CPSC 516 Problem Set 2 29 Sep 2022
W. Evans Due: 13 Oct 2022

Remember: Acknowledge any collaboration or resource.

1. Suppose the input to Graham's Scan is the (circular) list of vertices of a simple polygon (in ccw order around the polygon). If Graham's Scan uses this order, starting at the lowest vertex, rather than sorting the vertices by angle, then the algorithm would take $O(n)$ time.
   (a) Give an example of a simple polygon for which this $O(n)$ time algorithm fails to produce the convex hull. [Try to find a six vertex example.]
   (b) Describe an algorithm that finds the convex hull of a simple polygon in $O(n)$ time. [Extra Credit - hard]

2. (from Problem 3 in 3.2.3. Exercises from "Computational Geometry in C" by J. O'Rourke) *Min supporting line* Design an algorithm that given a set of n points P (in the plane), finds a line L that
   (a) has all the points of P on one side, and
   (b) minimizes the sum of the distances of the points in P to L. (The distance of a point p to L is the length of the line segment, perpendicular to L, from p to L.)

   Your algorithm should run in time $O(n \log n)$. Hint: Use the convex hull.

3. (from Zurich Exercise 4.31) Consider k convex polygons $P_1, \ldots, P_k$, for some *constant* $k \in \mathbb{N}$, where each polygon is given as a list of its vertices in counterclockwise orientation. Show how to construct the convex hull of $P_1 \cup \cdots \cup P_k$ in $O(n)$ time, where n is the sum of the number of vertices in P_i over all $1 \leq i \leq k$.

4. Given n real numbers $S = \langle x_1, x_2, \ldots, x_n \rangle$, we would like to determine if these numbers are *evenly-spaced*, that is, the difference between the ith and $i+1$st smallest in S is the same for all $1 \leq i \leq n-1$. Show that any algorithm in the algebraic decision tree model requires $\Omega(n \log n)$ time to solve this problem.

5. Given n real numbers $S = \langle x_1, x_2, \ldots, x_n \rangle$ and a real number g, we would like to determine if the *maximum space* between these numbers is g, that is, the maximum difference between the ith and $(i+1)$st smallest in S over all $1 \leq i \leq n-1$ is g. Show that any algorithm in the algebraic decision tree model requires $\Omega(n \log n)$ time to solve this problem. [Hint: Use a reduction.]