CPSC 516 Computational Geometry

8 Sept 2022

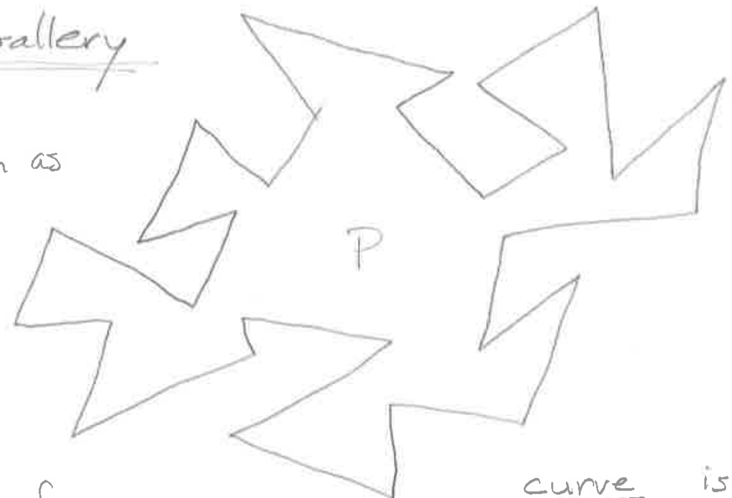
project on screen

Course webpage and overview

Scribing: No scribe today but sign up for some day on the sheet. <Maybe 2 days>

from Zurich Ch. 3

Guarding an Art Gallery



Represent floorplan as a SimplePolygon

A region of the plane that is bounded by a simple closed curve made up of n (a finite number) line segments

curve is a continuous map $\gamma : [0,1] \to \mathbb{R}^2$

closed $\gamma(0) = \gamma(1)$

simple $\gamma(x) \neq \gamma(y)$ for $x \neq y$ unless $\{x,y\} = \{0,1\}$

Polygon Δtion

Find a set $\mathcal{T}$ of triangles (polygons with $n=3$) such that

① $\bigcup_{T \in \mathcal{T}} T = P$

② $\bigcup_{T \in \mathcal{T}} V(T) = V(P)$

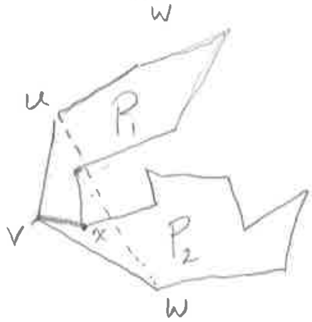
③ $T_1 \cap T_2$ is a vertex, edge, or empty for $T_1 \neq T_2$ in $\mathcal{T}$

Thm Every simple polygon has a triangulation.

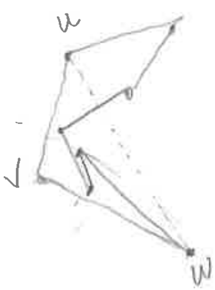
proof (by induction on # vertices) If $n=3$ done (base case)

If $n>3$, pick the leftmost vertex v of P (lowest if ties)

→ If $\overline{uw} \subset P$ then create $\triangle uvw$ and add it to triangulation of $P - v$ (which exists by induction)



→ if $\overline{uw} \not\subset P$ then split P by adding segment $\overline{vx}$ where x is the vertex of P in interior of $\triangle uvw$ farthest from $\overline{uw}$



(does closest to v work?)

Split P into two smaller polygons P_1 & P_2 by adding $\overline{vx}$, which both have triangulations τ_1 and τ_2 (by induction). Then $\tau_1 \cup \tau_2$ is a Δtion of P

$\boxtimes$

How many triangles in triangulation of n vertex polygon?

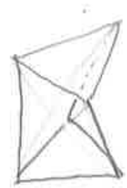
$n-2$ triangles

$2n-3$ edges

Does every polyhedron (3D polygon) have a tetrahedralization?

NO

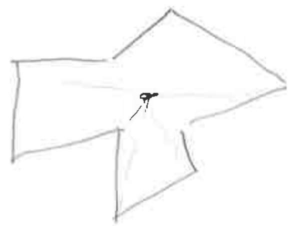
twist top triangle



Schönhardt polyhedron 1928

Art Gallery Problem

Victor Klee   How many guards are necessary and sufficient to guard the walls of an art gallery with n walls?



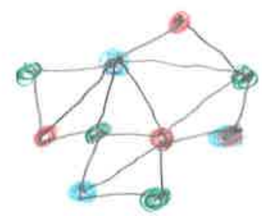
one?

two?

Chvátal $\lfloor n/3 \rfloor$

Fisk's proof "from the book"

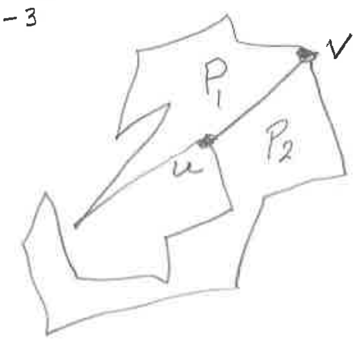
(A) Every triangulation of a simple polygon is 3-colorable



Use induction. $n=3$ done

$n>3$ Split polygon using one of the $2n-3-n$ interior edges. (uv) 3 color both pieces.

$= n-3$



Assume u is red and v is blue in P_1 coloring.

Call whatever color is used in P_2 ~~for~~ for u red and for v blue

(B) Some color appears at most $\lfloor n/3 \rfloor$ times   why?

(C) Put a guard at all vertices with this color.