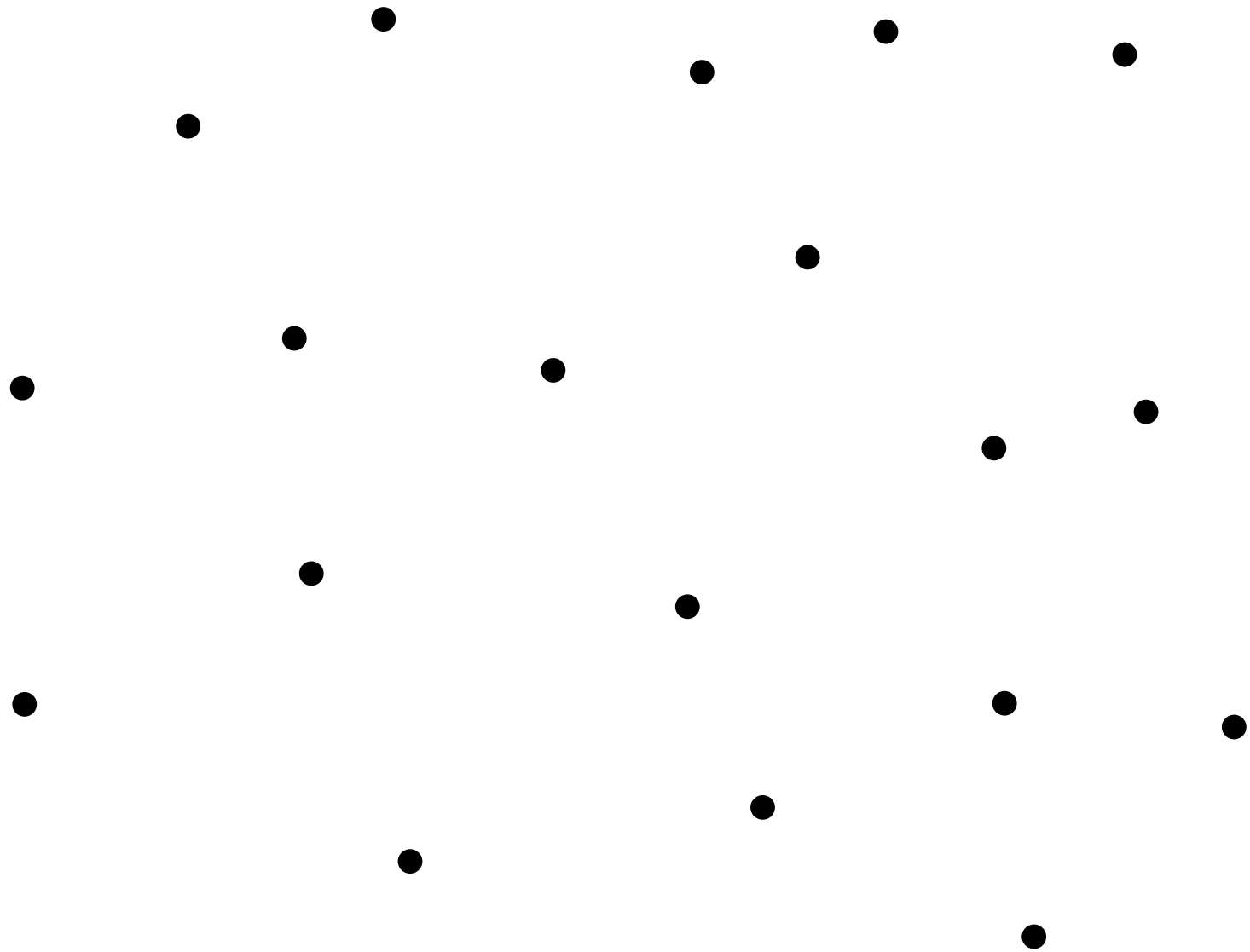
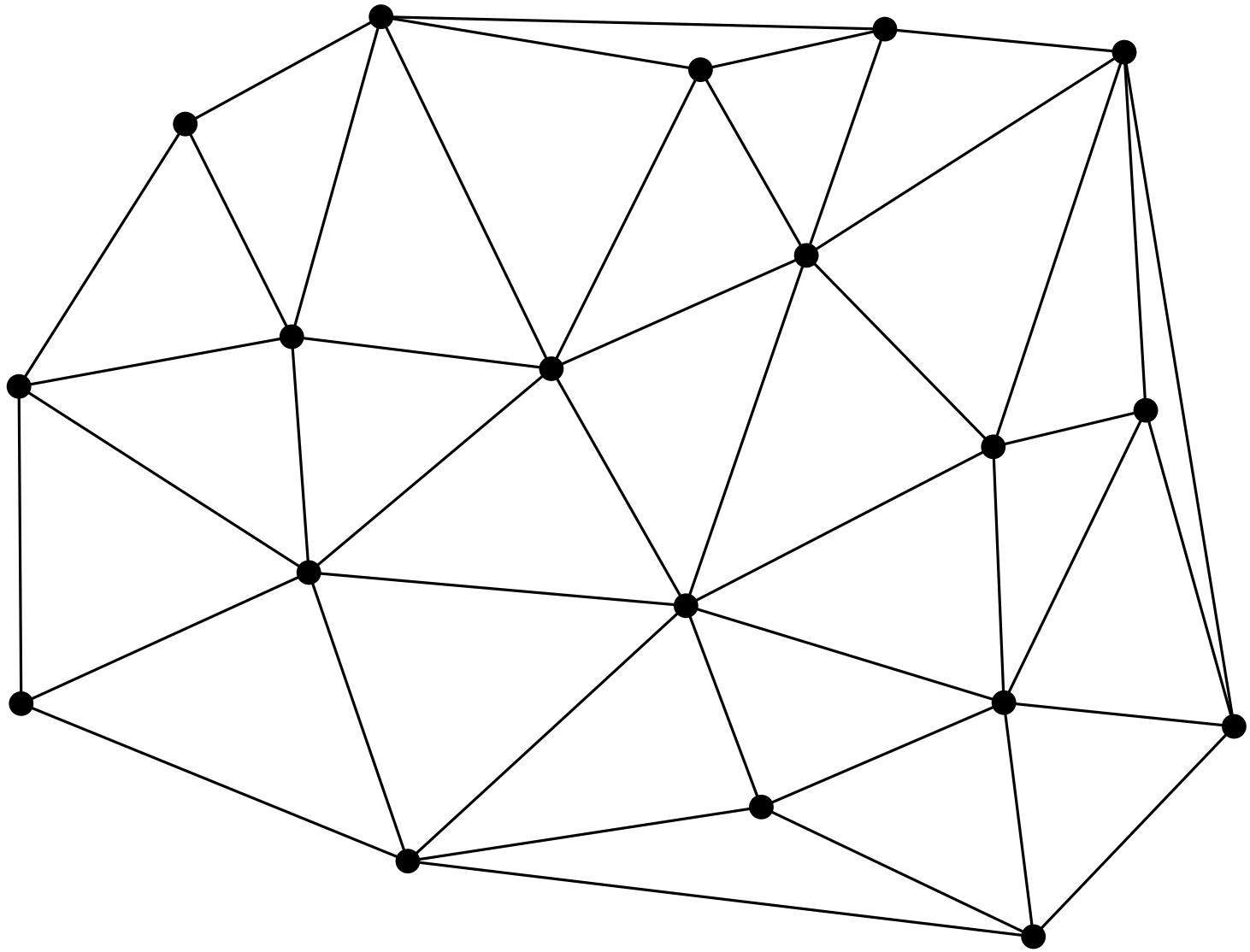
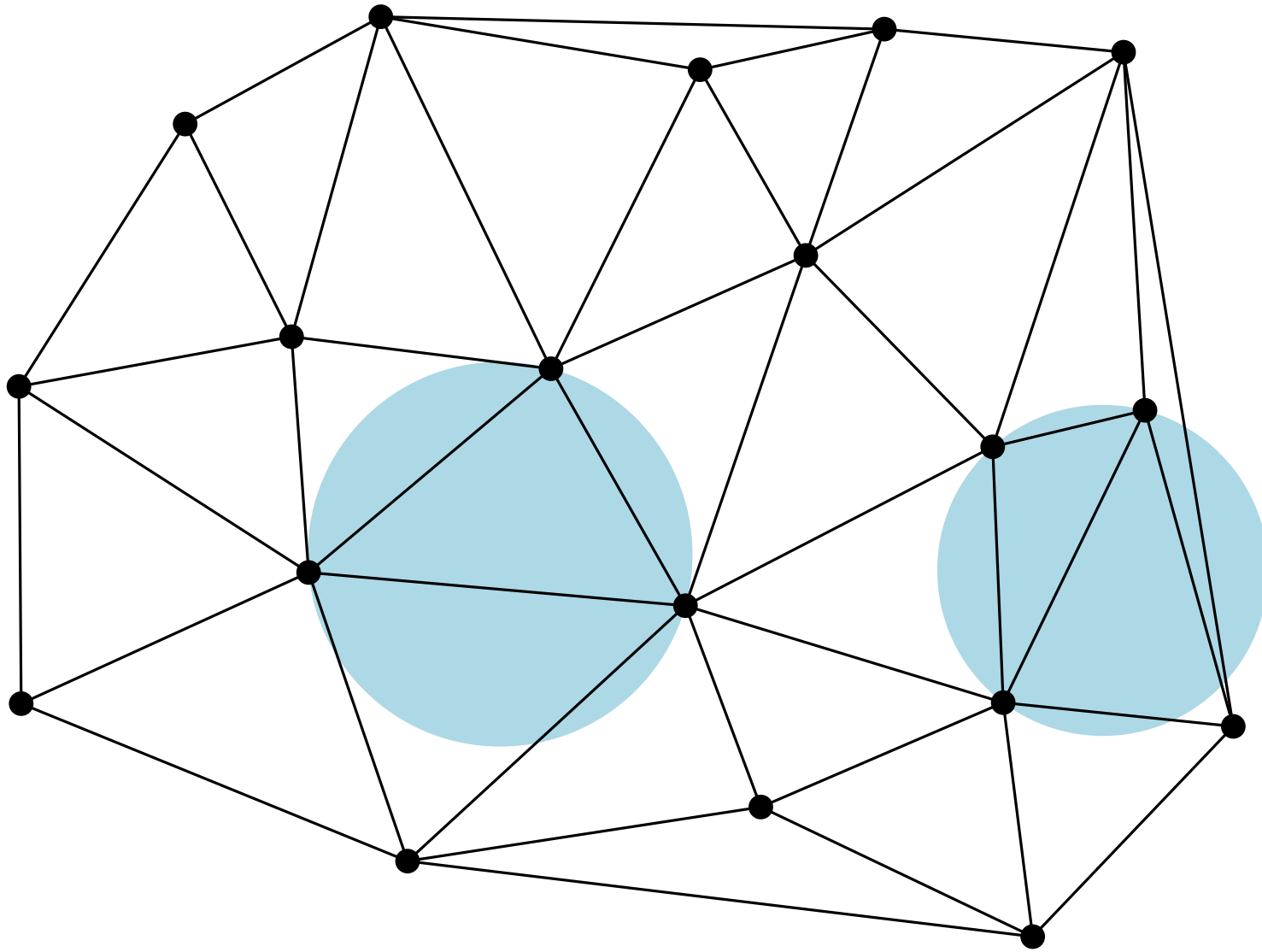
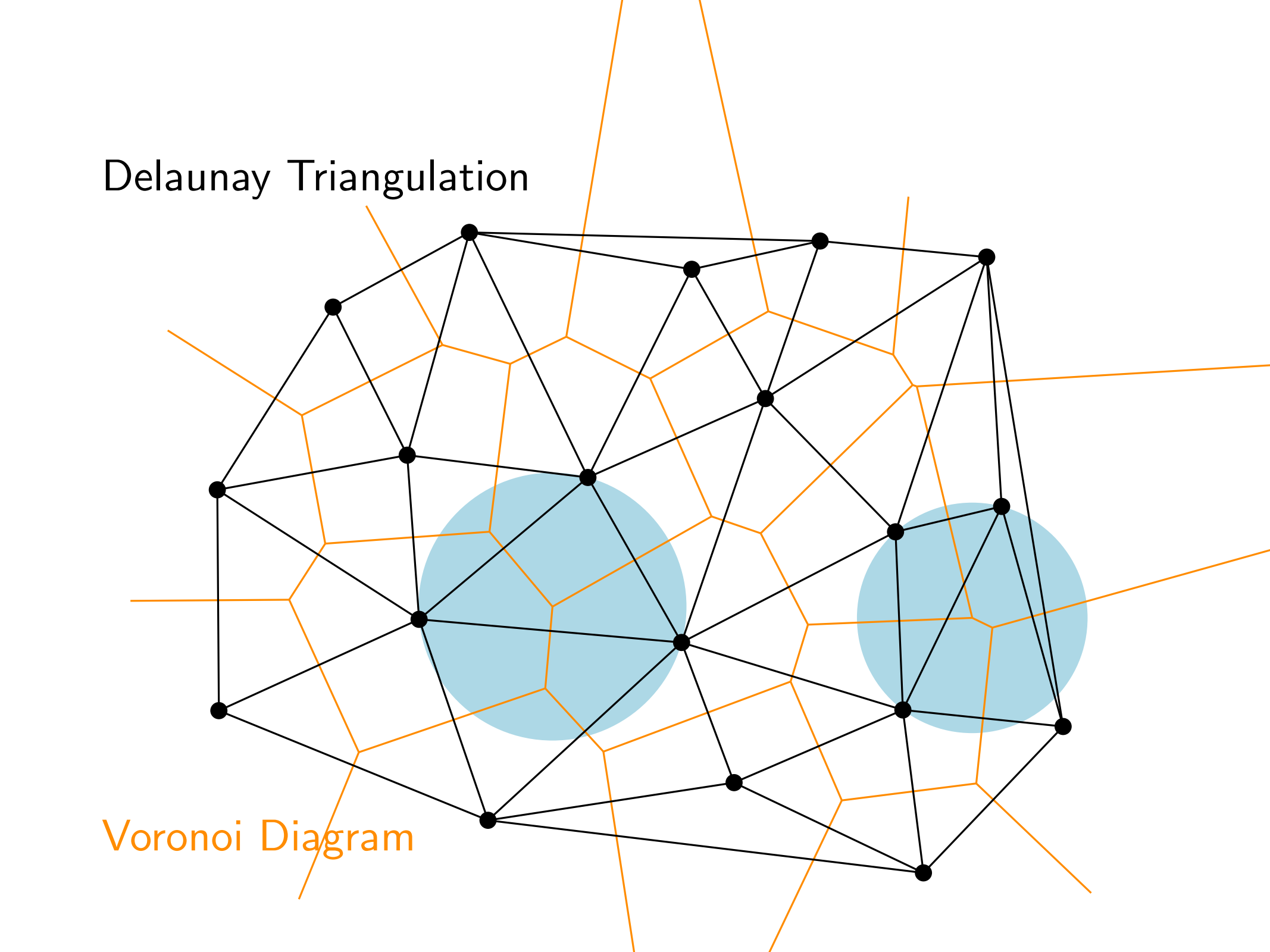


Delaunay Triangulation

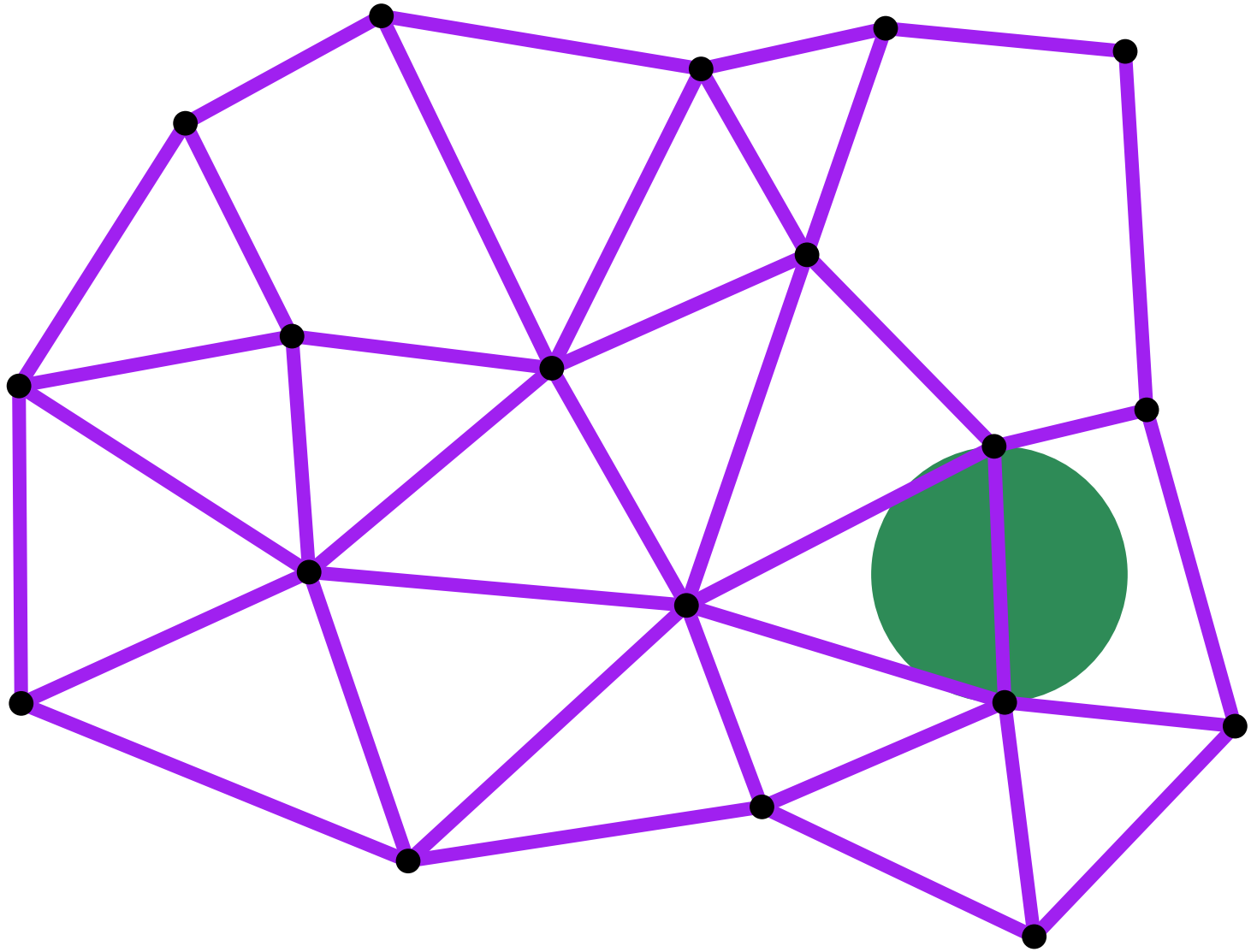


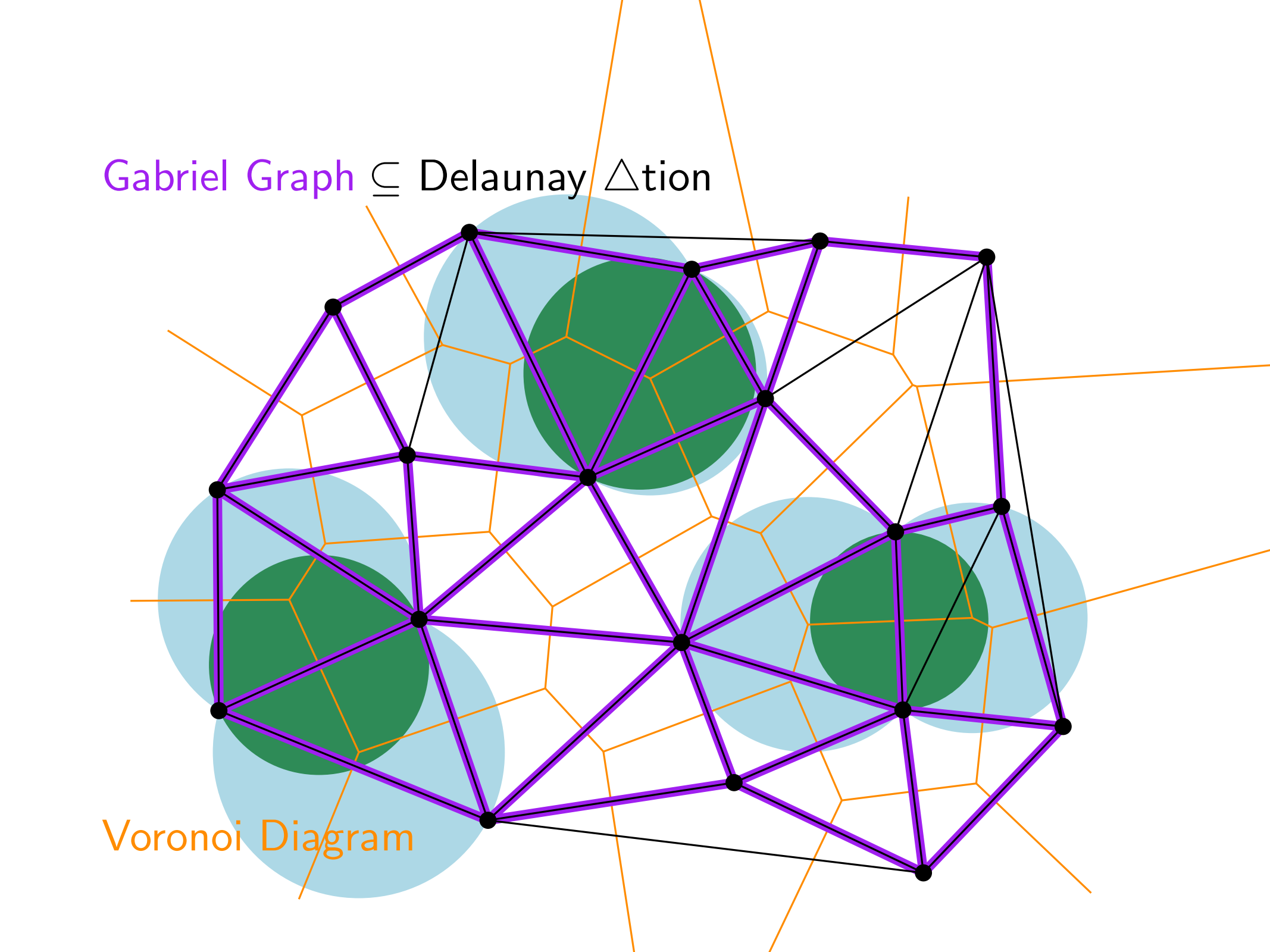
Delaunay Triangulation



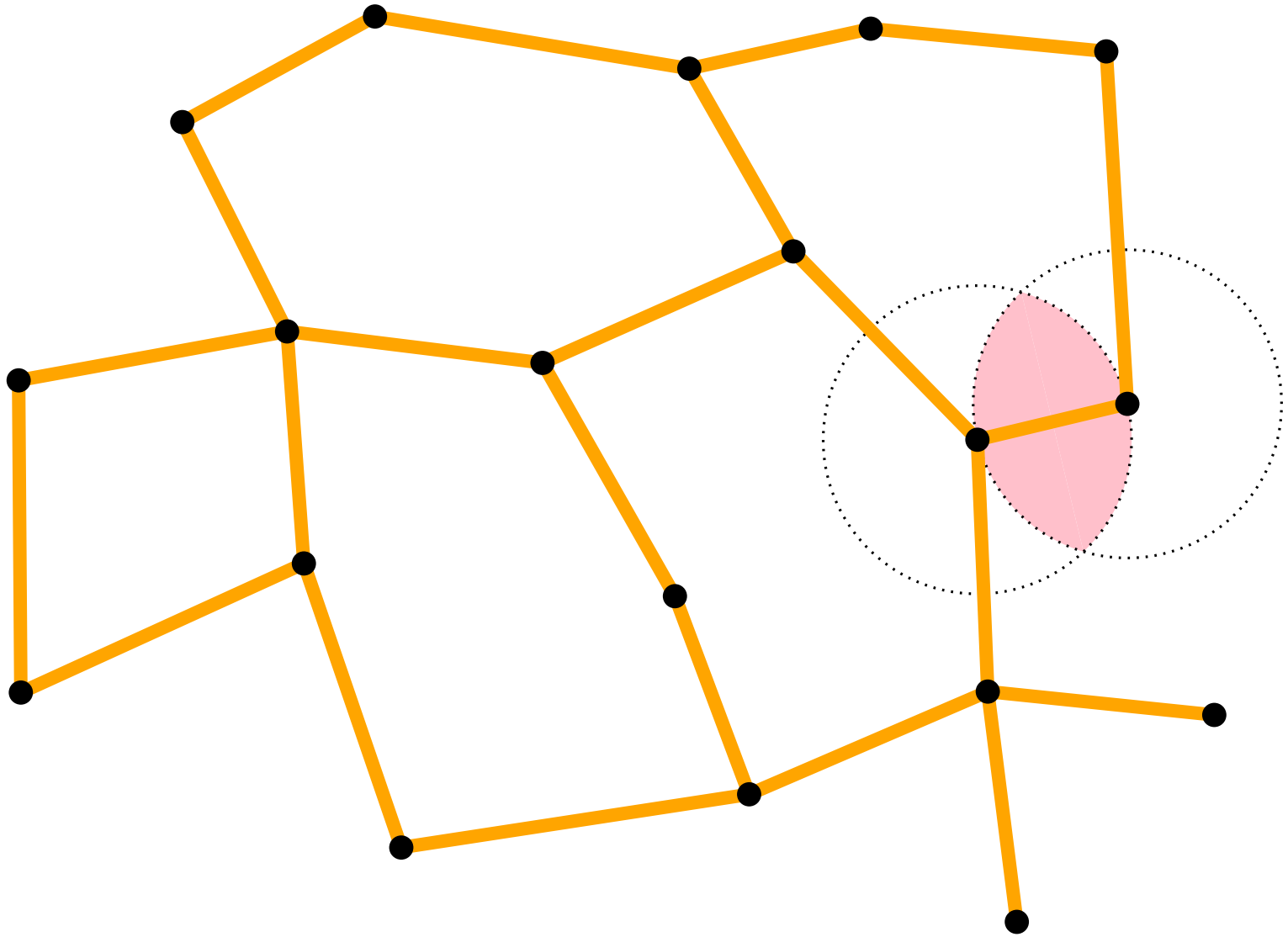

Delaunay Triangulation
Voronoi Diagram

Gabriel Graph

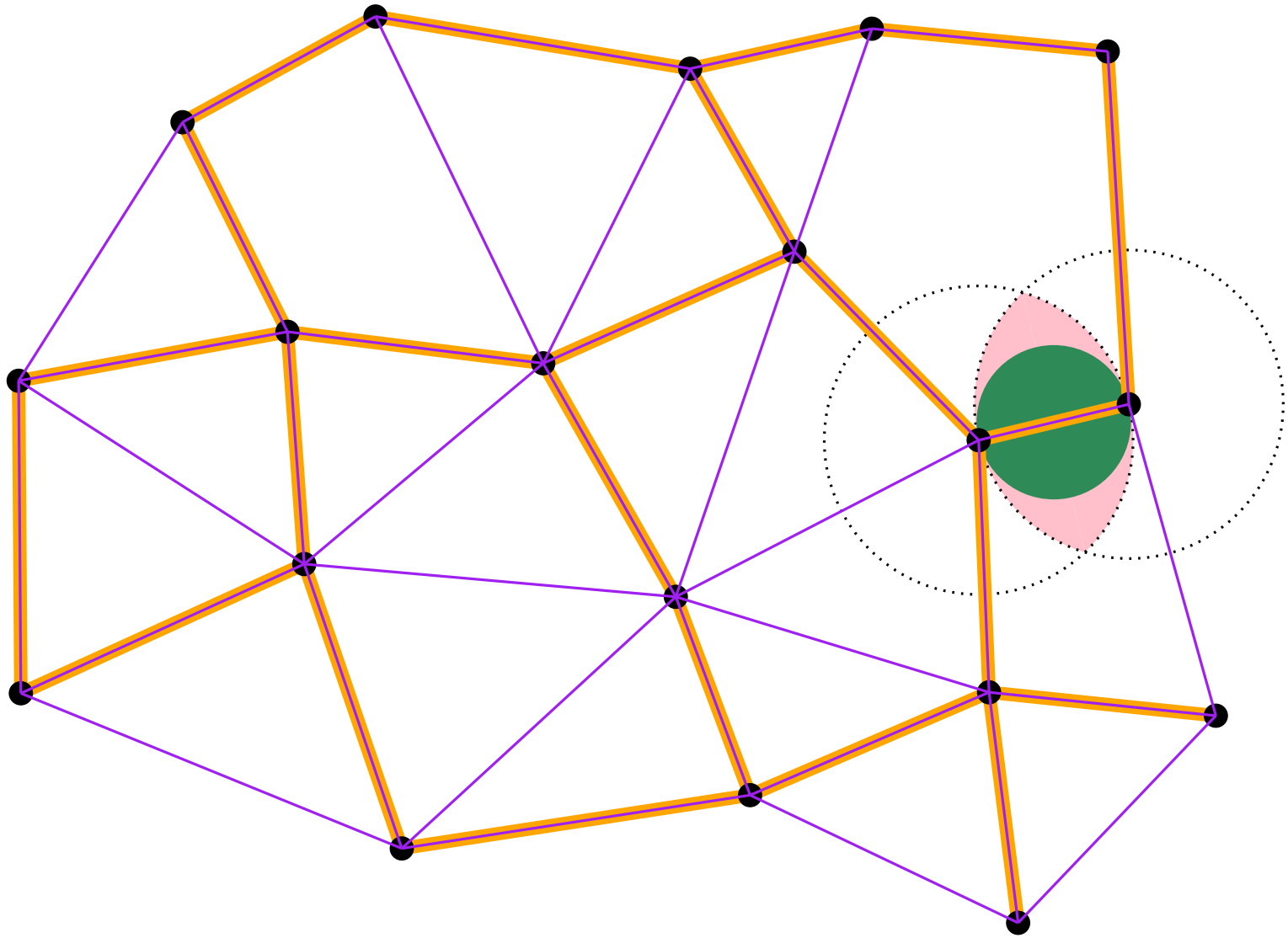



Gabriel Graph ⊆ Delaunay △tion
Voronoi Diagram

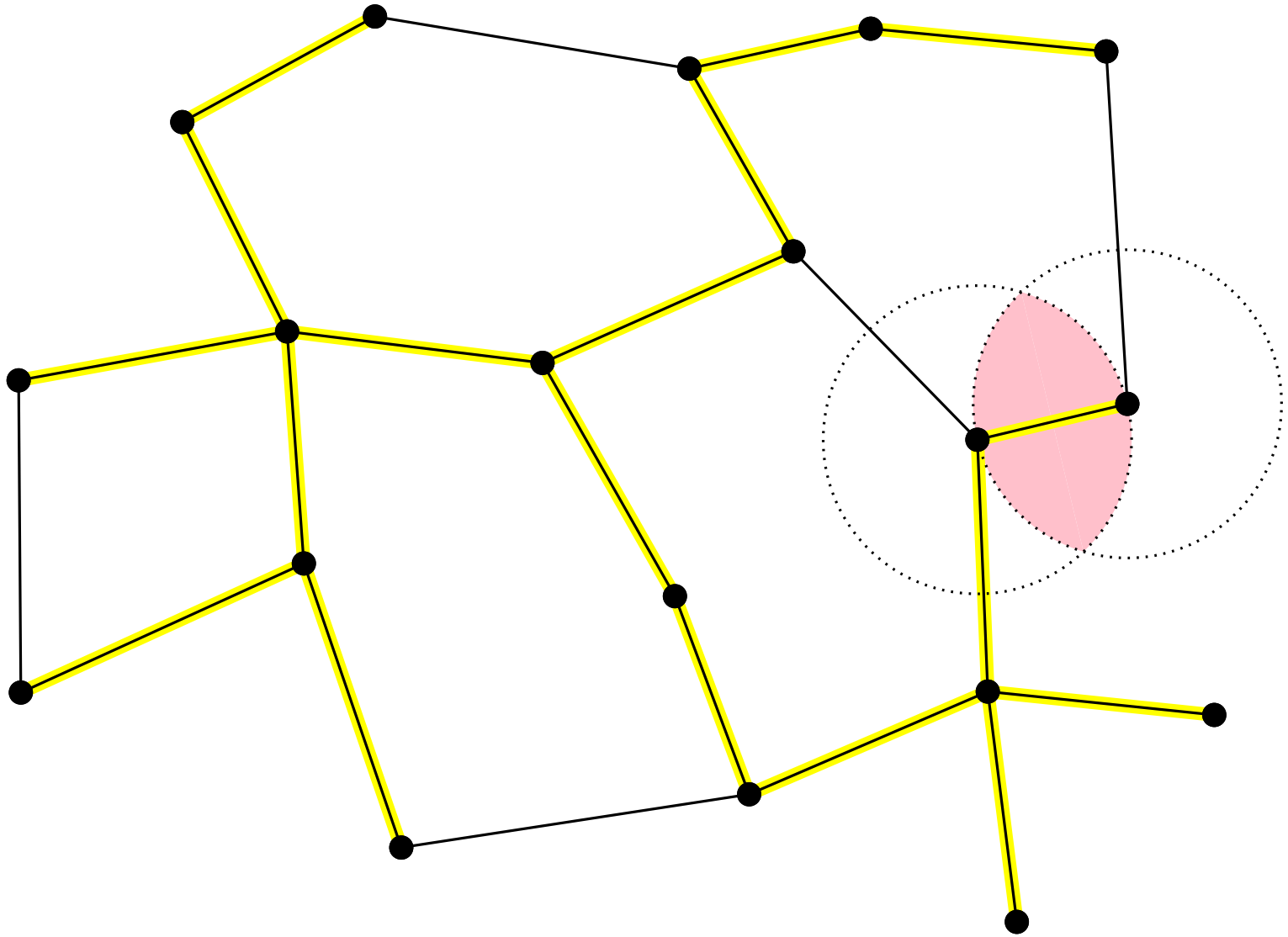
Relative neighborhood graph



Relative neighborhood graph $\subseteq$ Gabriel Graph

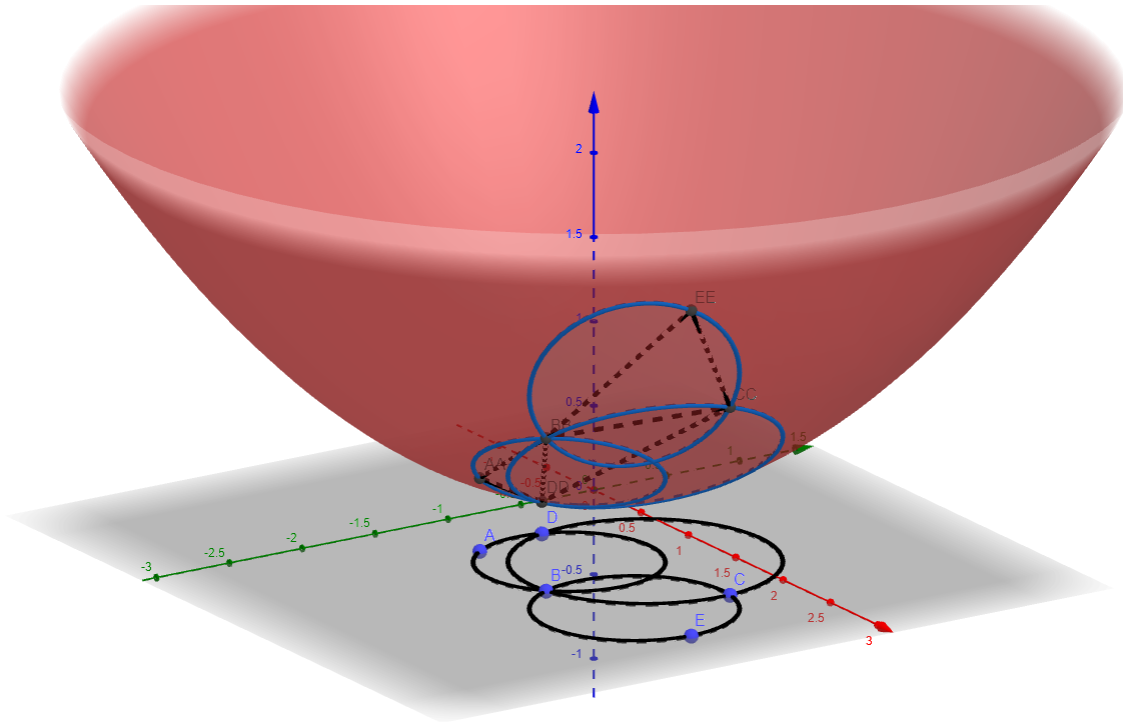


Min Spanning Tree $\subseteq$ Relative neighborhood graph

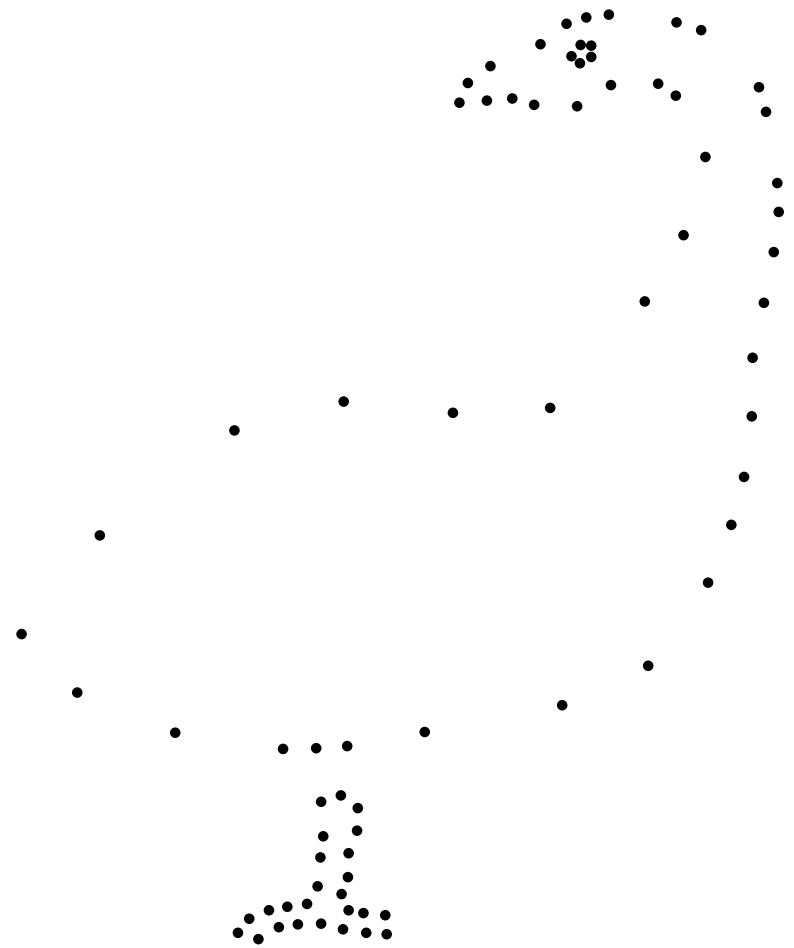


$$\mathrm{MST}(P) \subseteq \mathrm{RNG}(P) \subseteq \mathrm{GG}(P) \subseteq \mathrm{DT}(P)$$

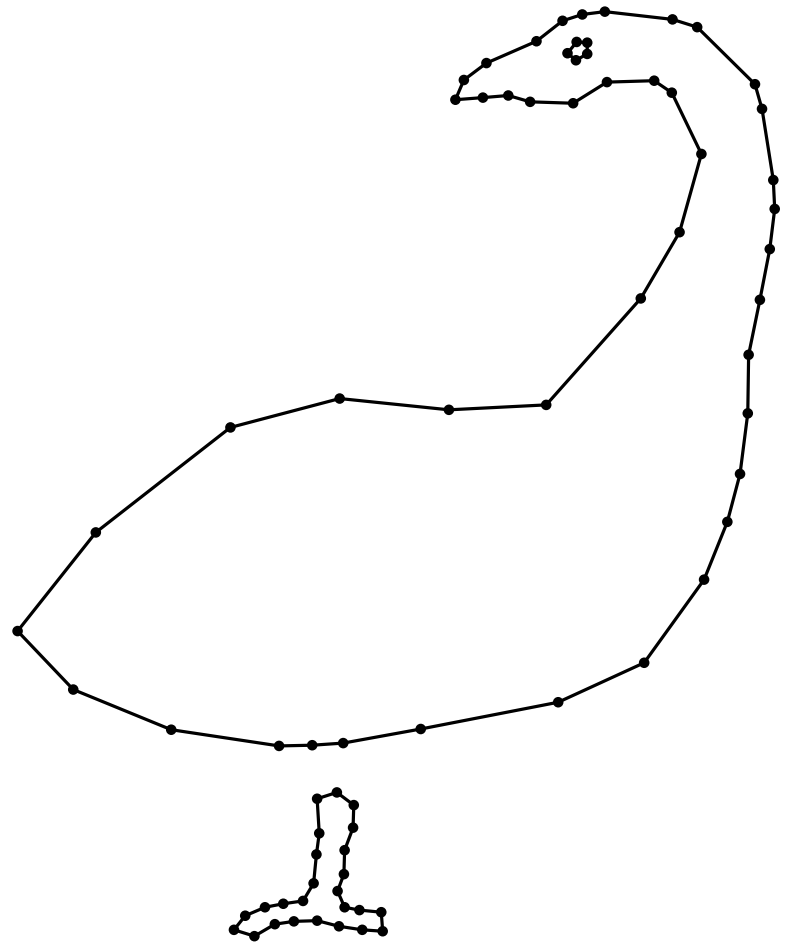
Delaunay Triangulation from 3D Convex hull



The Crust and the β-skeleton [Amenta, Bern & Eppstein 98]



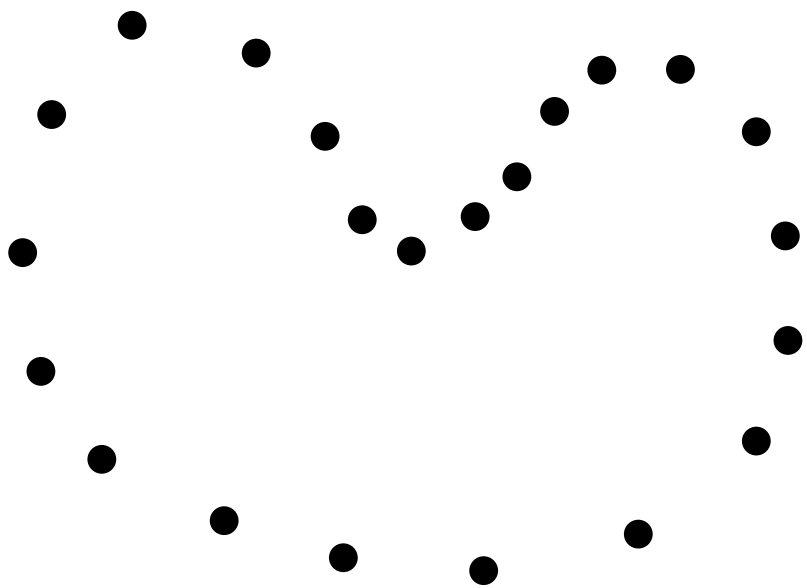
The Crust and the β-skeleton [Amenta, Bern & Eppstein 98]



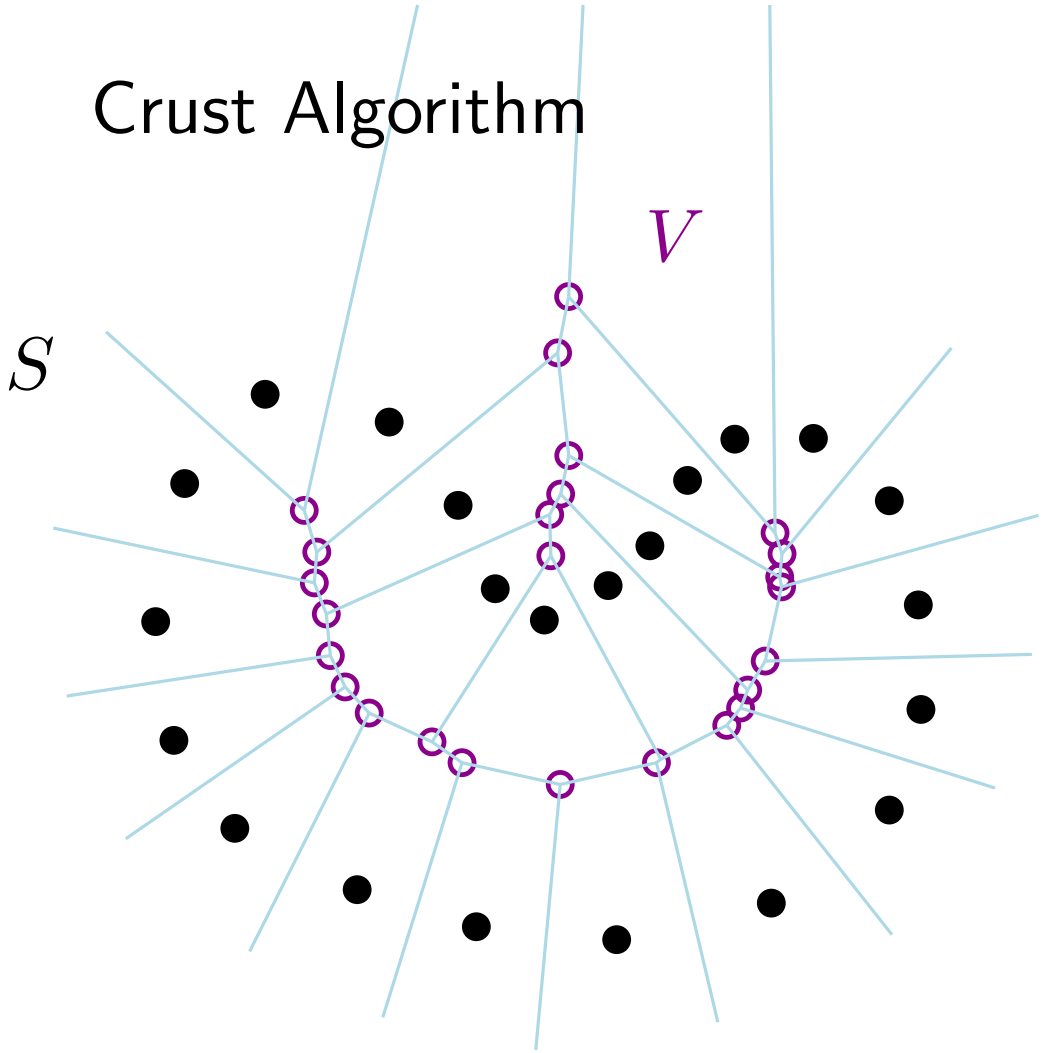
Crust Algorithm

Let S be sample points.

S



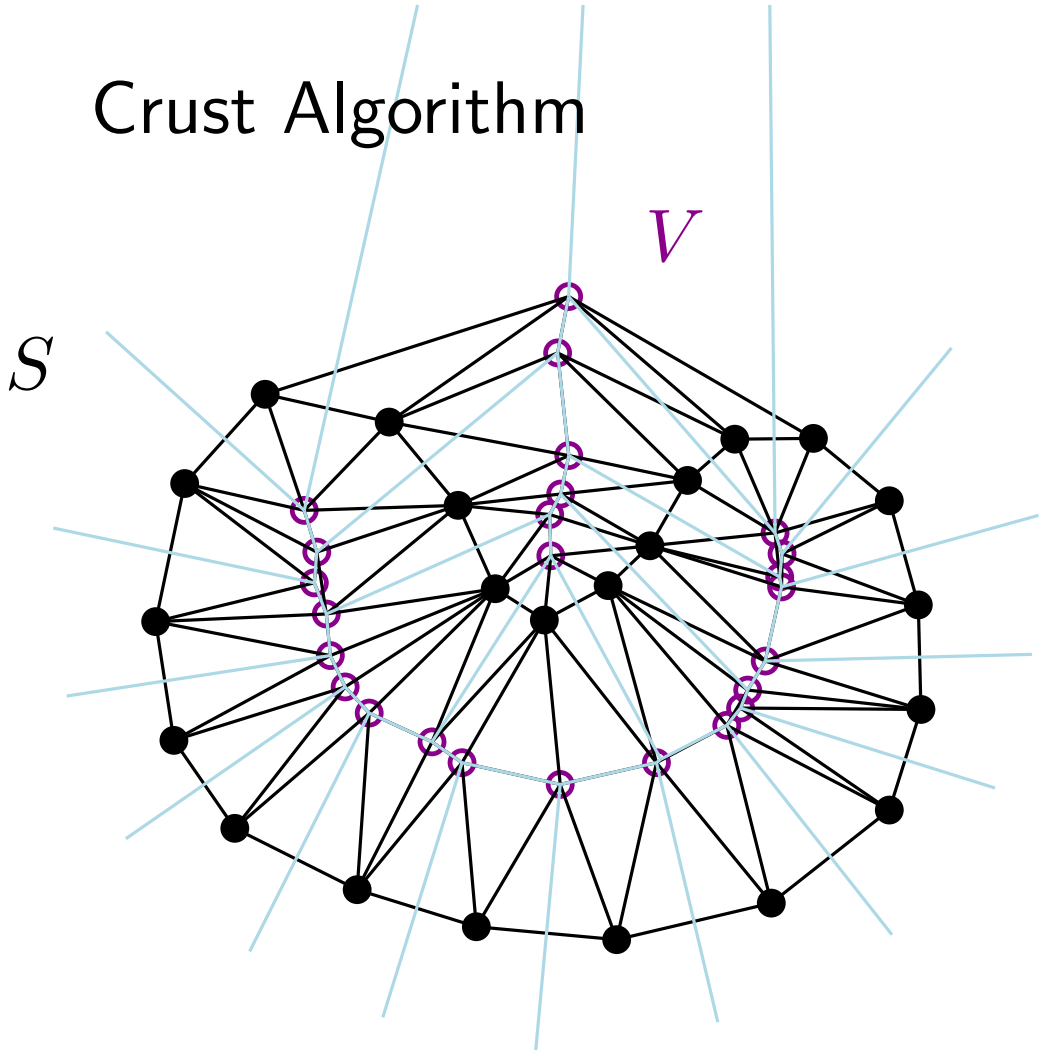
Crust Algorithm



Let S be sample points.

Let V be Voronoi vertices in $\mathrm{VD}(S)$

Crust Algorithm

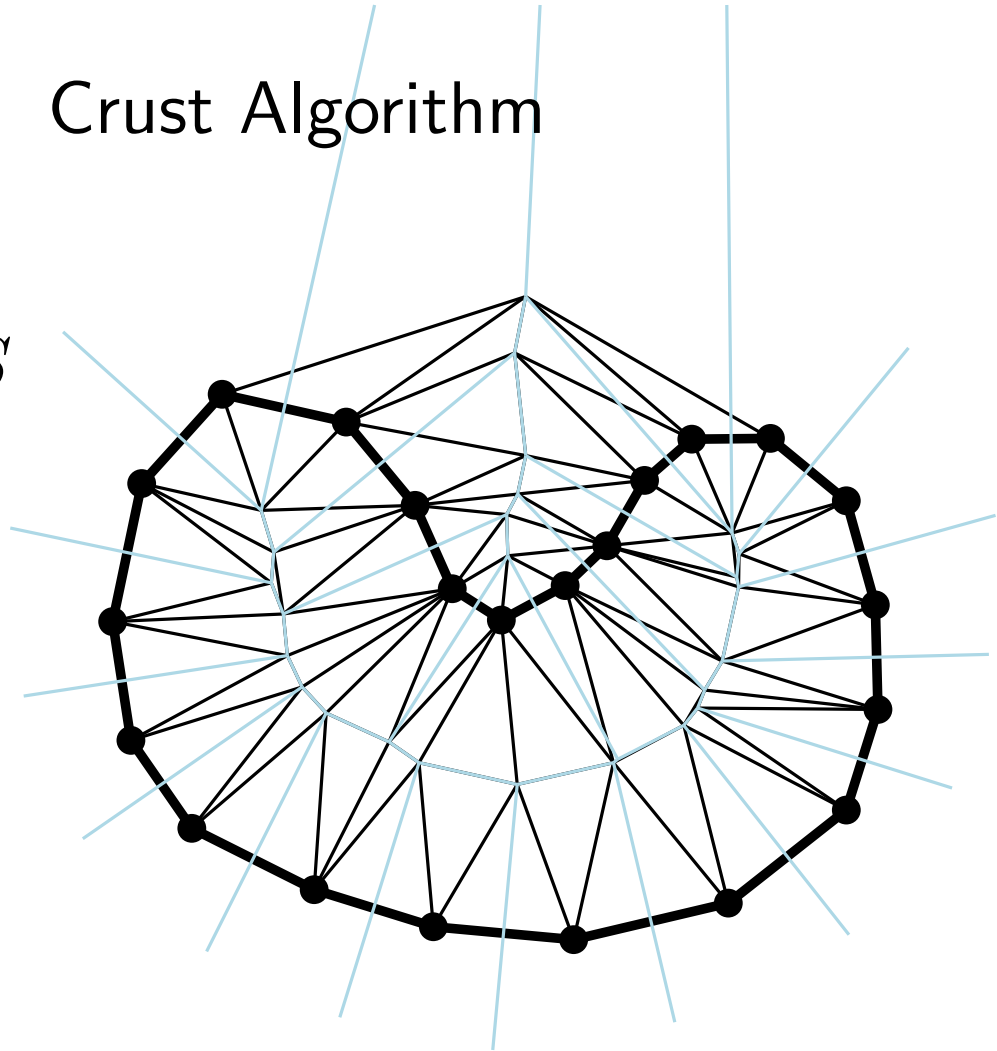


Let S be sample points.

Let V be Voronoi vertices in $\mathrm{VD}(S)$

Connect $p, q \in S$ if $\overline{pq}$ is an edge of $\mathrm{DT}(S \cup V)$

Crust Algorithm

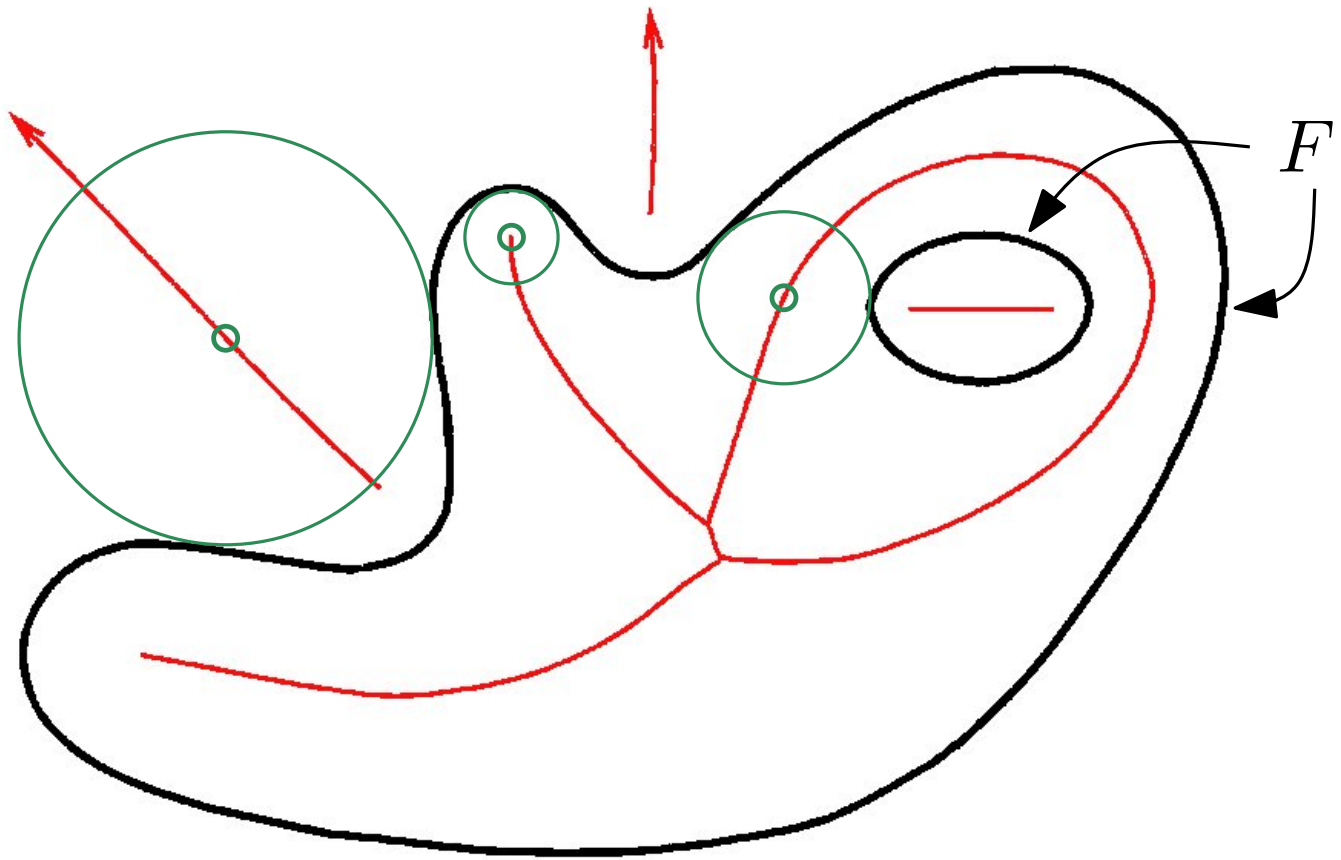


Let S be sample points.

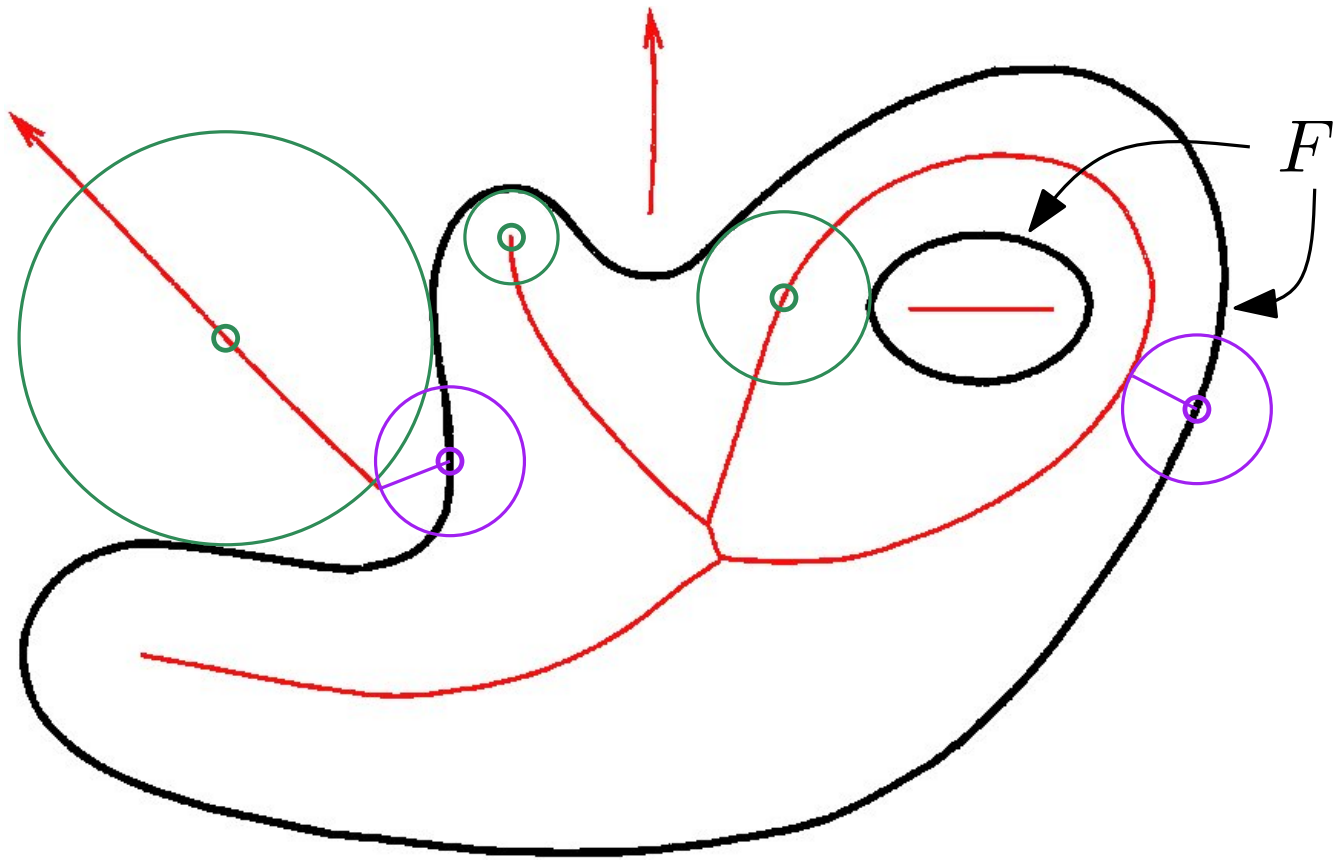
Let V be Voronoi vertices in $\mathrm{VD}(S)$

Connect $p, q \in S$ if $\overline{pq}$ is an edge of $\mathrm{DT}(S \cup V)$

The **medial axis** of F is the set of points in $\mathbb{R}^2$ with two or more closest points in F.



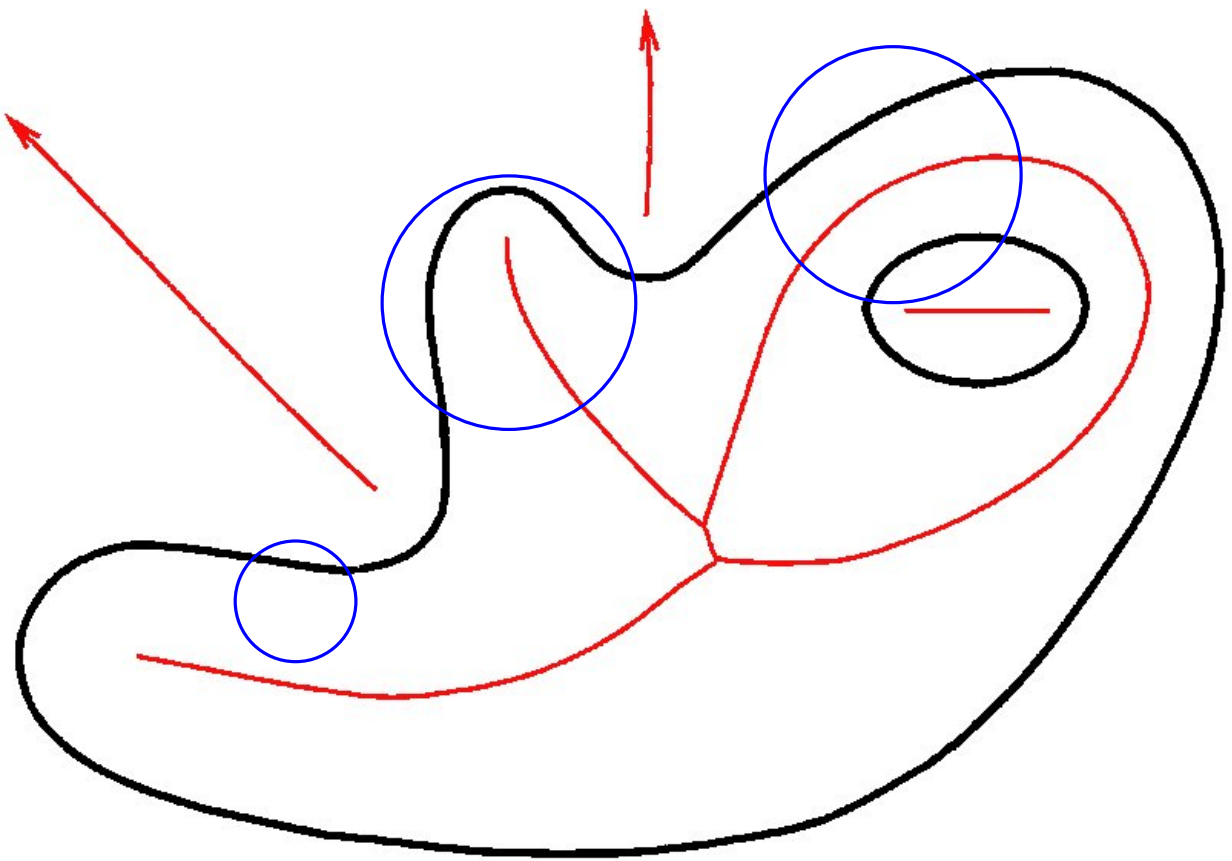
The **feature size at** $p \in F$ is the distance from p to the medial axis of F.



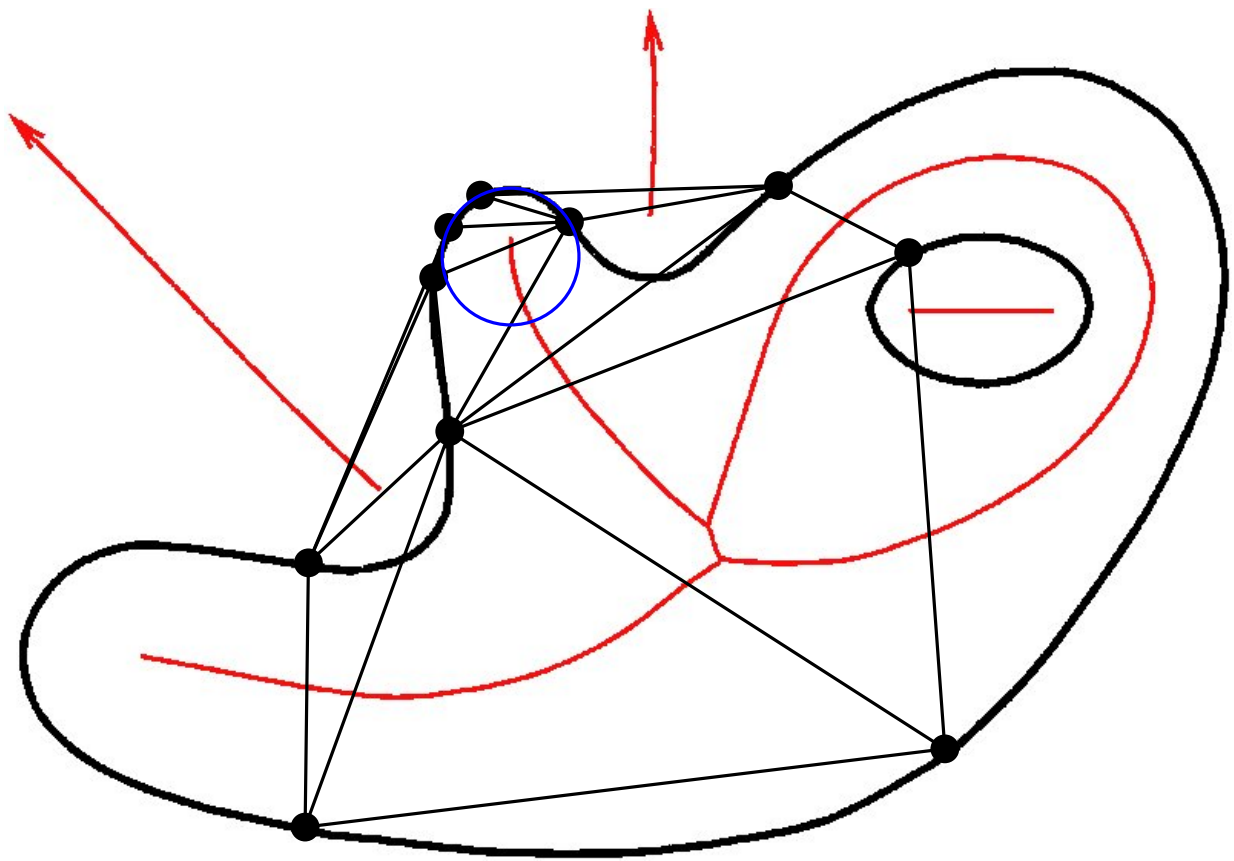
r-sampling condition: The distance from $p \in F$ to the nearest sample $s \in S$ is at most a constant r times the feature size at p.

Theorem The crust of an r-sampled smooth curve contains an edge between every pair of adjacent samples for $r < 0.4$.

Lemma 1. Any (Euclidean) disk B containing at least two points of a smooth curve F in the plane either intersects the curve in a single curve segment or contains a point of the medial axis (or both).

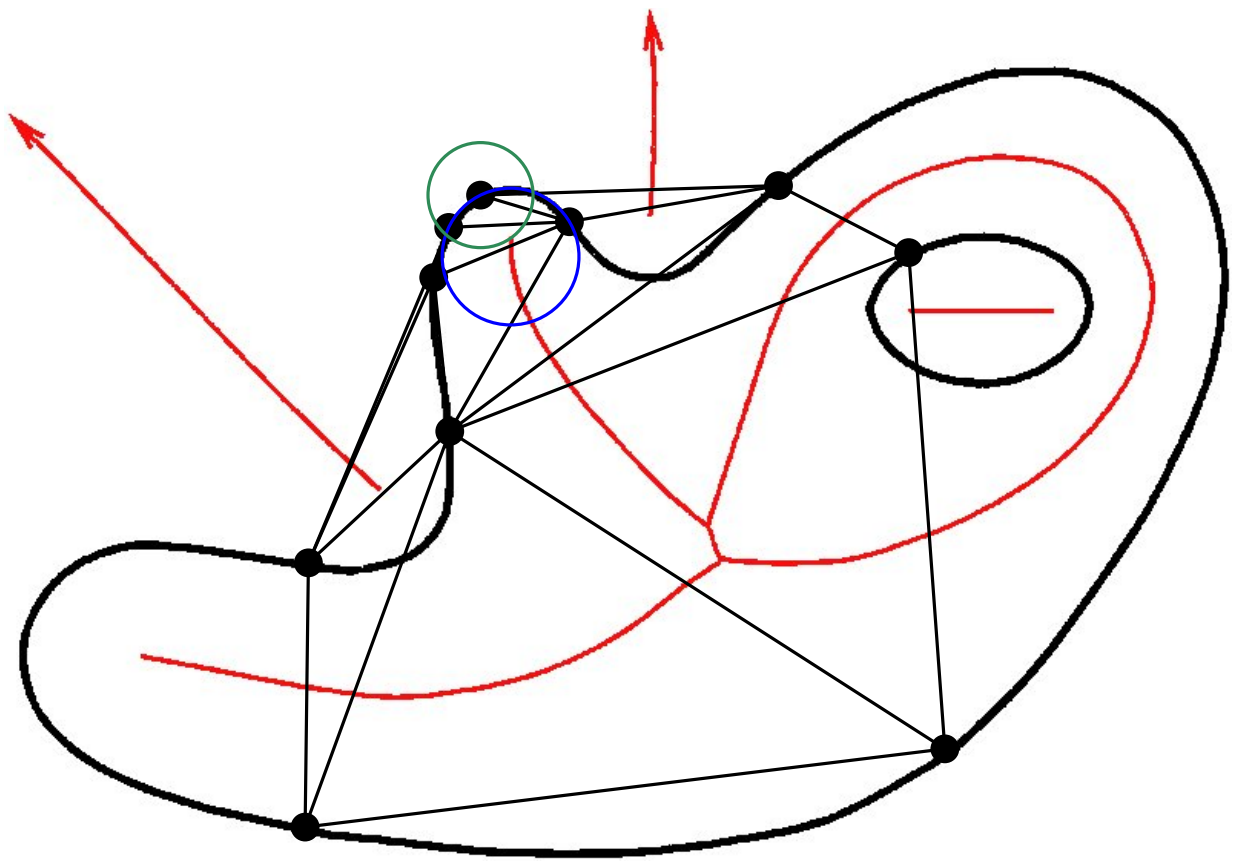


Lemma 2. In the plane, any Voronoi disk B of a finite set $S \subseteq F$, where F is a smooth curve, contains a point of the medial axis of F.



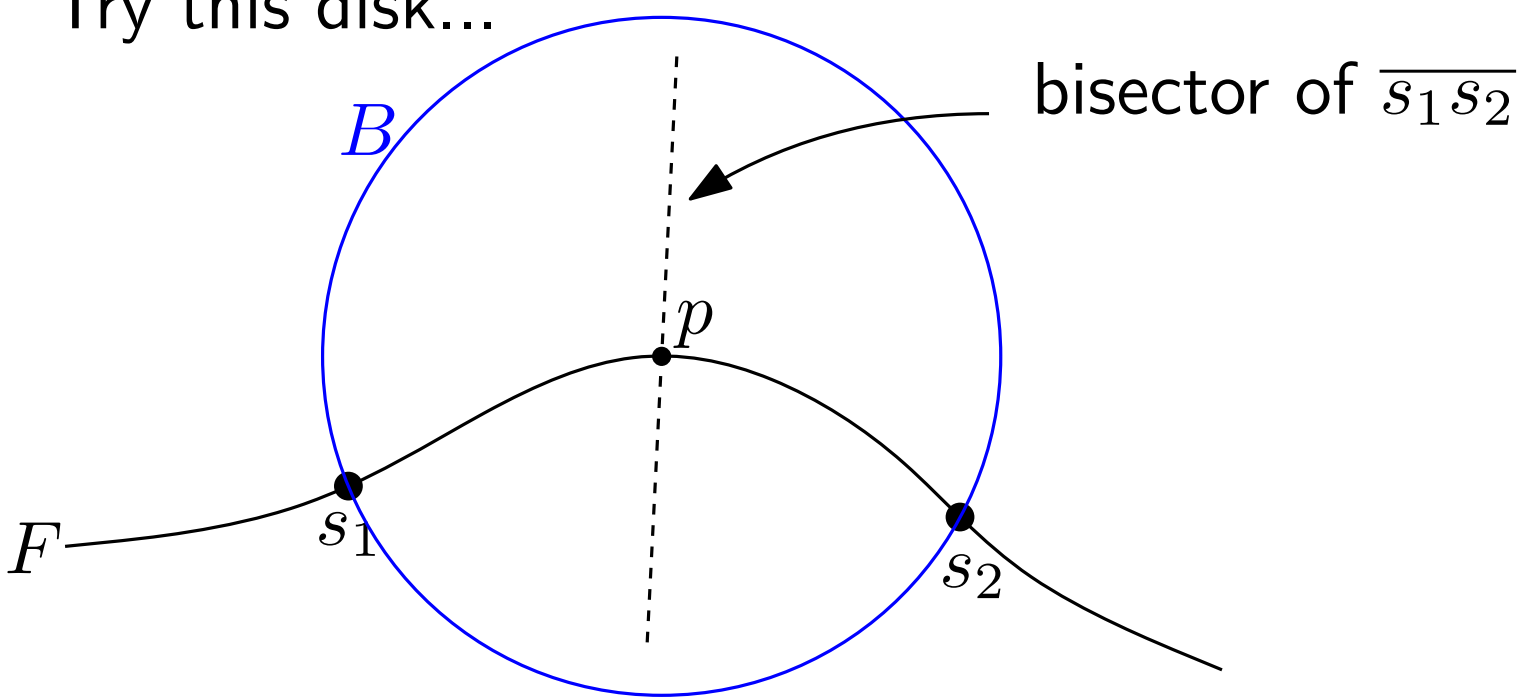
Lemma 2. In the plane, any Voronoi disk B of a finite set $S \subseteq F$, where F is a smooth curve, contains a point of the medial axis of F.

Cor. A disk centred at $p \in F$ with radius $\leq \mathrm{LFS(p)}$ contains one curve segment.



Lemma 3. Let F be an r-sampled smooth curve in the plane, $r \leq 1$. There is a disk touching each pair of adjacent samples that is empty of samples.

Try this disk...



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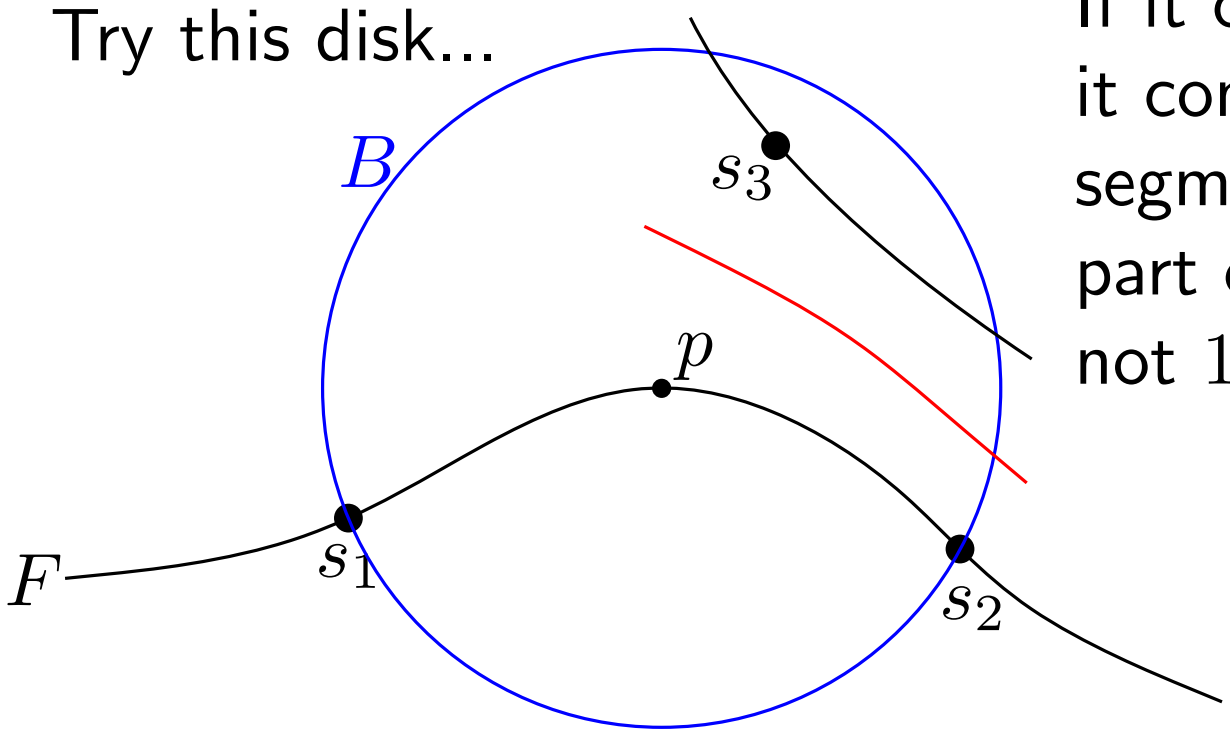
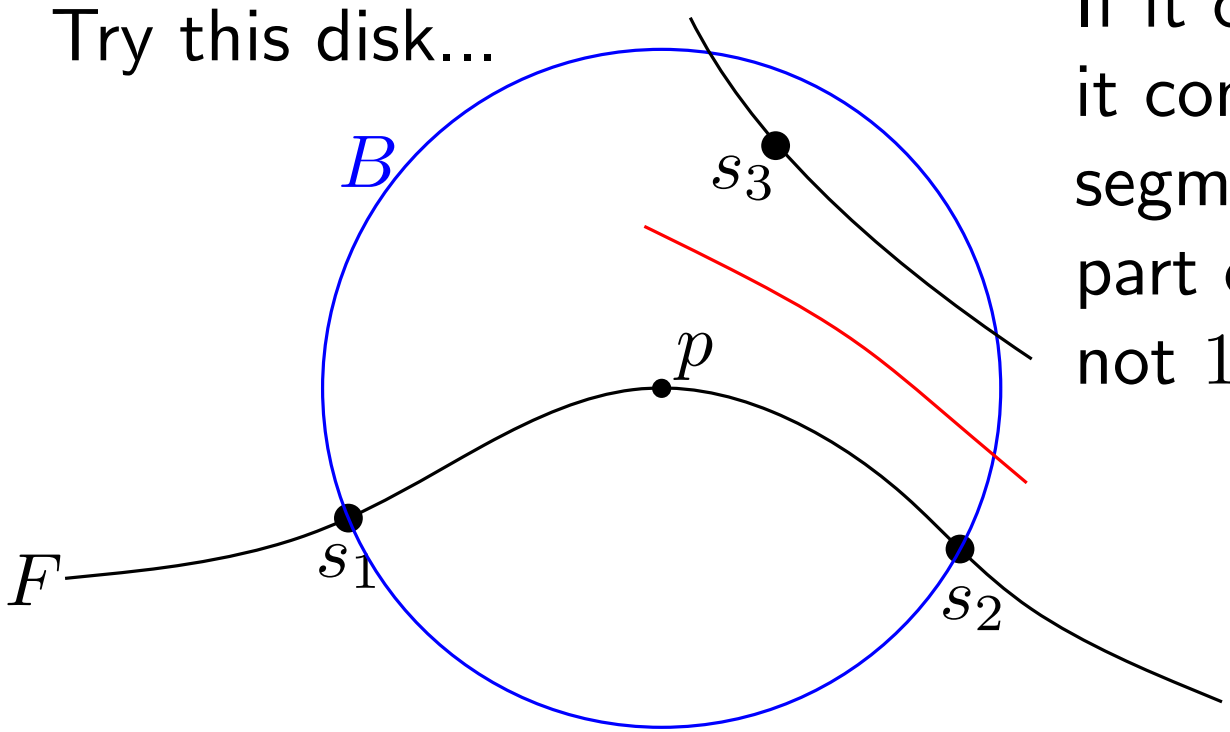


If it contains a sample, it contains > 1 curve segment $\Rightarrow$ it contains part of medial axis $\Rightarrow$ F not 1-sampled at p.$\Rightarrow\Leftarrow$

Lemma 3. Let F be an r-sampled smooth curve in the plane, $r \leq 1$. There is a disk touching each pair of adjacent samples that is empty of samples.

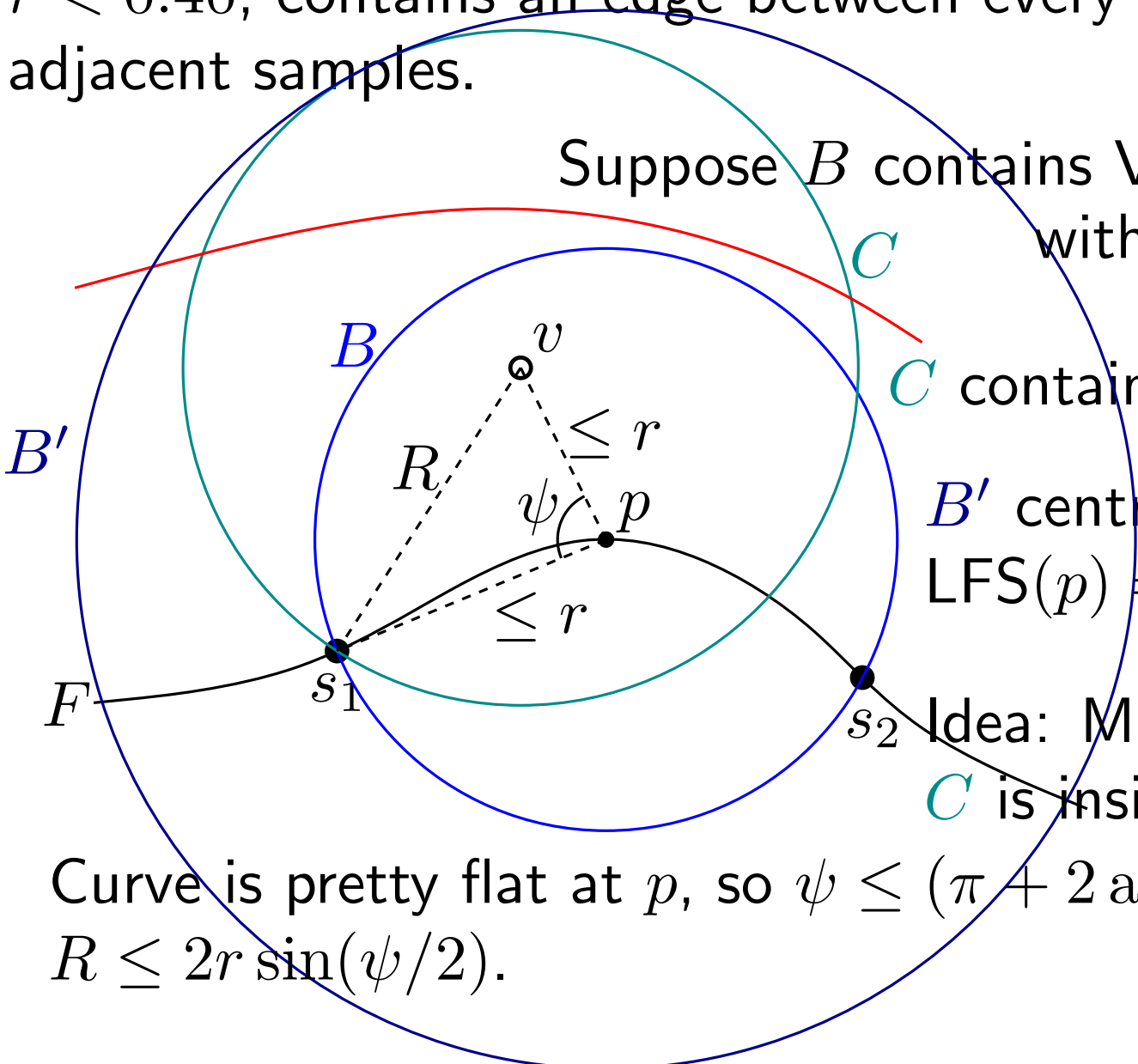
Try this disk...



If it contains a sample, it contains > 1 curve segment $\Rightarrow$ it contains part of medial axis $\Rightarrow$ F not 1-sampled at p.$\Rightarrow\Leftarrow$

Cor. Every pair of adjacent samples is connected by a Delaunay edge in $\mathrm{DT}(S)$.

Theorem The crust of an r-sampled smooth curve, $r < 0.40$, contains an edge between every pair of adjacent samples.



Suppose B contains Voronoi vertex v. with Voronoi disk C

C contains part of medial axis

B' centred at p with radius $\mathrm{LFS}(p) = 1$ *cannot*.

Idea: Make $r + R \leq 1$ so C is inside B'

Curve is pretty flat at p, so $\psi \leq (\pi + 2\arcsin(r/2))/2$ and $R \leq 2r\sin(\psi/2)$.

Theorem The crust of an r-sampled smooth curve does not contain any edge between non-adjacent samples for $r < 0.252$.