

# Scalable Drawing of Trees and Graphs

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joint work with Dan Archambault, David Auber,  
Francois Guimbretiere, Kristian Hildebrand, James  
Slack, Serdar Tasiran, Li Zhang, and Yunhong Zhou

## Outline

- Introduction
- H3
- TreeJuxtaposer
- TopoLayout
- Conclusion

## Scalable Visualization

- Visual representation of node-link graphs useful in many domains
  - many real-world datasets are very large
- Designing for scalability
  - graphics issues
    - guaranteed frame rate
  - interaction issues
    - guidance on where to look next
  - cognitive issues
    - maintain orientation
      - see details in context
      - guaranteed visibility of landmarks

## H3

- H3: Laying Out Large Directed Graphs in 3D Hyperbolic Space
  - Tamara Munzner. Proc. InfoVis 97, pp 2-10.
- Drawing Large Graphs with H3Viewer and Site Manager
  - Tamara Munzner. Proc. Graph Drawing 98, pp 384-393.
- video, free software available from <http://graphics.stanford.edu/~munzner/h3>



## H3 Features

- 3D hyperbolic geometry shows large local neighborhood
  - single focus
  - fisheye distortion
    - understanding graph topological structure does not require judging distances
  - details for dozens of nodes, aggregate information for thousands of nodes
- uses spanning tree as backbone for layout
  - explore non-tree links through interaction
  - appropriate for quasi-hierarchical graphs

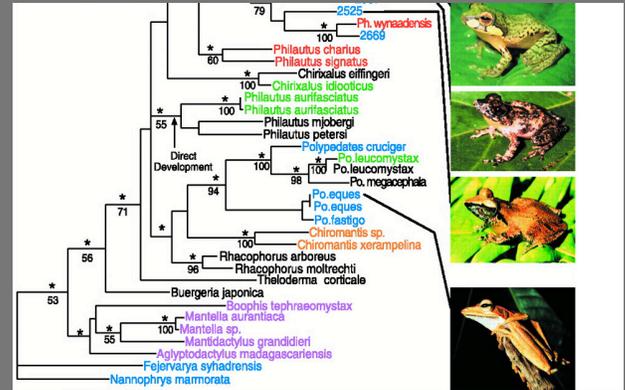
## H3 Limitations

- see large neighborhood but not global overview
  - can still get lost
- only single focus
  - intrinsic to hyperbolic geometry

## TreeJuxtaposer

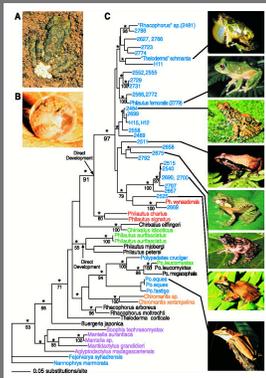
- TreeJuxtaposer: Scalable Tree Comparison using Focus+Context with Guaranteed Visibility
  - Tamara Munzner, Francois Guimbertiere, Serdar Tasiran, Li Zhang, and Yunhong Zhou. SIGGRAPH 2003, pp 453--462
  - side by side comparison of evolutionary trees

## Phylogenetic/Evolutionary Tree



M Meegaskumbura et al., Science 298:379 (2002)

## Common Dataset Size Today



M Meegaskumbura et al., Science 298:379 (2002)

## Future Goal: 10M Node Tree of Life



David Hillis, Science 300:1687 (2003)

## Paper Comparison: Multiple Trees

focus

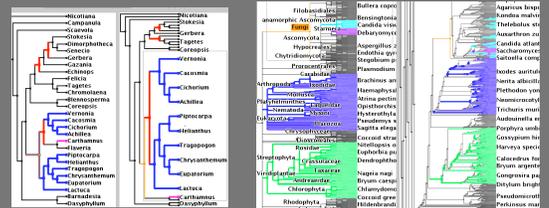


context



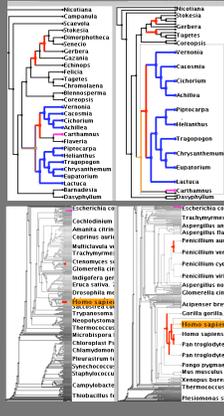
## TreeJuxtaposer

- side by side comparison of evolutionary trees
- [video]
  - video and free software downloadable from <http://olduvai.sf.net/tj>



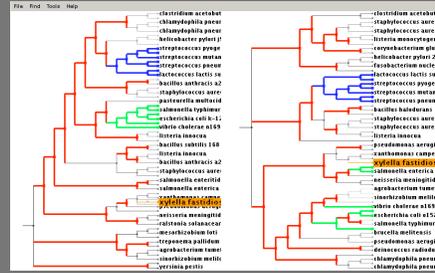
## Accordion Drawing

- rubber-sheet navigation
  - stretch out part of surface, the rest squishes
  - borders nailed down
  - Focus+Context technique
    - integrated overview, details
  - old idea
    - [Sarkar et al 93], [Robertson et al 91]
- guaranteed visibility
  - marks always visible
  - important for scalability
  - new idea
    - [Munzner et al 03]



## Guaranteed Visibility

- marks are always visible
- easy with small datasets



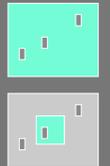
## Guaranteed Visibility Challenges

- hard with larger datasets
- reasons a mark could be invisible



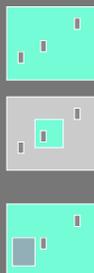
## Guaranteed Visibility Challenges

- hard with larger datasets
- reasons a mark could be invisible
  - outside the window
    - AD solution: constrained navigation



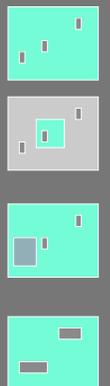
## Guaranteed Visibility Challenges

- hard with larger datasets
- reasons a mark could be invisible
  - underneath other marks
    - AD solution: avoid 3D



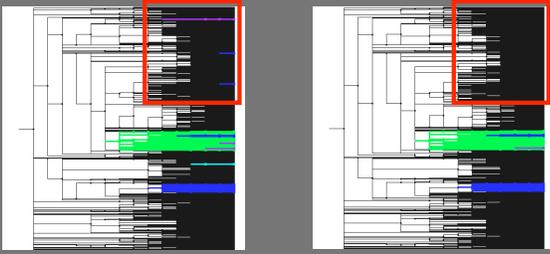
## Guaranteed Visibility Challenges

- hard with larger datasets
- reasons a mark could be invisible
  - underneath other marks
    - AD solution: avoid 3D
  - smaller than a pixel
    - AD solution: smart culling



## Guaranteed Visibility: Small Items

- Naïve culling may not draw all marked items



Guaranteed visibility  
of marks

No guaranteed visibility

## TJ Contributions

- first interactive tree comparison system
  - automatic structural difference computation
  - guaranteed visibility of marked areas
- scalable to large datasets
  - 250,000 to 500,000 total nodes
  - all preprocessing subquadratic
  - all realtime rendering sublinear
- scalable to large displays (4000 x 2000)
- introduced
  - guaranteed visibility, accordion drawing

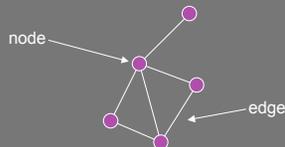
## Further Work

- Partitioned Rendering Infrastructure for Scalable Accordion Drawing (Extended Version)
  - James Slack, Kristian Hildebrand, and Tamara Munzner. Information Visualization 5(2), pp 137-151, 2006
  - generic and efficient rendering
    - handles trees over 4,000,000 nodes
- Composite Rectilinear Deformation for Stretch and Squish Navigation
  - James Slack and Tamara Munzner. Proc. InfoVis06, to appear
  - generic navigation
- SequenceJuxtaposer: Fluid Navigation For Large-Scale Sequence Comparison In Context
  - James Slack, Kristian Hildebrand, Tamara Munzner, and Katherine St. John. German Conference on Bioinformatics 2004, pp 37-42
  - accordion drawing for gene sequences

## TopoLayout

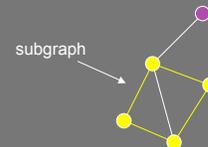
- TopoLayout: Multi-Level Graph Layout by Topological Features
  - Dan Archambault, Tamara Munzner, David Auber
  - Trans. Visualization and Computer Graphics, to appear
- Emphasis on offline computation of best possible static layout, vs. interactive frame rates

## Graph



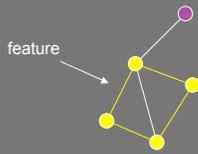
Graph:  $G(V, E)$  set  $V$  of nodes and set  $E$  of edges such that  $E$  is subset of  $V \times V$

## Subgraph and Feature



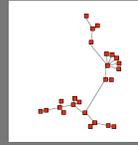
**Subgraph:** subset of these nodes and subset of the edges between them

## Subgraph and Feature

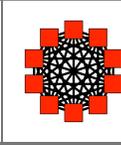


**Feature:** any subgraph of interest

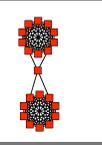
## Topological Features



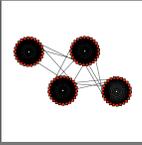
Tree



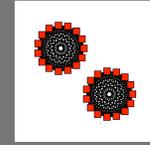
Complete Graph



Biconnected Component



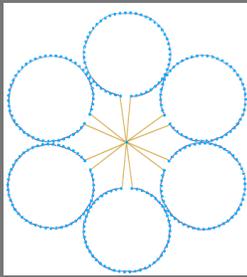
Cluster



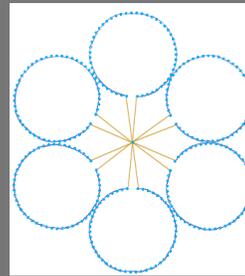
Connected Components

- Interconnection beyond direct adjacency
- Not hole counting in meshes

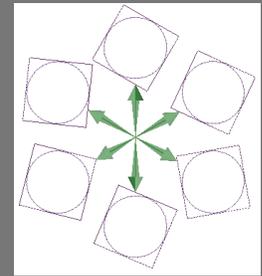
## Multi-level Structure



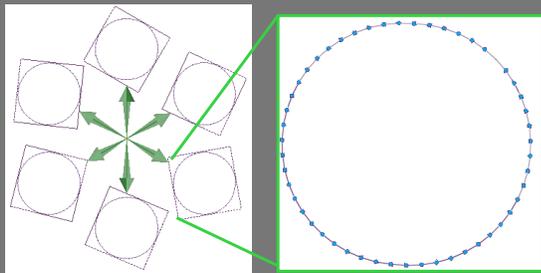
## High-Level Structure



- Small tree

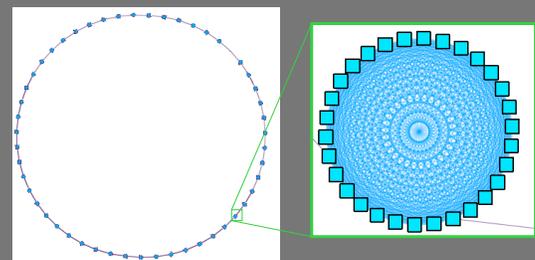


## Mid-level Structure



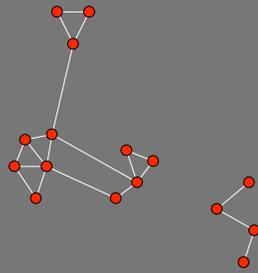
- Multiple levels of mid-level structure possible
  - Loop

## Low-level Structure

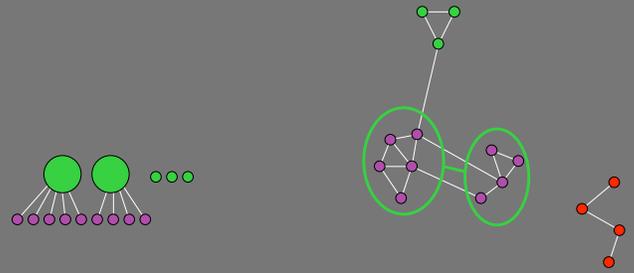


- Features involving original nodes and edges
- Lowest level is direct adjacencies, paths
  - Complete subgraph

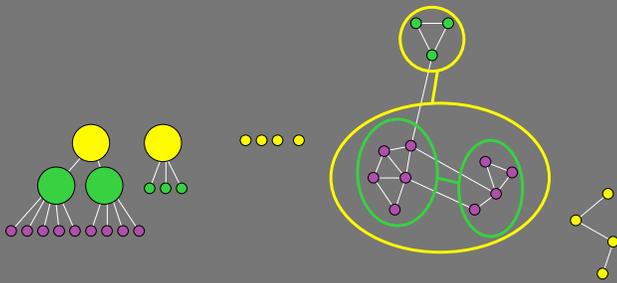
# Multi-Level Hierarchy



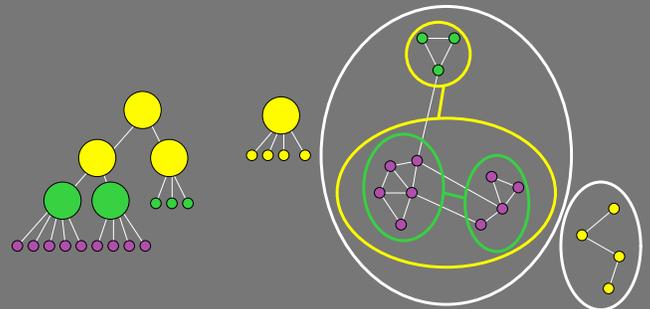
# Multi-Level Hierarchy



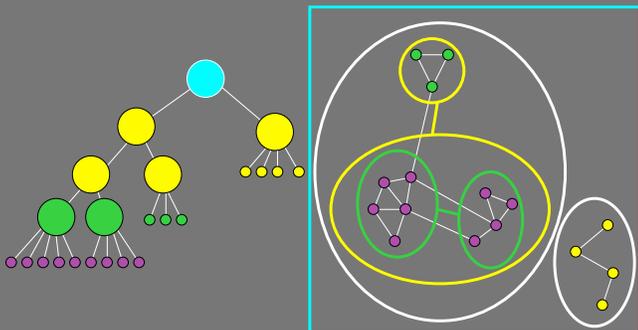
# Multi-Level Hierarchy



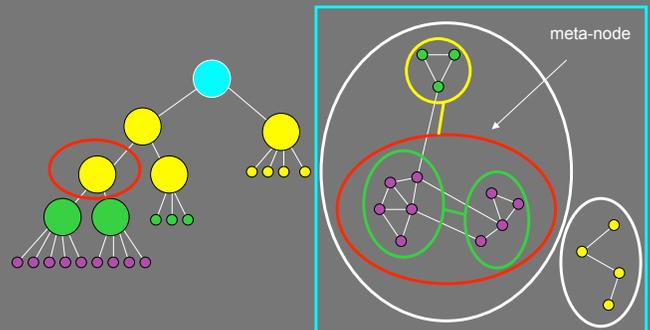
# Multi-Level Hierarchy



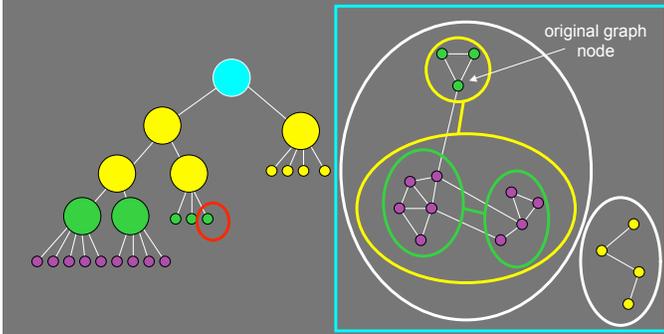
# Multi-Level Hierarchy



# Multi-Level Hierarchy

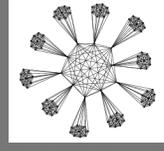
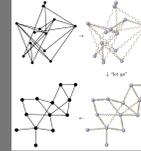


## Multi-Level Hierarchy



## Previous: Force-Directed Approaches

- Spring-Electrical: nodes repel, edges attract
  - Eades 1984, Fruchterman and Reingold 1991, Frick et al. (GEM) 1995
- Energy-Based: maxima/minima of energy function
  - Kamada and Kawai 1989, Davidson and Harel 1996, Noack 2003
- Limitation:  $O(|V|^3)$  complexity



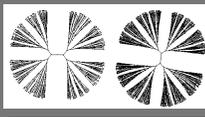
Frick et al. (GEM)

## Previous: Multi-Level Approaches

- Recursively coarsen into hierarchy
- Limitations
  - Lowest level features
  - Force-directed each level
- FM<sup>3</sup> current state of the art
  - Provable  $O(|V|\log|V| + |E|)$  complexity

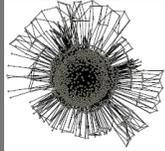


Walshaw 2000

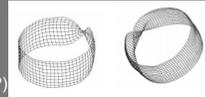


Harel and Koren 2000

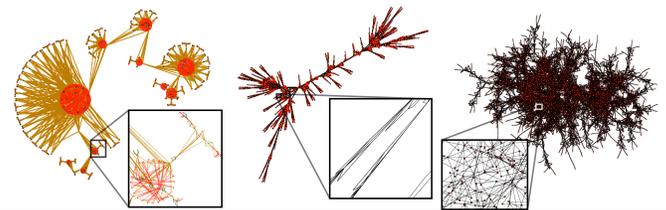
Hachul and Junger 2004 (FM<sup>3</sup>)



Gajer et al 2002 (GRIP)



## Results



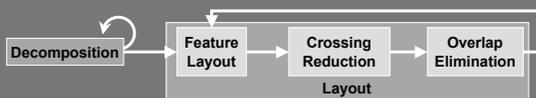
TopoLayout  
14 seconds

FM<sup>3</sup>  
12 seconds

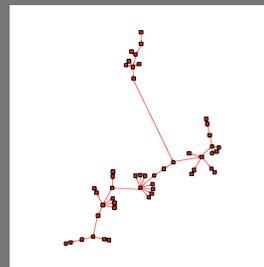
GRIP  
1 second

## TopoLayout Phases

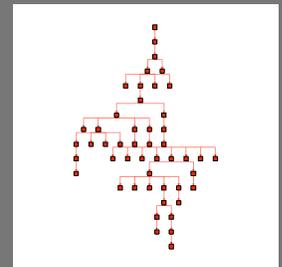
- Recursively decompose by feature
  - detectors
- Lay out each piece with appropriate algorithm
- Refine: reduce crossings, eliminate overlaps



## Trees

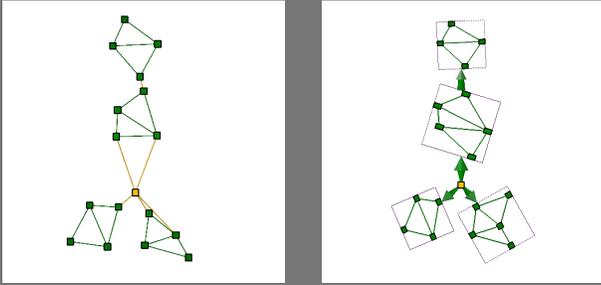


Bubble Tree, Bushy



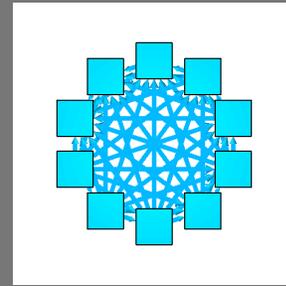
Reingold and Tilford, Deep

## Biconnected



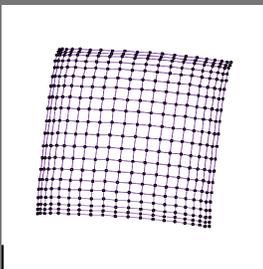
Higher level structure biconnected is a tree

## Complete Graphs



Circular Layout

## HDE Components

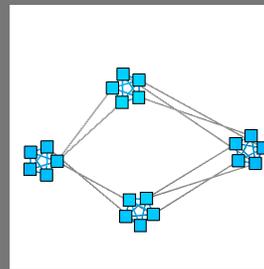


HDE

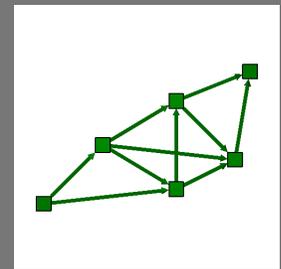
Mesh-like

Detected using eigenvalues

## Cluster and Unknown



Clusters

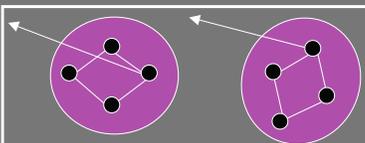


Unknown

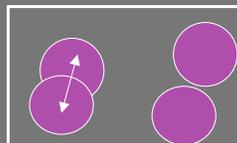
Force-directed layout - GEM

## Crossing Reduction and Overlap Resolution

- Crossing reduction
  - Rotate features to reduce edge crossings
  - Novel algorithm described in paper
- Overlap resolution
  - No overlapping pairs features
  - Use Dwyer *et al.*  $O(|V|\log|V|)$  approach

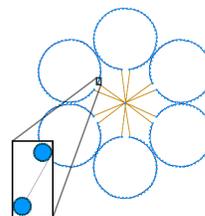


Crossing Reduction

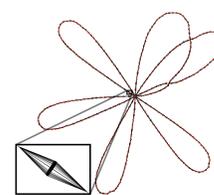


Overlap Elimination

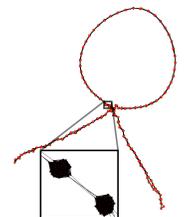
## Results



TopoLayout  
70 seconds

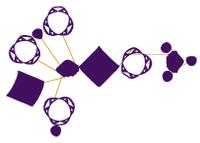


FM<sup>3</sup>  
11 seconds

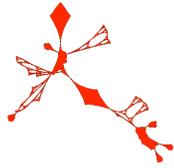


GRIP  
4 seconds

## Results



TopoLayout  
26 seconds



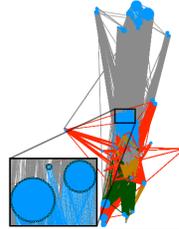
FM<sup>3</sup>  
134 seconds



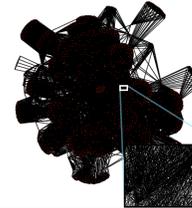
HDE  
1 second

- GRIP unable to produce drawing

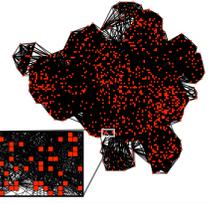
## Results



TopoLayout  
76 seconds



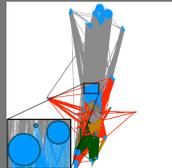
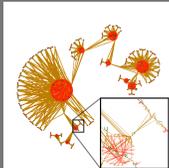
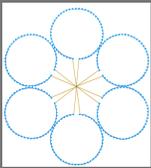
FM<sup>3</sup>  
3 seconds



GRIP  
1 second

## Benefits and Limitations

- Benefits
  - Faster and/or better visual quality showing high-level and low-level structure
- Limitations
  - Some mid-level structure still hidden
  - Running time and visual quality degrade when no detected features are present



## Outline

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- TopoLayout
- **Conclusion**

## Challenges

- determining appropriate information density
  - clutter vs. wasted space
- automatic detection of when given layout algorithm is appropriate
- scalability along different dimensions
  - addressed here
    - dataset size, display size
  - not addressed
    - heterogeneous vs. homogeneous datasets