CPSC 516 Problem Set 3 27 Oct 2022

W. Evans Due: 10 Nov 2022

Remember: Acknowledge any collaboration or resource.

1. Given $n-1$ real numbers, $x_1 < x_2 < \cdots < x_{n-1}$, describe a linear time algorithm to test if these numbers could be the x-coordinates of the boundaries between the Voronoi regions of some set of n sites on the x-axis.

2. The *furthest point Voronoi diagram* of a set S of sites (points in 2D) groups together points in the plane that share a common *furthest* site (rather than *closest* site as in the traditional Voronoi diagram).

   (a) Prove that a site has a non-empty furthest point Voronoi region associated with it if and only if $p \in S$ is a vertex of the convex hull of S.

   (b) The planar graph dual of the *furthest* point Voronoi diagram is the *furthest* point Delaunay triangulation. Design an $O(n \log n)$ algorithm that given a set S of n sites constructs the furthest point Delaunay triangulation.

3. (from O'Rourke 5.3.3.6) *Pitteway triangulations.* A triangulation of a set of points P is called a *Pitteway triangulation* if, for each triangle $T = (a, b, c)$, every point in T has one of a, b, or c as its nearest neighbor among points of P.

   (a) Show by example that not every Delaunay triangulation is a Pitteway triangulation.

   (b) Characterize those Delaunay triangulations that are Pitteway triangulations. (Hint: Consider a joint property of the Voronoi diagram *and* the Delaunay triangulation of P.)

4. (CGAA Exercise 8.4) Let L be a set of n lines in the plane. Give an $O(n \log n)$ time algorithm to compute an axis-parallel rectangle that contains all the vertices of the arrangement of L in its interior.