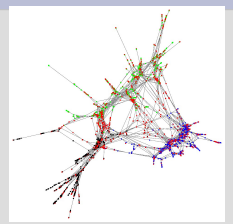
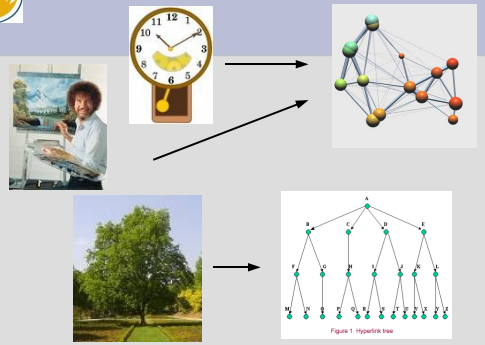
Graph Drawing Using Physical Models



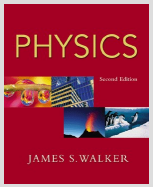
A Cody Robson Presentation

Graph Drawing In General



Graph Drawing In General

Ø →



Graph Drawing Using Physical Models

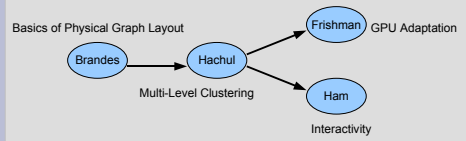
Four Papers

Drawing on Physical Analogies Brandes 2001

Drawing large graphs with a potential-field-based multilevel algorithm Hachul et al 2004

Multi-Level Graph Layout on the GPU Frishman et al 2007

Interactive Visualization of Small World Graphs Ham et al 2005



Drawing on Physical Analogies

General Idea:

Avoid Clutter **REPEL**

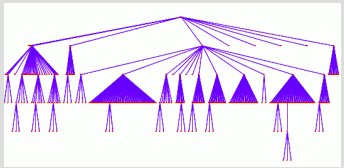
Vertices should spread out

Avoid Distortion **ATTRACT**

Adjacent vertices should be close

Why a Physical Model?

Trees can have distortion!



Maybe it's what we want?

Physical based models try to strike a balance of minimizing both – making no assumptions about the data.

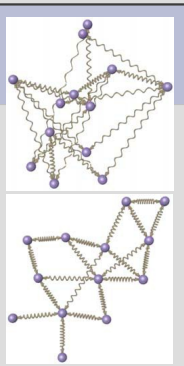
Analogy Time!

REPEL

"Charged Particles"
(Vertices)

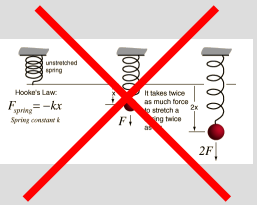
ATTRACT

"Springs"
(Edges)



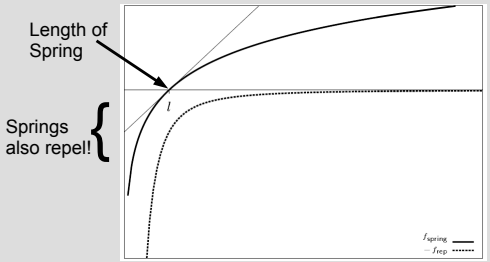
Charged Particles and Springs

Hooke's law tells us springs scale linearly



Charged Particles and Springs

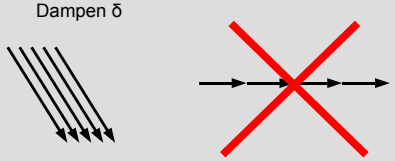
Spring force scales logarithmically!



Charged Particles and Springs

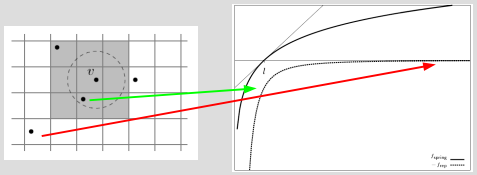
Basic Algorithm

while(δ (FORCE) > 0):
  For each Vertex:
    FORCE = Σ (attraction) + Σ (repulsion)
  For each Vertex:
    Move δ (FORCE)
  Dampen δ



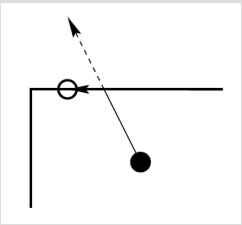
Whats the problem so far?

FORCE = Σ (attraction of neighbors) + Σ (repel of others)
(my edges) (other vertices)



More Improvements!

Vertices shouldn't fly off the monitor.



More Improvements!

Disconnected graphs shouldn't drift off into space.

Gravitational force pulls vertices towards barycenter of all vertices.

Use The Force

f_{rep} → For reducing vertex clutter.

f_{attr} → For keeping neighbors close.

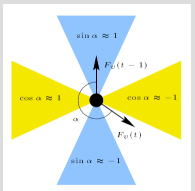
f_{grav} → For preventing disconnected graphs from drifting apart.

$f_{?}(?)$ → For good measure.

DELTA FORCE

Momentum → δ+

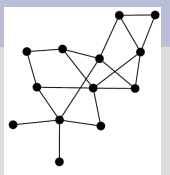
Rotation → δ-



δ-

Oscillation → δ-

Force-Directed Graph Drawing



Vertices ↑

Energy Based Placement

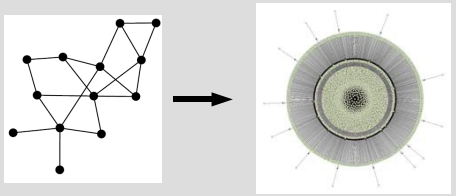
Use similar force model, build an energy equation.

Solve with typical optimization methods.

- Better overall placement with small graphs
- Scales *worse* than force-directed placement

Fast Multipole Multilevel Method (FM^3)

O(BAD) → O(V log V)



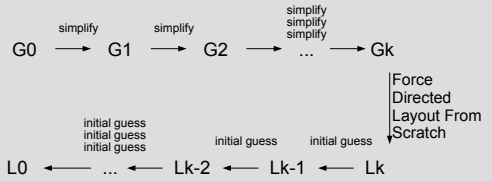
Fast Multipole Multilevel Method (FM^3)

Coarsening: Lets us compute smaller set.

Partitioning: Lets us approximate forces.

Multiple Levels

G : Our Huge Graph
L : Layout of our huge graph



Coarsening

Edge Collapse

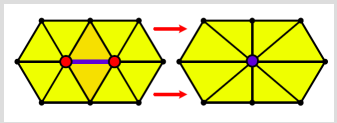
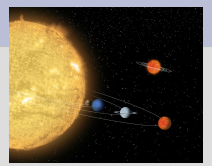


Image Courtesy of Prof Sheffer!

Analogy Time!

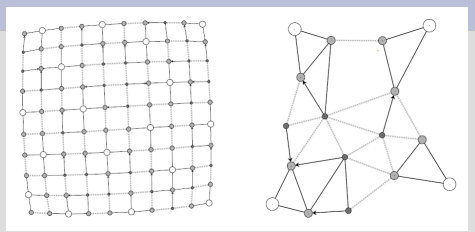
Group vertices into "solar systems"



Sun: Central vertex in a solar system.
Planet: Vertex neighbors of a sun.
Moon: Vertex neighbors of a planet.

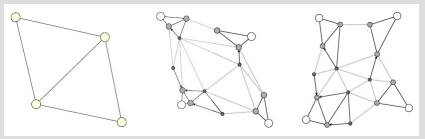
G1 is the graph of solar systems in G0
G2 is the graph of solar systems in G1
And so on...

Multiple Levels (Coarsening)



Gi Gi+1

Multiple Levels (Coarsening)



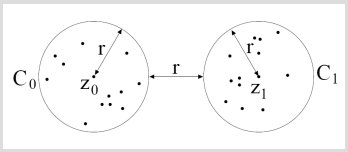
Layout the simple graph with traditional force directed layout.

Expand the solar systems roughly how we collapsed them.

Run force directed layout now that we're close to a solution.

Multipoles (Partitioning)

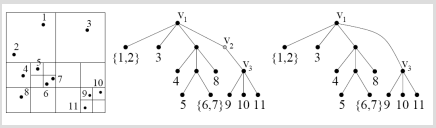
Repulsion force calculation is sped up with partitioning.



Approximate all force of particles in C1 acting on particles in C0

Multipoles (Parititioning)

Partitions created with "Reduced Quadtrees"



Multi-Level Graph Layout on the GPU

FM^3

GPU 101

A modern graphics card gives us two things:

Shader Processors

X 128

Memory *from the future!*

GPU 101

Writing a shader program...

Input Arrays (Texture) → Computation (Rendering) → Output Array (Texture)

Each texture element can hold four elements (Red, Green, Blue, Alpha). We can use multiple textures as input, but only one as output (four output values per computation).

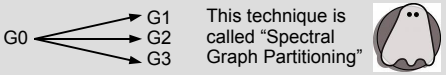
Multi-Level Graph Layout on the GPU

FM^3 constructs simplified levels iteratively.

G0 → G1 → G2 → G3

Errors propagate from stage to stage.

This approach uses the most detailed graph for each simplification.



This technique is called "Spectral Graph Partitioning"

Spectral Graph Partitioning

Idea: Partition the original graph into two equal sized pieces as many times as necessary.

Use a "Normalized Cut" to find the weakest link that can cut the graph into two equal sized pieces.

Same Normalized Cut from Image Processing!

Spectral Graph Partitioning

Problem: Normalized Cut partitioning takes way too long.

Use standard edge collapsing to coarsen the graph. Do normalize cut on coarse graph and use as initial guess for each finer iteration of the graph.

Seeing a pattern?

Big Graph → Coarsen (Edge Collapse) → Coarsen → Layout (Partitioning)

Coarsening and Partitioning

FM^3

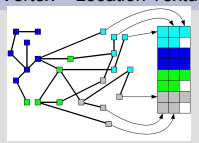
For Each LOD:
  Partition:
    Calculate forces, layout

GPU

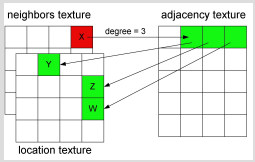
For Each LOD:
  Calculate Multi Level Partitions
For Each Partition:
  Calculate forces, layout

Multi-Level Graph Layout on the GPU

Vertex = Location Texture



Edge = Neighbors + Adjacency Textures



Multi-Level Graph Layout on the GPU

Two Partition Textures:

Partition Information Texture:
- Index of the location texture of the start of the partition
- Width and Height of partition block in location texture
- Number of nodes in the last row of the partition

Partition Center of Gravity Texture:
- Holds current (x,y) location of center of gravity of partition.

Multi-Level Graph Layout on the GPU

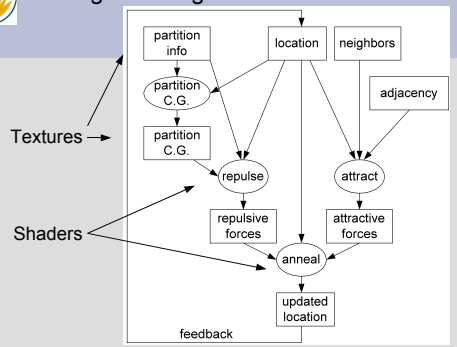
Force Calculation Output:

Attractive Force Texture

Repulsive Force Texture

Both hold 2D force for each vertex in the location texture.

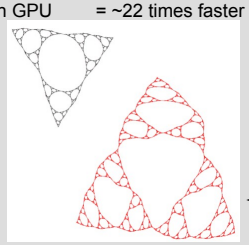
Putting It All Together...



Results

GPU sans GPU = 2-4 times faster than FM^3
GPU with CoreDuo = 3-6 times faster than FM^3
GPU with GPU = ~22 times faster than FM^3

FM^3



GPU

Interactive Visualization of Small World Graphs

"Small World"
- Small Average Edge Length
- High Degree of Clustering

+ Use a better force model for small world graphs.

+ Use clustering for layout as well as visualization!

Small World Forces

Problem: Springs do not discriminate.

Ideally, we'd like springs in our cluster to mean more than springs linking us to other clusters.

Strangely, using *constant force* for springs seems to work pretty well!

$f_{attr} = 1$

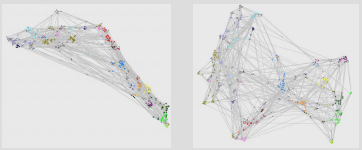
Allows Intra-Cluster edges to shrink while inter-cluster edges grow.

Small World Forces

Too good to be true?

This force model is *highly* susceptible to bad looking local minima.

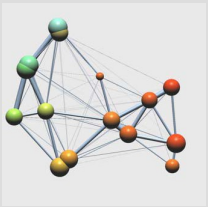
Use another approach first, use results as initial guess!



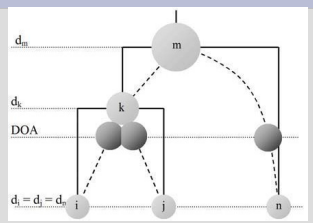
Small World Visualization

Clustering done by same edge-collapse process. Clusters drawn as constant-sizes spheres.

Clustering can be dynamically updated by a Degree of Abstraction variable.

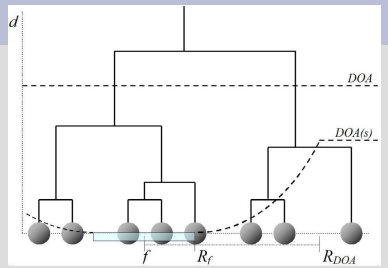


Degree of Abstraction



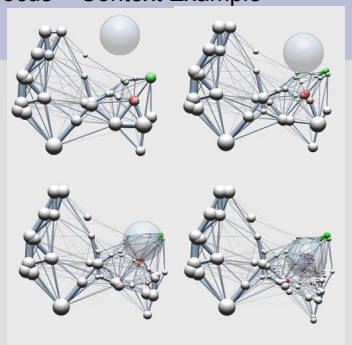
Linear versus Exponential Interpolation

Degree of Abstraction



Focus + Context Returns!

Focus + Context Example



Formal Conclusion

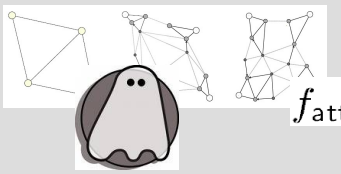
In the absence of high-level information of our data...

Physical-based graph layout models produce good results and scale well with recent methods!

Informal Conclusion

If what you want is too slow...

...use something that works as an initial guess!



$f_{attr} = 1$