University of British Columbia
CPSC 414 Computer Graphics
Viewing and Projections
Mon 22 Sep 2003

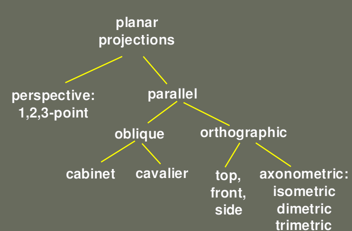
- project 1 solution demo
- recap: projections 1
- projections 2

News

- Project 1 solution executable available
  - idea of what's expected
  - no need to copy look and feel exactly

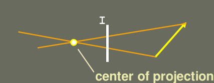
Projection Taxonomy recap

- from 3D to 2D



Projection recap

perspective



parallel : center of projection at ∞

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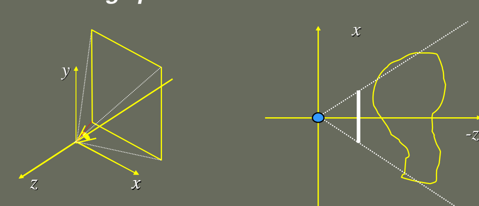
Projections 2

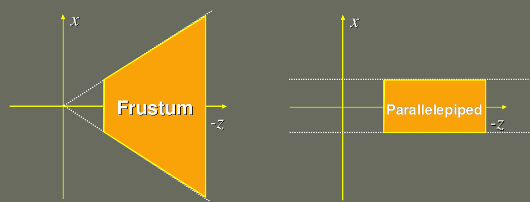
Perspective Projection

- project all geometry
  through a common ***center of projection*** (***eye point***)
  onto an ***image plane***



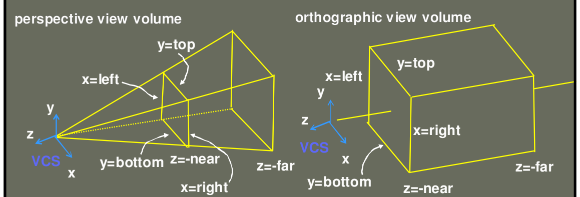
Projective Transformations

- transformation of space
  - center of projection moves to infinity
  - view volume transformed
    - from frustum (truncated pyramid) to parallelepiped (box)



View Volumes

- specifies field-of-view, used for clipping
- restricts domain of ***z*** stored for visibility test

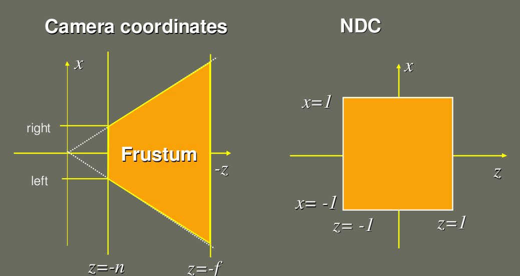


View Volume

- convention
  - viewing frustum mapped to specific parallelepiped
    - Normalized Device Coordinates (NDC)
    - same as clipping coords
  - only objects inside the parallelepiped get rendered
  - which parallelepiped?
    - depends on rendering system

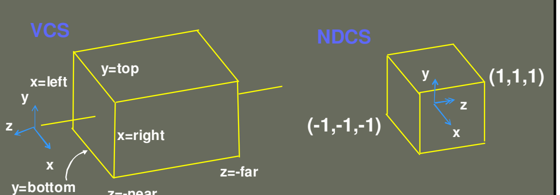
Normalized Device Coordinates

left/right x =+/- 1, top/bottom y =+/- 1, near/far z =+/- 1



Understanding Z

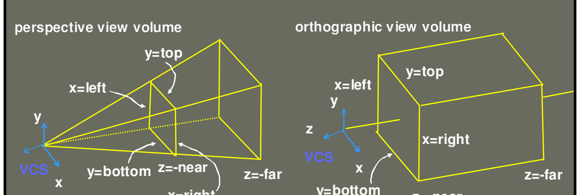
- z axis flip changes coord system handedness
  - RHS before projection (eye/view coords)
  - LHS after projection (clip, norm device coords)



Understanding Z

- near, far always positive in OpenGL calls

glOrtho(left,right,bot,top,near,far);
glFrustum(left,right,bot,top,near,far);
glPerspective(fovy,aspect,near,far);

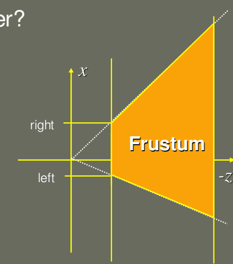


Understanding Z

- why near and far plane?
  - near plane:
    - avoid singularity (division by zero, or very small numbers)
  - far plane:
    - store depth in fixed-point representation (integer), thus have to have fixed range of values (0…1)
    - avoid/reduce numerical precision artifacts for distant objects

Asymmetric Frusta

- our formulation allows asymmetry
  - why bother?



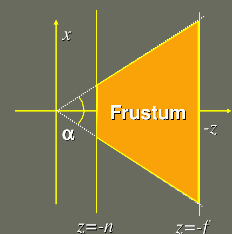
Simpler Formulation

- left, right, bottom, top, near, far
  - nonintuitive
  - often overkill
- look through window center
  - symmetric frustum
- constraints

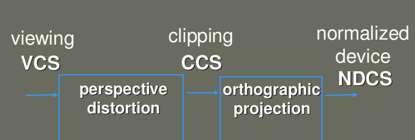
  left = -right, bottom = -top

Field-of-View Formulation

- FOV in one direction + aspect ratio (w/h)
  - determines FOV in other direction
  - also set near, far (reasonably intuitive)

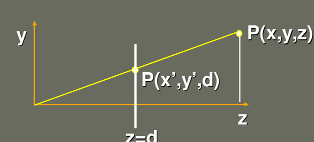


Projection Normalization



- distort such that orthographic projection of distorted objects is desired persp projection
  - convenient coord sys: clipping, hidden surfaces

Basic Perspective Projection



similar triangles

$\frac{y'}{d} = \frac{y}{z} \rightarrow y' = \frac{y \cdot d}{z}$ also $x' = \frac{x \cdot d}{z}$ but $z' = d$

- nonuniform foreshortening
  - not affine

Basic Perspective Projection

- can express as homogenous 4x4 matrix!

$$\begin{bmatrix} x \\ y \\ z \\ z/d \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \\ z/d \end{bmatrix} \xrightarrow{/\,w} \begin{bmatrix} x \cdot d/z \\ y \cdot d/z \\ d \end{bmatrix}$$

Projective Transformations

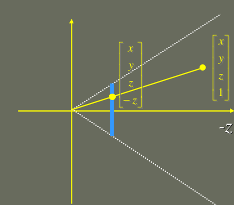
- can express as homogeneous 4x4 matrices!
- 16 matrix entries
  - multiples of same matrix all describe same transformation
  - 15 degrees of freedom
  - mapping of 5 points uniquely determines transformation

Projective Transformations

- determining the matrix representation
  - need to observe 5 points in general position, e.g.
    - $[left,0,0,1]^T \rightarrow [1,0,0,1]^T$
    - $[0,top,0,1]^T \rightarrow [0,1,0,1]^T$
    - $[0,0,-f,1]^T \rightarrow [0,0,1,1]^T$
    - $[0,0,-n,1]^T \rightarrow [0,0,0,1]^T$
    - $[left*f/n,top*f/n,-f,1]^T \rightarrow [1,1,1,1]^T$
  - solve resulting equation system to obtain matrix

Perspective Projection

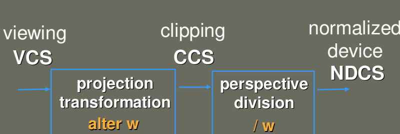
- specific example
  - assume image plane at $z = -1$
  - a point $[x,y,z,1]^T$ projects to $[-x/z,-y/z,-z/z,1]^T \equiv [x,y,z,-z]^T$



Perspective Projection

$$T\left(\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \\ -z \end{bmatrix} \equiv \begin{bmatrix} -x/z \\ -y/z \\ -1 \\ 1 \end{bmatrix}$$

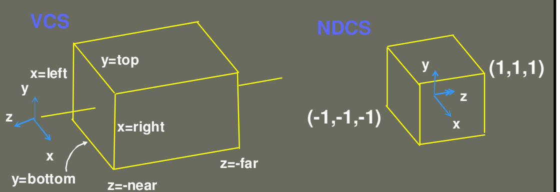
Projection Normalization



- distort such that orthographic projection of distorted objects is desired persp projection
  - separate division from standard matrix multiplies
  - division: normalization

Orthographic Derivation

- scale, translate, reflect for new coord sys



Orthographic Derivation

- scale, translate, reflect for new coord sys

$$P' = \begin{bmatrix} \frac{2}{right-left} & 0 & 0 & -\frac{right+left}{right-left} \\ 0 & \frac{2}{top-bot} & 0 & -\frac{top+bot}{top-bot} \\ 0 & 0 & \frac{-2}{far-near} & -\frac{far+near}{far-near} \\ 0 & 0 & 0 & 1 \end{bmatrix} P$$

Orthographic Derivation

$$y' = a \cdot y + b \qquad y = top \rightarrow y' = 1$$
$$y = bot \rightarrow y' = -1$$

solving for a and b gives:

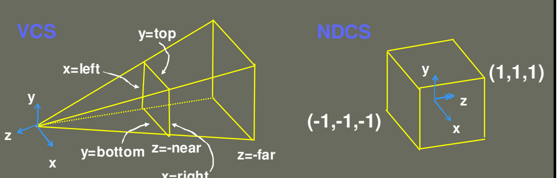
$$a = \frac{2}{top-bot} \qquad b = \frac{-(top+bot)}{top-bot}$$

same idea for right/left, far/near

Orthographic OpenGL

```
glMatrixMode(GL_PROJECTION);
glLoadIdentity();
glOrtho(left,right,bot,top,near,far);
```


Perspective Derivation



Perspective Derivation

earlier:

$$\begin{bmatrix} x \\ y \\ z \\ z/d \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

complete: shear, scale, projection-normalization

$$\begin{bmatrix} x' \\ y' \\ z' \\ h' \end{bmatrix} = \begin{bmatrix} E & 0 & A & 0 \\ 0 & F & B & 0 \\ 0 & 0 & C & D \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Perspective Derivation

$$\begin{bmatrix} x' \\ y' \\ z' \\ w' \end{bmatrix} = \begin{bmatrix} E & 0 & A & 0 \\ 0 & F & B & 0 \\ 0 & 0 & C & D \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$x' = Ex + Az \qquad y' = Fy + Bz \qquad z' = Cz + D \qquad w' = -z$$

$$x = left \;\to\; x'/w' = 1$$
$$x = right \to x'/w' = -1$$
$$y = top \;\to\; y'/w' = 1$$
$$y = bottom \to y'/w' = -1$$
$$z = near \;\to\; z'/w' = 1$$
$$z = far \to z'/w' = -1$$

$$y' = Fy + Bz, \quad \frac{y'}{w'} = \frac{Fy + Bz}{w'}, \quad 1 = \frac{Fy + Bz}{w'}, \quad 1 = \frac{Fy + Bz}{-z},$$

$$1 = F\frac{y}{-z} + B\frac{z}{-z}, \quad 1 = F\frac{y}{-z} - B, \quad 1 = F\frac{top}{-(-near)} - B,$$

$$1 = F\frac{top}{near} - B$$

Perspective Derivation

- similarly for other 5 planes
- 6 planes, 6 unknowns

$$\begin{bmatrix} \frac{2n}{r-l} & 0 & \frac{r+l}{r-l} & 0 \\ 0 & \frac{2n}{t-b} & \frac{t+b}{t-b} & 0 \\ 0 & 0 & \frac{-(f+n)}{f-n} & \frac{-2fn}{f-n} \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

Perspective OpenGL

```
glMatrixMode(GL_PROJECTION);
glLoadIdentity();

glFrustum(left,right,bot,top,near,far);
  or
glPerspective(fovy,aspect,near,far);
```


Perspective Example

view volume

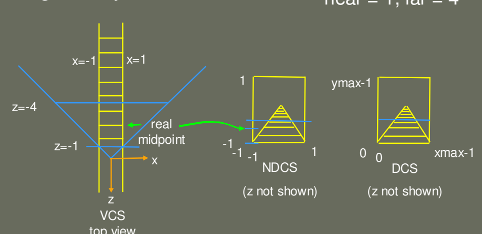
- left = -1, right = 1
- bot = -1, top = 1
- near = 1, far = 4

$$\begin{bmatrix} \frac{2n}{r-l} & 0 & \frac{r+l}{r-l} & 0 \\ 0 & \frac{2n}{t-b} & \frac{t+b}{t-b} & 0 \\ 0 & 0 & \frac{-(f+n)}{f-n} & \frac{-2fn}{f-n} \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -5/3 & -8/3 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

Perspective Example

tracks in VCS:
left x=-1, y=-1
right x=1, y=-1

view volume
left = -1, right = 1
bot = -1, top = 1
near = 1, far = 4



Perspective Example

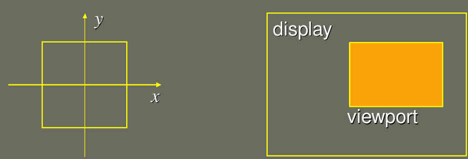
$$\begin{bmatrix} 1 \\ -1 \\ -5z_{VCS}/3 - 8/3 \\ -z_{VCS} \end{bmatrix} = \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & -5/3 & -8/3 \\ & & -1 & \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ z_{VCS} \\ 1 \end{bmatrix}$$

/ w

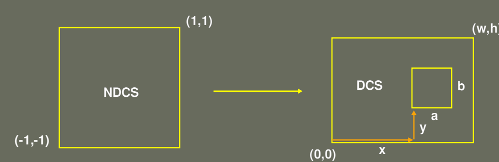
$$x_{NDCS} = -1/z_{VCS}$$
$$y_{NDCS} = 1/z_{VCS}$$
$$z_{NDCS} = \frac{5}{3} + \frac{8}{3z_{VCS}}$$

Viewport Transformation

- generate pixel coordinates
  - map x, y from range –1…1 (*normalized device coordinates*) to pixel coordinates on the display
  - involves 2D scaling and translation



Viewport Transformation



$$x_{DCS} = w\frac{(x_{NDCS}+1)}{2}$$

$$y_{DCS} = h\frac{(y_{NDCS}+1)}{2}$$

$$z_{DCS} = \frac{(z_{NDCS}+1)}{2}$$

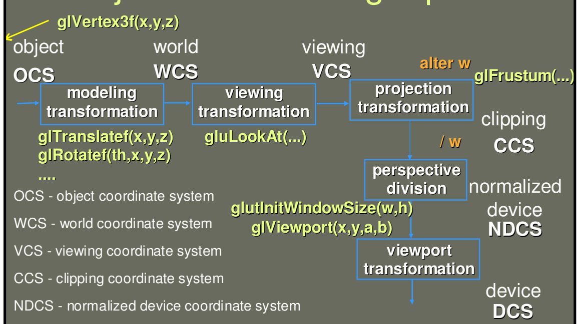
OpenGL

```
glViewport(x,y,a,b);
```

default:

```
glViewport(0,0,w,h);
```


Projective Rendering Pipeline



OCS - object coordinate system

WCS - world coordinate system

VCS - viewing coordinate system

CCS - clipping coordinate system

NDCS - normalized device coordinate system

DCS - device coordinate system