Animating an Object

Representing Orientations

Are these rotation matrices ?

$$\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & -1 \\ 1 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Representing Rotations

	numbers	constraints
1. 3x3 Rotation matrix		
2. Euler Angles		
3. Angle-Axis (exponential map)		
4. Quaternions		

Euler Angles

Angle-Axis

- Euler's Rotation Theorem (1776)

  Theorema. Quomodocunque sphaera circa centrum suum conuertatur, semper assignari potest diameter, cuius directio in situ translato conueniat cum situ initiali.

  When a sphere is moved around its centre it is always possible to find a diameter whose direction in the displaced position is the same as in the initial position.

$$R = \begin{bmatrix} \cos\theta + u_x^2\left(1-\cos\theta\right) & u_x u_y\left(1-\cos\theta\right) - u_z\sin\theta & u_x u_z\left(1-\cos\theta\right) + u_y\sin\theta \\ u_y u_x\left(1-\cos\theta\right) + u_z\sin\theta & \cos\theta + u_y^2\left(1-\cos\theta\right) & u_y u_z\left(1-\cos\theta\right) - u_x\sin\theta \\ u_z u_x\left(1-\cos\theta\right) - u_y\sin\theta & u_z u_y\left(1-\cos\theta\right) + u_x\sin\theta & \cos\theta + u_z^2\left(1-\cos\theta\right) \end{bmatrix}$$

Unit Quaternions

- desiderata

circle

sphere

A bit more on Euler ….

- Notations introduced:
- Historical context

Euler and Complex Exponentials

Multiplying Complex Numbers

Beyond complex numbers…

- commutative

- associative

- distributive

Quaternion Definitions

- form

- unit quaternion
- addition

- multiplication

Quaternion Multiplication

Quaternions

- angle-axis equivalent
- composition

Quaternion rotation of a point

Comments

- Rotation matrices
- Euler angles
- Angle-axis
- Quaternions

Conversion and Composition

Interpolation of Quaternions

SLERP

Interpolating Rotation Matrices

- Linear interpolation

- Matrix exponential