University of British Columbia

CPSC 414 Computer Graphics

Sampling

Week 7, Fri 17 Oct 2003

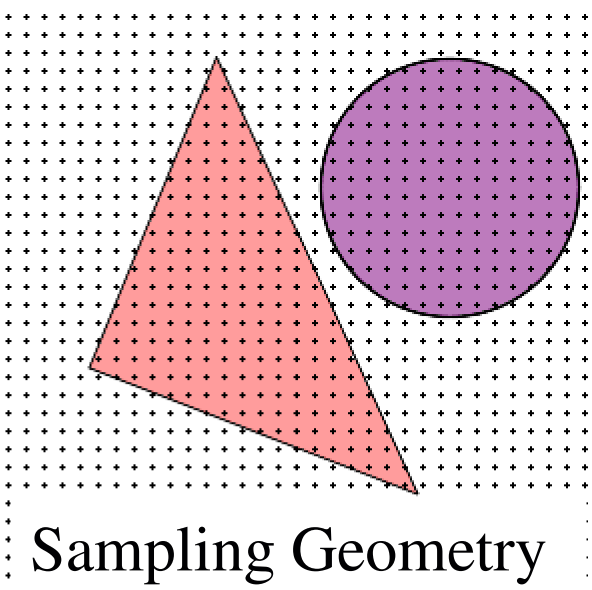
- p1 demos
- sampling

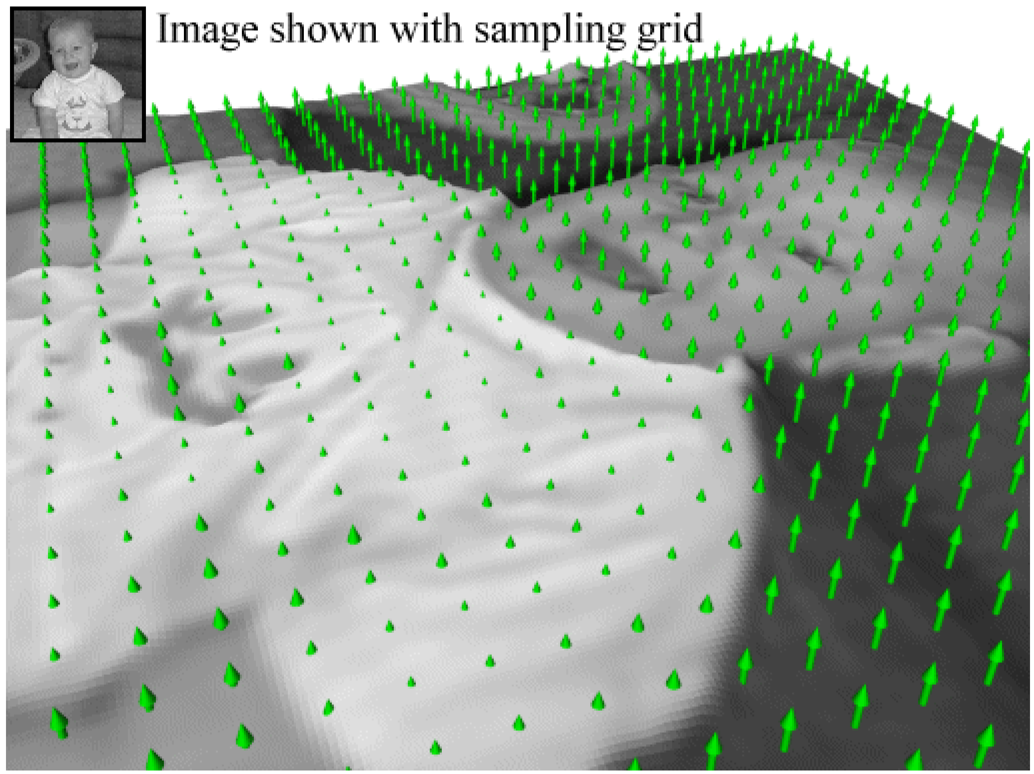
News

- hw 1 solutions out
  - no more accepted as of right now
- next week
  - Mon: midterm
    - no Mon office hours, I'm away at conferences
  - Wed: Prof. van de Panne on animation
  - Fri: TA Ahbijeet Ghosh on textures
- correct p1 grades posted on web site now
- project 1
  - finish hall of fame demos

Point Sampling

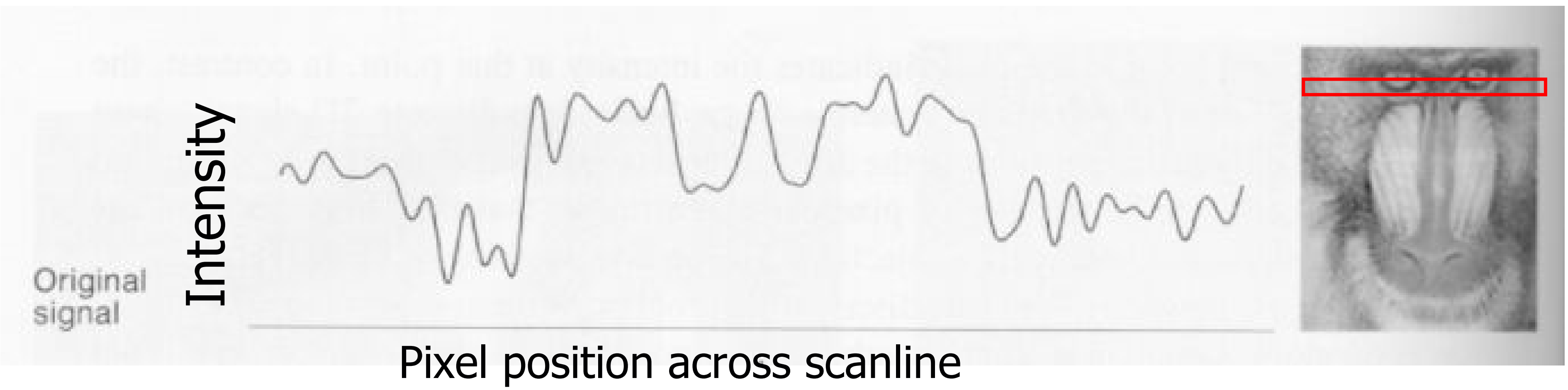
- multiply sample grid by image intensity to obtain a discrete set of points, or samples.





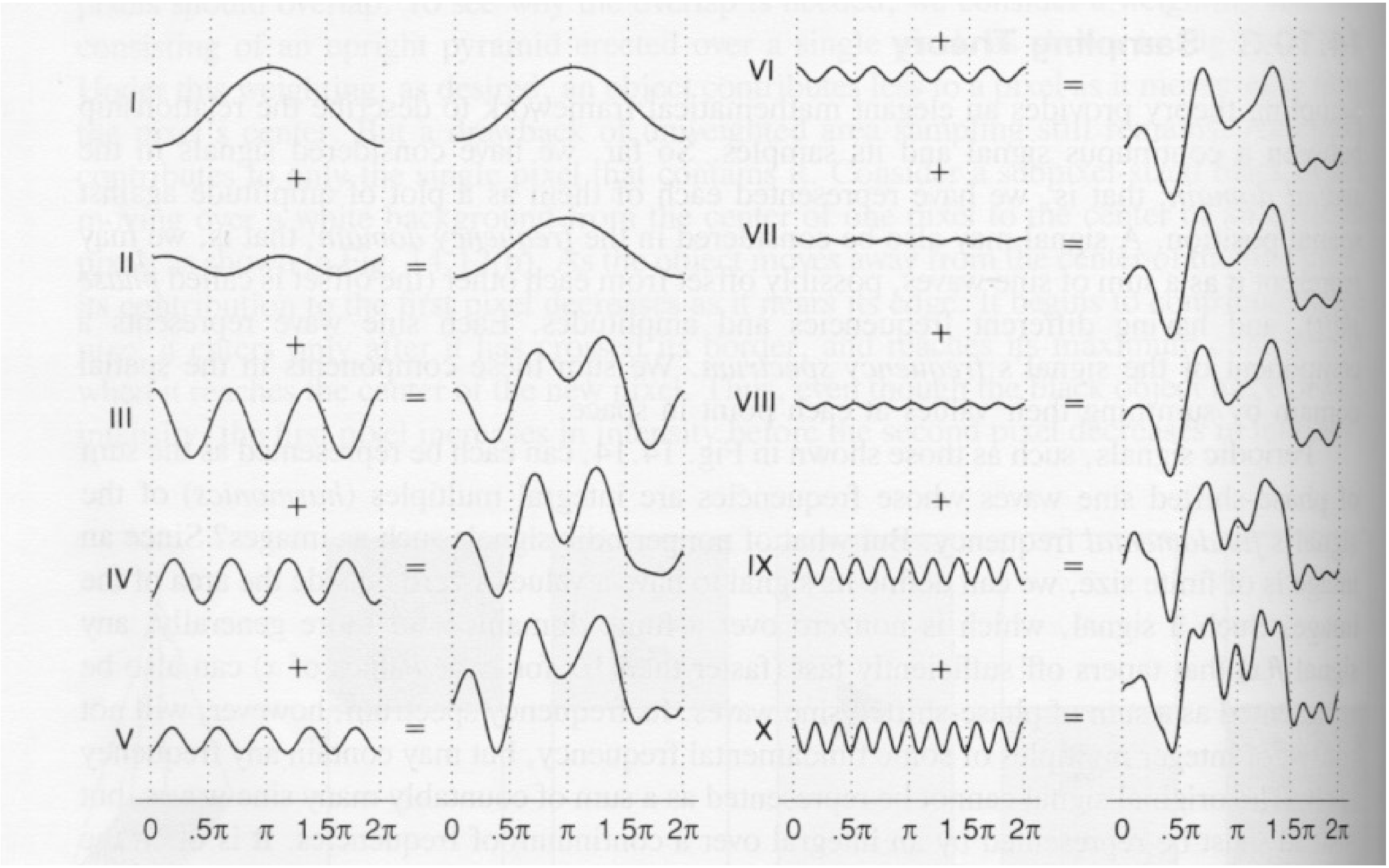
Spatial Domain

- image as spatial signal

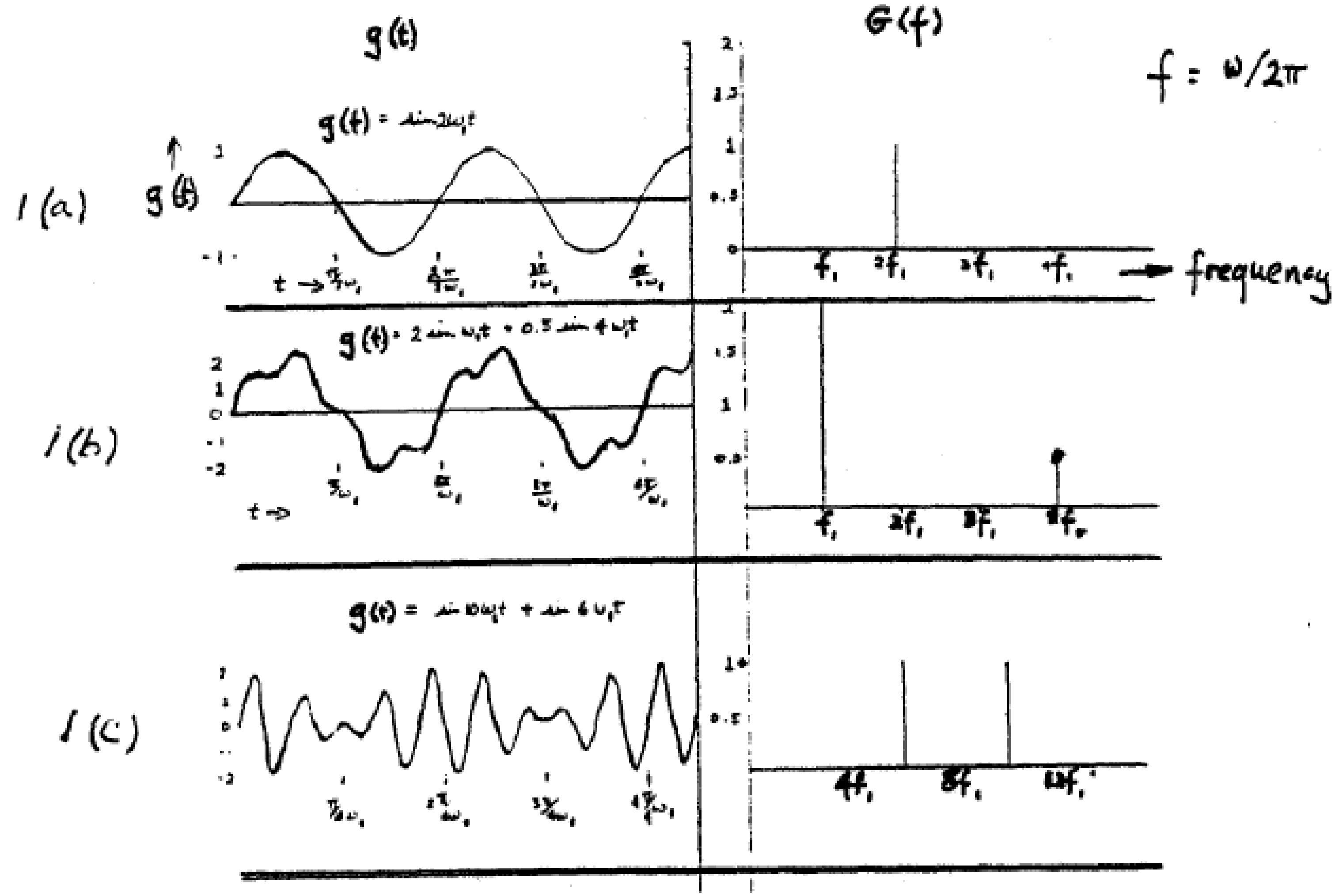


Examples from Foley, van Dam, Feiner, and Hughes

Spatial Domain: Summing Waves

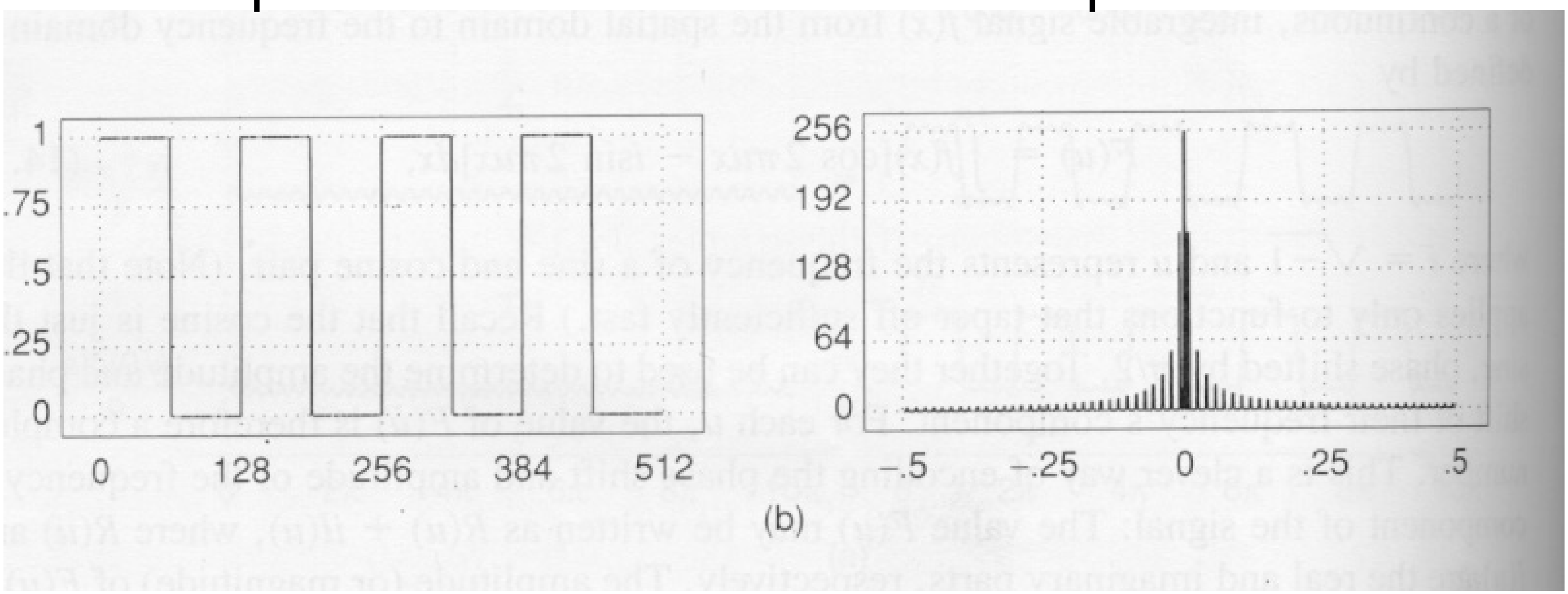


Frequencies: Summing Spikes



Frequency Domain

- position: frequency
- height: strength of each frequency
  - sine wave: impulse
  - square wave: infinite train of impulses

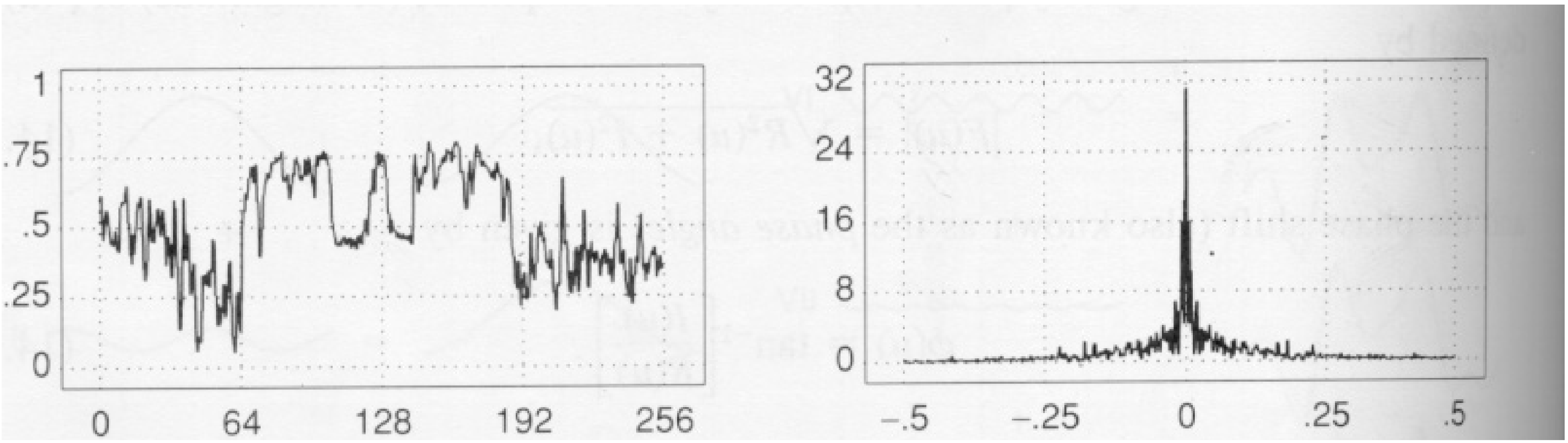


(b)

Fourier Transform Example

spatial domain

frequency domain



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Sampling

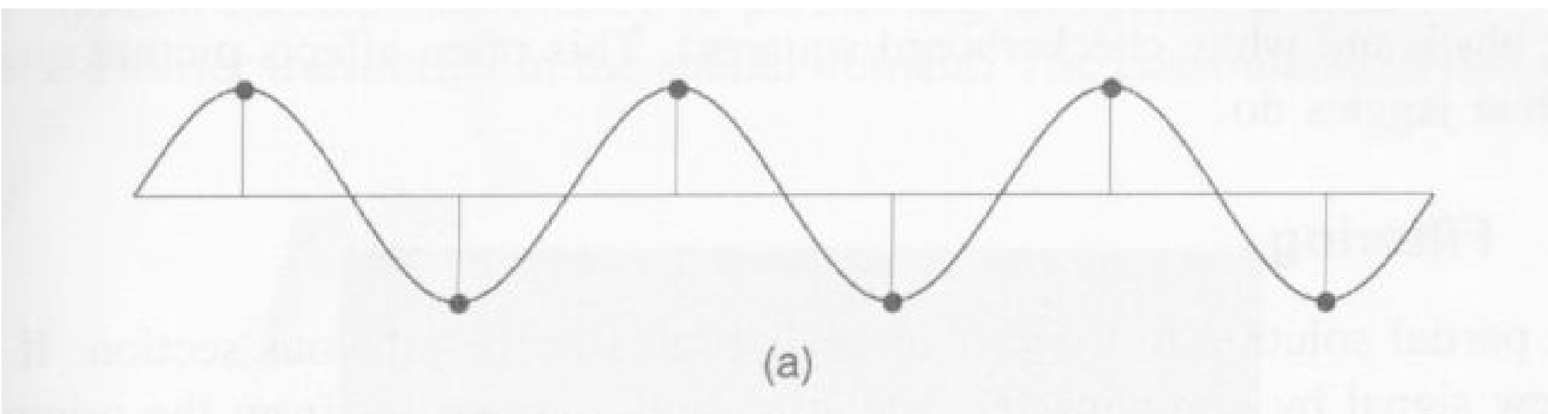
Sampling Theorem

continuous-time signal can be completely recovered from its samples iff the sampling rate is greater than twice the maximum frequency present in the signal.

- Claude Shannon

Nyquist Rate

- the lower bound on the sampling rate equals twice the highest frequency component in the image's spectrum
- this lower bound is the Nyquist Rate



Falling Below Nyquist Rate

- when sampling below Nyquist Rate, resulting signal looks like a lower-frequency one
  - this is aliasing!

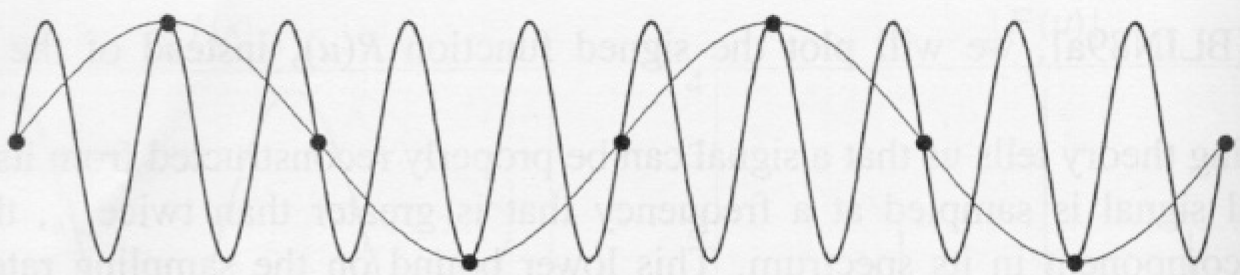


Fig. 14.17 Sampling below the Nyquist rate. (Courtesy of George Wolberg, Columbia University.)

Flaws with Nyquist Rate

- samples may not align with peaks

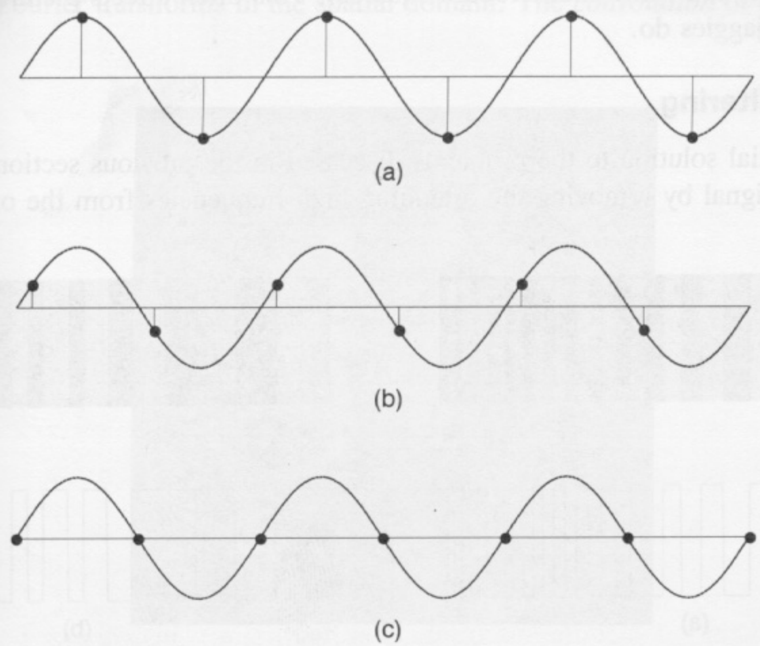
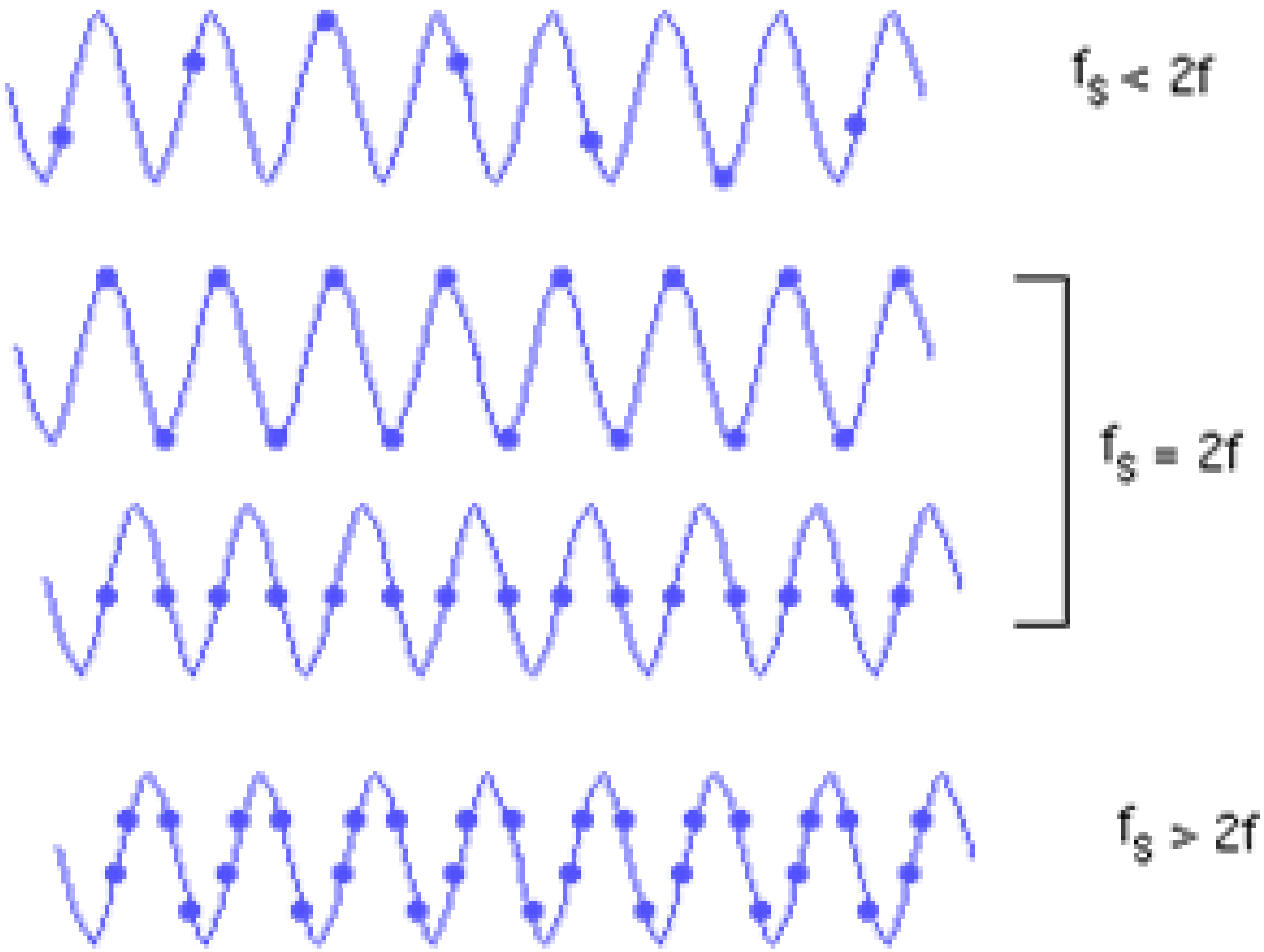


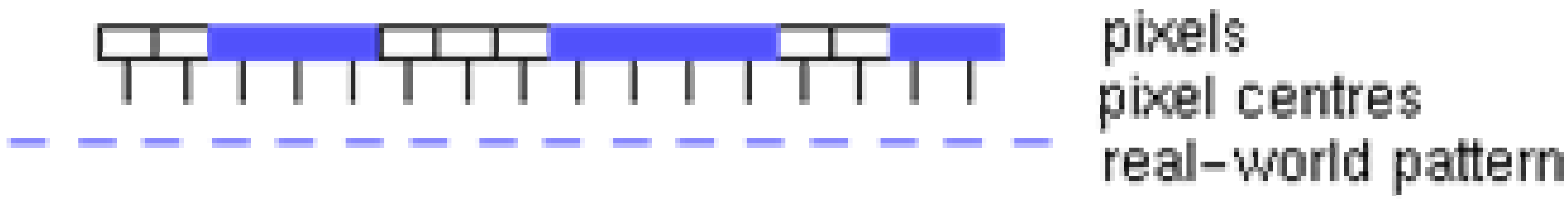
Fig. 14.16 Sampling at the Nyquist rate (a) at peaks, (b) between peaks, (c) at zero crossings. (Courtesy of George Wolberg, Columbia University.)

Nyquist Rate

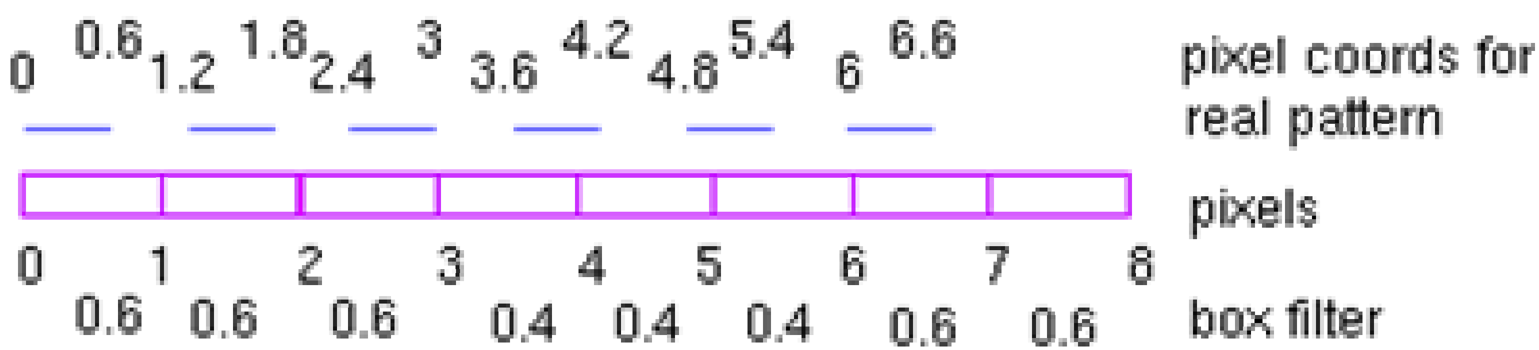


Nyquist and Checkerboards

- point sampled 1D checkerboard: aliases



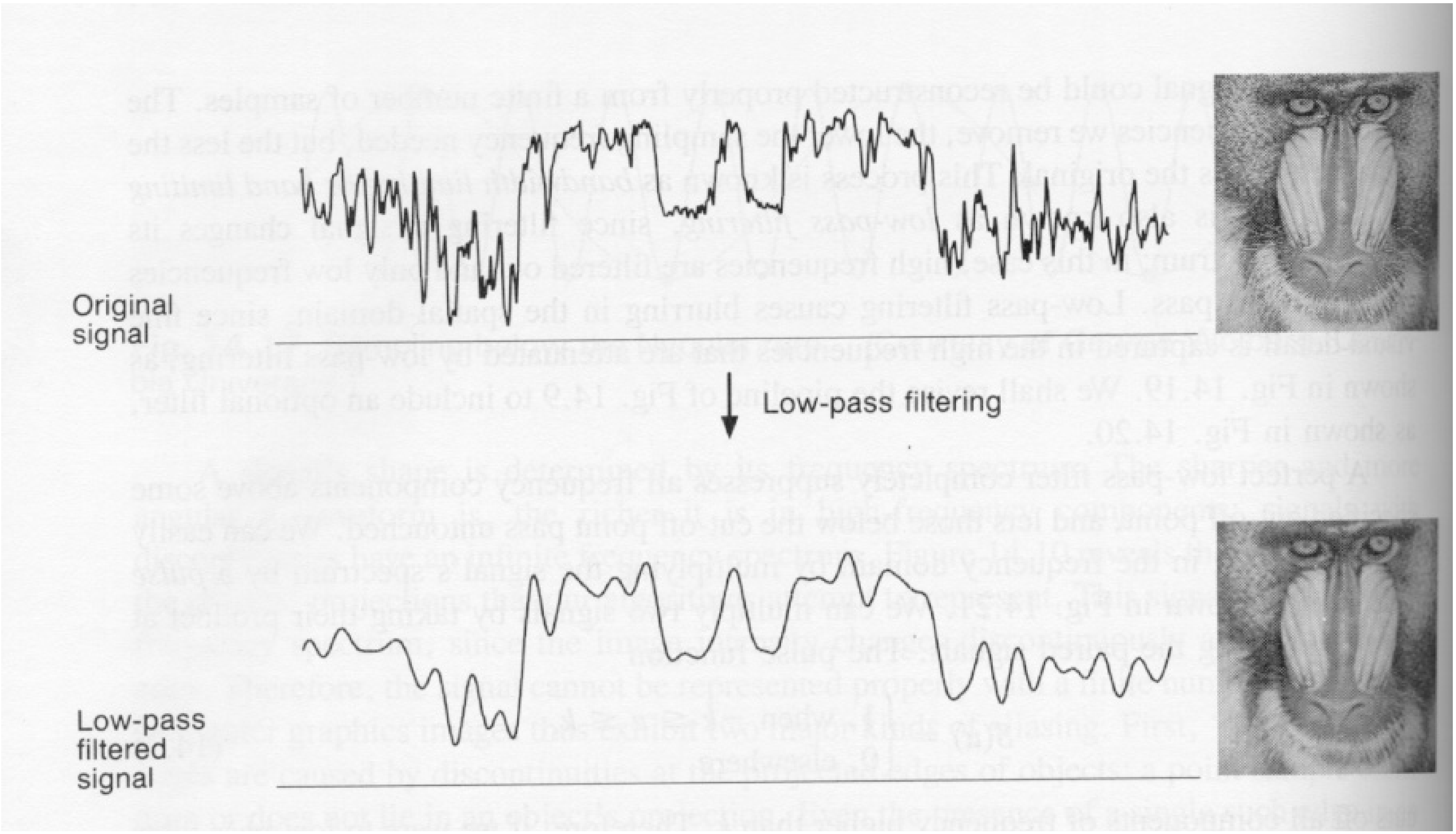
- unweighted area sample: still have aliasing



Band-limited Signals

- if you know a function contains no components of frequencies higher than *x*
  - band-limited implies original function will not require any ideal functions with frequencies greater than x
  - facilitates reconstruction
  - avoids Nyquist Limit mistakes
- to lower Nyquist rate, remove high frequencies from image: *low-pass filter*
  - only low frequencies remain: band-limited

Low-Pass Filtering



Low-Pass Filtering

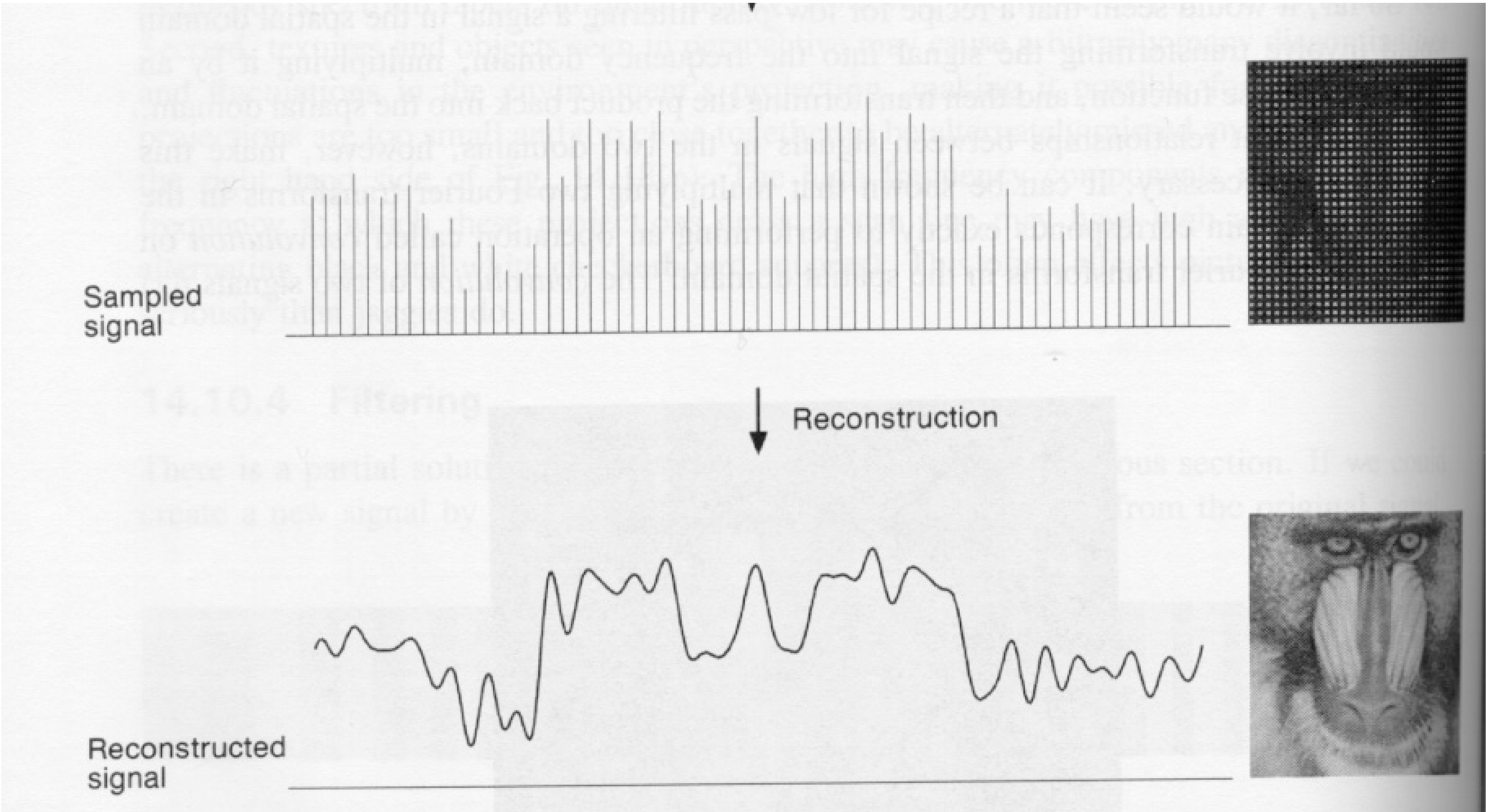
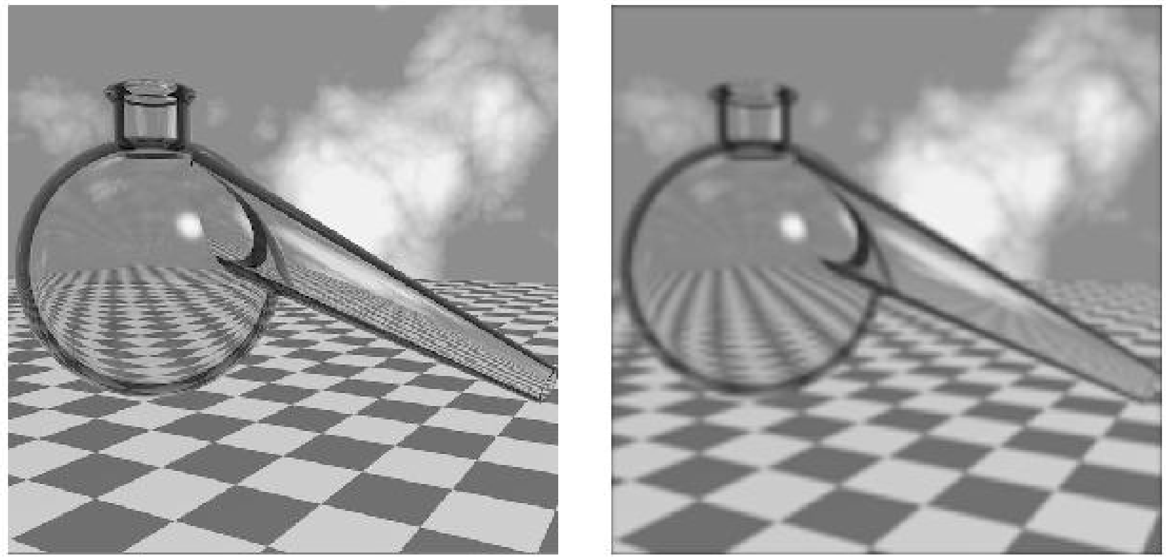


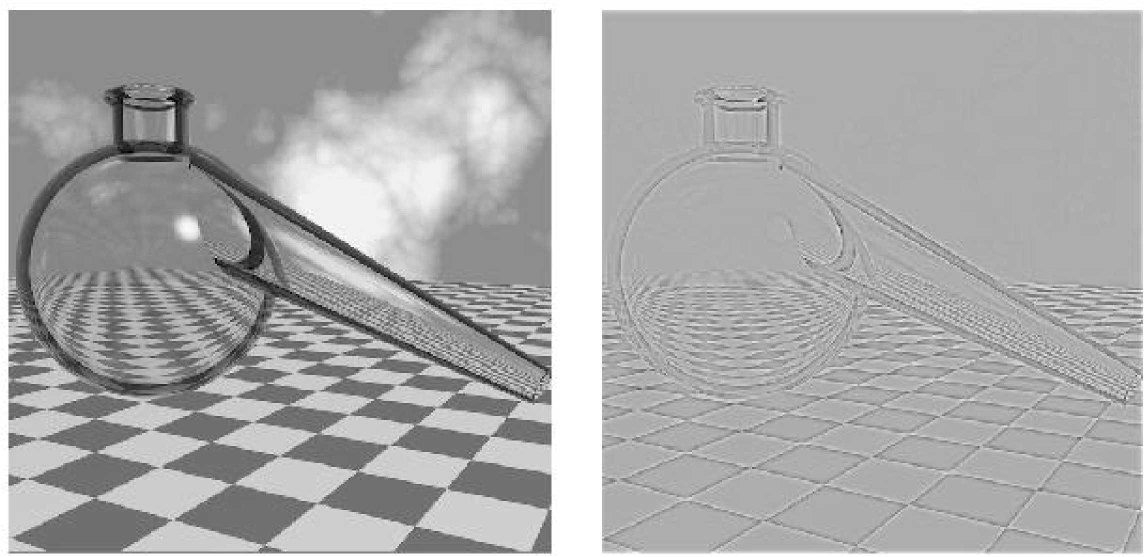
Fig. 14.20 The sampling pipeline with filtering. (Courtesy of George Wolberg, Columbia University.)

Filtering

- low pass
  - blur

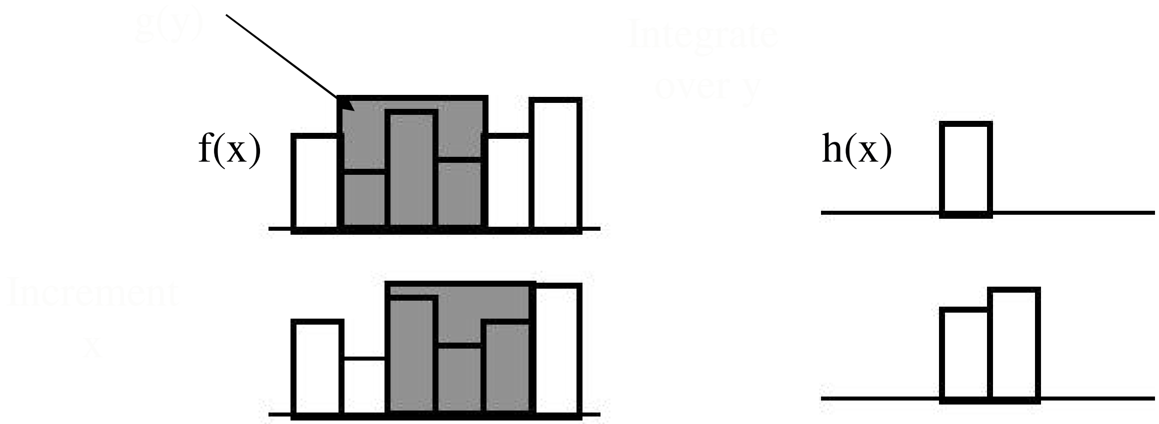


- high pass
  - edge finding



Filtering in Spatial Domain

- blurring or averaging pixels together



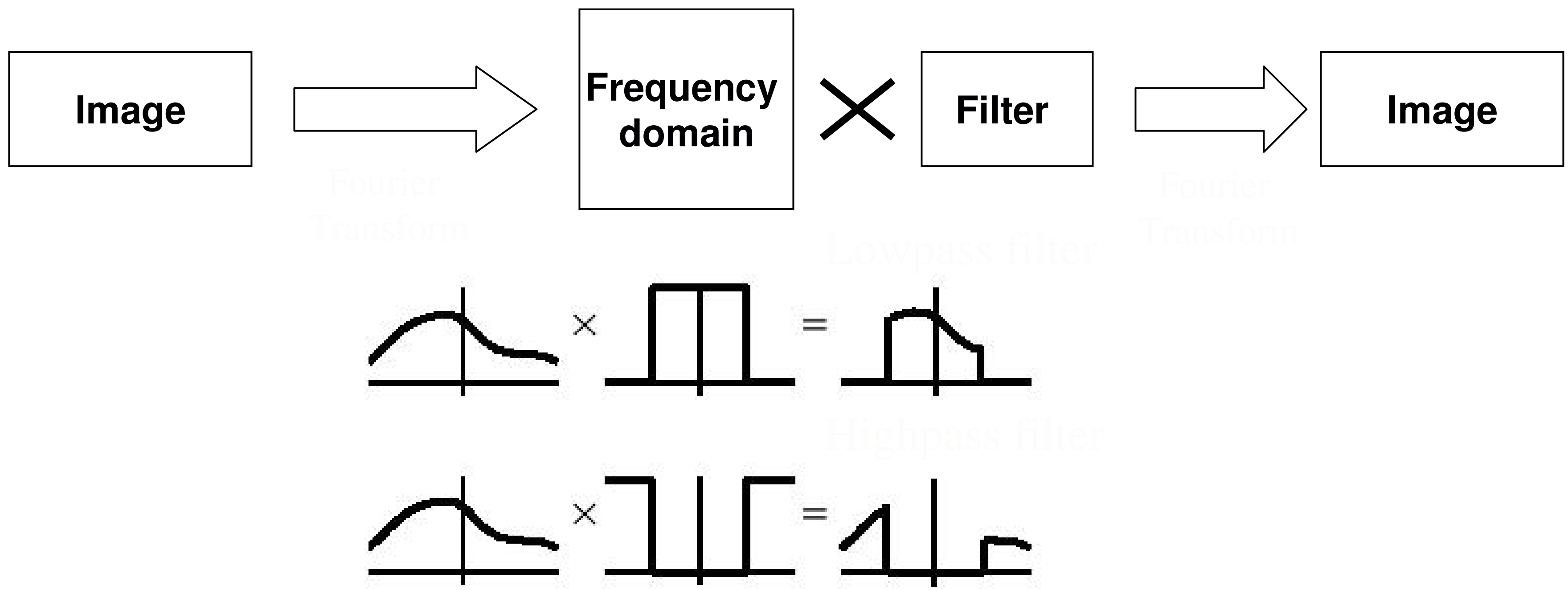
Calculate integral of one function, f(x) by a sliding second function g(x-y).

Known as *Convolution*.

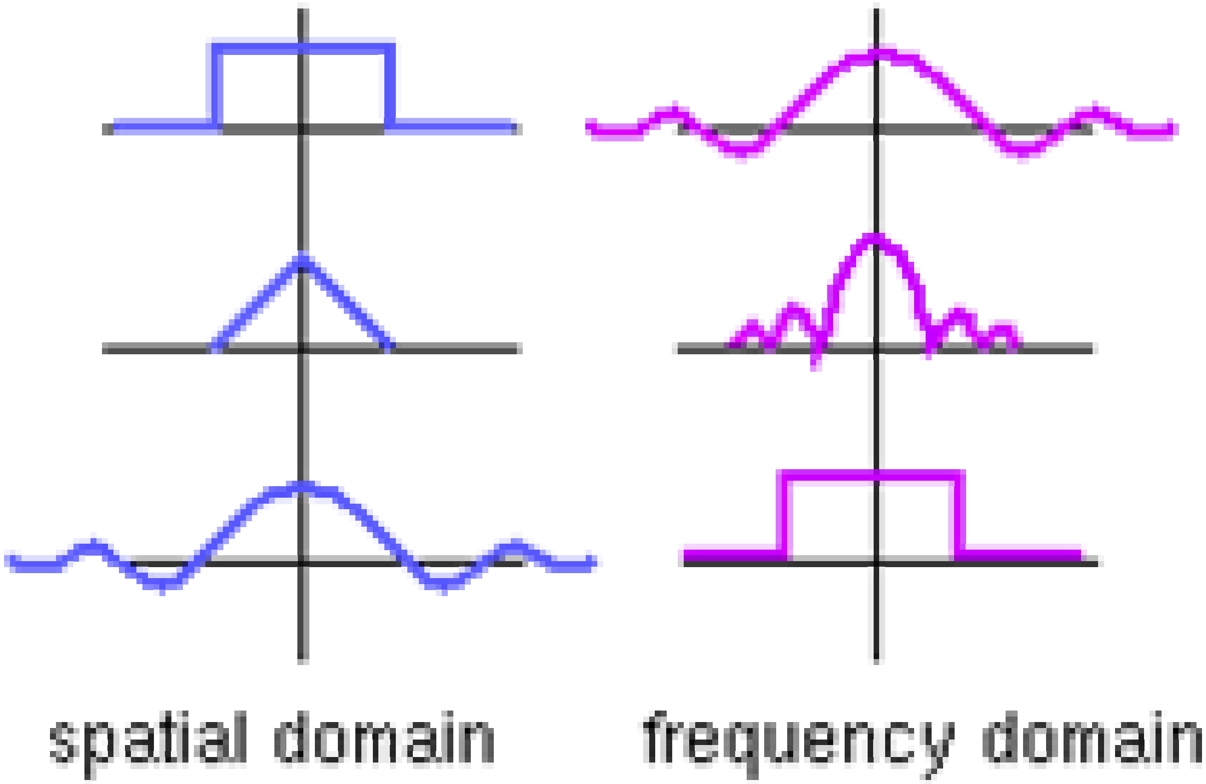
$$h(x) = f \otimes g = \int f(x) g(x-y) dy$$

Filtering in Frequency Domain

- multiply signal's spectrum by pulse function

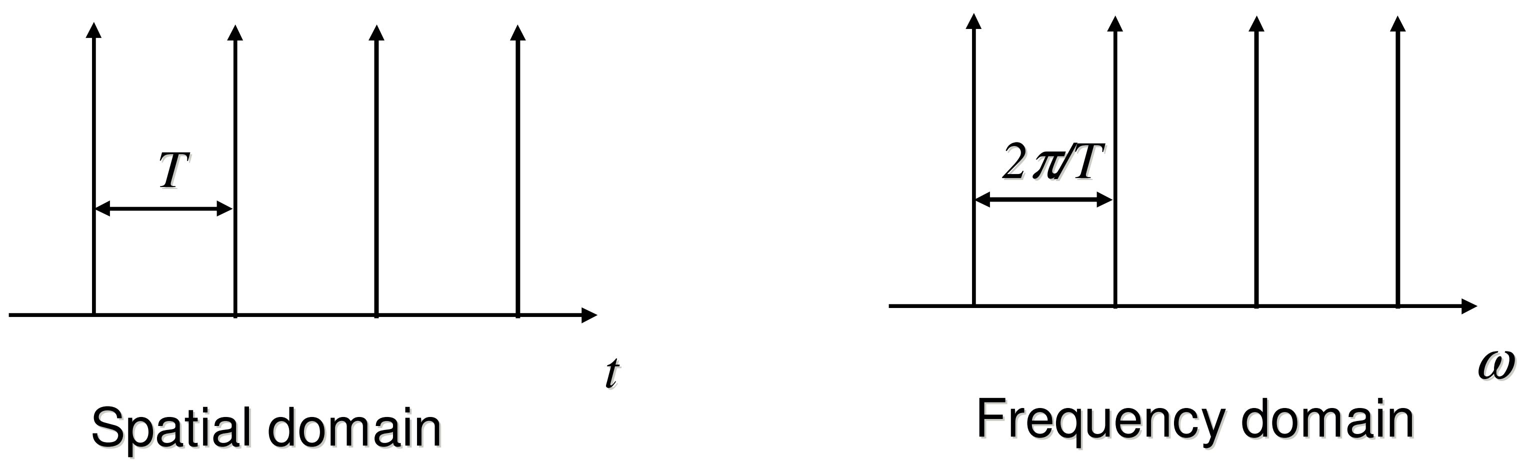


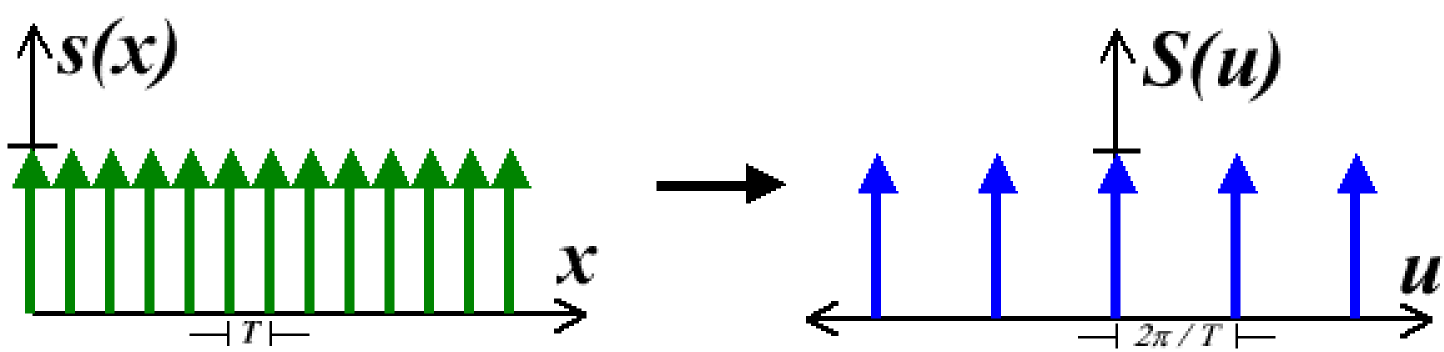
Common Filters



Dualities

– inverse relationship between size
  - T large -> 2π/T small

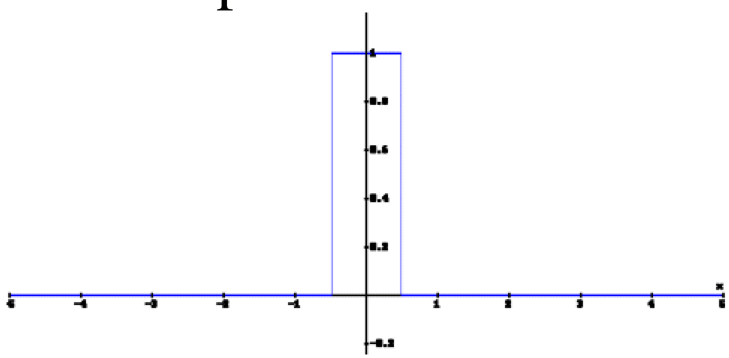




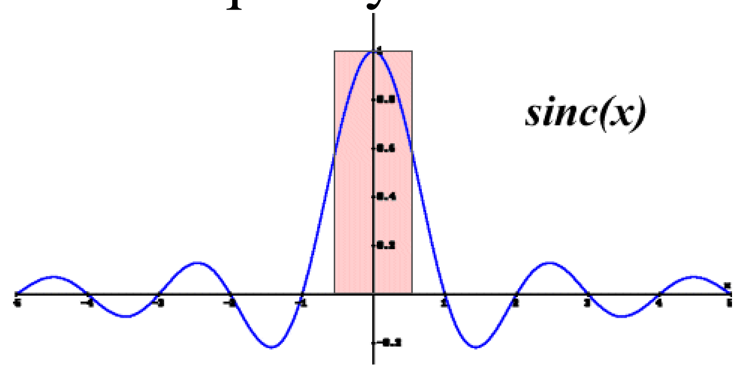
Sinc Function

- sinc (pulse) function is common filter:
  - sinc(x) = sin (πx)/πx
  - infinite in frequency domain

Spatial Domain

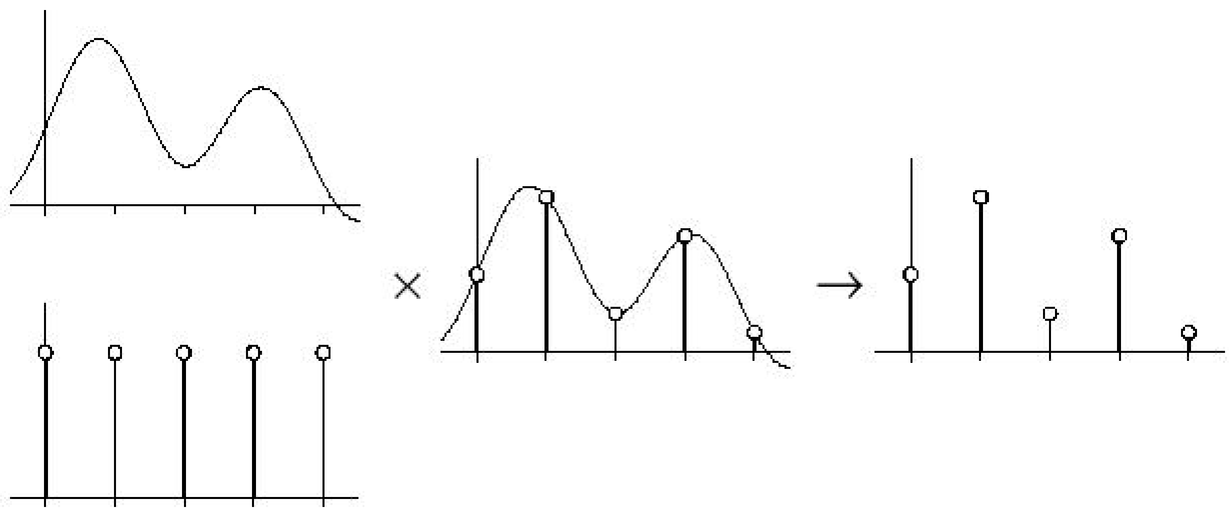


Frequency Domain



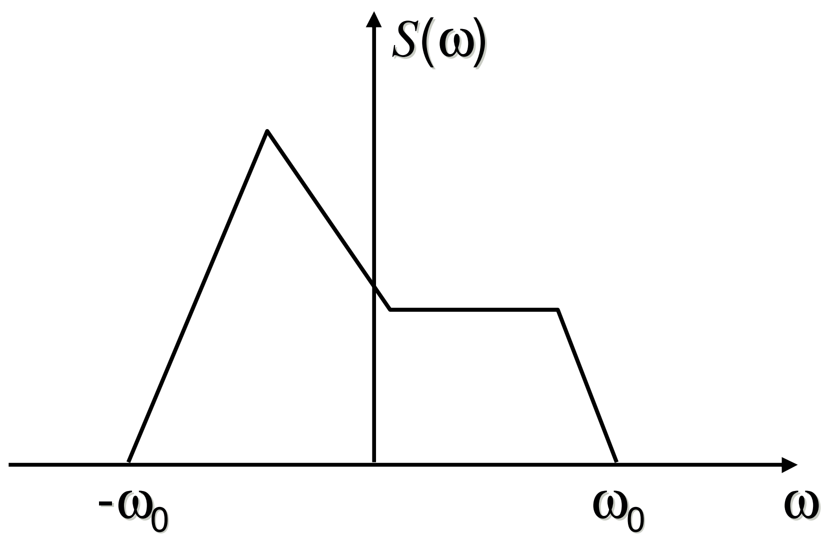
Sampling in Spatial Domain

- Q: what is sampling (i.e. evaluating a continuous function at evenly spaced points)?
- A: multiplication of the sample with a regular train of delta functions (spikes).



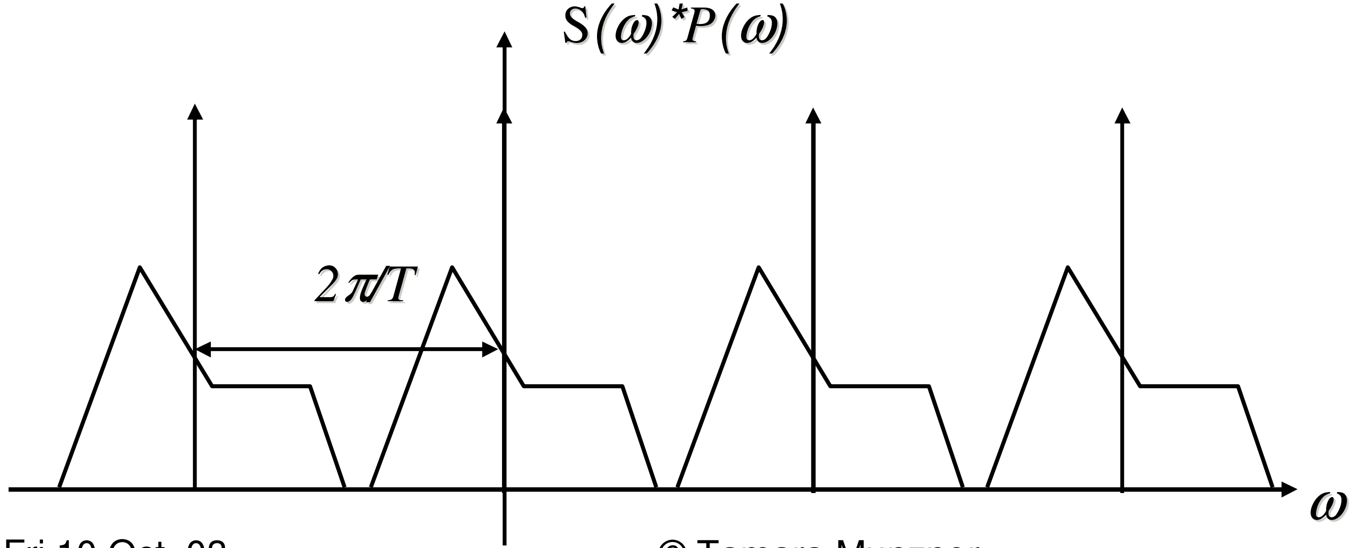
Sampling in Frequency Domain

- multiple copies of spectrum
- example: given spectrum $S(\omega)$ of a signal $s(t)$



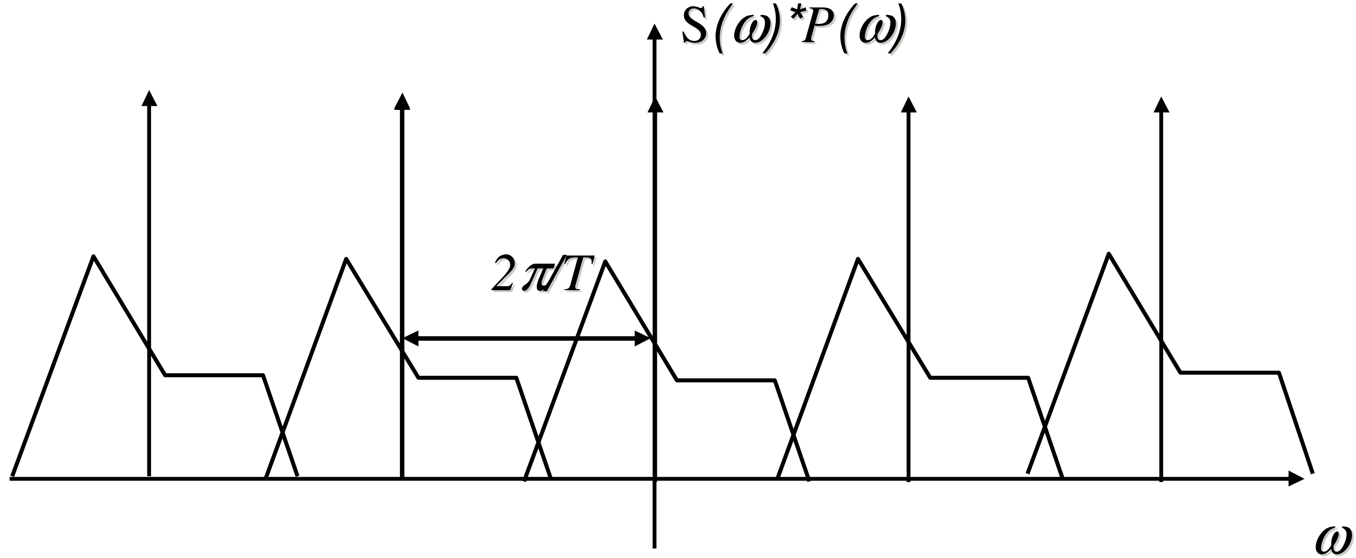
Sampling in Frequency Domain

- multiple shifted copies of $S(\omega)$ are added up during sampling
- if $2\pi/T$ is **large** enough (T is **small** enough)
  - individual spectrum copies do not overlap
  - *depends on maximum frequency* ω_0 in s(t)

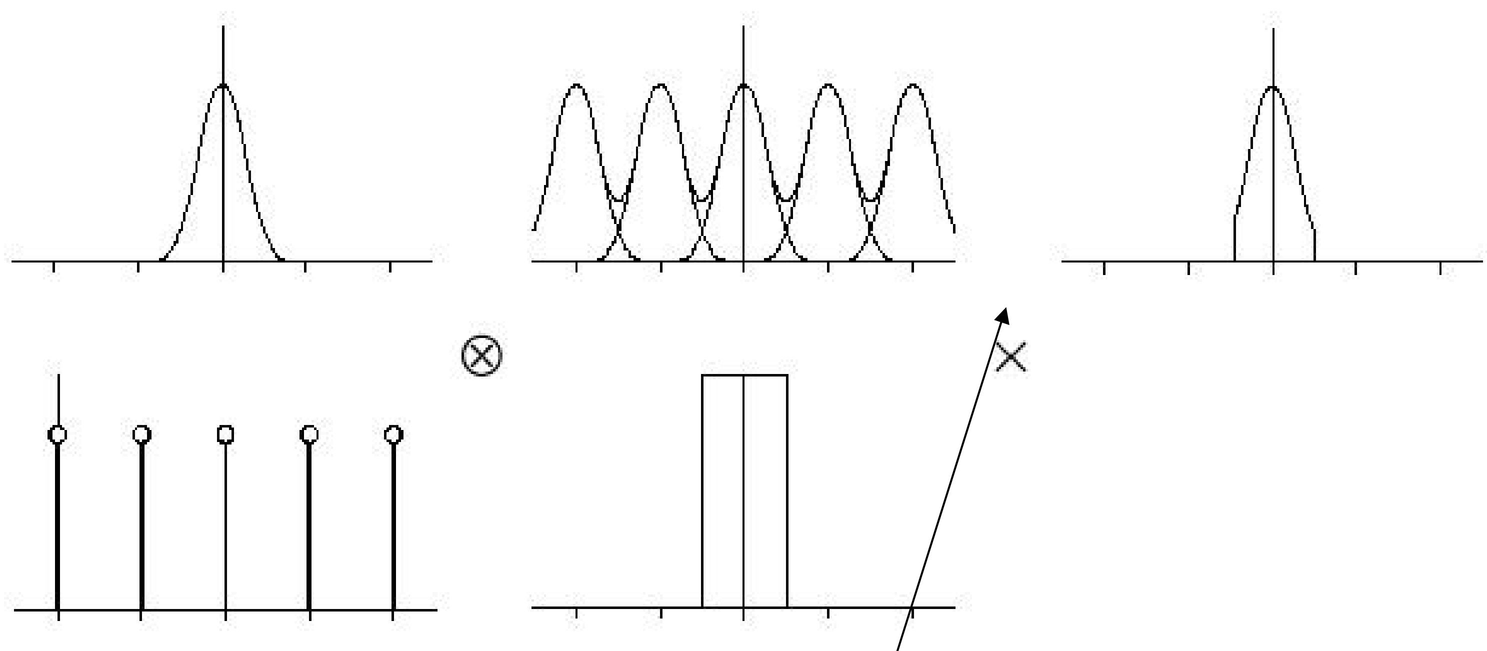


Sampling in Frequency Domain

- if T is too **large** ($2\pi/T$ is **small**), overlap occurs
  - this is **aliasing**



Undersampling leads to aliasing.

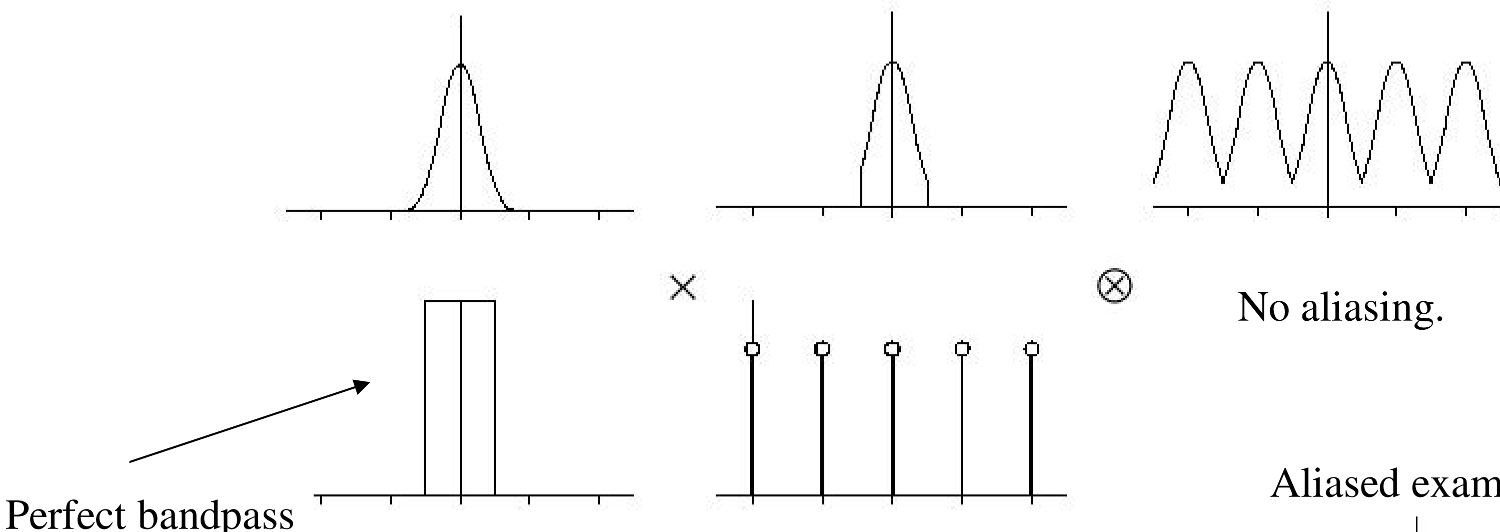


Samples are too close together in f.

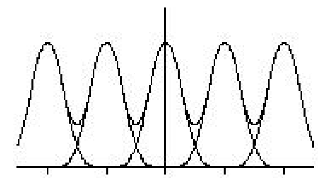
Spurious components : Cause of aliasing.

How do we remove aliasing ?

- perfect solution - prefilter with perfect bandpass filter.



Aliased example



How do we remove aliasing ?

- perfect solution - prefilter with perfect bandpass filter.
  - difficult/Impossible to do in frequency domain
- convolve with sinc function in space domain
  - optimal filter - better than area sampling.
  - sinc function is infinite !!
  - computationally expensive

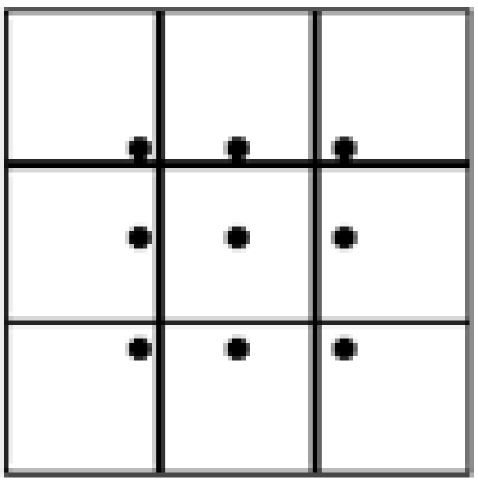
How do we remove aliasing ?

- cheaper solution : take multiple samples for each pixel and average them together → supersampling.
- can weight them towards the centre → weighted average sampling
- stochastic sampling
- importance sampling

Removing aliasing is called *antialiasing*

Weighted Sampling

- multiple samples per pixel



3x3 Bartlett

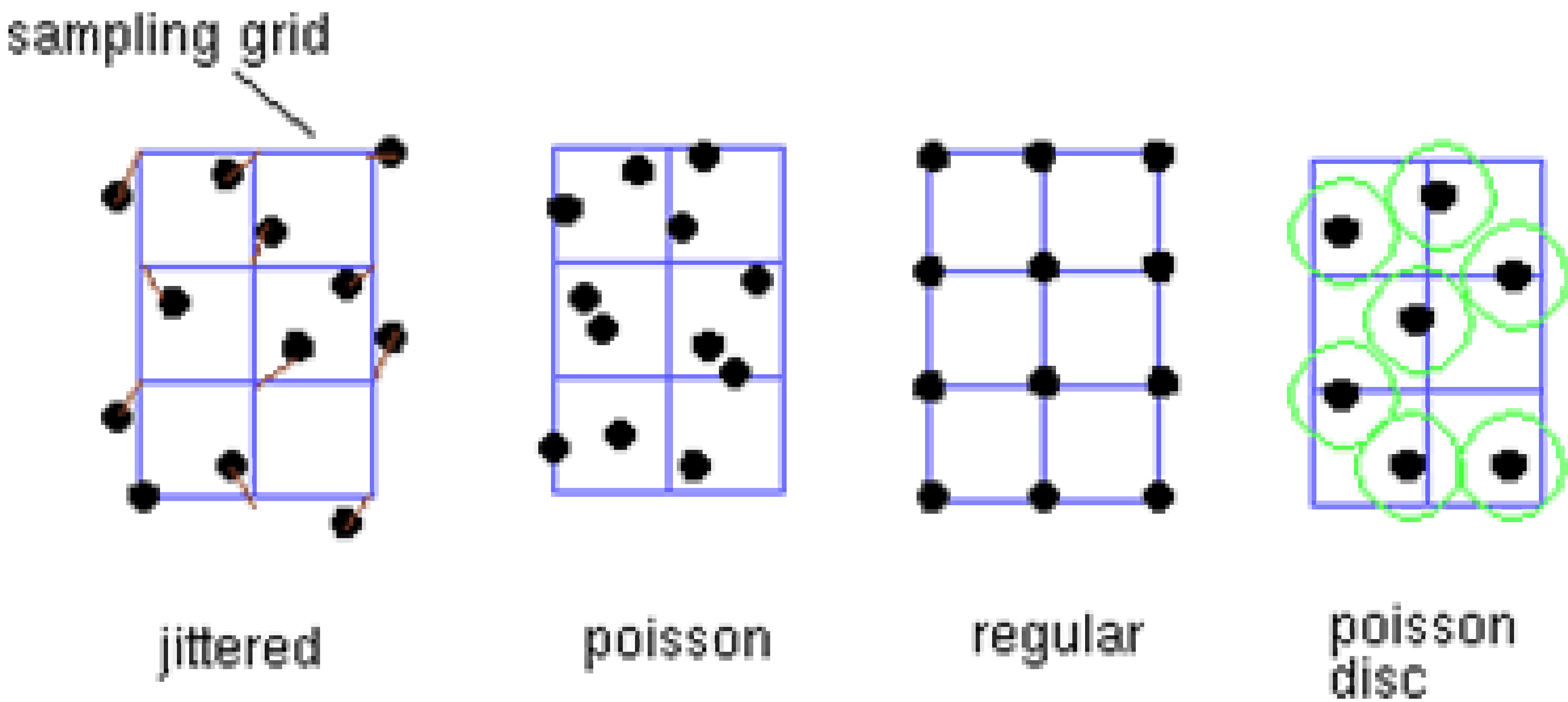
1	2	1
2	4	2
1	2	1

5x5 Bartlett

1	2	3	2	1
2	4	6	4	2
3	6	9	6	3
2	4	6	4	2
1	2	3	2	1

Stochastic Supersampling

- high frequency noise preferable to aliases



Importance Sampling