Course: [webpage] and overview

Scribing: No scribe today but sign up for some day: on the sheet. (Maybe 2 days.)

Guarding an Art Gallery

Represent floor plan as a Simple Polygon

A region of the plane that is bounded by a simple closed curve made up of $n$ (a finite number) line segments.

Polygons with $n=3$

Find a set $T$ of triangles such that

1. $\bigcup T = P$
2. $\bigcup V(T) = V(P)$
3. $T_1 \cap T_2$ is a vertex, edge, or empty for $T_1 \neq T_2$ in $\mathcal{C}$

Curve is a continuous map $\gamma: [0,1] \to \mathbb{R}^2$

Closed $\gamma(0) = \gamma(1)$

Simple $\gamma(x) \neq \gamma(y)$ for any unless $x,y \in \{0,1\}$
Theorem: Every simple polygon has a triangulation.

Proof (by induction on # vertices): If \( n = 3 \), done (base case).
If \( n > 3 \), pick the leftmost vertex \( V \) of \( P \) (if ties, does closest to \( V \) work?).

- If \( uw \not\subset P \) then create \( \Delta uvw \) and add it to triangulation of \( P - V \) (which exists by induction).

- If \( uw \subset P \) then split \( P \) by adding segment \( \overline{vx} \), where \( x \) is the vertex of \( P \) in interior of \( \Delta uvw \) farthest from \( uw \) (does closest to \( V \) work?).

Split \( P \) into two smaller polygons \( P_1 \) and \( P_2 \) by adding \( \overline{vx} \), which both have triangulations \( \Gamma_1 \) and \( \Gamma_2 \) (by induction).

Then \( \Gamma_1 \cup \Gamma_2 \) is a triangulation of \( P \).

How many triangles in triangulation of \( n \)-vertex polygon?

- \( n - 2 \) triangles
- \( 2n - 3 \) edges

Does every polyhedron (3D polygon) have a tetrahedralization?

No

Schoenhardt polyhedron 1928
Art Gallery Problem

Victor Klee How many guards are necessary and sufficient to guard the walls of an art gallery with n walls?

Chvatal \[ \lfloor \frac{n}{3} \rfloor \]

Fisk's proof "from the book"

(A) Every triangulation of a simple polygon is 3-colorable

Use induction. \( N=3 \) done
\( n>3 \) Split polygon using one of the \( 2n-3-n \) interior edges. \( uv \) 3-color both pieces.

\[ n = n-3 \]

Assume \( u \) is red and \( v \) is blue in \( P_1 \) coloring.
Call whatever color is used in \( P_2 \) for \( u \) red and for \( v \) blue.

(B) Some color appears at most \( \lfloor \frac{n}{3} \rfloor \) times. Why?

(C) Put a guard at all vertices with this color.