s drawing on an integer grid requires area that is exponential in the number of vertices for some planar graphs. (You may specify the external face and the placement of its vertices in convex position.)

3. What is the Laplacian, \( L \), of the complete graph \( K_n \) on \( n \) vertices? What are the eigenvalues and eigenvectors of \( L \)? How many spanning trees does \( K_n \) contain? (There’s a variant of Kirchhoff’s theorem that says the number of spanning trees is \( 1/n \) times the product of the non-zero eigenvalues of the Laplacian.)

4. Use Kirchhoff’s theorem to calculate the number of spanning trees in the following graph:

   ![Graph](image)

5. ⋆ Describe your course project in at most one page.

   This will involve deciding on a project and talking with me about what you want to do. The final project will involve writing a less than 10 page paper (five pages is o.k., commented code can count for some of it) and presenting your work to the class (20 minutes with questions). The deadline for the paper and presentation will be April 10 or 12.