An Introduction to Computer Animation

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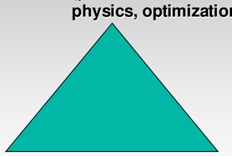
Overview

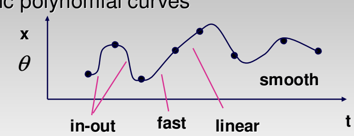
(1) Creating Animations
(2) Representing Rotations

(1) Creating Animations

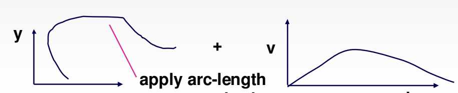


Representing motion

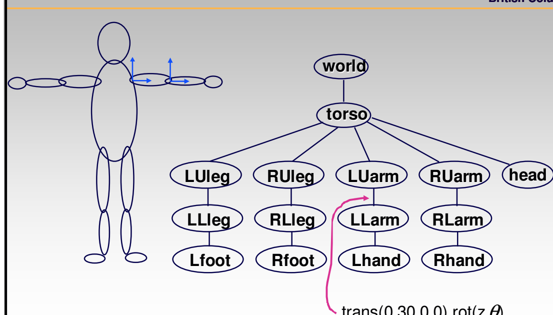
- DOF vs time
- cubic polynomial curves



- alternative for motion through space:

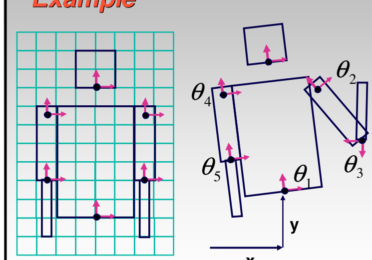


(2) Representing Rotations



Transformation Hierarchies

Example




```
glTranslate3f(x,y,0);
glRotatef(θ1,0,0,1);
DrawBody();
glPushMatrix();
  glTranslate3f(0,7,0);
  DrawHead();
glPopMatrix();
glPushMatrix();
  glTranslate(2.5,5.5,0);
  glRotatef(θ2,0,0,1);
  DrawUArm();
  glTranslate(0,-3.5,0);
  glRotatef(θ3,0,0,1);
  DrawLArm();
glPopMatrix();
... (draw other arm)
```

Rotation DOFs

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- 2D: 1 DOF
- 3D: 3 DOF
- 4D: 6 DOF

Transformations

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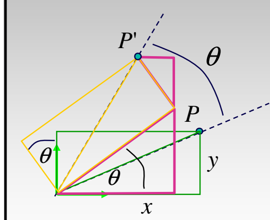
Rotation



$$Rotate(z,\theta)$$

$$x' = x\cos\theta - y\sin\theta$$

$$y' = x\sin\theta + y\cos\theta$$

$$z' = z$$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & & \\ \sin\theta & \cos\theta & & \\ & & 1 & \\ & & & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

glRotatef(angle,x,y,z);
glRotated(angle,x,y,z);

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3x3 Rotation Matrix

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$$\begin{bmatrix} x' \\ y' \\ z' \\ h' \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ m_{41} & m_{42} & m_{43} & m_{44} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ h \end{bmatrix}$$

$$\begin{bmatrix} m_{11} & m_{12} & m_{13} & T_x \\ m_{21} & m_{22} & m_{23} & T_y \\ m_{31} & m_{32} & m_{33} & T_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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3x3 Rotation Matrix

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- 9 elements
- 6 constraints
- renormalization algorithms
- extracting pure rotational component (polar decomp)

$$\begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix}$$

$$R = \begin{bmatrix} \vec{a} & \vec{b} & \vec{c} \end{bmatrix}$$

$$R^{-1} = R^T$$

$$a \bullet b = 0 \qquad |a| = 1$$

$$b \bullet c = 0 \qquad |b| = 1$$

$$a \bullet c = 0 \qquad |c| = 1$$

... and determinant = 1

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Rotations

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SO(3)

- rotations do not commute $A \cdot B \neq B \cdot A$
- require at least 4 parameters for a smooth parameterization
  - *analogy: surface of the earth*
    - 2D surface, 3 params
- combing the hairy ball
  - *camera orientation: view object from any dir*

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Rotations

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- orientation vs rotation?
- how to specify?
- how to interpolate?
- 2D vs 3D

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Fixed Angle Representations

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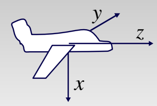
- fixed sequence of 3 rotations
  - *RPY orientation:* z, y, x



roll pitch yaw

$$R_{RPY} = Rot(z,\alpha)\,Rot(y,\beta)\,Rot(x,\gamma)$$

- can use many ordering of axes
- Euler angles: z, x, z

Euler's Rotation Theorem

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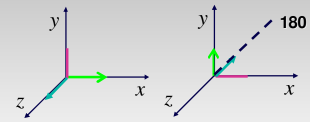
- can always go from one orientation to another with one rotation about a single axis



$$Rot(\vec{k},\theta) = \begin{bmatrix} k_x^2 v + c & k_x k_y v - k_z s & k_x k_z v + k_y s \\ k_x k_y v + k_z s & k_y^2 v + c & k_y k_z v - k_x s \\ k_x k_z v - k_y s & k_y k_z v + k_x s & k_z^2 v + c \end{bmatrix}$$

where

$c = \cos\theta$

$v = 1 - \cos\theta$

$s = \sin\theta$

Quaternions

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- review of complex numbers

$$i^2 = -1$$

$$z = a + bi$$

- quaternions

$$q = w + xi + yj + zk$$

$$[w \quad x \quad y \quad z] = (s, \vec{v})$$

s $\vec{v}$

$$Rot(\vec{k},\theta) = (\cos\frac{\theta}{2}, \sin\frac{\theta}{2}\vec{k})$$

Quaternions

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- rotation of a vector

$$\vec{v}' = Rot(\vec{k},\theta)\,\vec{v} = q \cdot \tilde{v} \cdot \overline{q}$$

$$\tilde{v} = (0, \vec{v}) \qquad \overline{q} = (s, -\vec{v})$$

- two successive rotations

$$q_2(q_1 \cdot \tilde{v} \cdot \overline{q}_1)q_2$$

Quaternions

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$$i^2 = -1 \qquad i \cdot j = -j \cdot i = k$$

$$j^2 = -1 \qquad j \cdot k = -k \cdot j = i$$

$$k^2 = -1 \qquad k \cdot i = -i \cdot k = j$$

RH rule

- unit quaternions

$$w^2 + x^2 + y^2 + z^2 = 1$$

- addition $(s_1, v_1) + (s_2, v_2) = (s_1 + s_2, v_1 + v_2)$
- multiplication

$$(s_1, v_1) \cdot (s_2, v_2) = (s_1 \cdot s_2 - v_1 \bullet v_2, s_1 \cdot v_1 + s_2 \cdot v_2 + v_1 \times v_2)$$