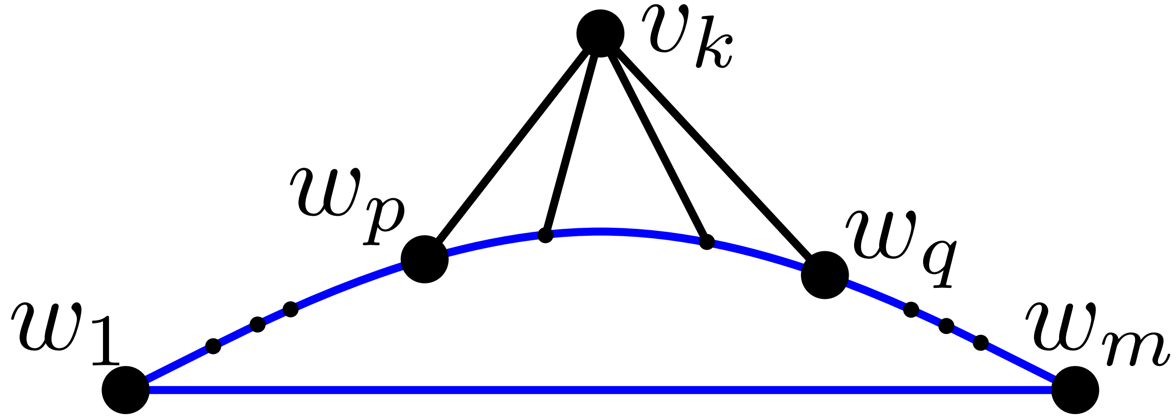
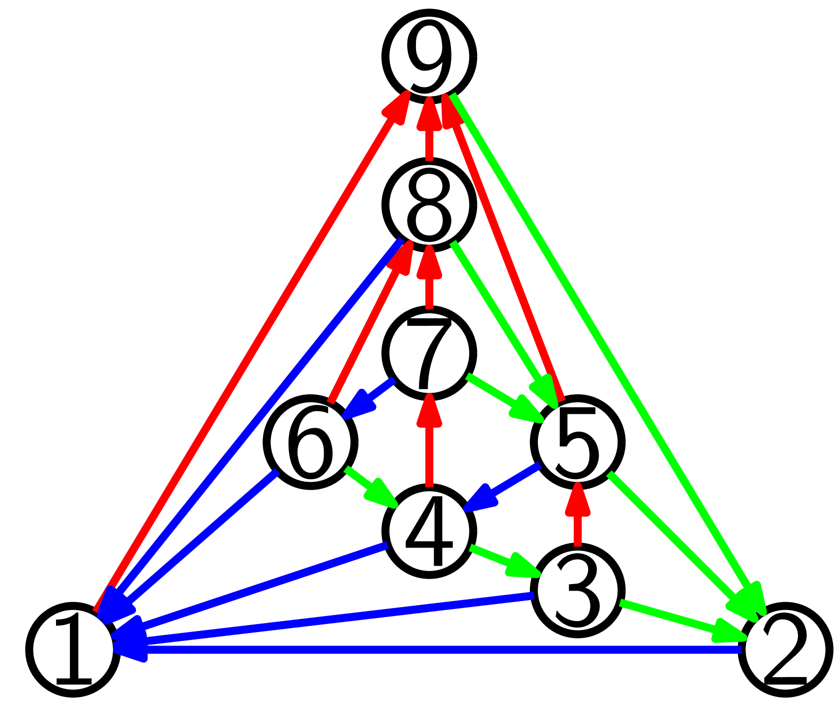
Canonical Ordering $\Rightarrow$ Planar Straight-line Drawing

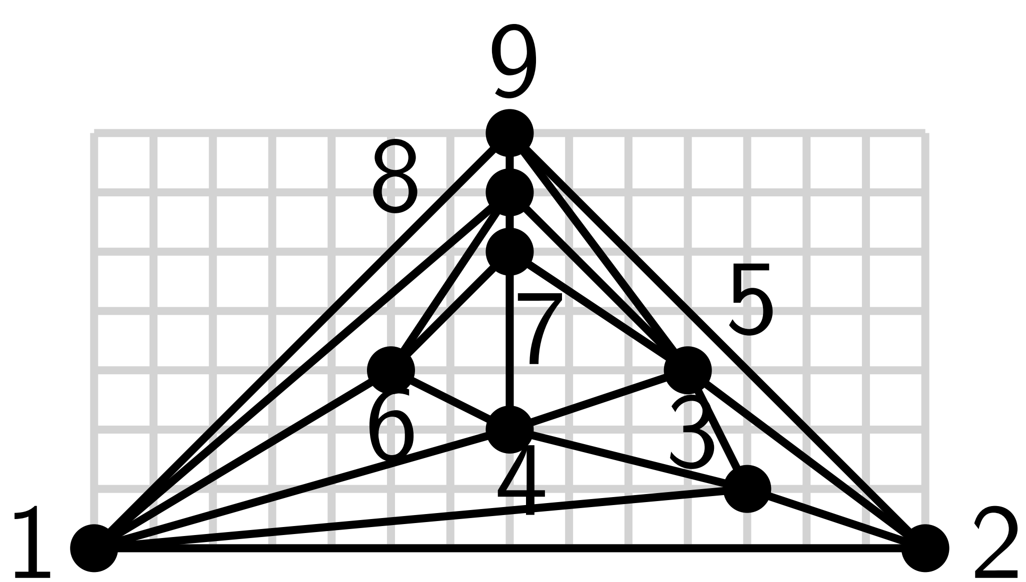




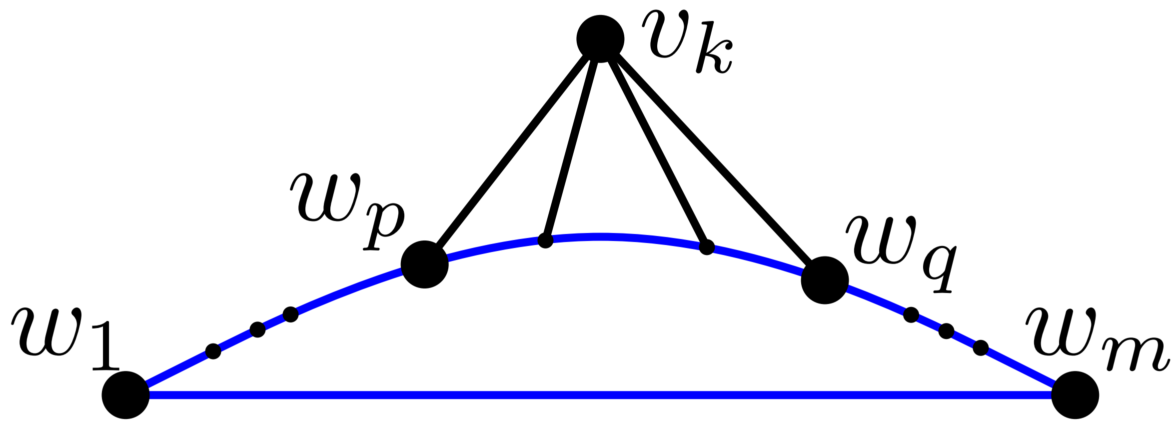
If $v \in \bigcup_{i=q}^{m} L(w_i)$ then $x(v) {+}{=}\ 2$.

If $v \in \bigcup_{i=p+1}^{q-1} L(w_i)$ then $x(v) {+}{=}\ 1$.

$L(v_k) = \{v_k\} \cup \bigcup_{i=p+1}^{q-1} L(w_i)$.



Canonical Ordering ⇒ Planar Straight-line Drawing



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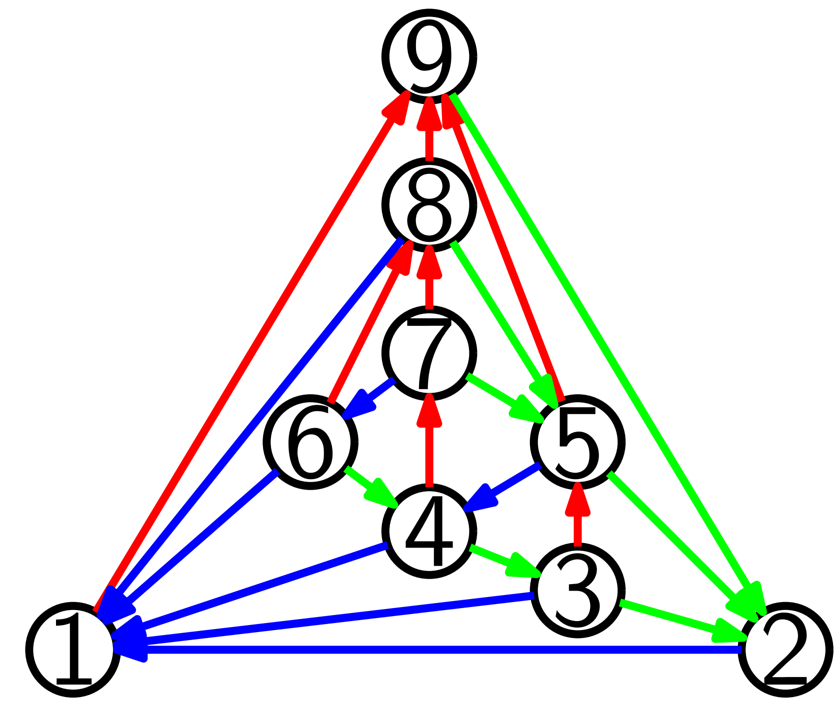
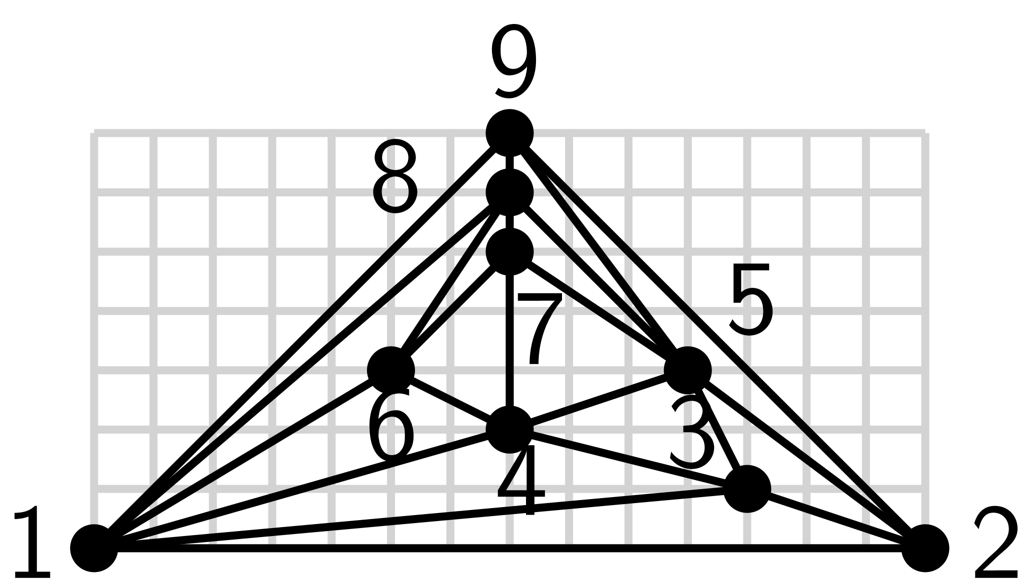


Schnyder Wood



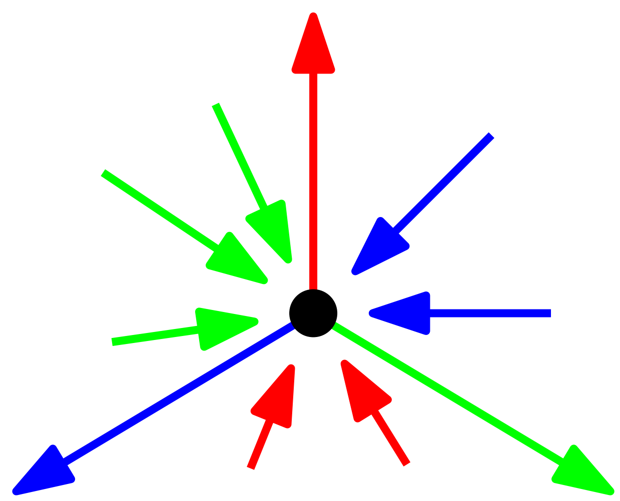
Schnyder Woods

Given a plane triangulation $G = (V, E)$

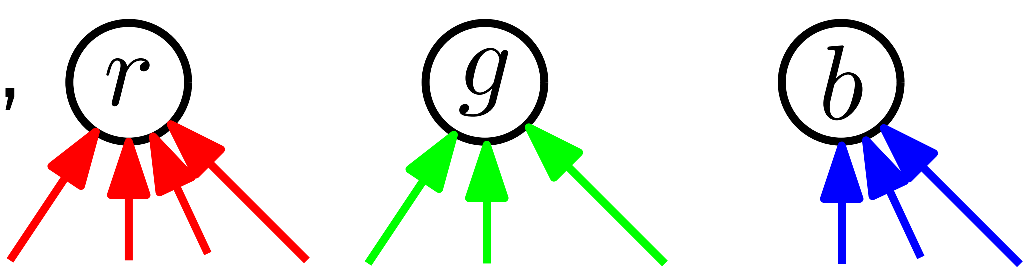
with vertices r, g, b on the outer face

a Schnyder wood is a coloring and orientation of the interior edges of G such that:

For every interior vertex,

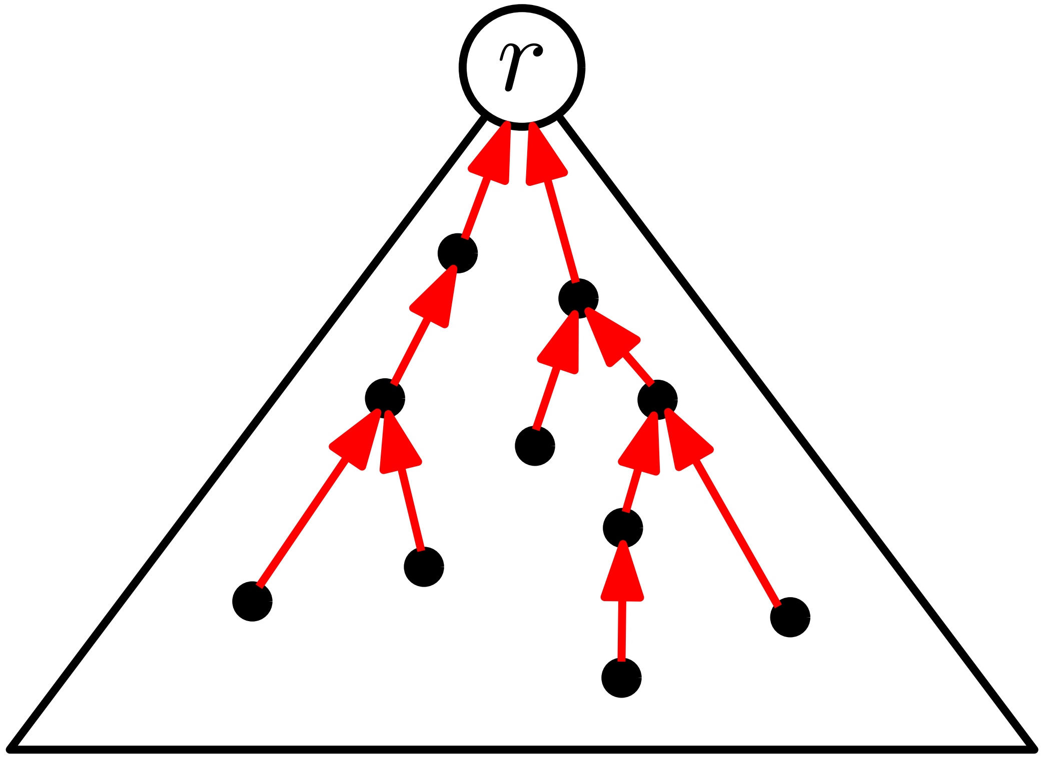


For exterior vertices,



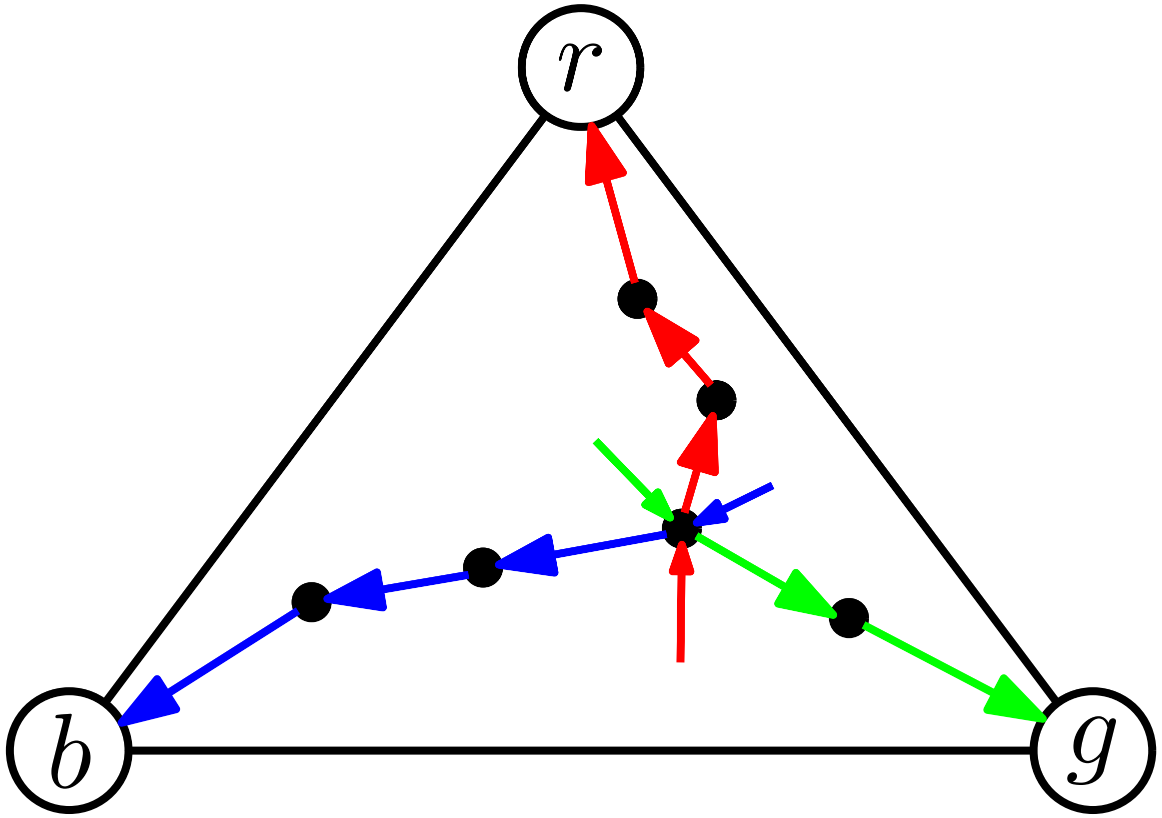
Schnyder Trees

The edges in one color class form a tree



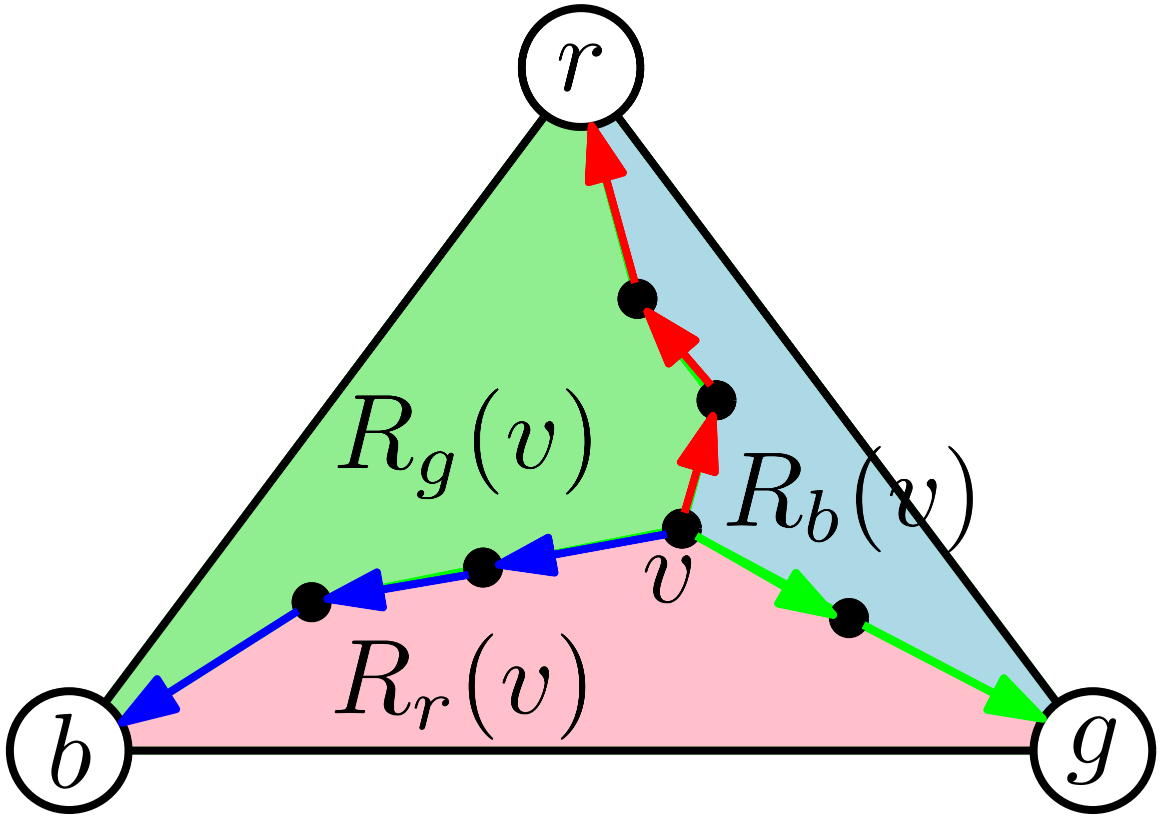
Schnyder Trees

Different colored ^directed^ paths share at most one vertex.



Schnyder Trees

Every vertex has three regions.



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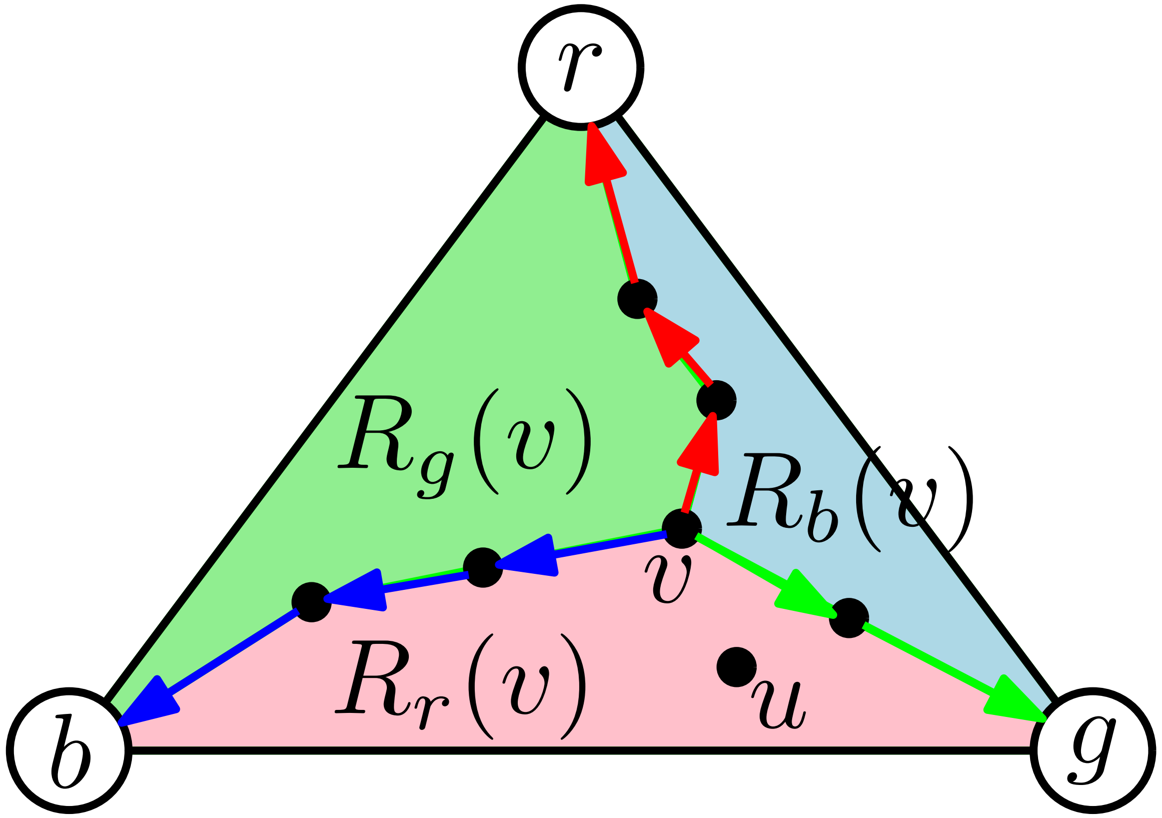
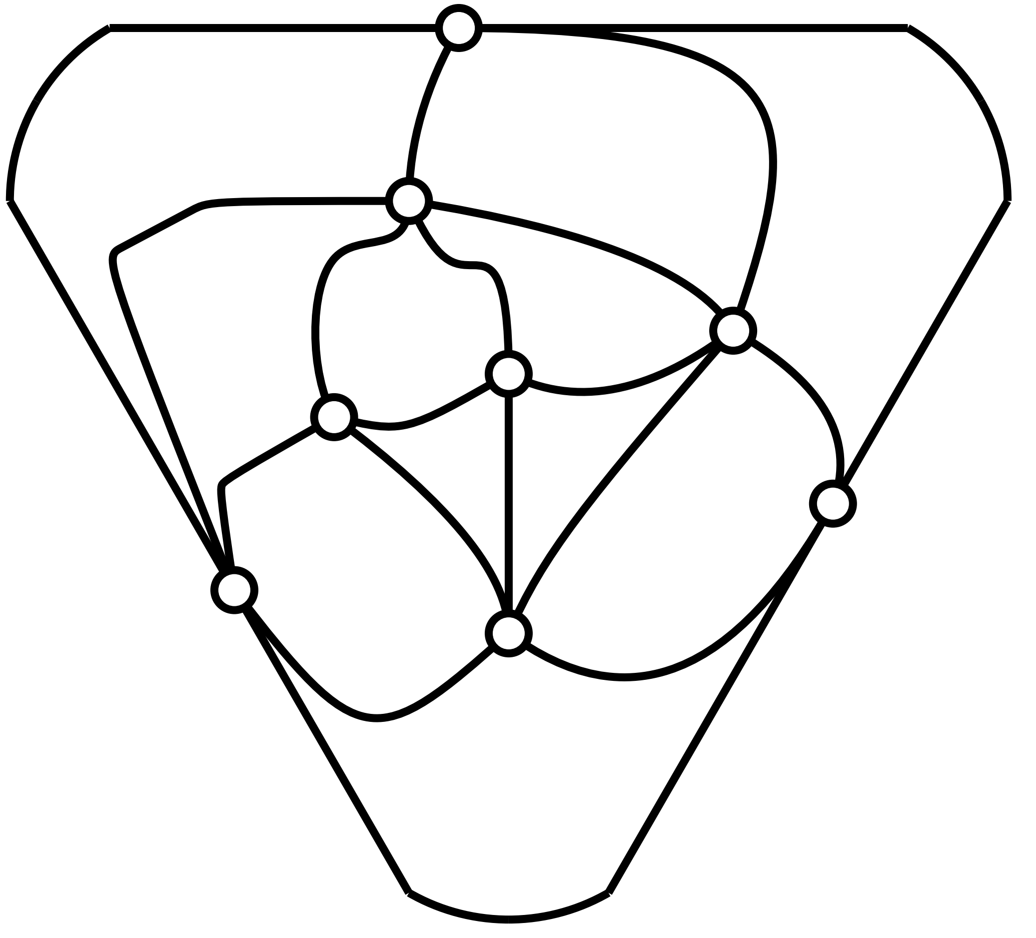


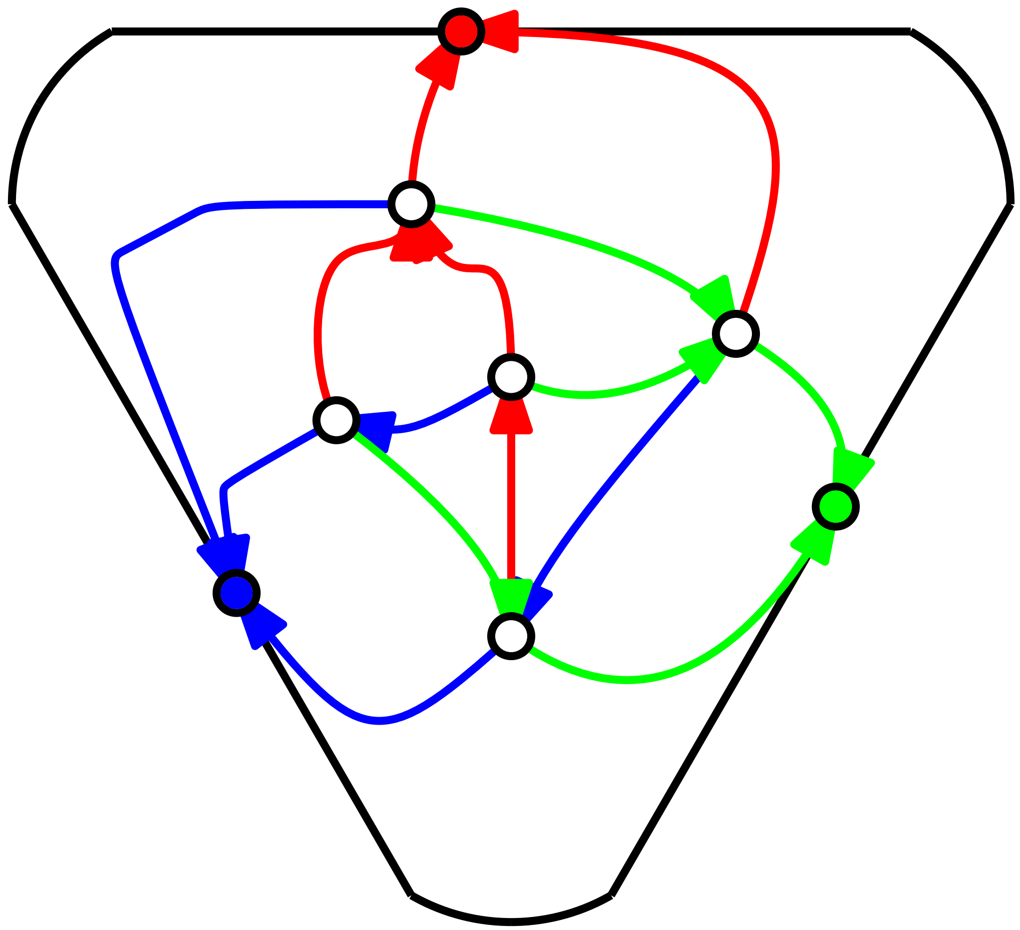
If $u \in R_c(v)$ then $R_c(u) \subset R_c(v)$.

To draw a plane triangulation...



To draw a plane triangulation...

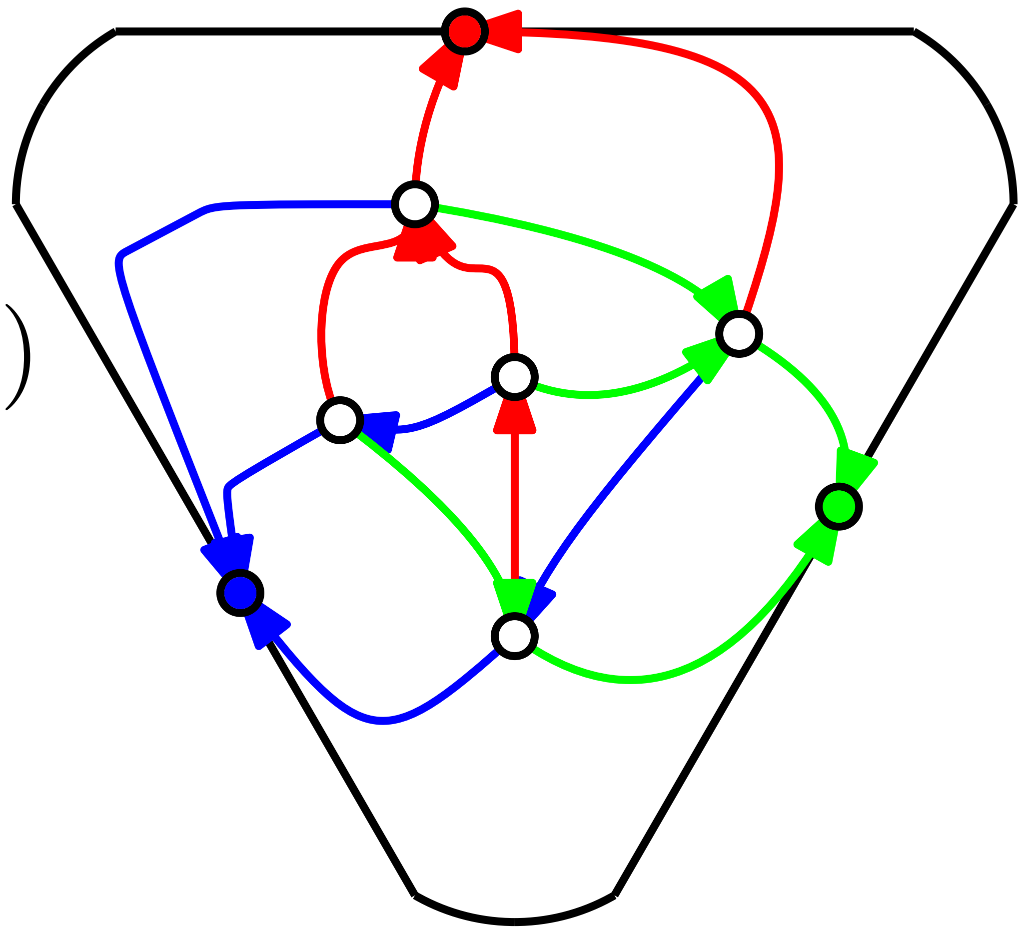
Make a Schnyder Wood



To draw a plane triangulation...

Make a Schnyder Wood

$\phi_c(v) = \#$ faces in $R_c(v)$

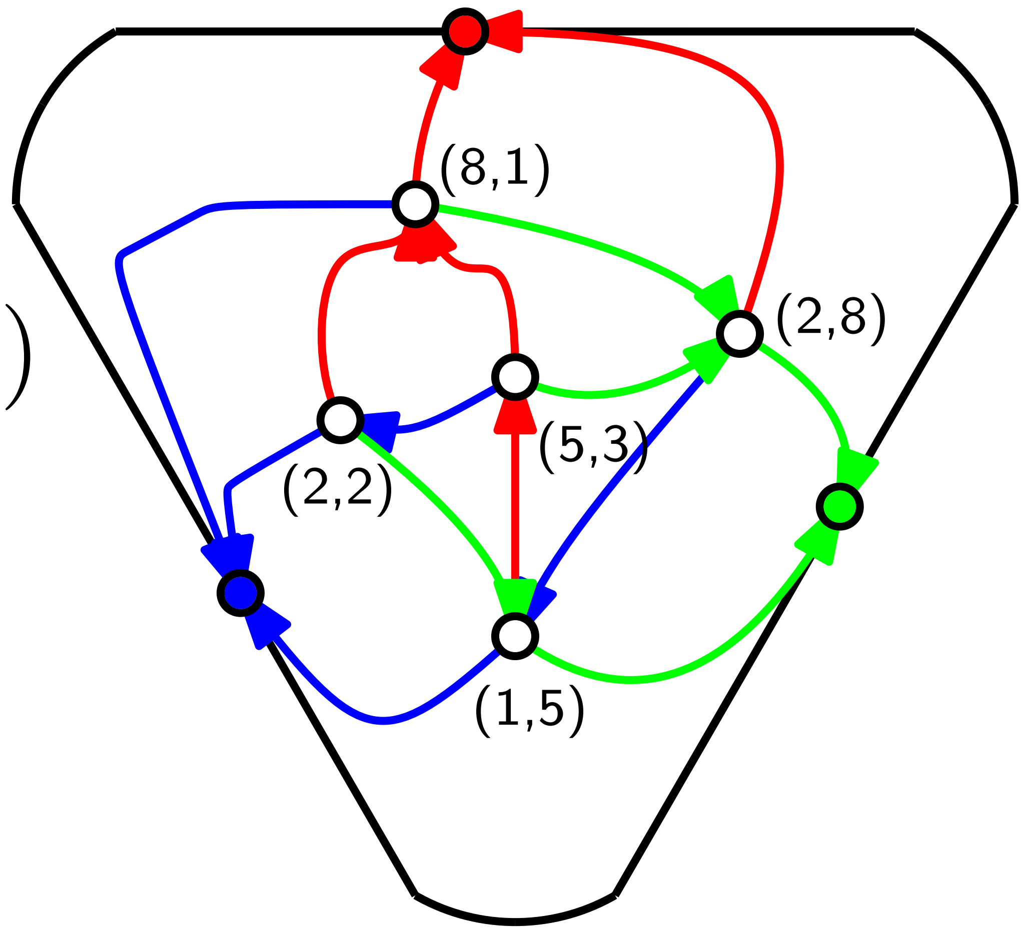


To draw a plane triangulation...

Make a Schnyder Wood

$\phi_c(v) = \#$ faces in $R_c(v)$

Draw v at $(\phi_r(v), \phi_g(v))$



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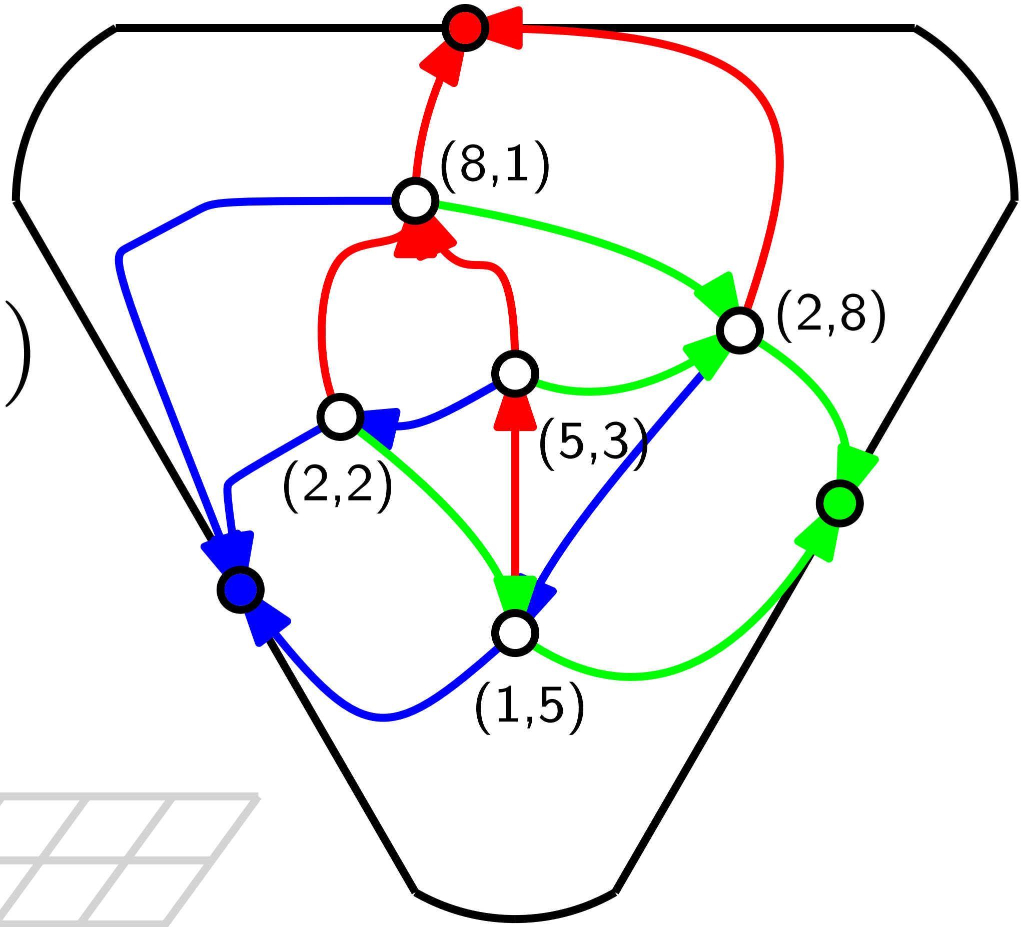

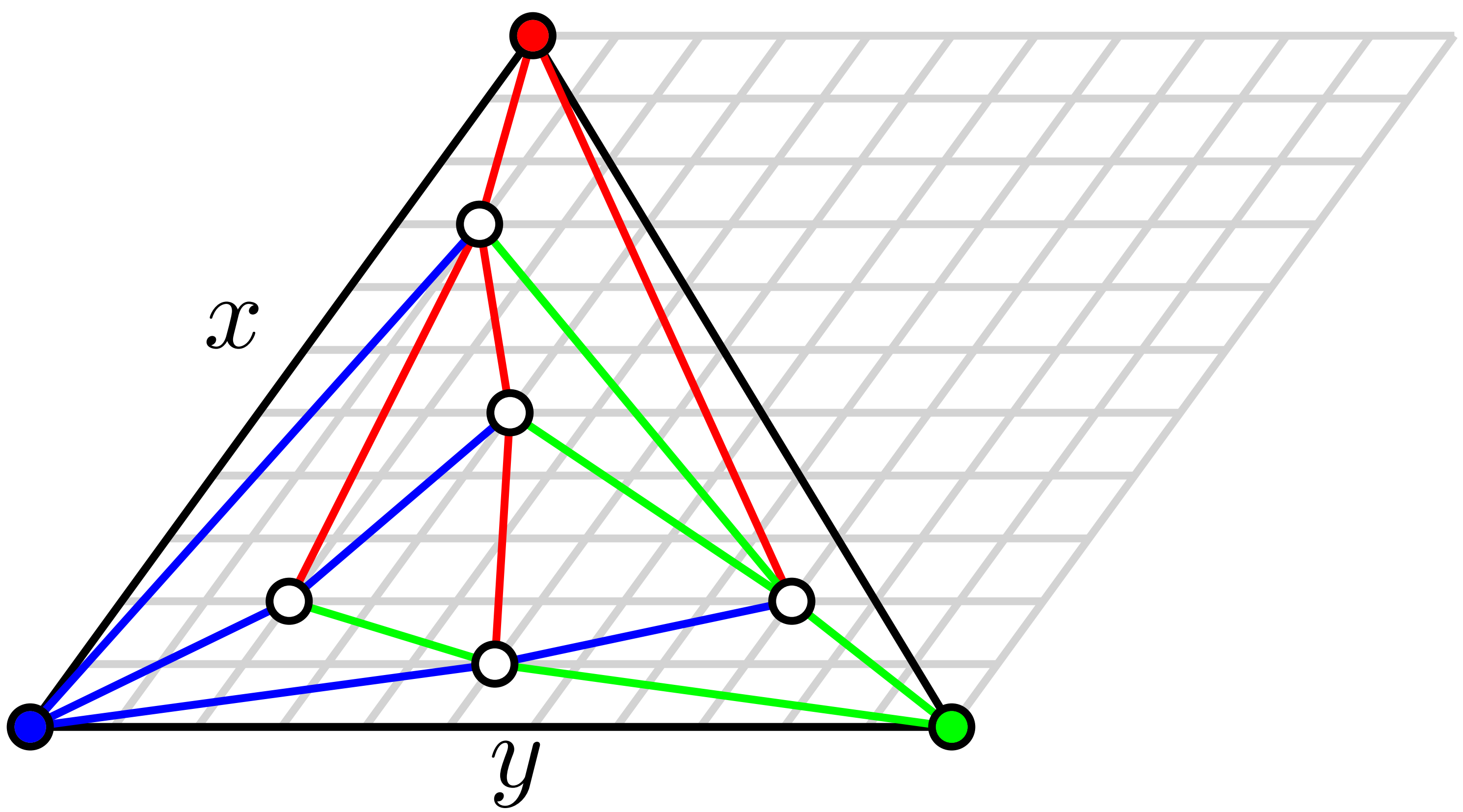

NICE CONTACT REPRESENTATIONS OF PLANAR GRAPHS

with Farzad Fallahi

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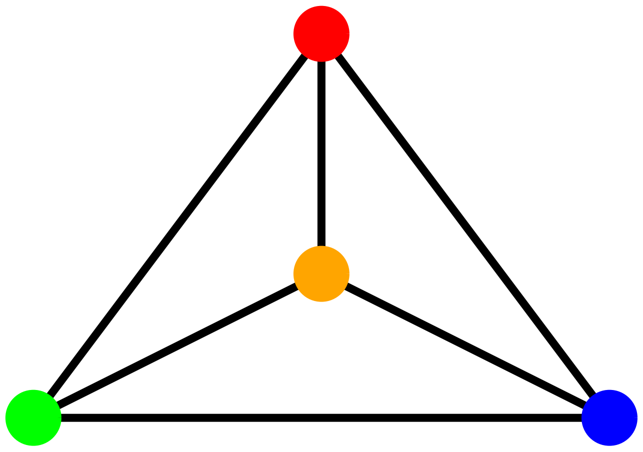
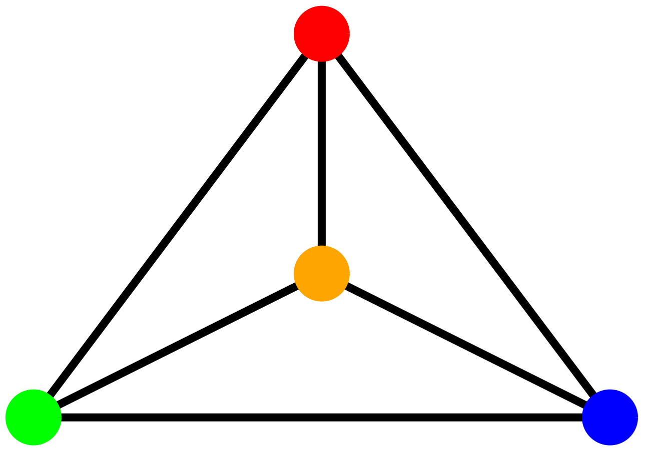
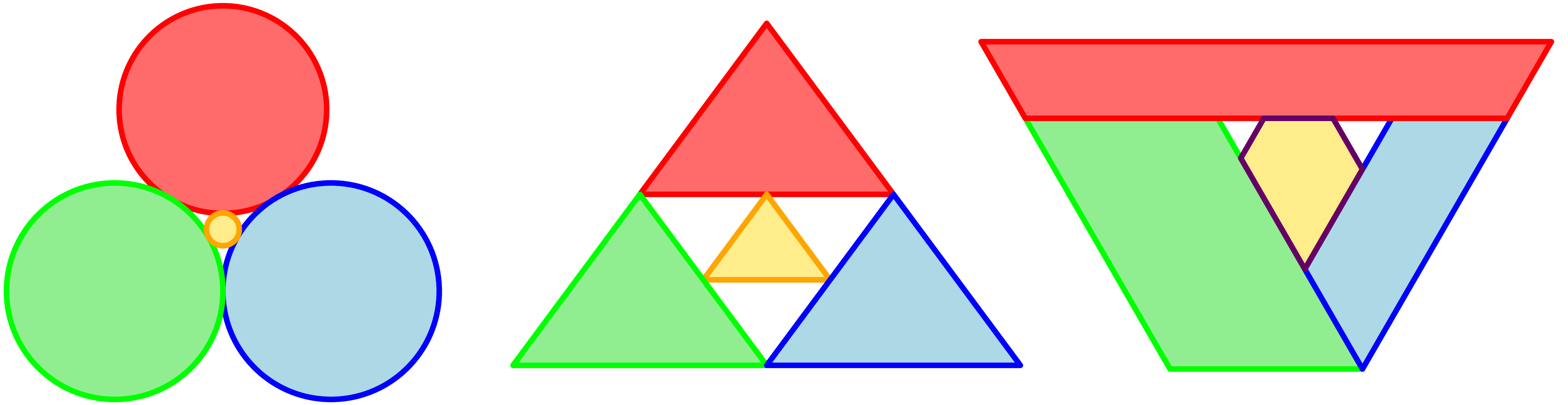


NICE CONTACT REPRESENTATIONS OF PLANAR GRAPHS

with Farzad Fallahi

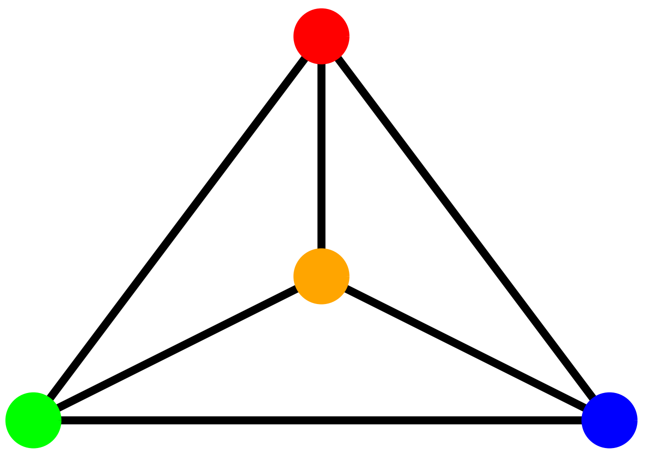


Contact Representations



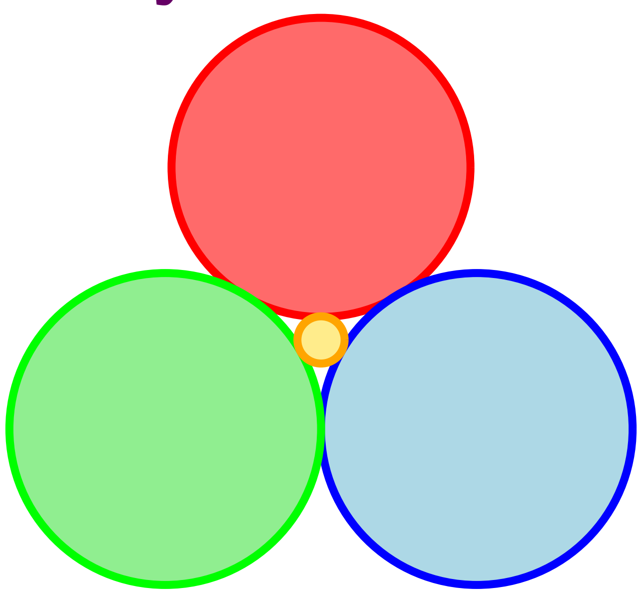
NICE CONTACT REPRESENTATIONS OF PLANAR GRAPHS

with Farzad Fallahi

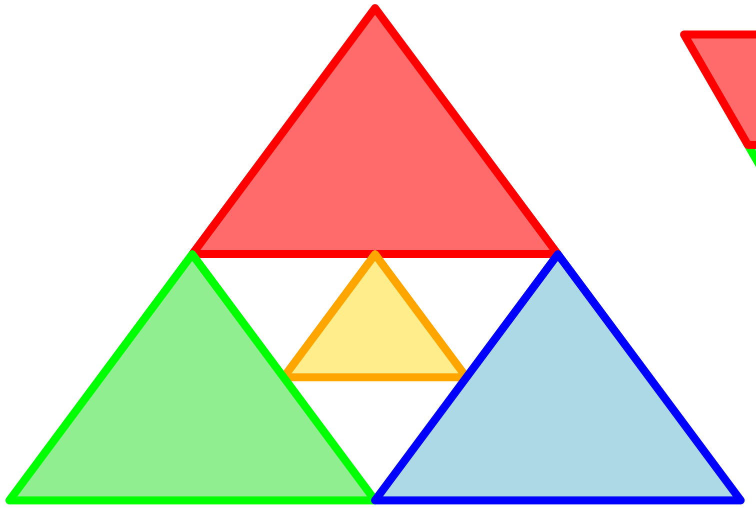


Contact Representations

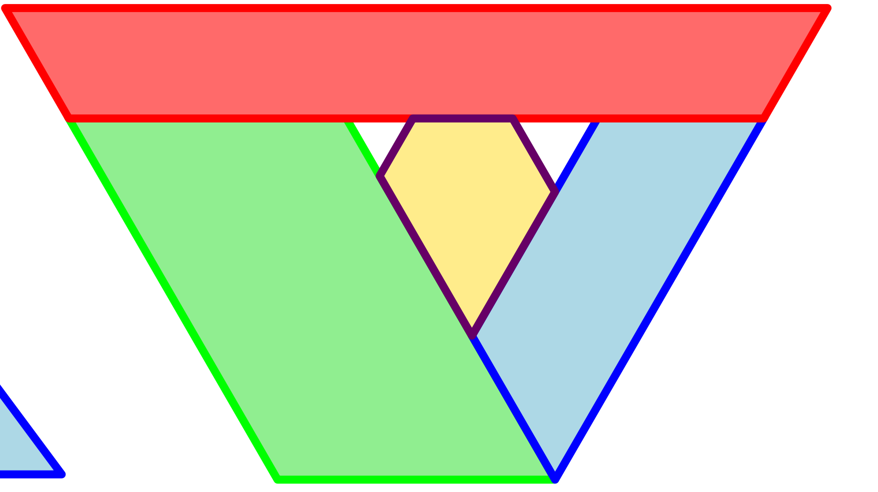
Symmetric



Simple



Face-to-face



Facts of Life

To represent all planar graphs with Face-to-Face contact, you need 6-sided polygons.

How Symmetric can they be?

Facts of Life

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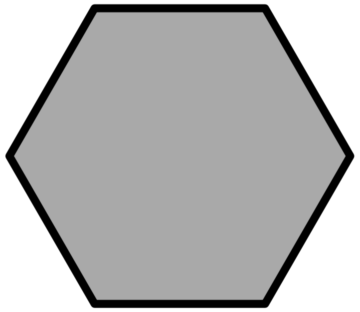
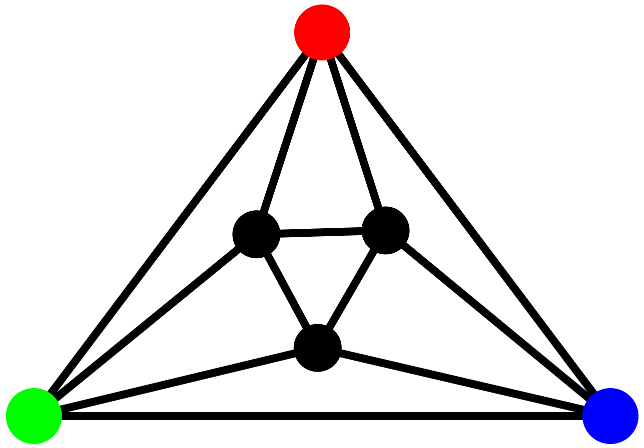
How Symmetric can they be?

Regular Hexagon



Too Symmetric



Facts of Life

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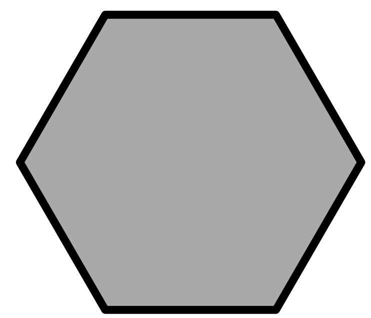
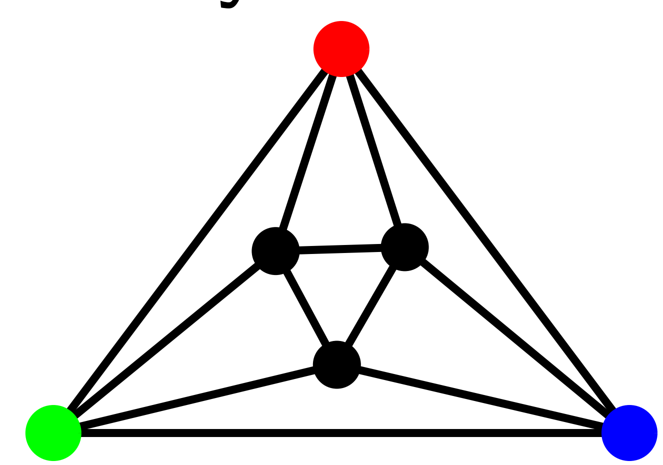
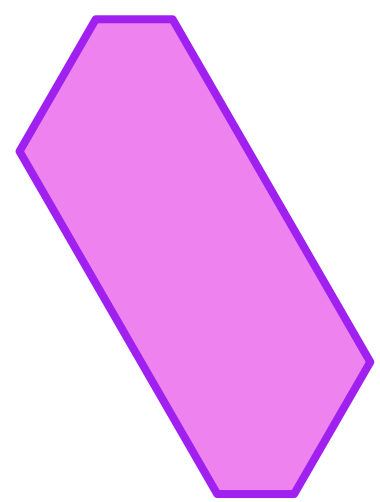


Too Symmetric

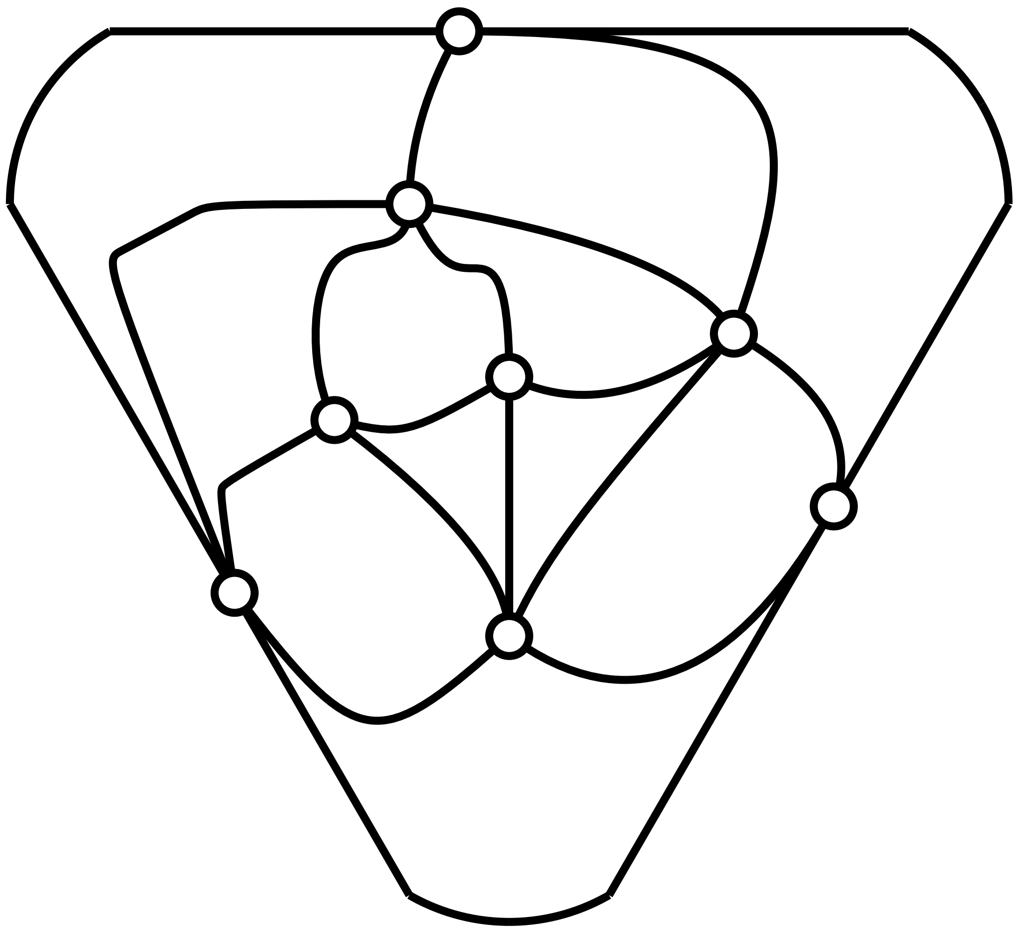


Equi-Parallel Hexagon



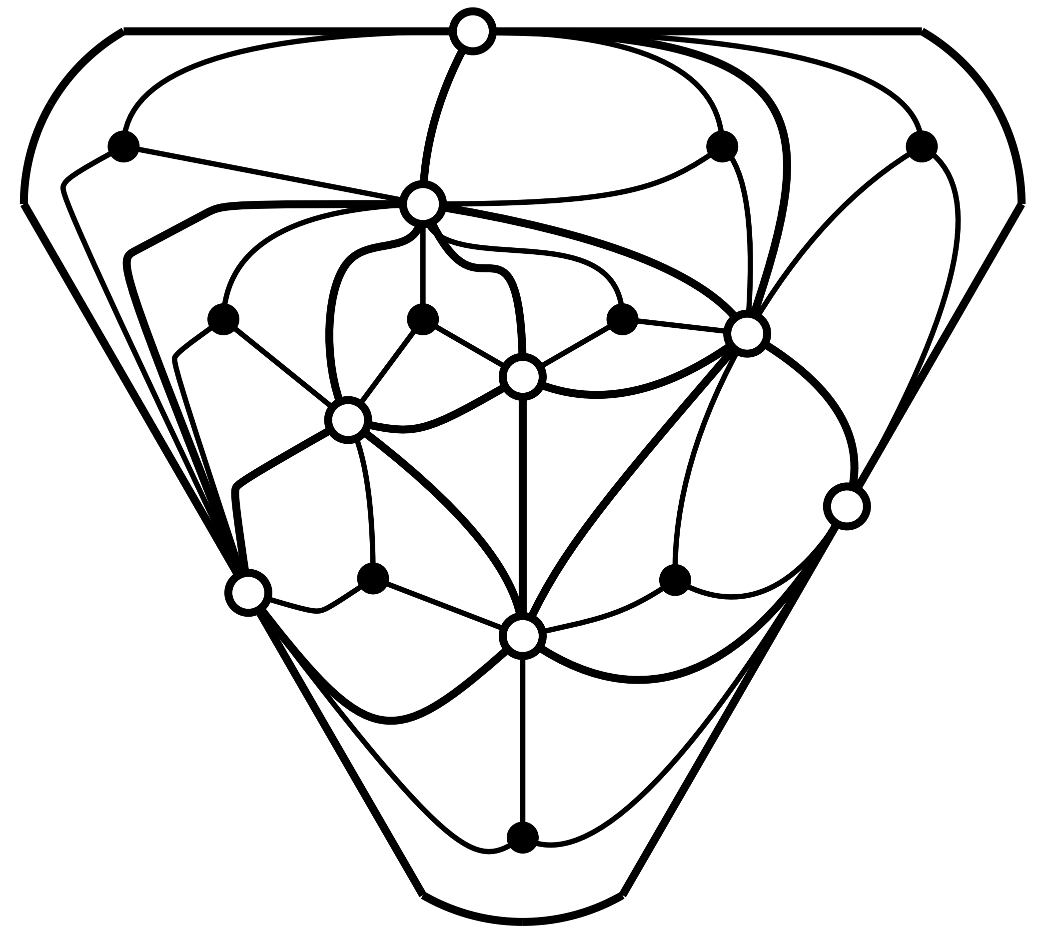
Just Symmetric enough.

Given a planar graph...



Given a planar graph...

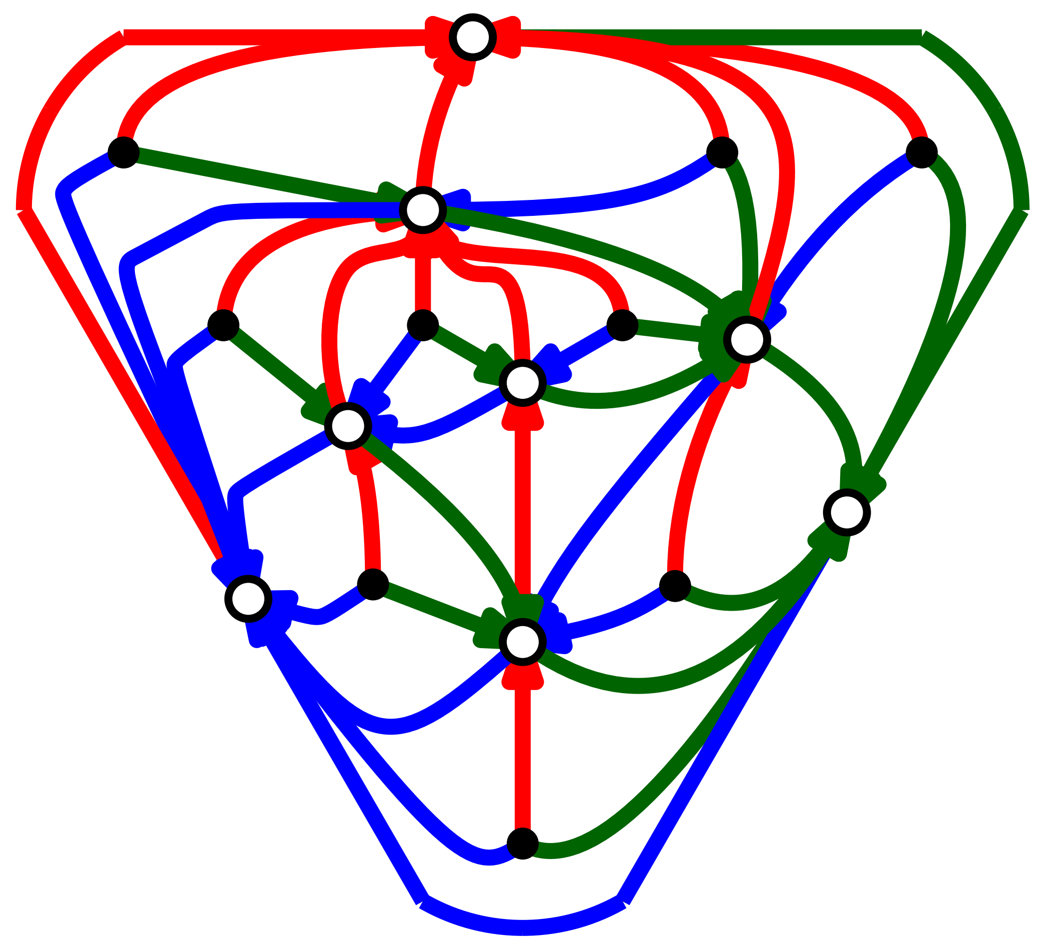
add dummy vertex in each face,



Given a planar graph...

add dummy vertex in each face,

color and direct edges to form a Schnyder wood,

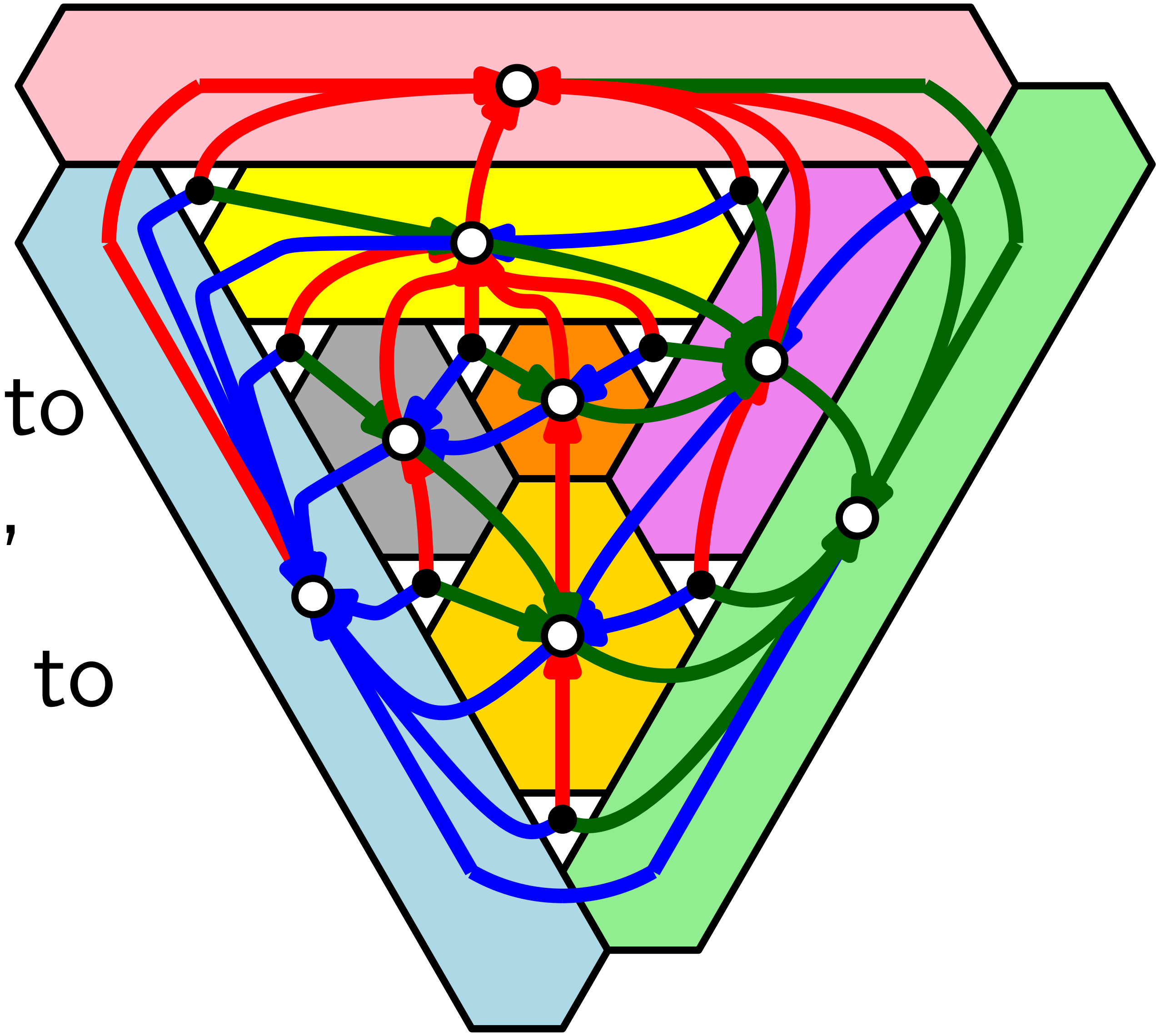


Given a planar graph...

add dummy vertex in each face,

color and direct edges to form a Schnyder wood,

and solve a 3-way flow to find side lengths.



Given a planar graph...

add dummy vertex in each face,

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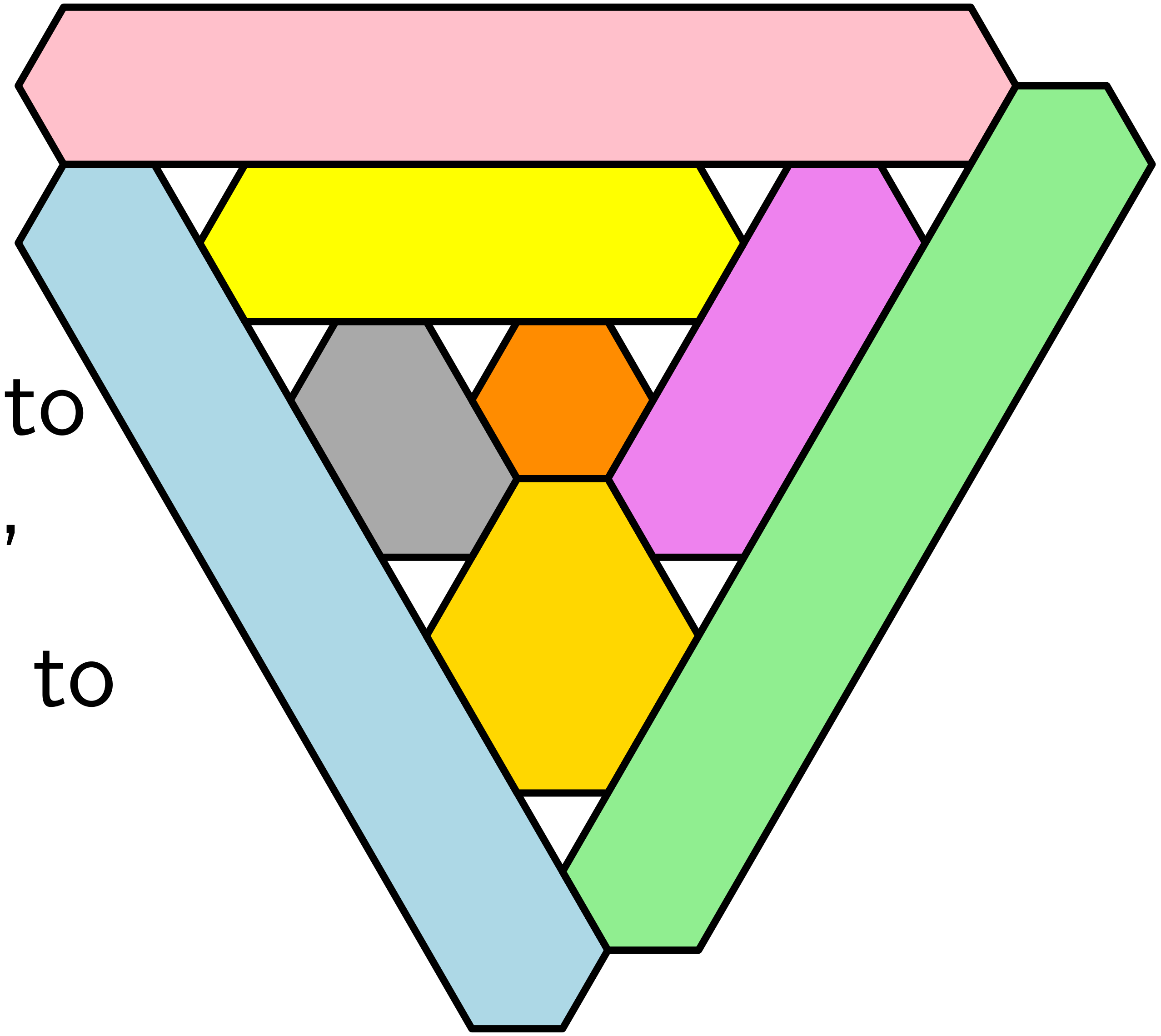
Ta da!



CONTACT REPRESENTATIONS OF NON-PLANAR GRAPHS IN 3D

with

Md. Jawaherul Alam

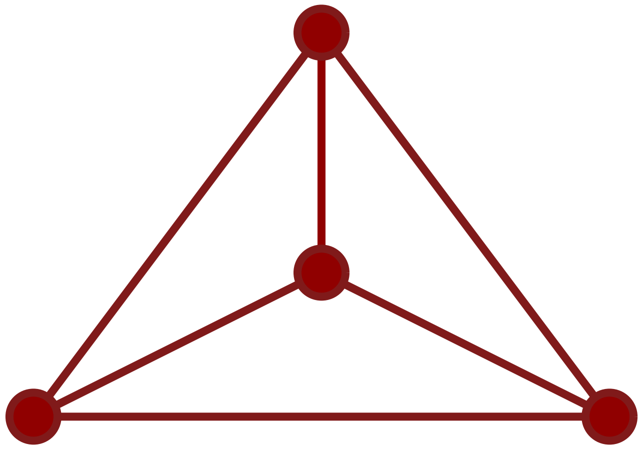
Stephen Kobourov

Sergey Pupyrev

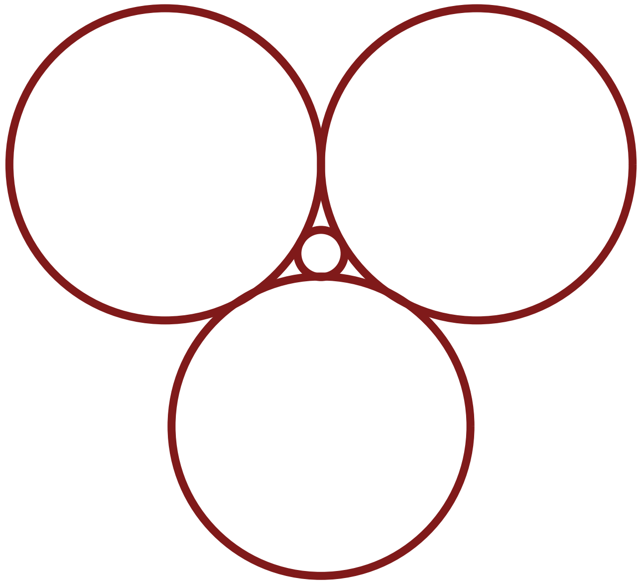
Jackson Toeniskoetter

Torsten Ueckerdt

Contact Representation

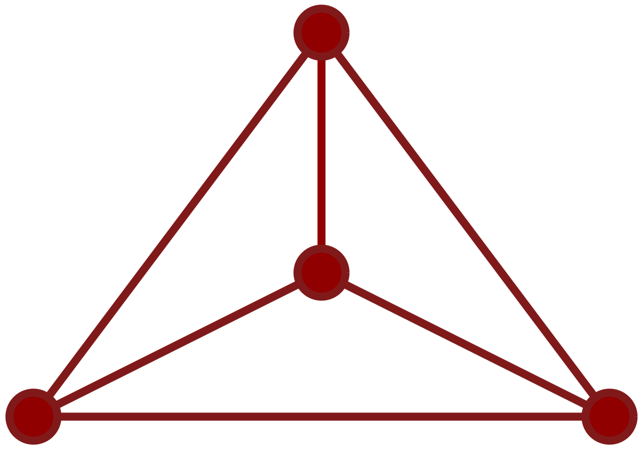


Contact Representation

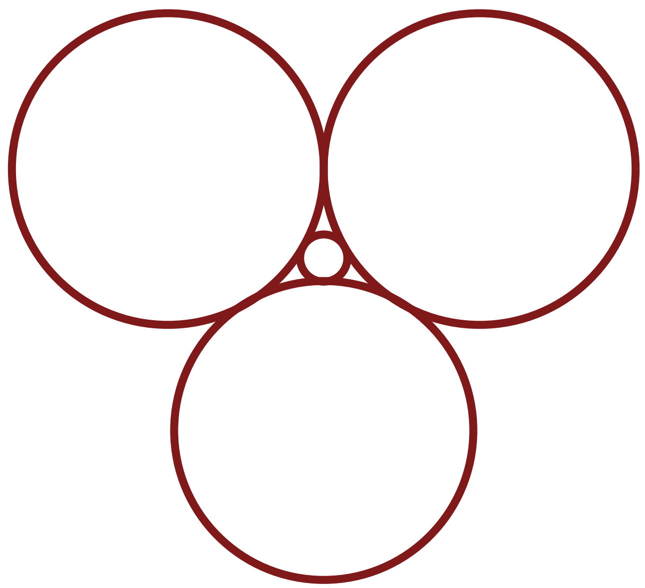


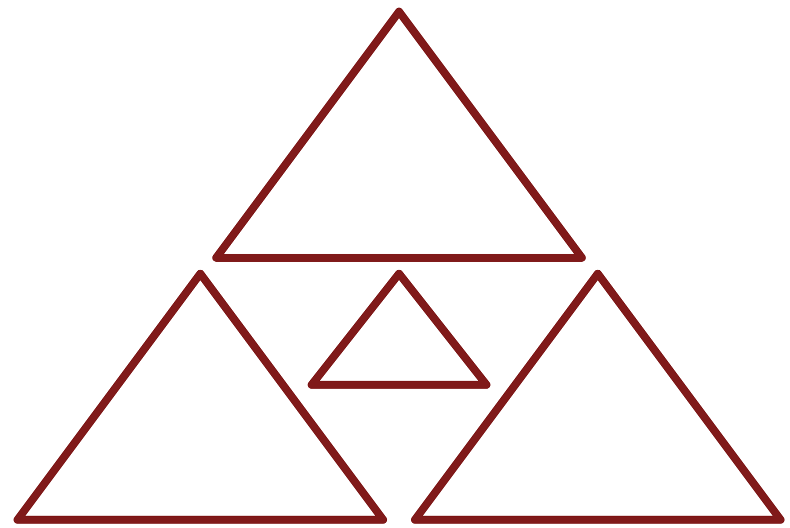
Vertices = Interior disjoint objects

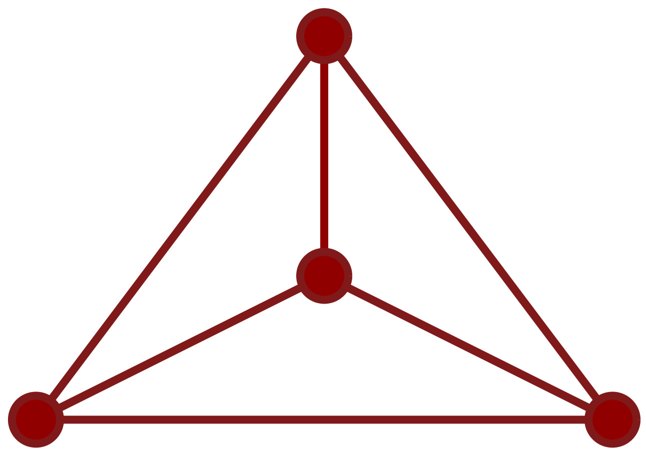
Edges = Contact



Contact Representation







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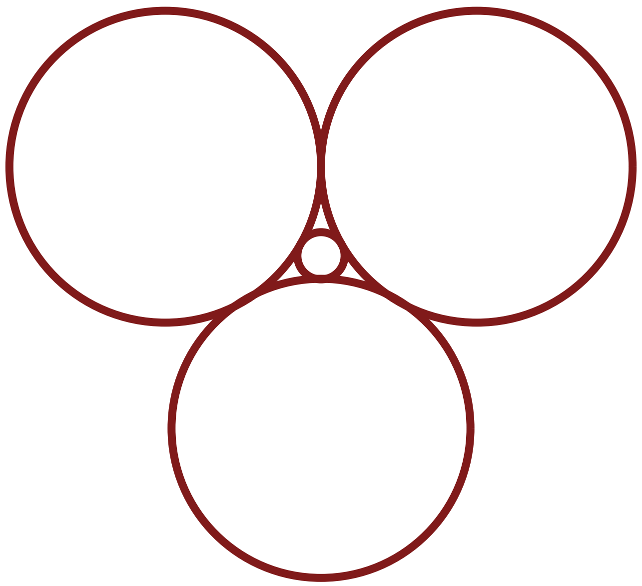
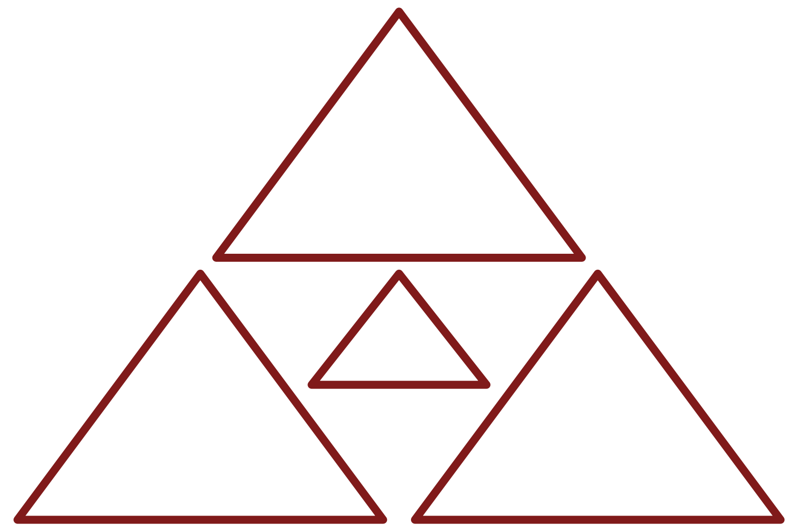
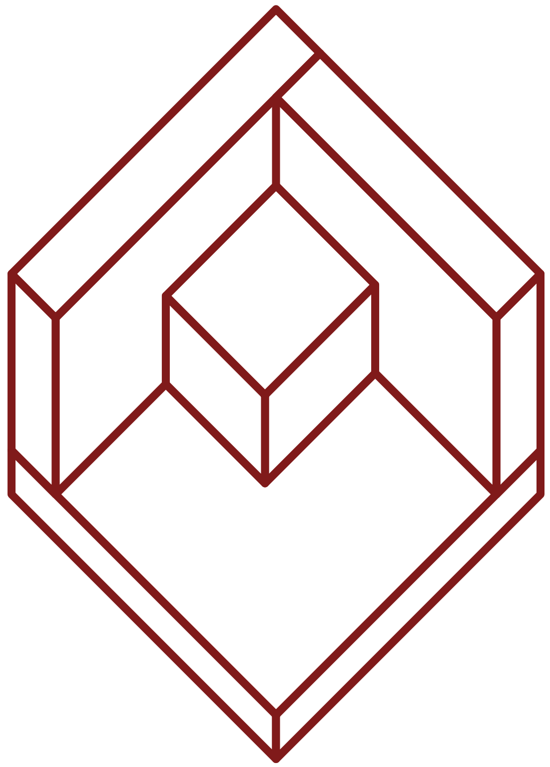
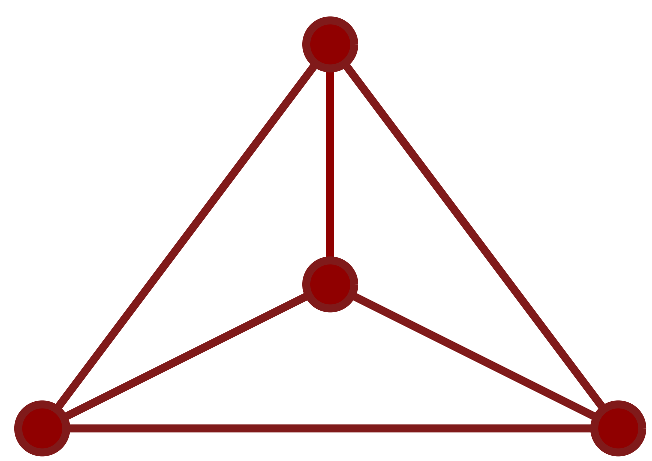
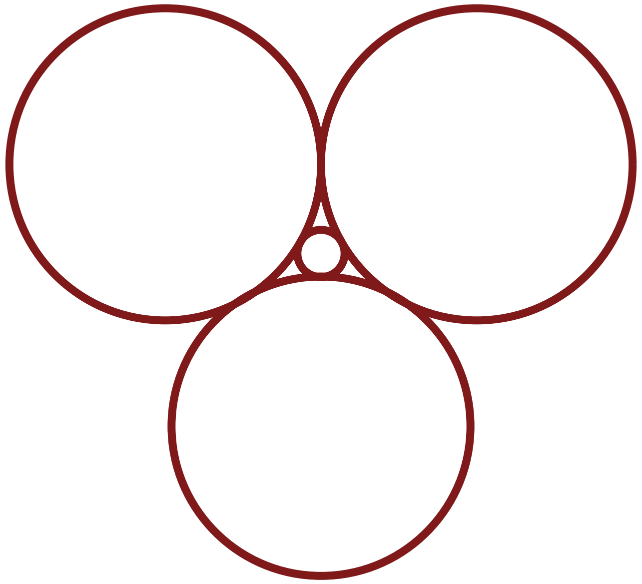


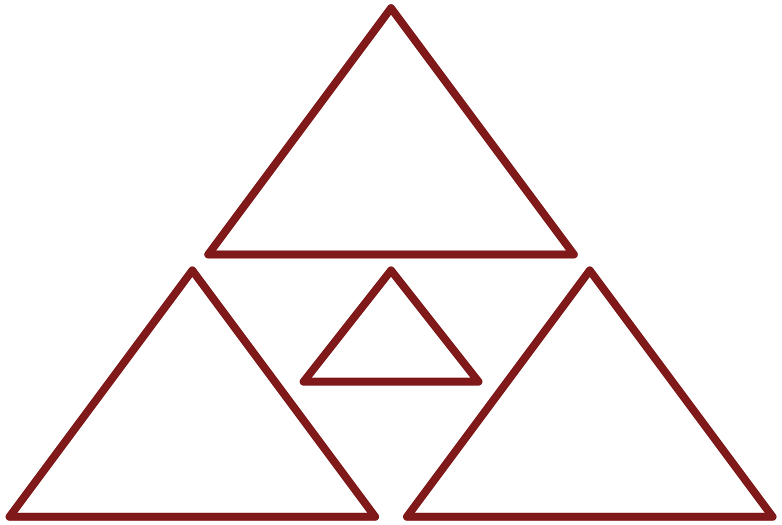


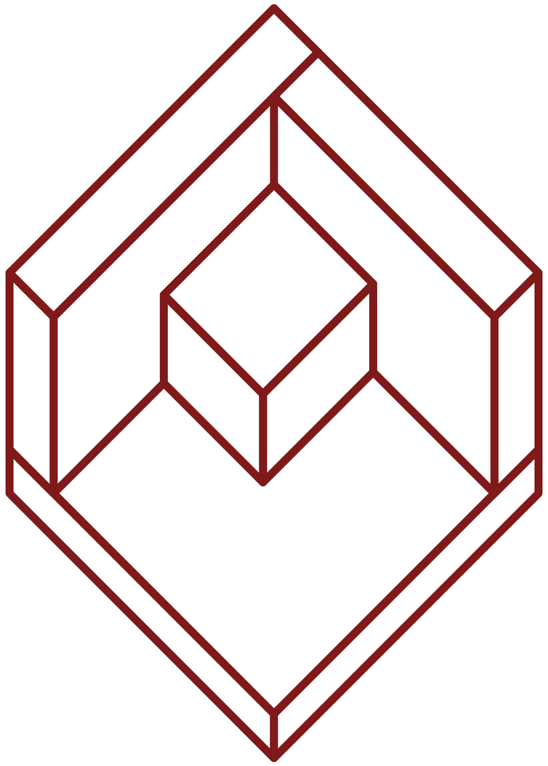




What graphs can be represented?







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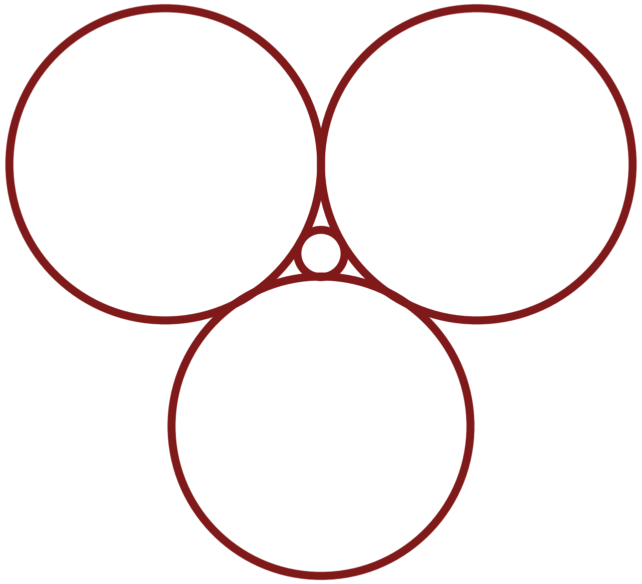
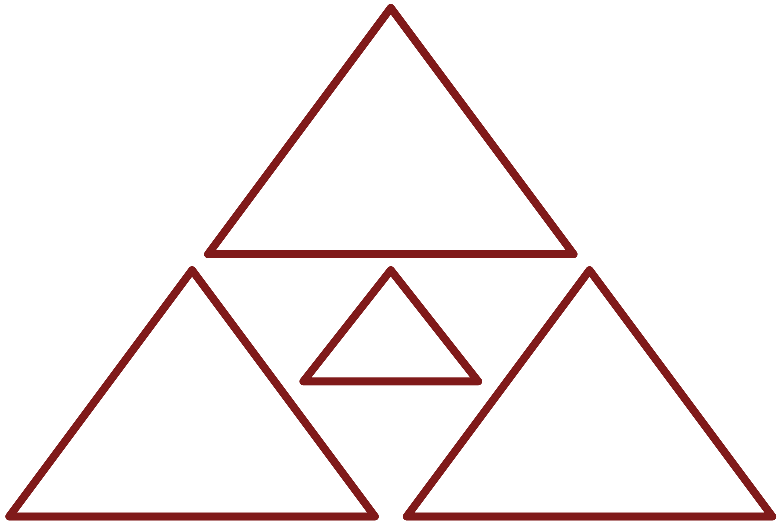
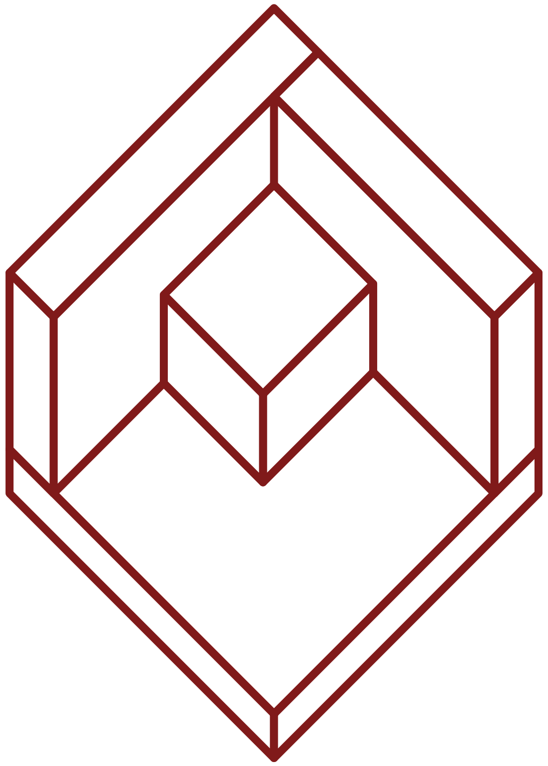


Planar Graphs
Kobe 36





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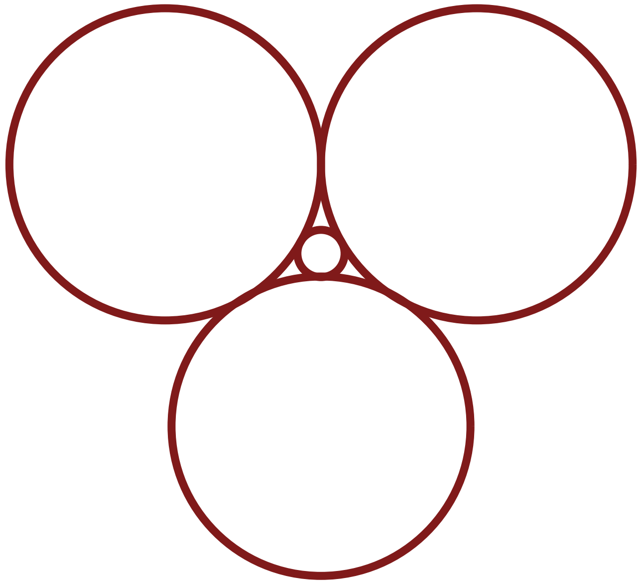

Planar Graphs
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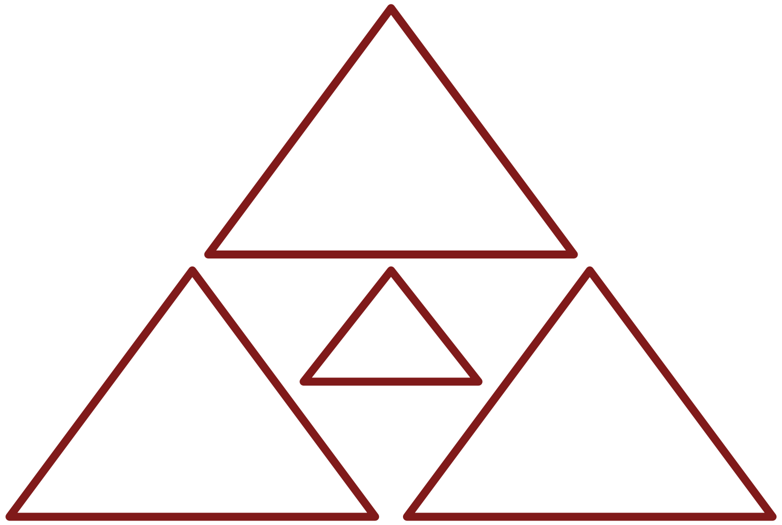
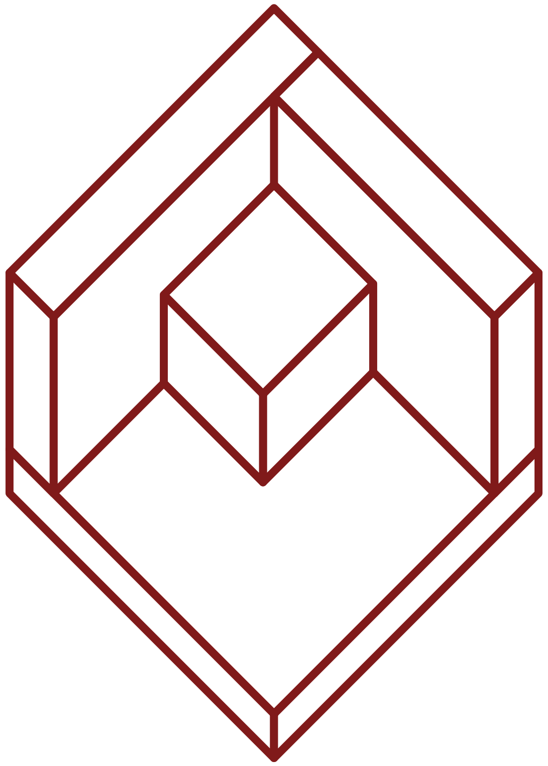


Planar Graphs
De Fraysseix et al. 94



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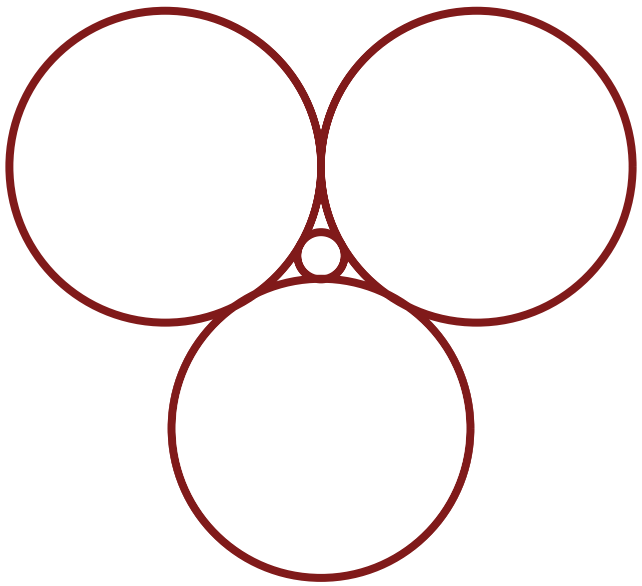
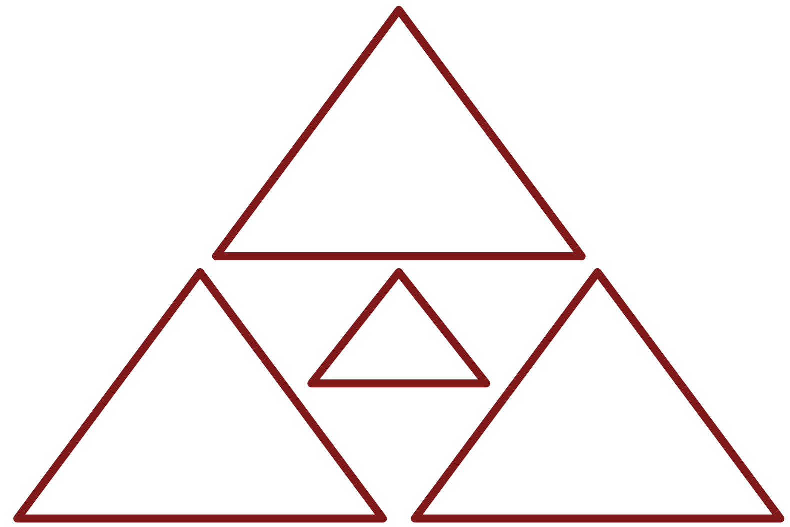
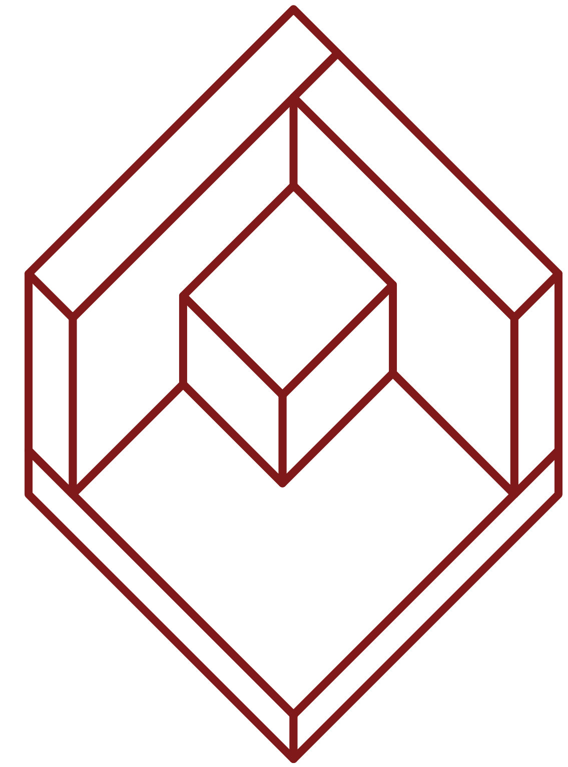


Planar Graphs
Kobe 36



Planar Graphs
De Fraysseix et al. 94



Planar Graphs
Thomassen 86

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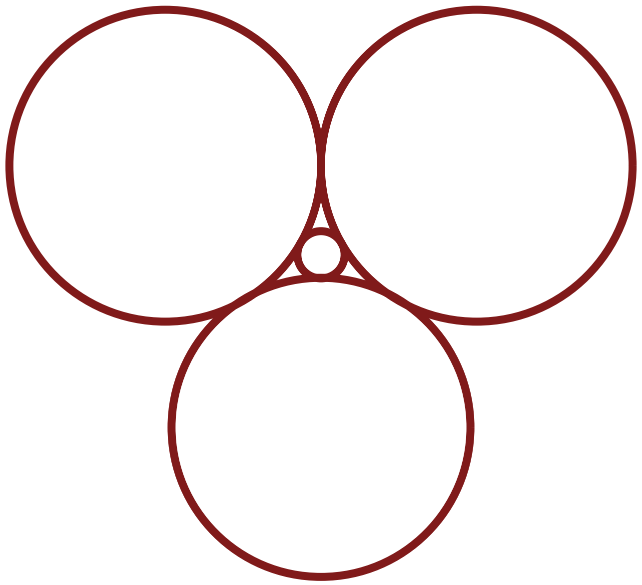
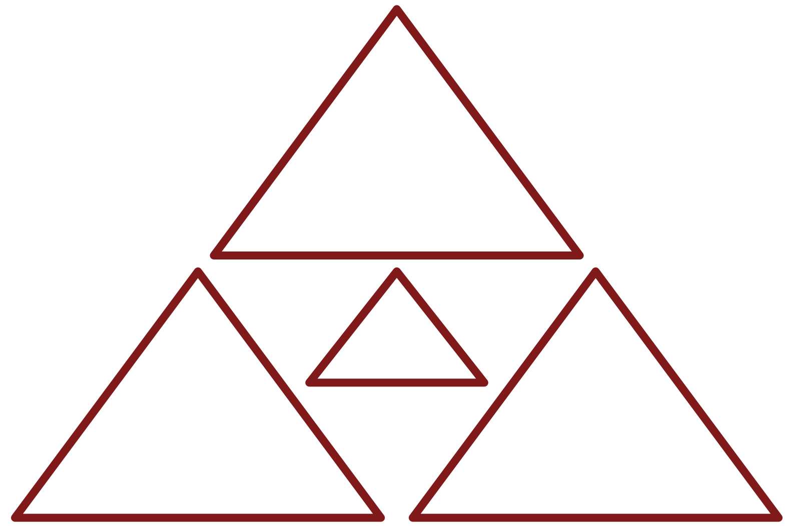
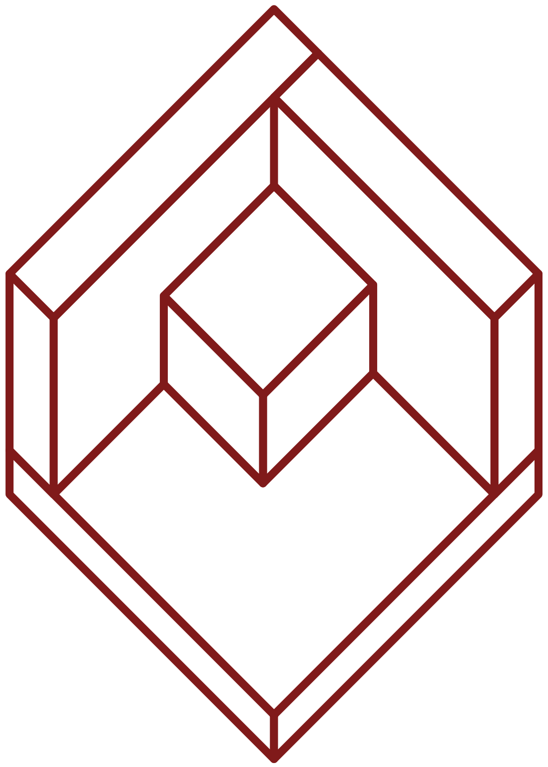


Planar Graphs
Kobe 36

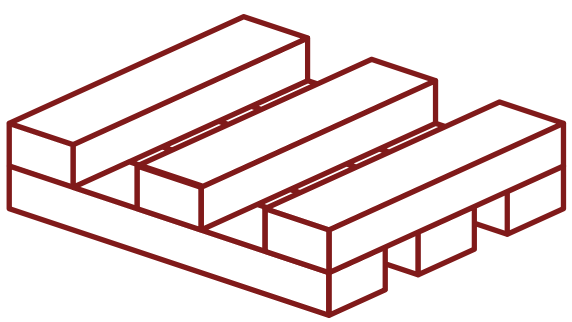


Planar Graphs
De Fraysseix et al. 94



Planar Graphs
Thomassen 86

... and more.



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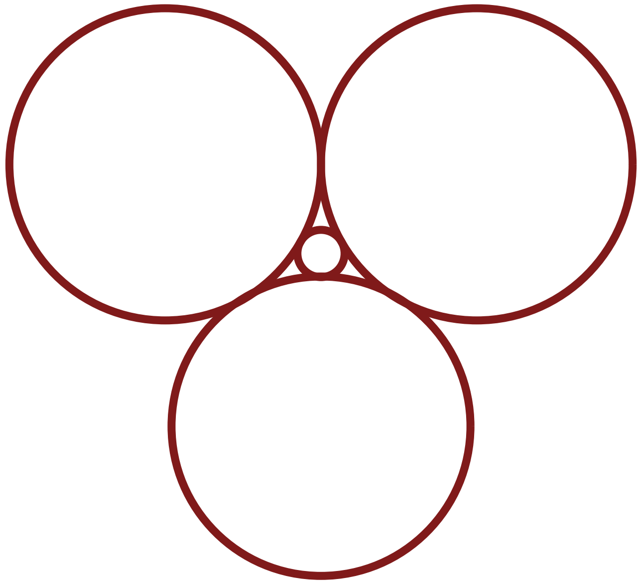
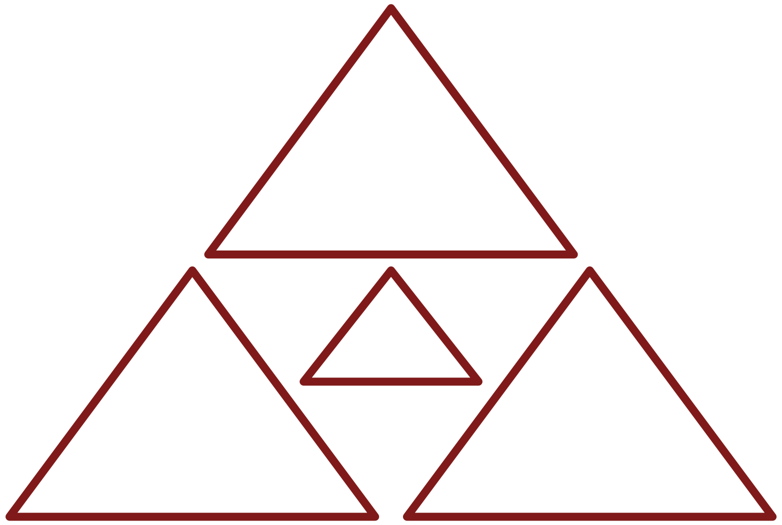


Planar Graphs
Kobe 36



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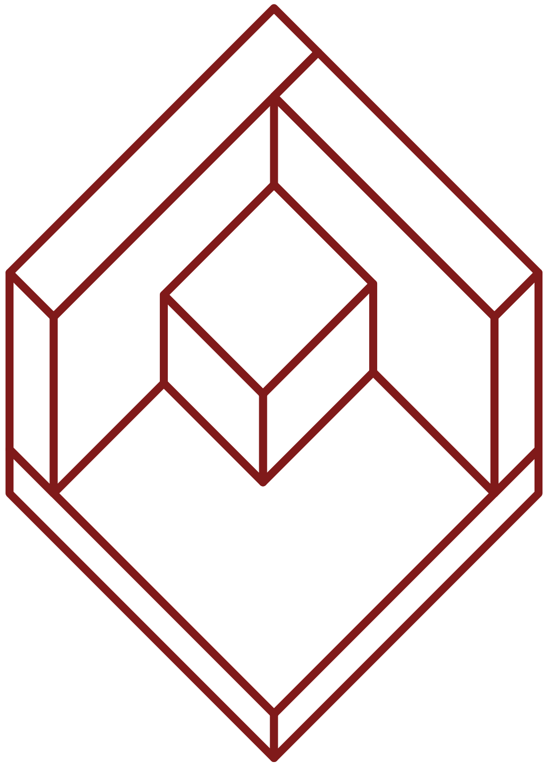
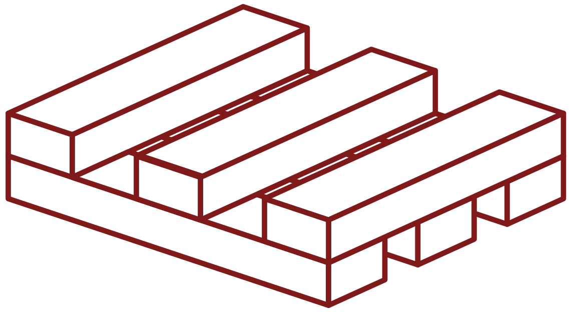


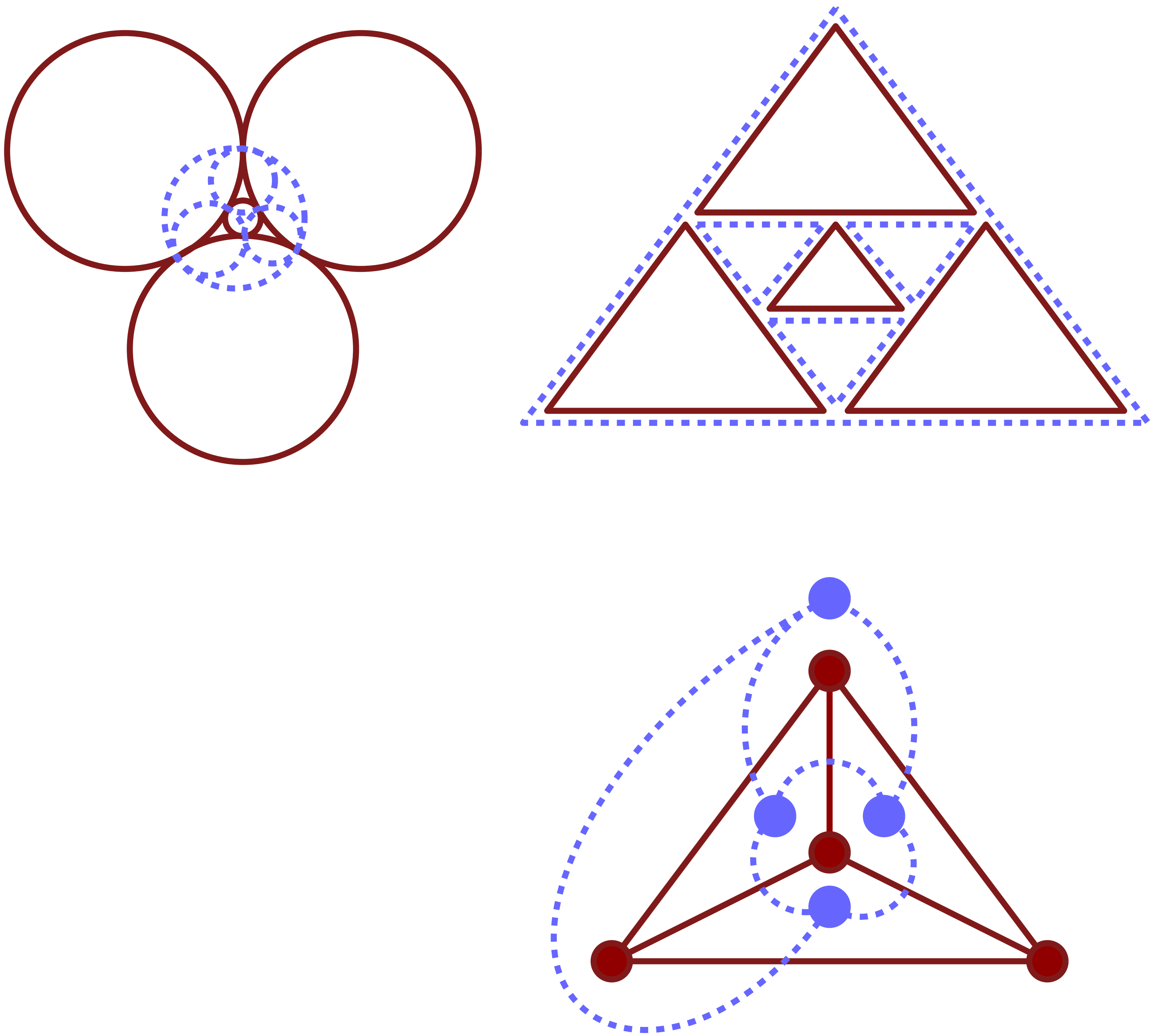
Planar Graphs
Thomassen 86

... and more.

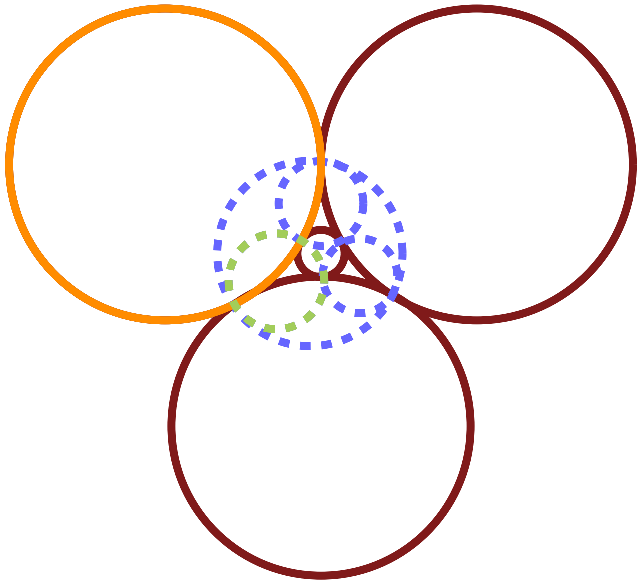
How much more?

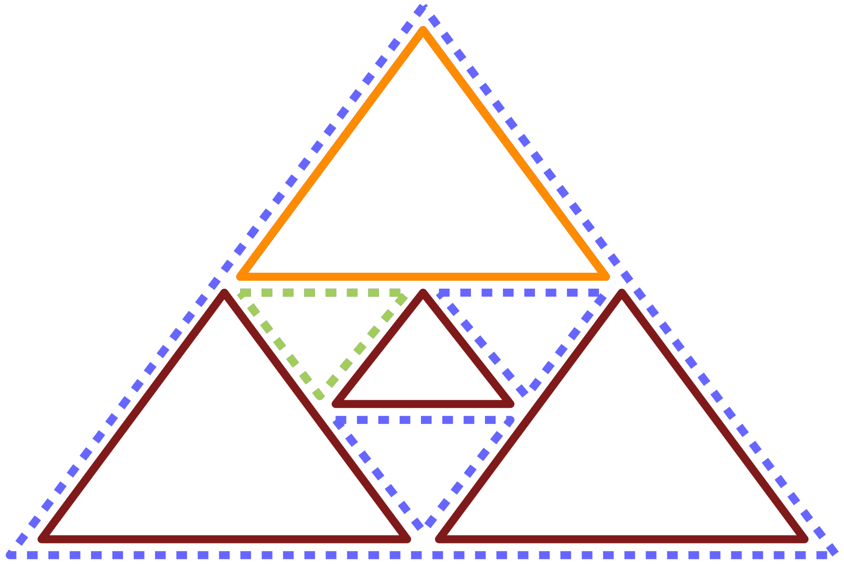


Simultaneous Primal-Dual Contact Representation

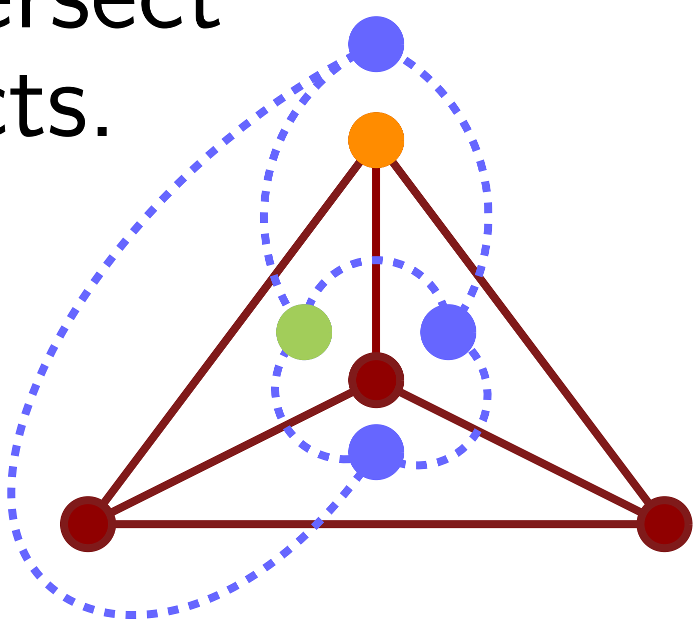


Simultaneous Primal-Dual Contact Representation

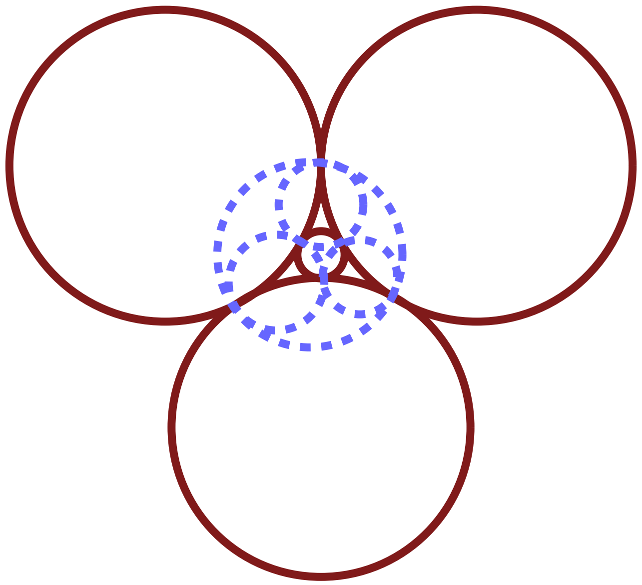




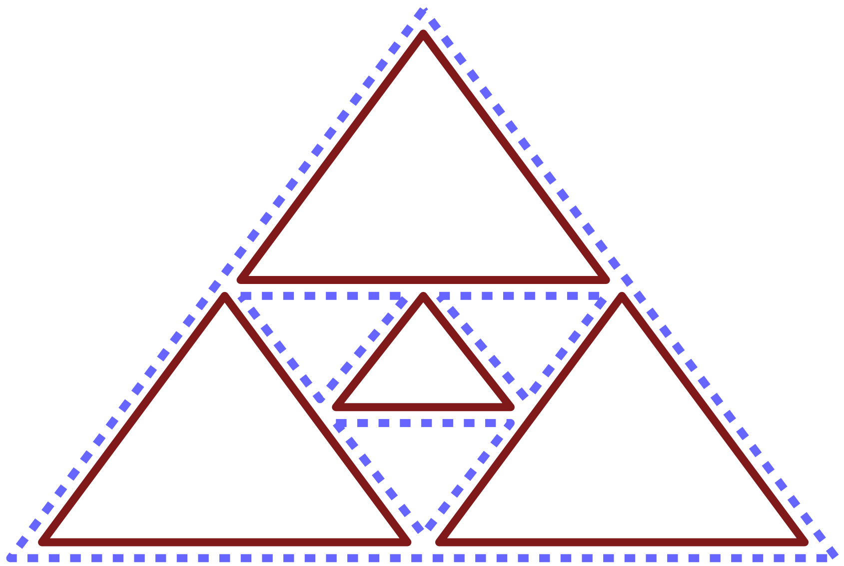
Vertex objects intersect incident face objects.



Simultaneous Primal-Dual Contact Representation



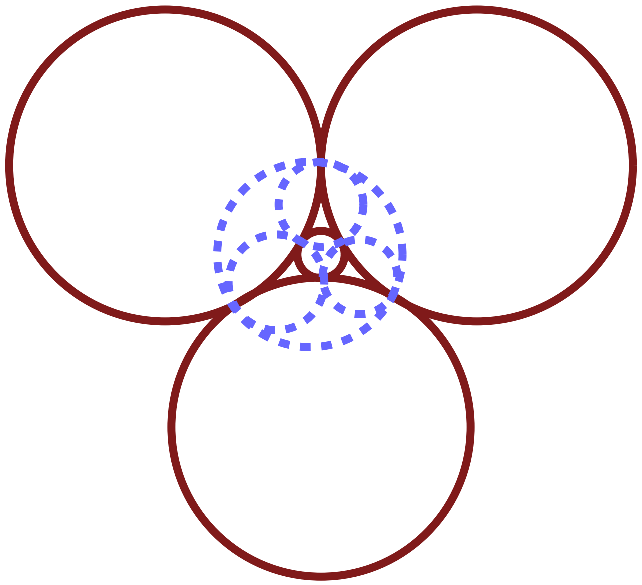
Andreev 70



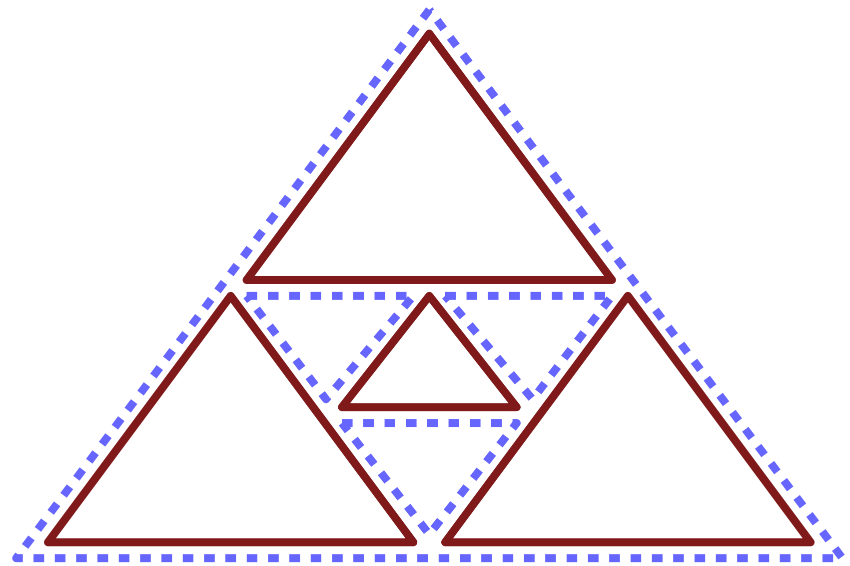
Gonçalves 12

3-connected planar graph & dual

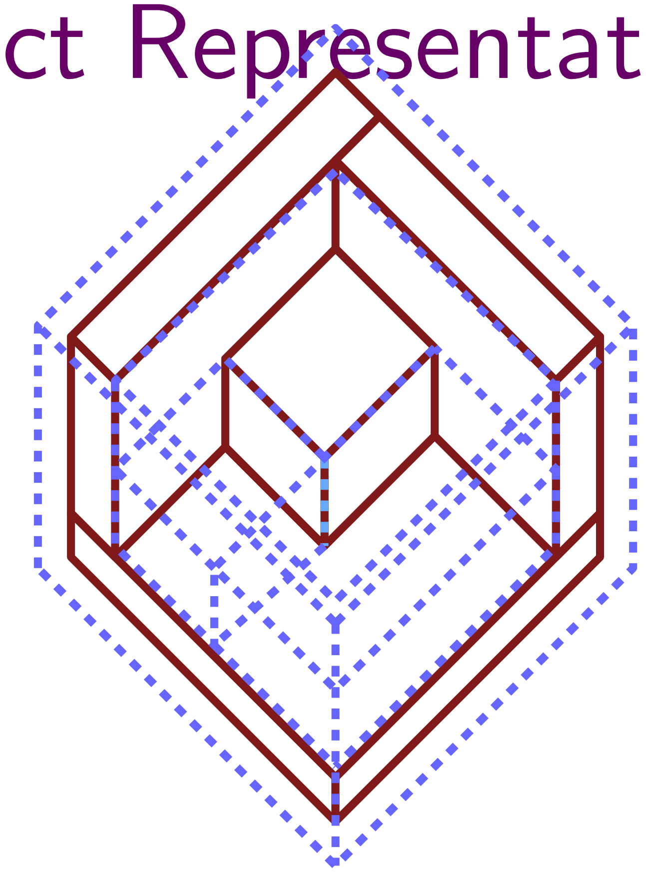
Simultaneous Primal-Dual Contact Representation



Andreev 70



Gonçalves 12

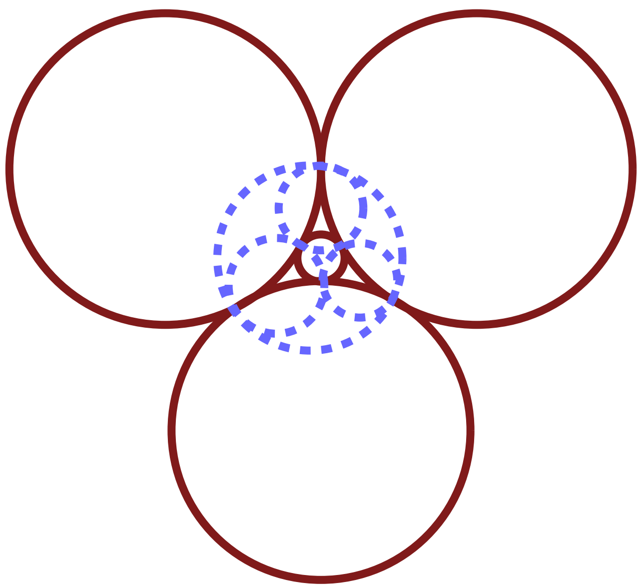


This paper

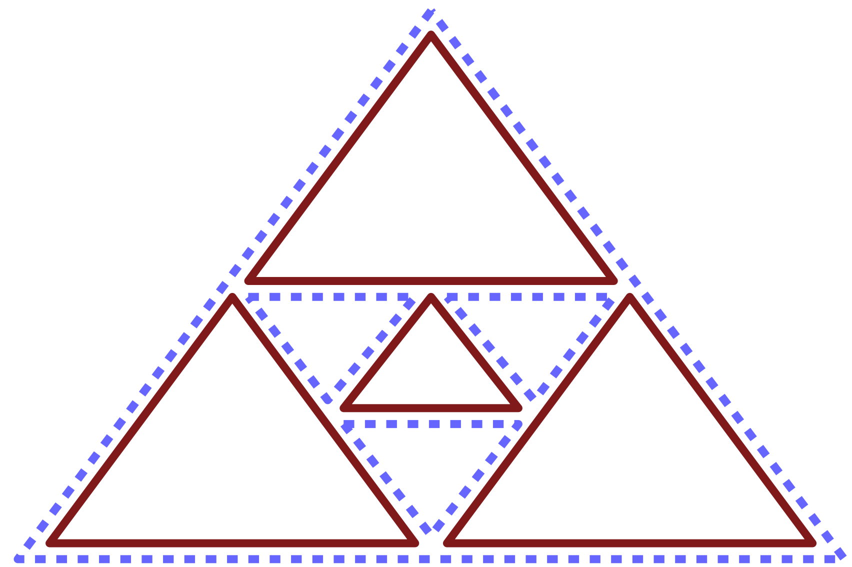
3-connected planar graph & dual

Thm 1 Every 3-connected planar graph admits a proper primal-dual 3D box-contact representation.

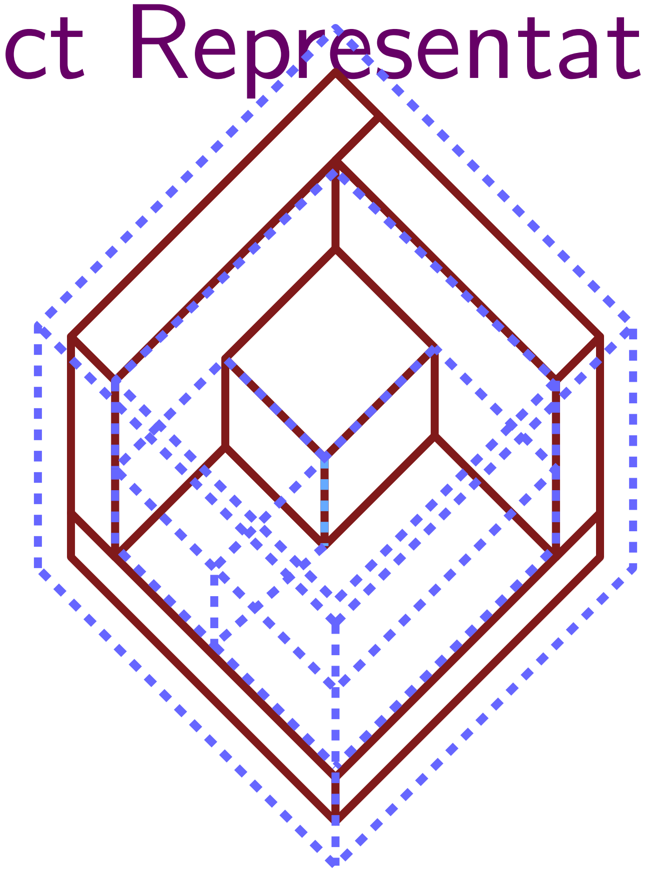
Simultaneous Primal-Dual Contact Representation



Andreev 70



Gonçalves 12



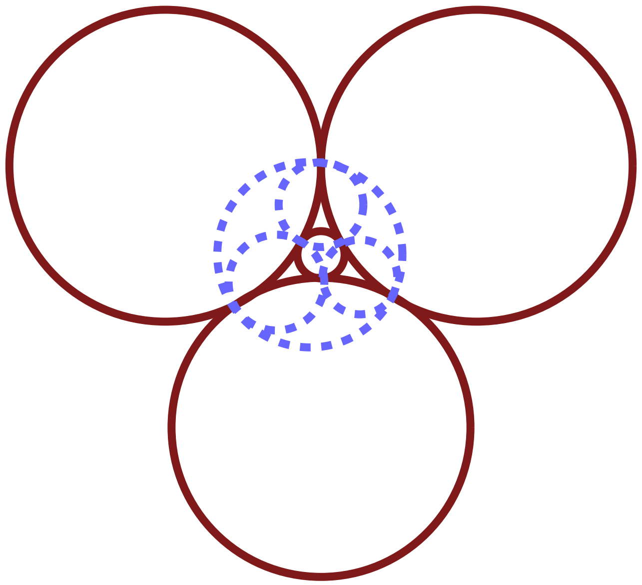
This paper

3-connected planar graph & dual

Thm 1 Every 3-connected planar graph admits a <u>proper</u> primal-dual 3D box-contact representation.

proper: face-to-face contact

Simultaneous Primal-Dual Contact Representation



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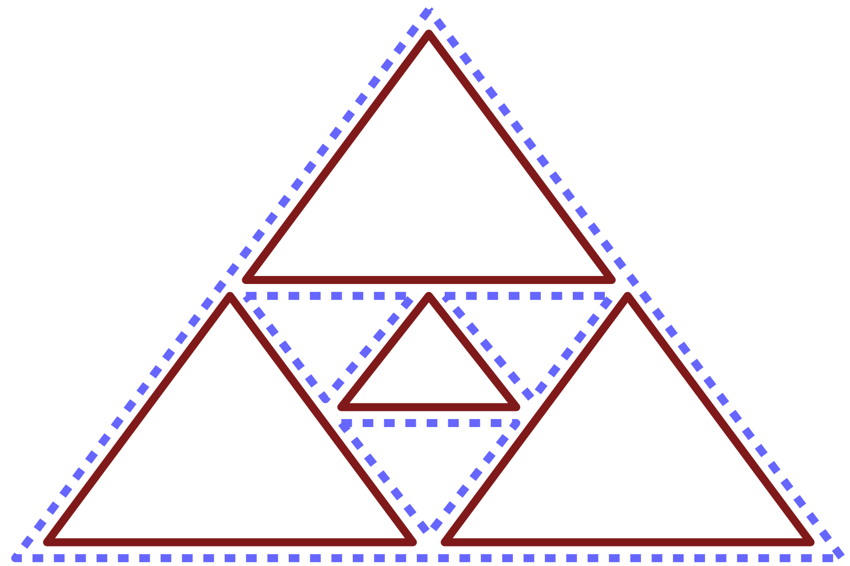

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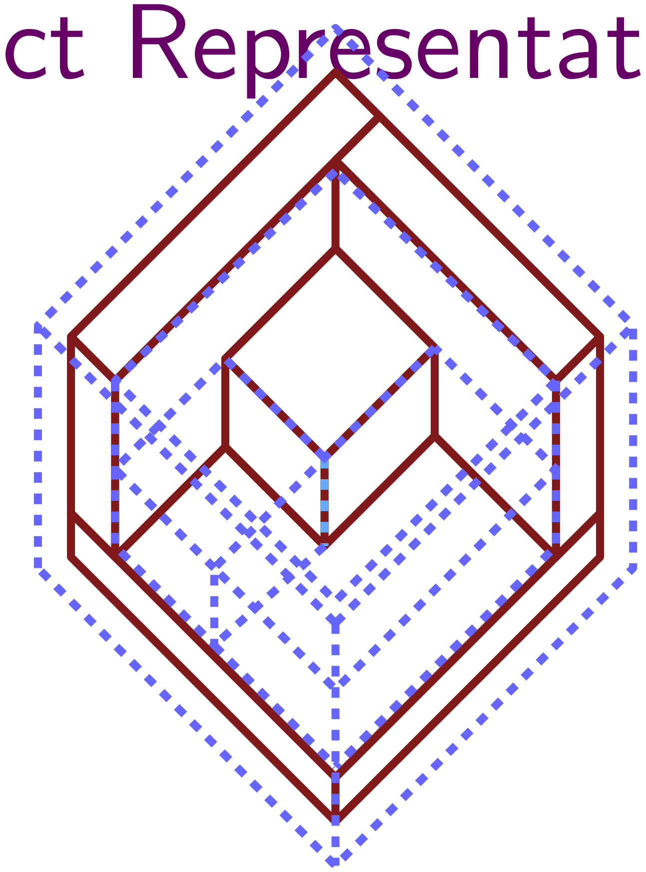


This paper

3-connected planar graph & dual

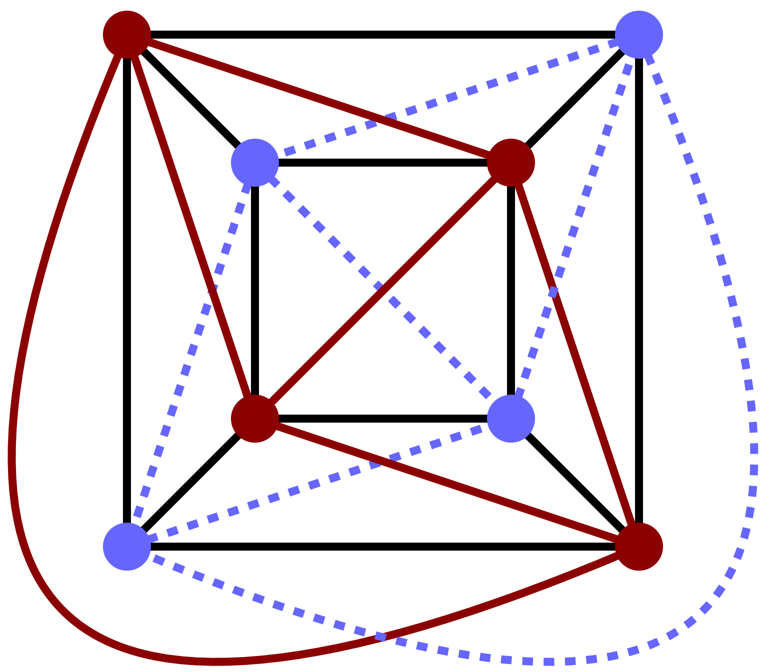
Thm 1 Every 3-connected planar graph admits a proper primal-dual 3D box-contact representation.

And it can be computed in linear time.

Primal-Dual to Non-planar Representation

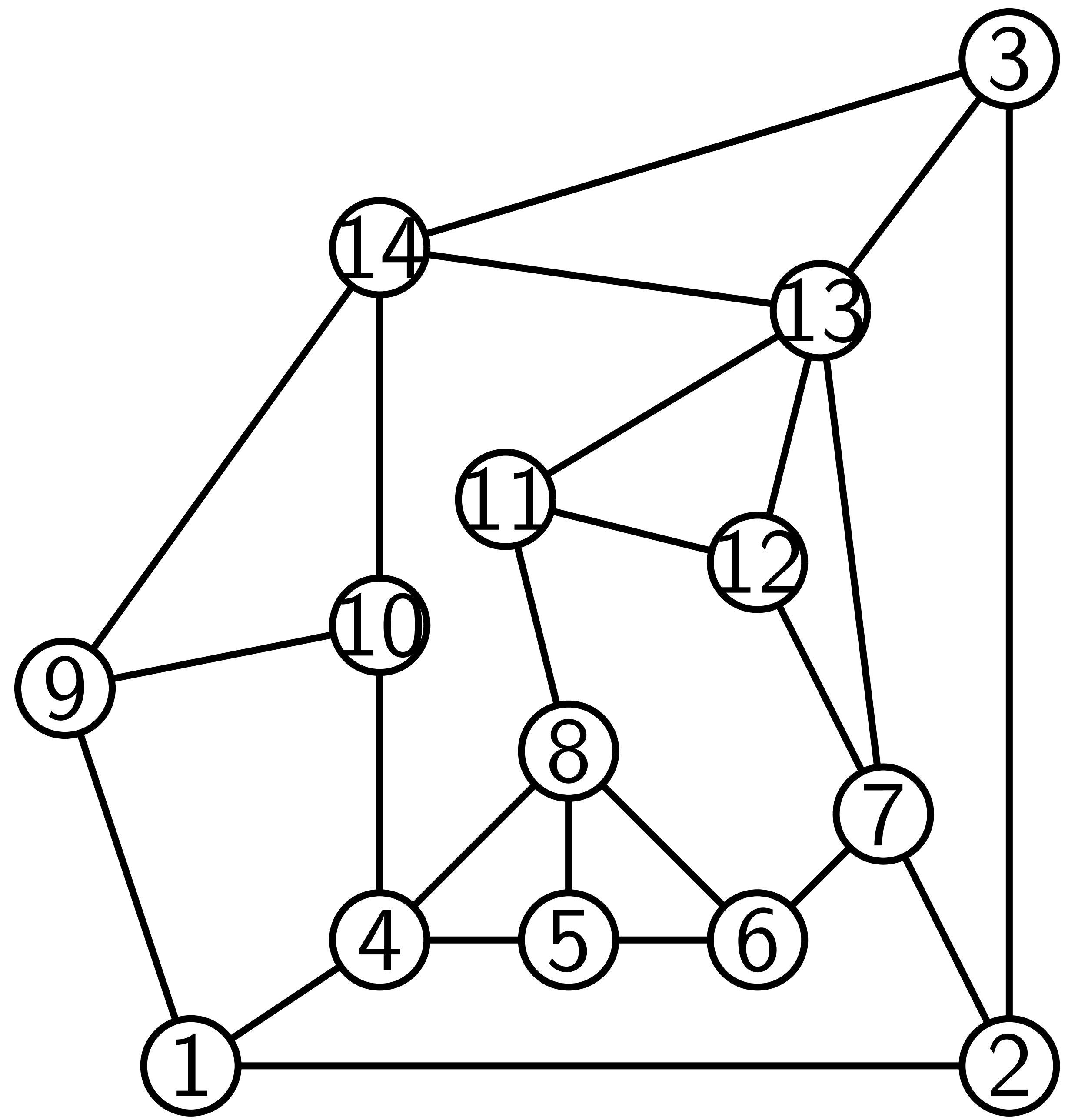
Thm 1 Every 3-connected planar graph admits a proper primal-dual 3D box-contact representation.

Cor Every prime 1-planar graph has a proper shelled 3D box-contact representation.



Schnyder Wood

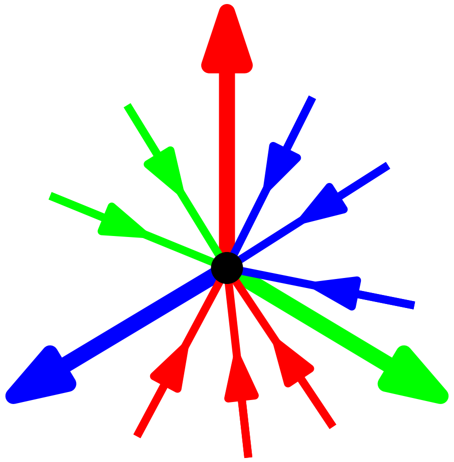
Edge orientation and coloring of 3-connected planar graph using 3 colors so that



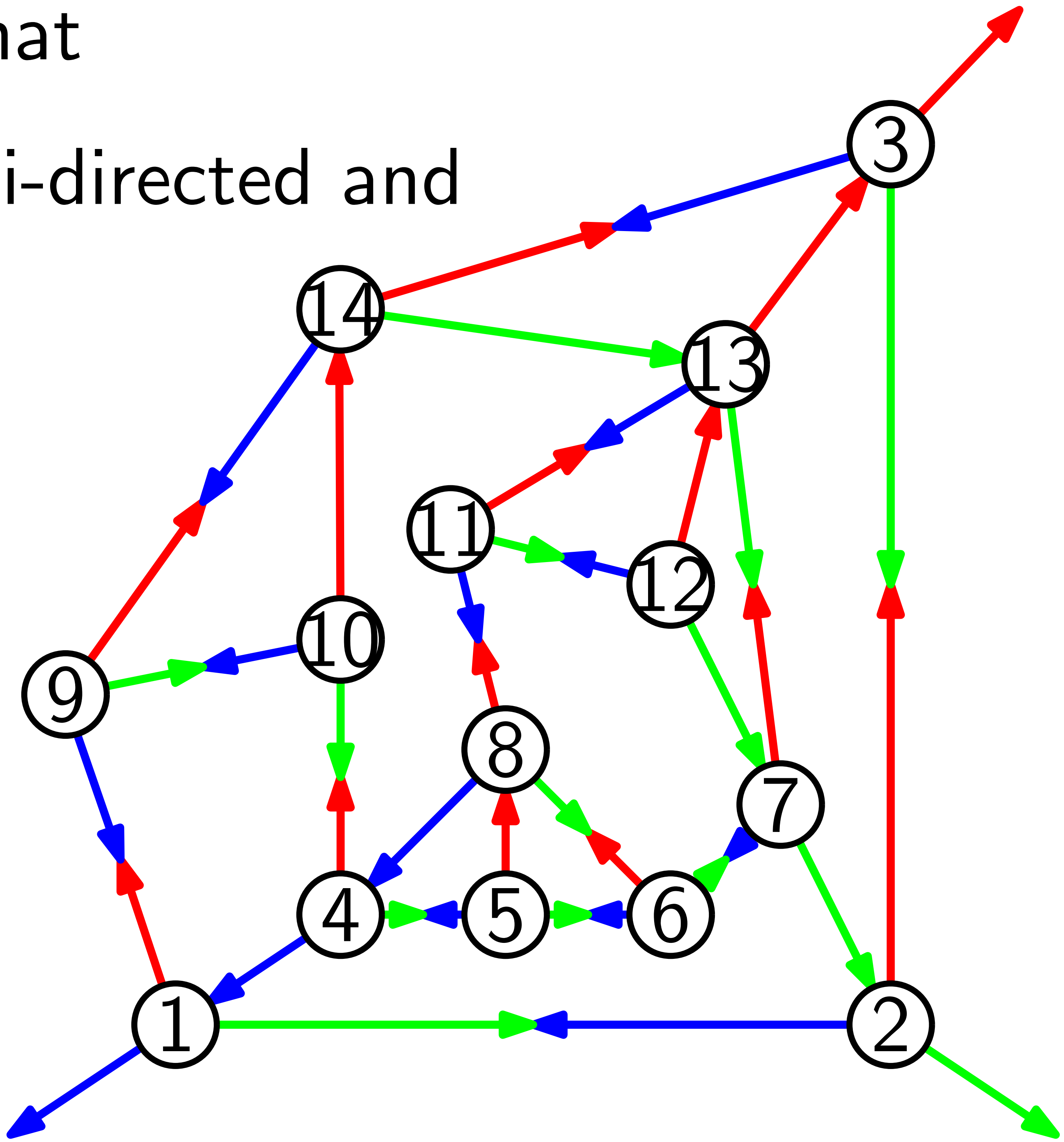
Schnyder Wood

Edge orientation and coloring of 3-connected planar graph using 3 colors so that

1. Every edge is uni- or bi-directed and each direction colored.

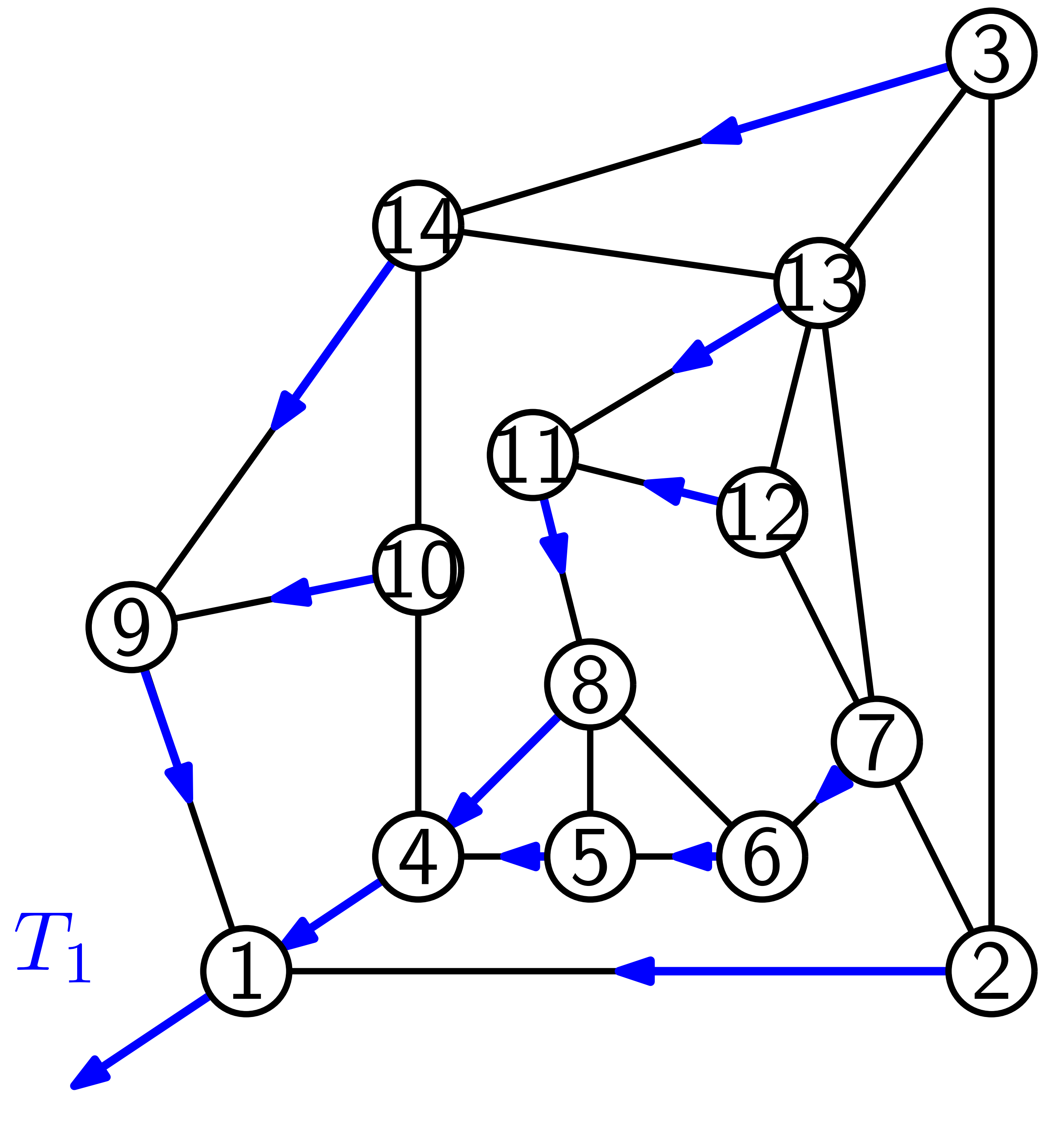
2. 

3. No cycle in one color.



Schnyder Wood

Each color class forms a tree.



Schnyder Wood

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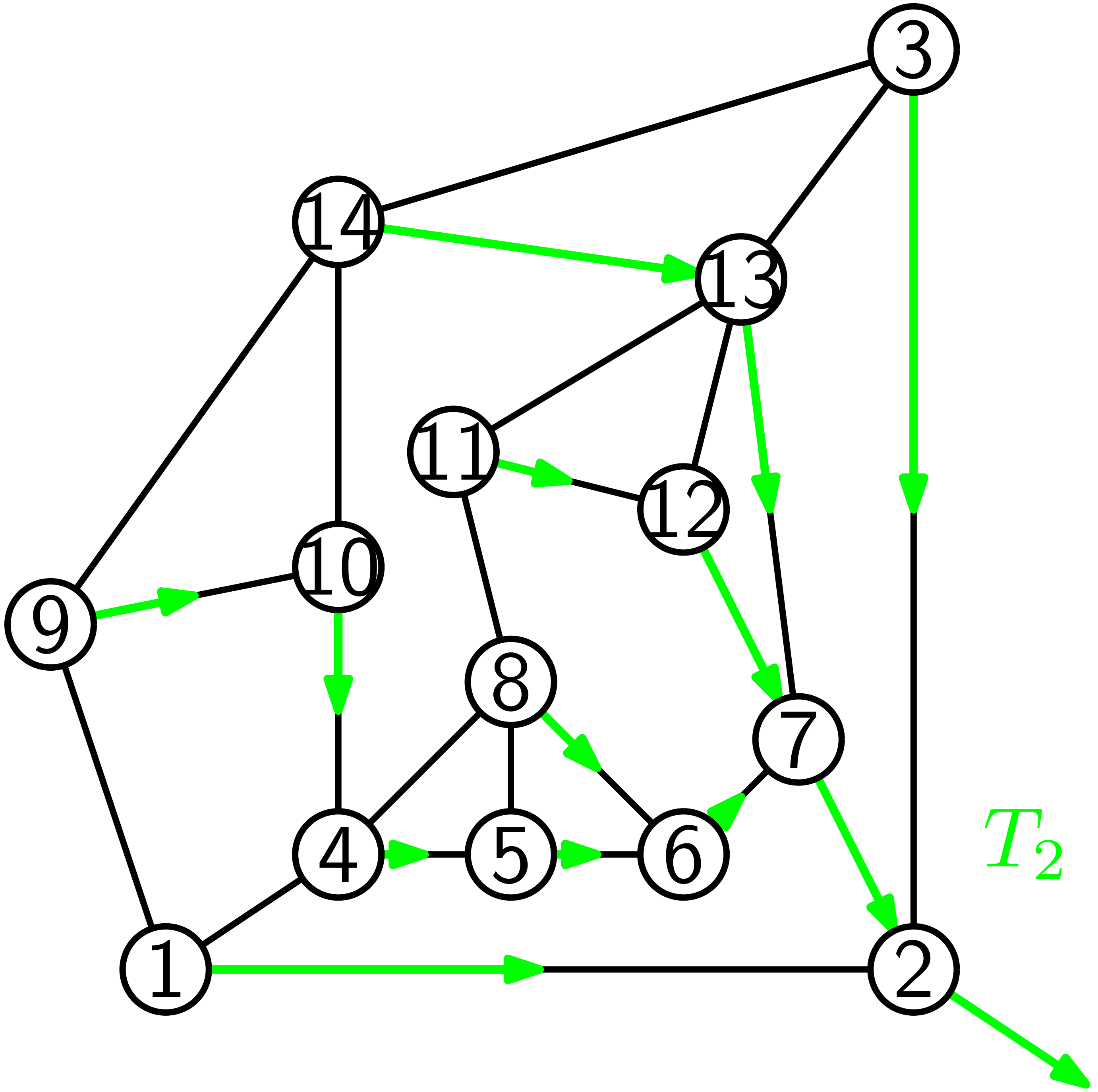

Schnyder Wood

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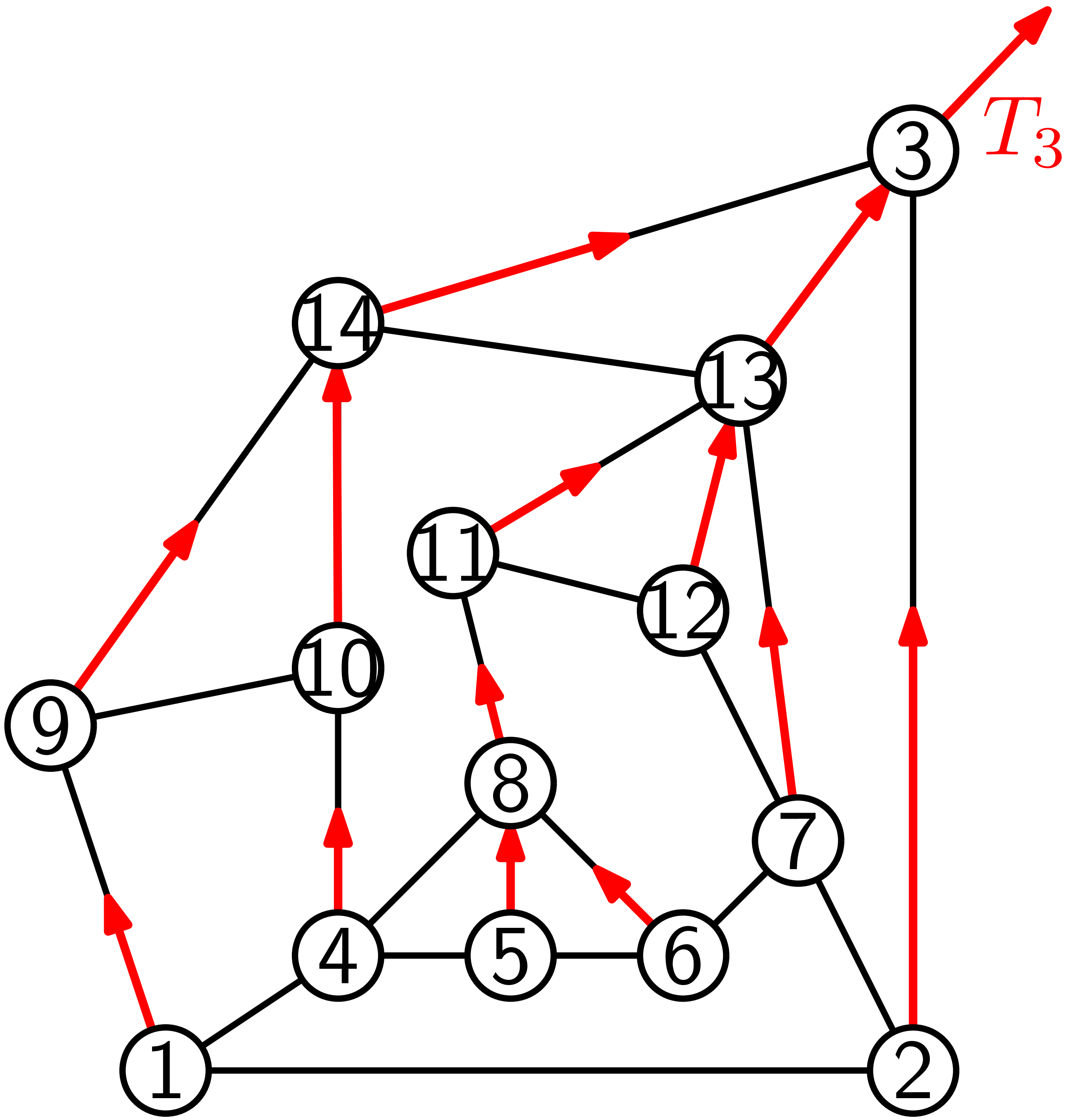
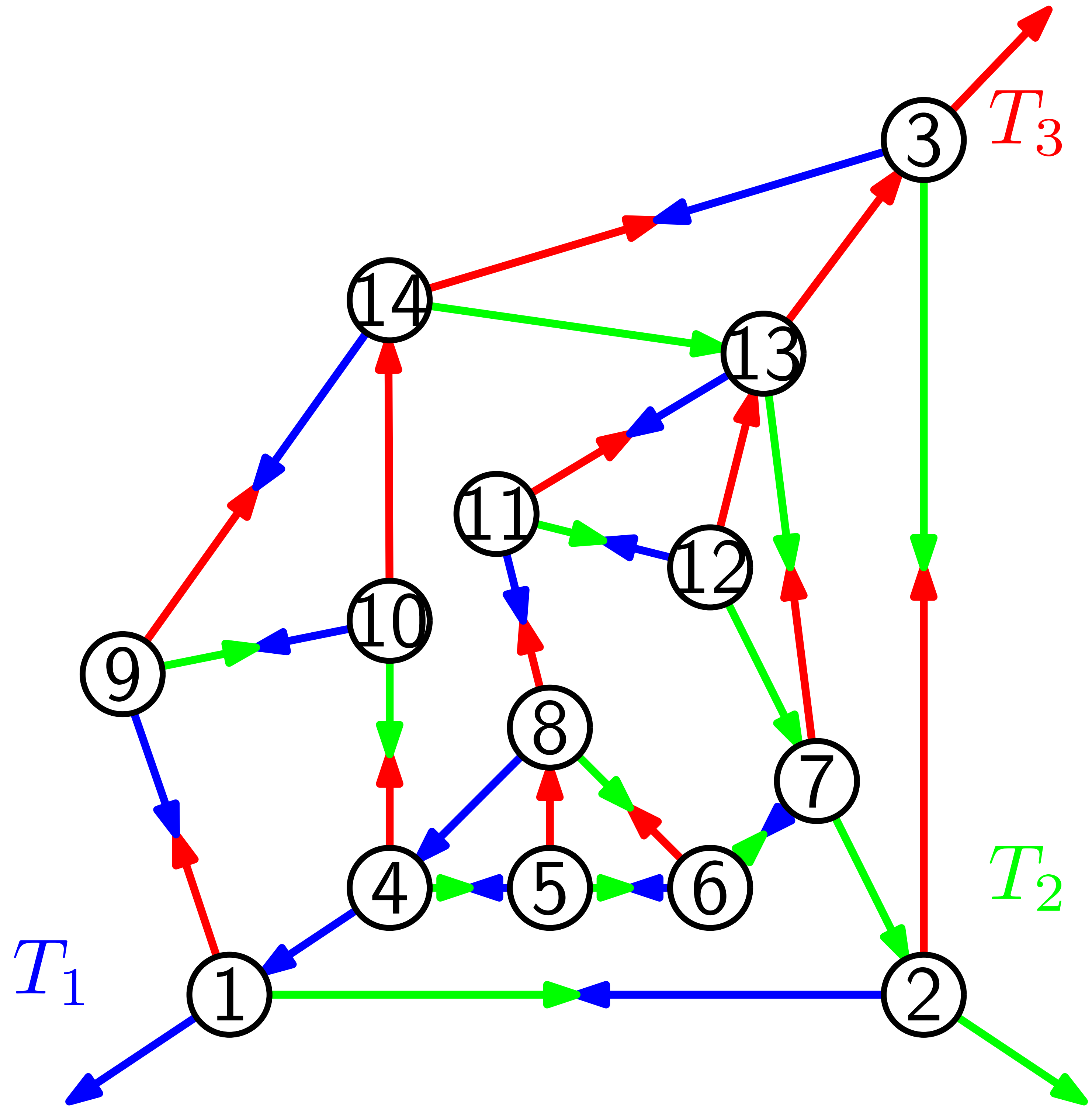


Schnyder Wood and Ordered Path Partition

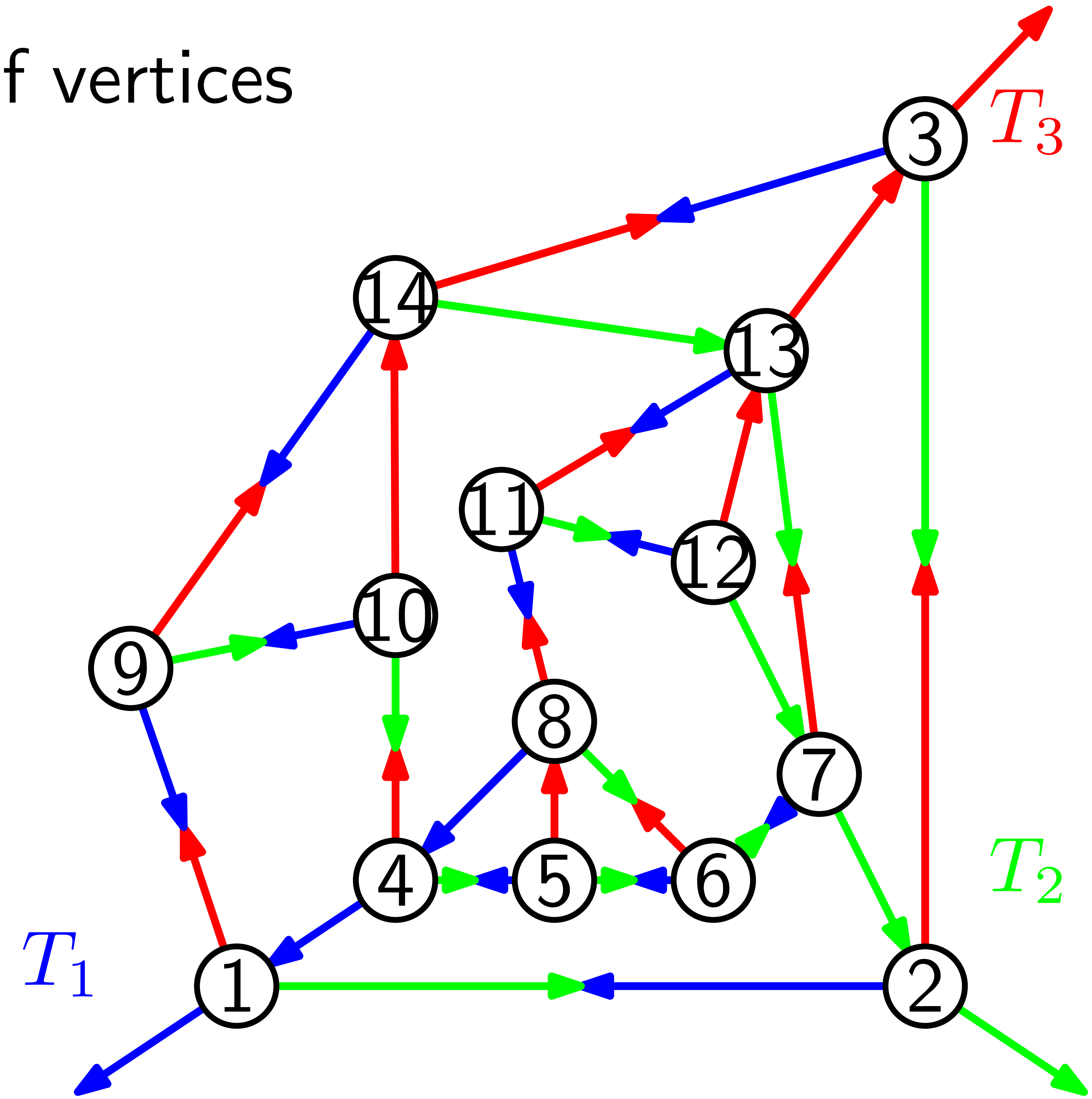
Partition graph into paths according to T_1 T_2 T_3



Schnyder Wood and Ordered Path Partition

Partition graph into paths according to T_1 T_2 T_3

Partially order groups of vertices to respect T_1^{-1} T_2^{-1} T_3



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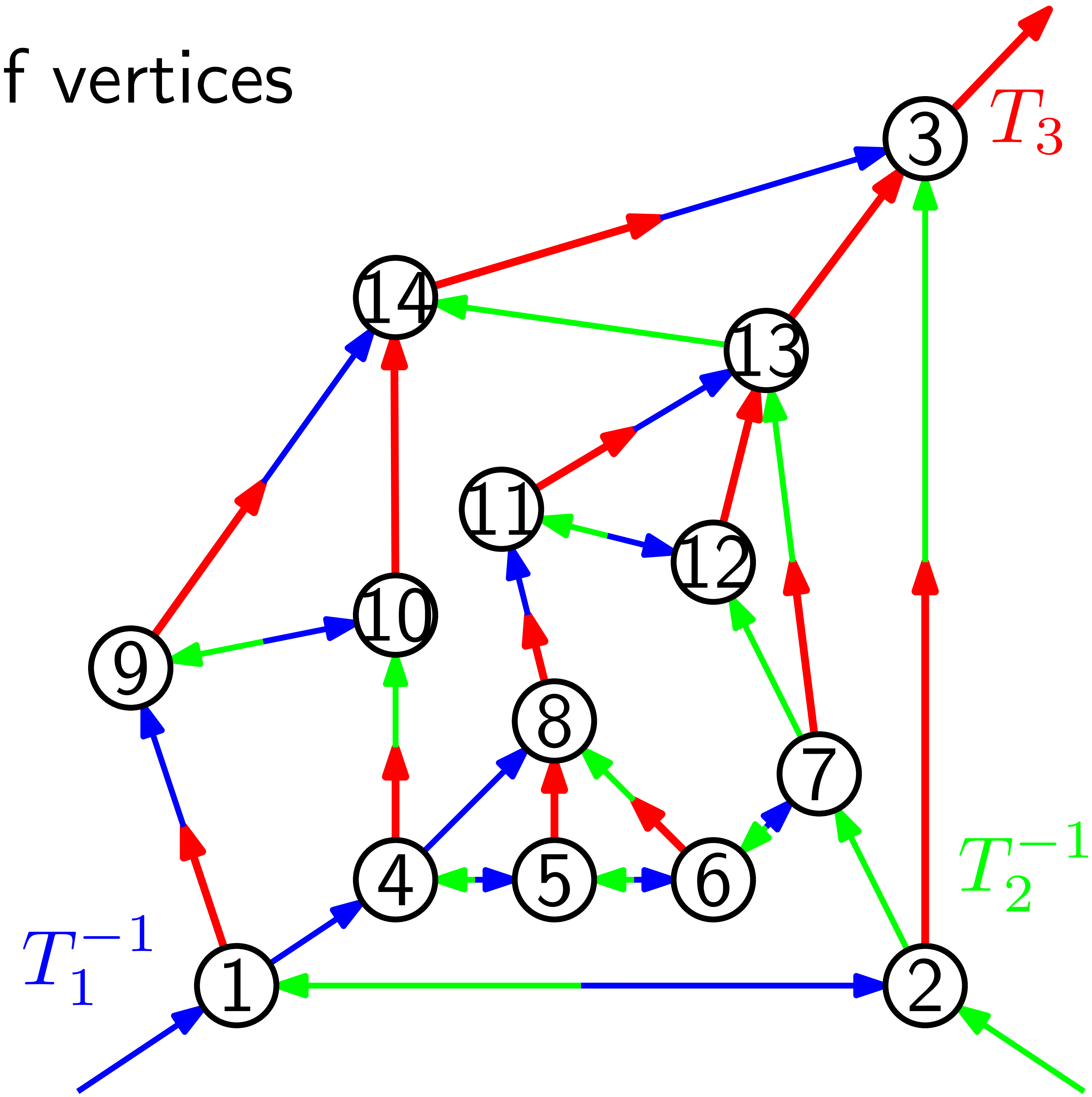
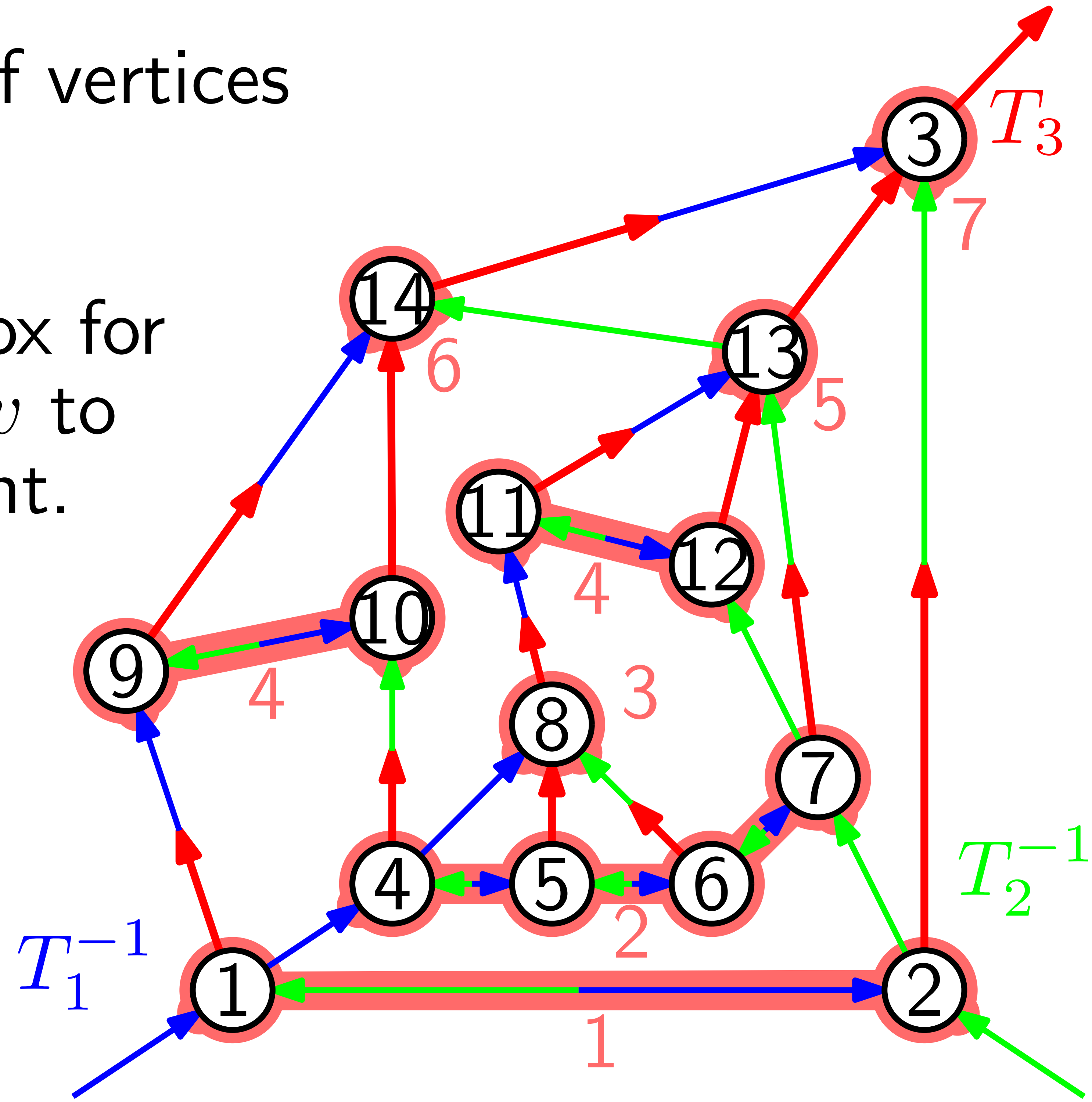


Schnyder Wood and Ordered Path Partition

Partition graph into paths according to T_1 T_2 T_3

Partially order groups of vertices to respect T_1^{-1} T_2^{-1} T_3

The z-interval of the box for vertex v is the level of v to the level of v's T_3 parent.



Schnyder Wood and Ordered Path Partition

Partition graph into paths according to T_1 T_2 T_3

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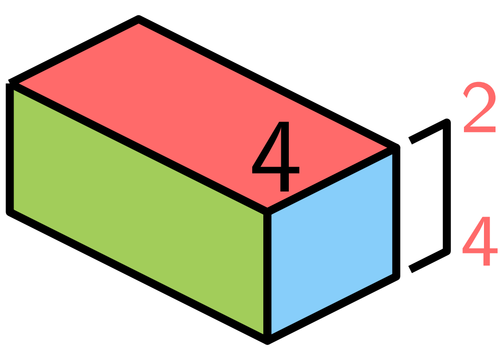
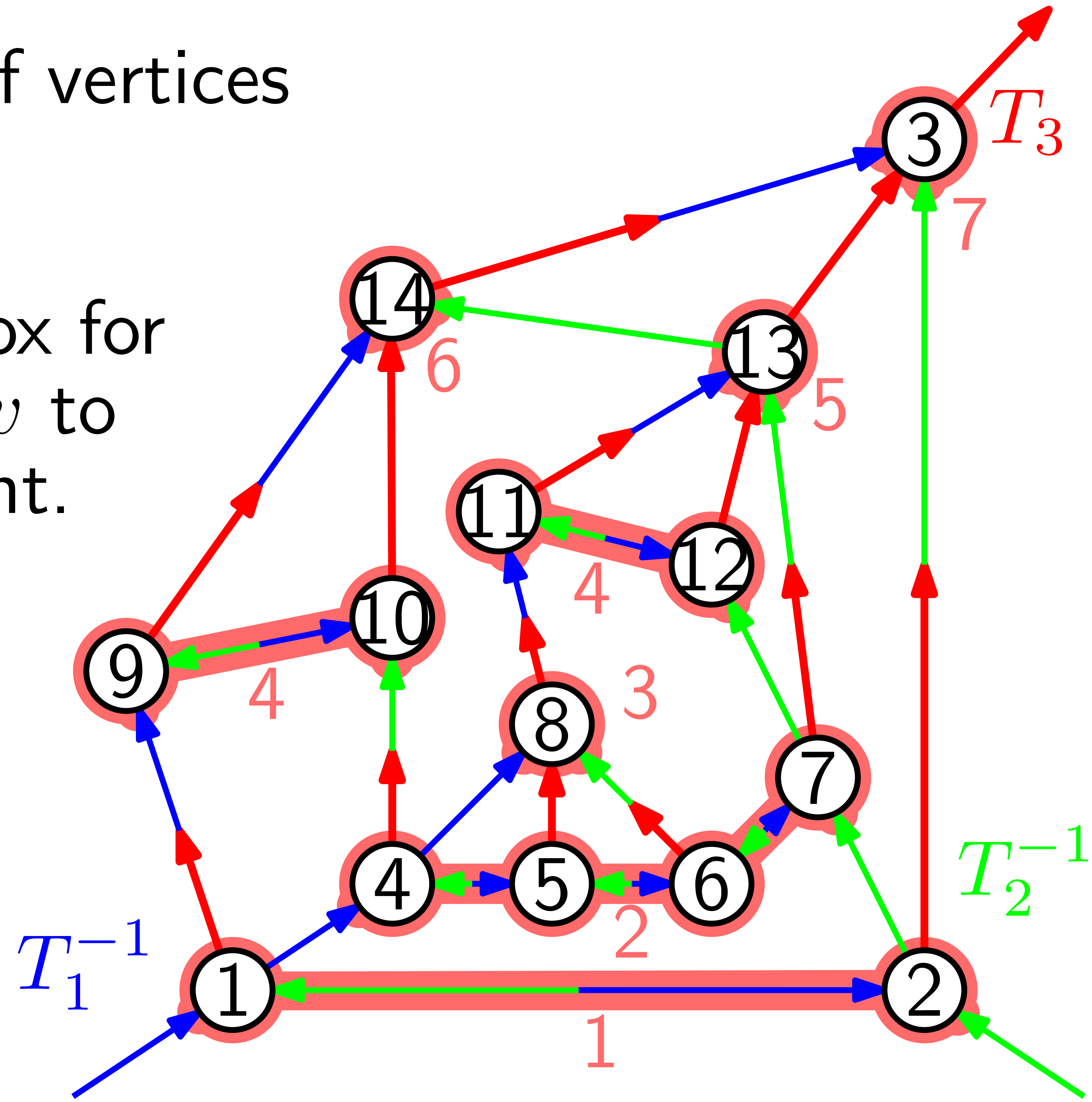
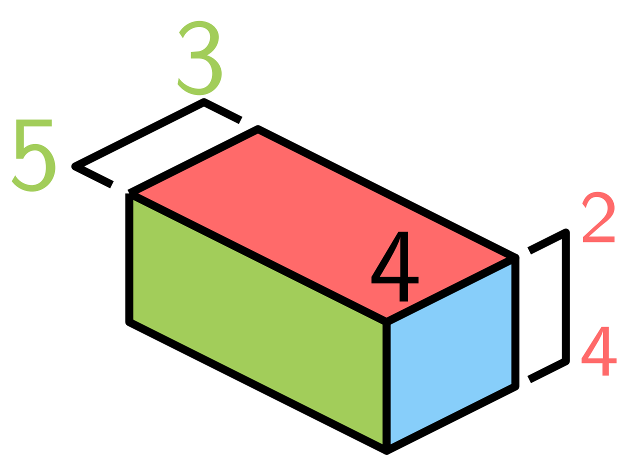


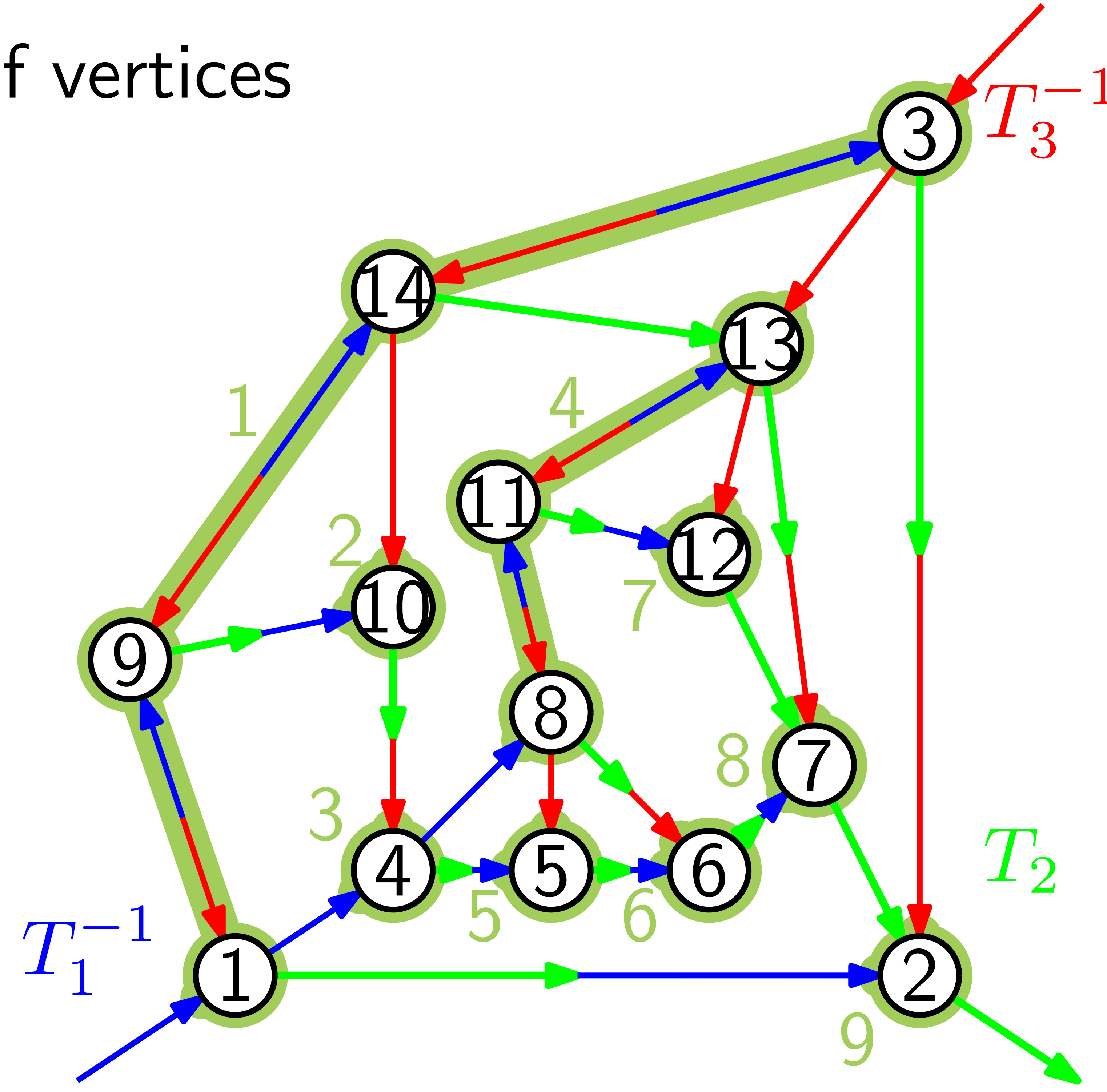


Schnyder Wood and Ordered Path Partition

Partition graph into paths according to T_1 T_2 T_3

Partially order groups of vertices to respect T_1^{-1} T_2 T_3^{-1}

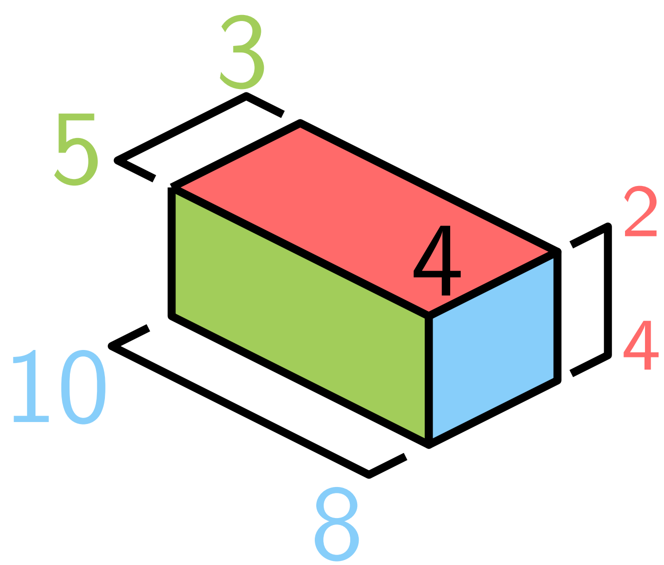


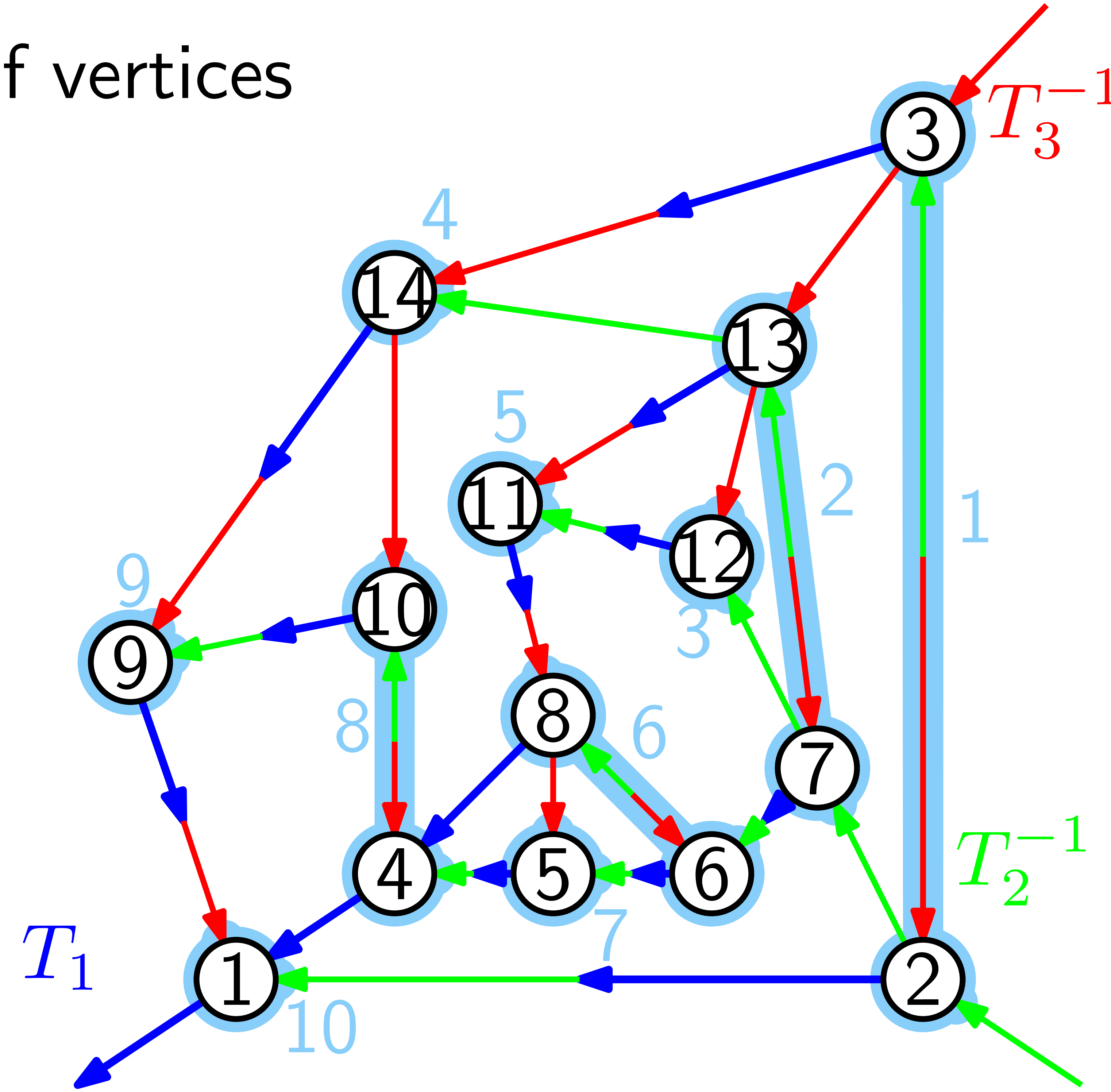


Schnyder Wood and Ordered Path Partition

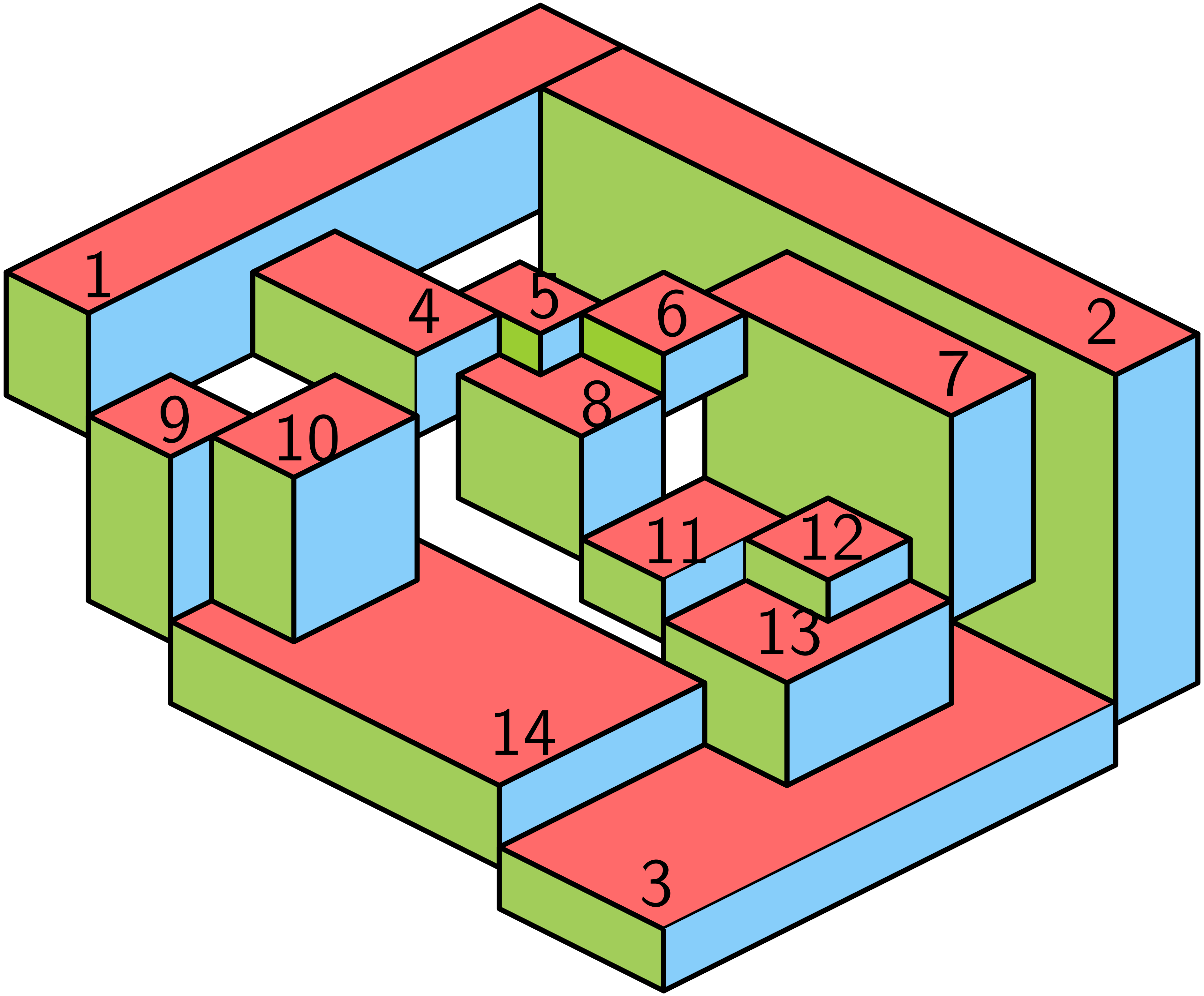
Partition graph into paths according to T_1 T_2 T_3

Partially order groups of vertices to respect T_1 T_2^{-1} T_3^{-1}



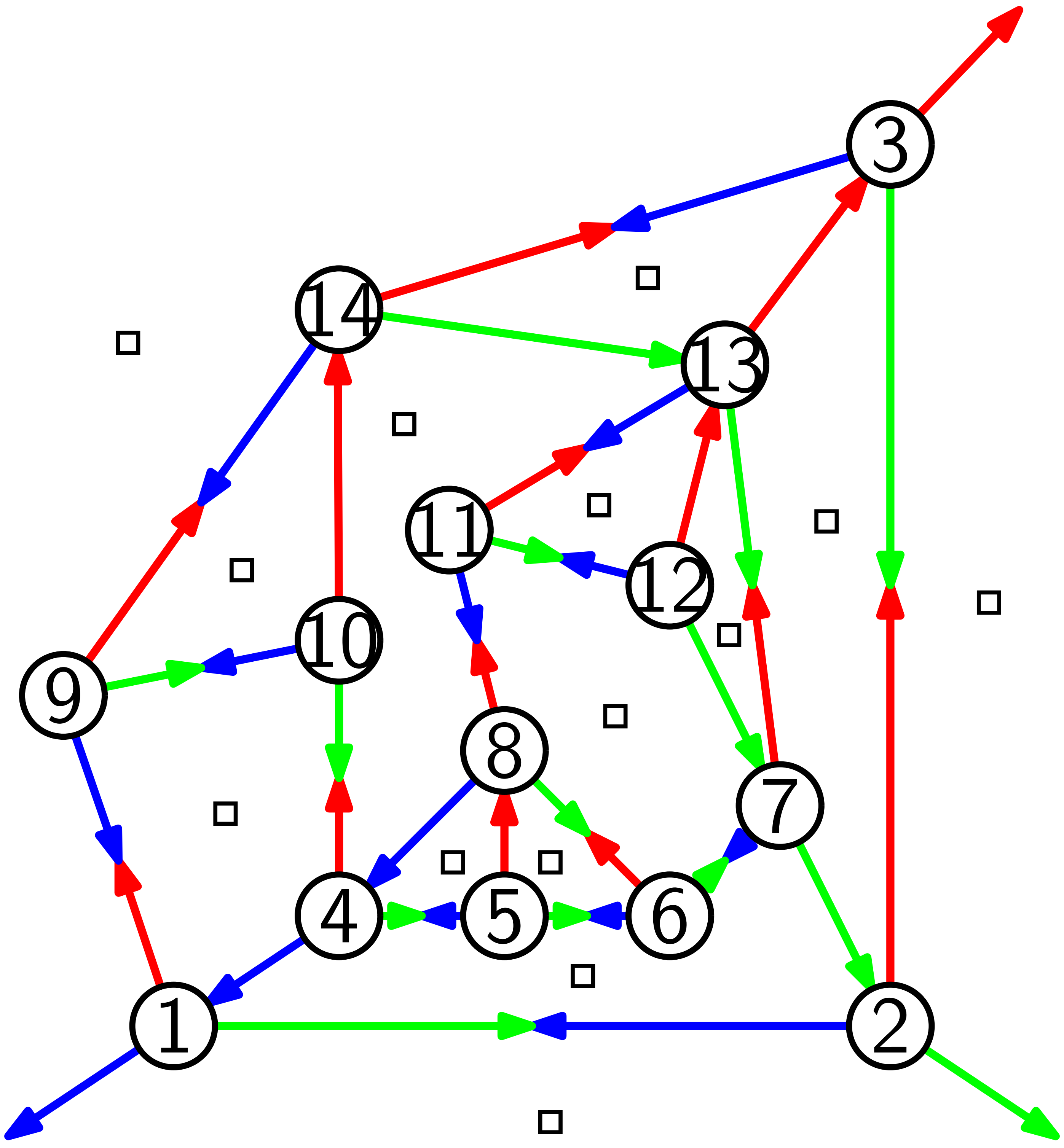


Box Contact Representation



Compatible Dual Schnyder Wood

Between an edge and its dual, all 3 colors appear.



Compatible Dual Schnyder Wood

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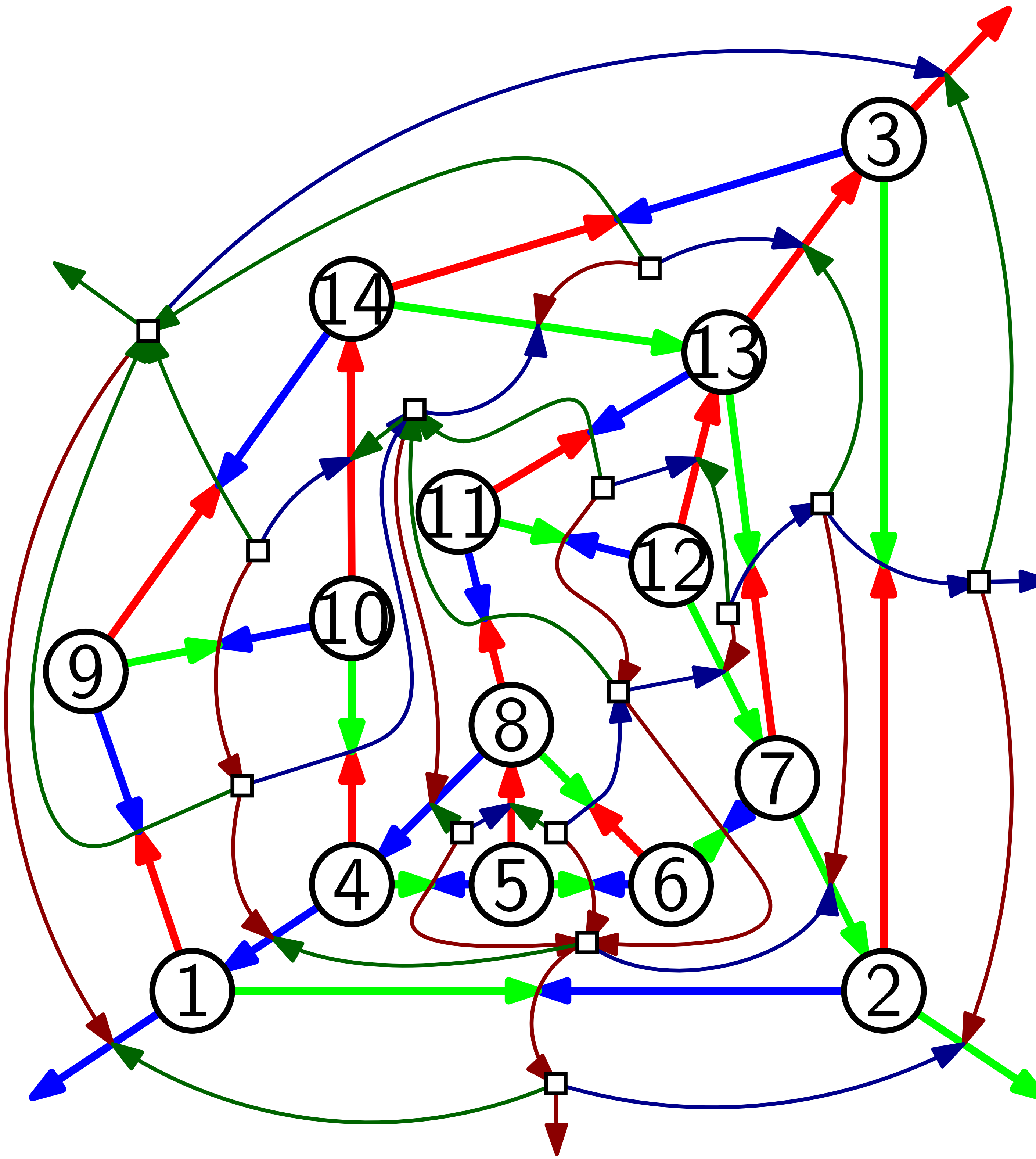
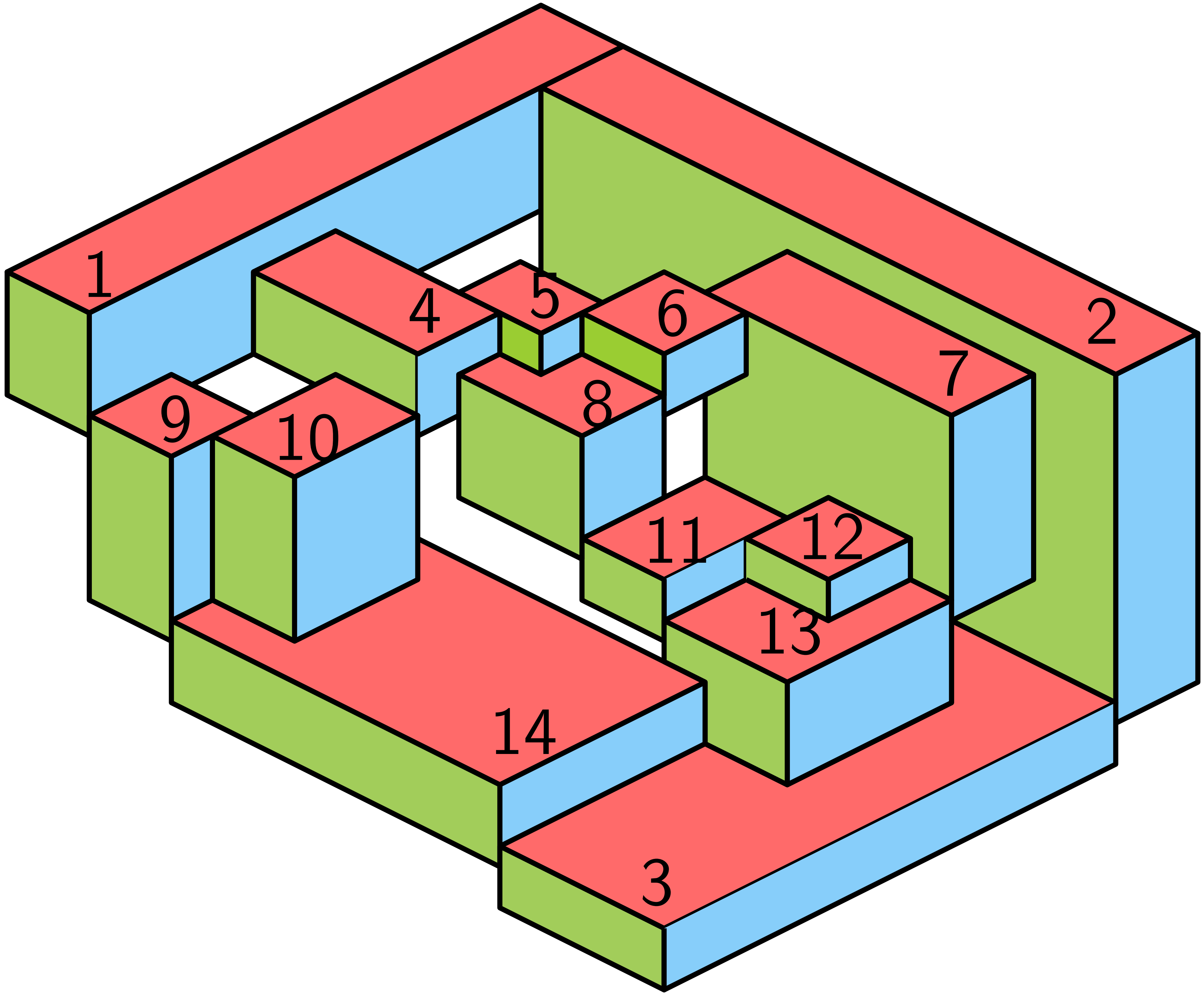
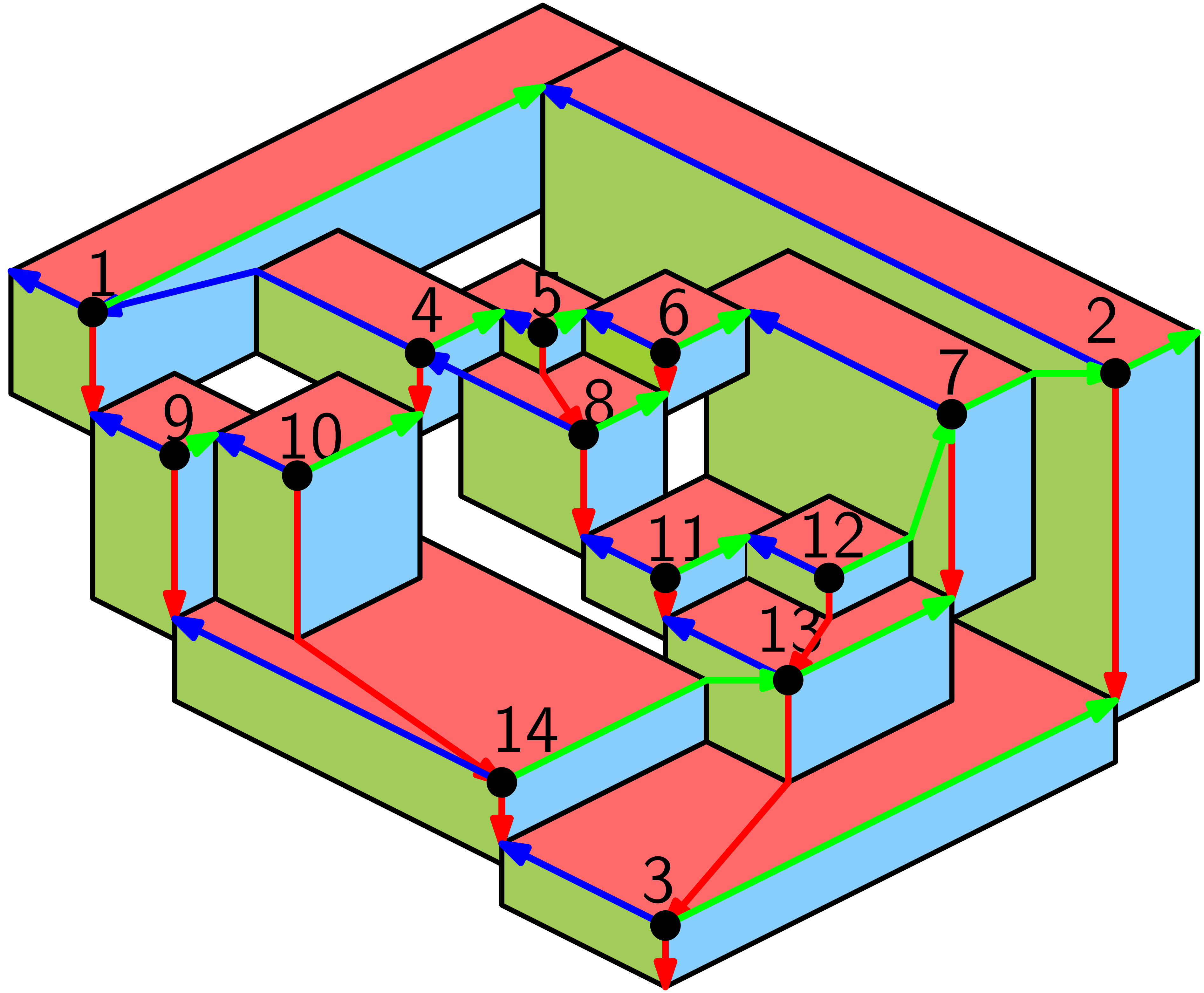


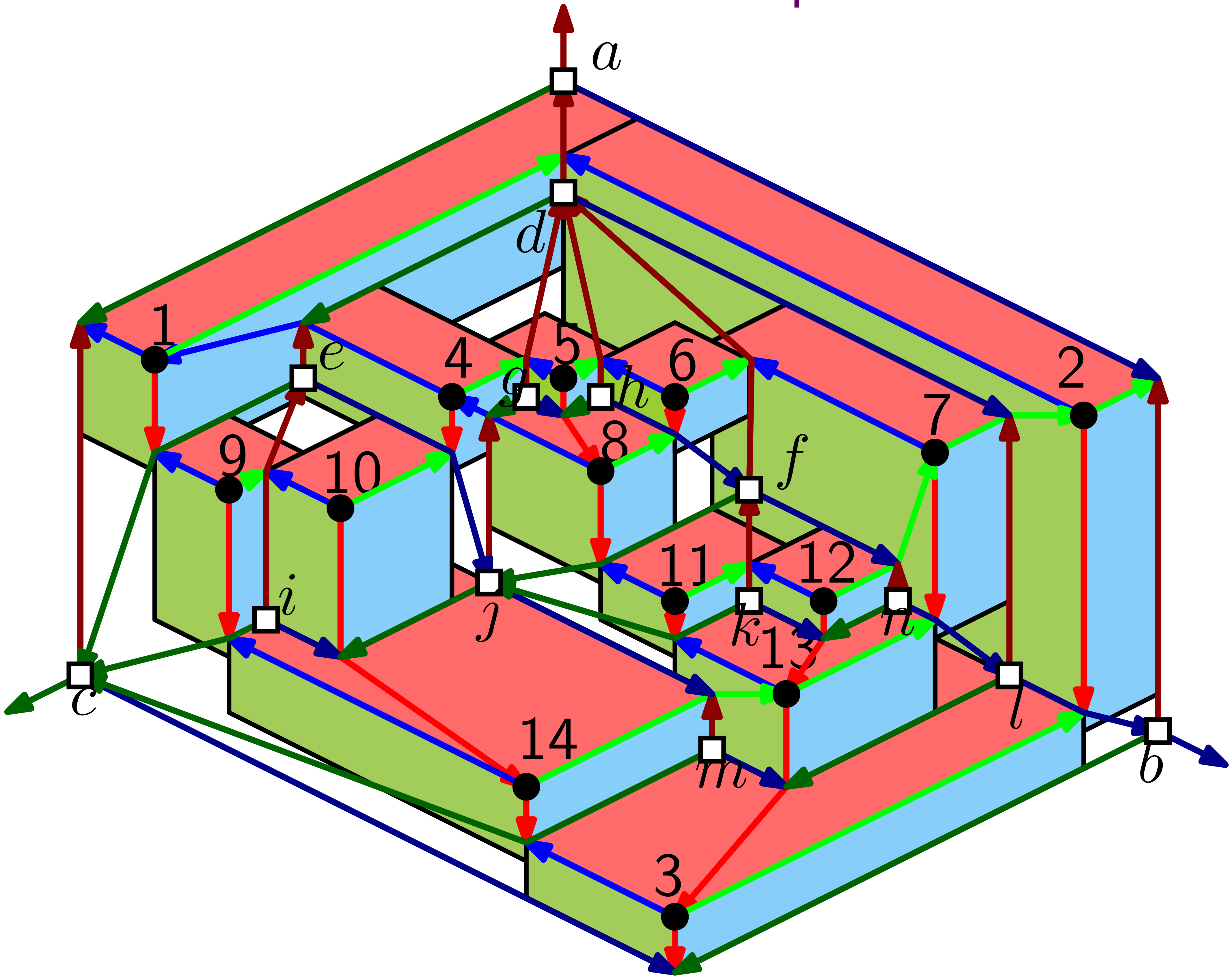
Primal-Dual Box Contact Representation



Primal-Dual Box Contact Representation



Primal-Dual Box Contact Representation



Primal-Dual to Non-planar Representation

Thm 1 Every 3-connected planar graph admits a proper primal-dual 3D box-contact representation.

Cor Every prime 1-planar graph has a proper shelled 3D box-contact representation.

prime: optimal, no separating 4-cycle

Thm 2 Every optimal 1-planar graph has a proper shelled 3D L-contact representation.

Open Problems

What graphs have 3D box-contact representations?

Do all planar graphs have proper 3D cube-contact representations?

Do all 1-planar graphs have proper 3D L-contact representations?