University of British Columbia
CPSC 314 Computer Graphics
May-June 2005

Tamara Munzner

Compositing, Clipping, Curves

Week 3, Thu May 26

http://www.ugrad.cs.ubc.ca/~cs314/Vmay2005

News

- extra lab coverage: Mon 12-2, Wed 2-4

- P2 demo slot signup sheet
- handing back H1 today
- we'll try to get H2 back tomorrow
  - we will put them in bin in lab, next to extra handouts
  - solutions will be posted

- you don't have to tell us you're using grace days
  - only if you're turning it in late and you do *not* want to use up grace days
  - grace days are integer quantities

Homework 1 Common Mistakes

- Q4, Q5: too vague
  - don't just say "rotate 90", say around which axis, and in which direction (CCW vs CW)
  - be clear on whether actions are in old coordinate frame or new coordinate frame
- Q8: confusion on push/pop and complex operations
  - wrong: object drawn in wrong spot!

```
glPushMatrix();
glTranslate(..a..);
glRotate(..);
  draw things
glPop();
```

  - correct: object drawn in right spot
  - both: nice modular function that doesn't change modelview matrix

```
glPushMatrix();
glTranslate(..a..);
glRotate(..);
glTranslate(..-a..);
  draw things
glPop();
```

Schedule Change

- HW 3 out Thu 6/2, due Wed 6/8 4pm

Poll

- which do you prefer?
  - P4 due Fri, final Sat
  - final Thu in-class, P4 due Sat

Midterm Logistics

- Tuesday 12-12:50
  - sit spread out: every other row, at least three seats between you and next person
  - you can have one 8.5x11" handwritten one-sided sheet of paper
    - keep it, can write on other side too for final
  - calculators ok

Midterm Topics

- H1, P1, H2, P2
- first three lectures
- topics
  - Intro, Math Review, OpenGL
  - Transformations I/II/III
  - Viewing, Projections I/II

Reading: Today

- FCG Chapter 11
  - pp 209-214 only: clipping
- FCG Chap 13
- RB Chap Blending, Antialiasing, ...
  - only Section Blending

Reading: Next Time

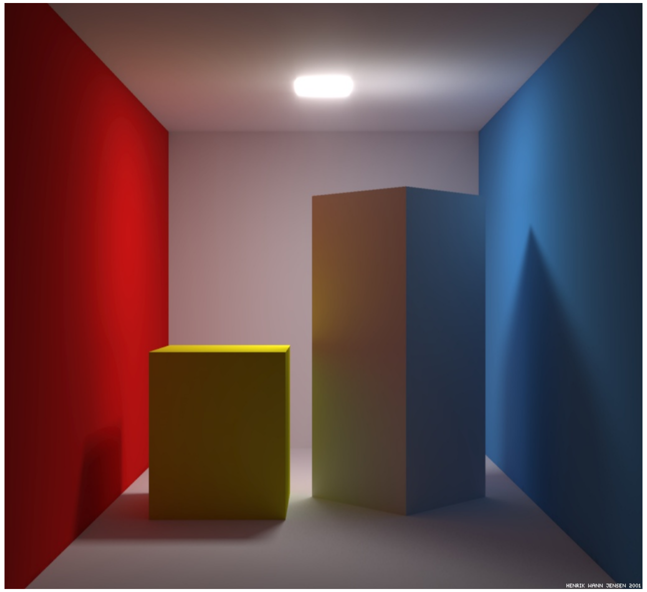
- FCG Chapter 7

Errata

- p 214
  - f(p) > 0 is “outside” the plane
- p 234
  - For quadratic Bezier curves, N=3
  - w_i^N(t) = (N-1)! / (i! (N-i-1)!)...

Review: Illumination

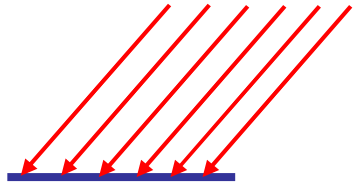
- transport of energy from light sources to surfaces & points
  - includes *direct* and *indirect illumination*



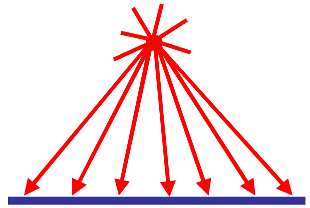
Images by Henrik Wann Jensen

Review: Light Sources

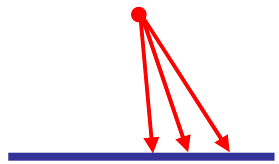
- directional/parallel lights
  - point at infinity: $(x,y,z,0)^T$



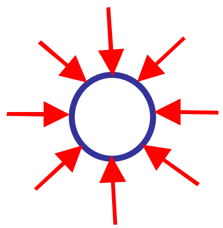
- point lights
  - finite position: $(x,y,z,1)^T$



- spotlights
  - position, direction, angle



- ambient lights

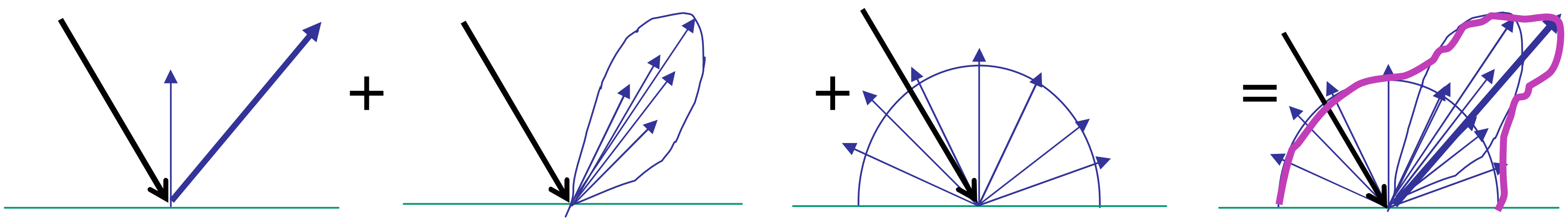


Review: Light Source Placement

- geometry: positions and directions
  - standard: world coordinate system
    - effect: lights fixed wrt world geometry
  - alternative: camera coordinate system
    - effect: lights attached to camera (car headlights)

Review: Reflectance

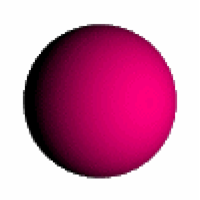
- *specular*: perfect mirror with no scattering
- *gloss*: mixed, partial specularity
- *diffuse*: all directions with equal energy

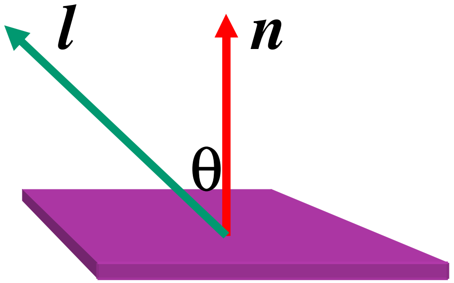


specular + glossy + diffuse =
reflectance distribution

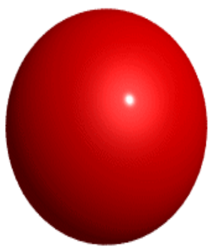
Review: Reflection Equations

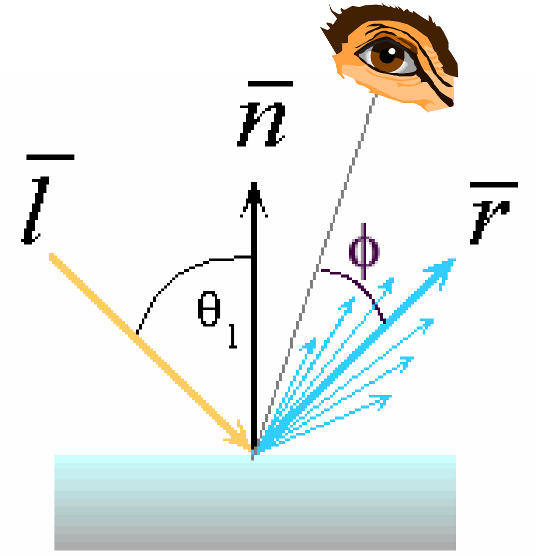
$$I_{diffuse} = k_d \, I_{light} \, (\mathbf{n} \bullet \mathbf{l})$$





$$I_{specular} = k_s I_{light} (\mathbf{v} \bullet \mathbf{r})^{n_{shiny}}$$

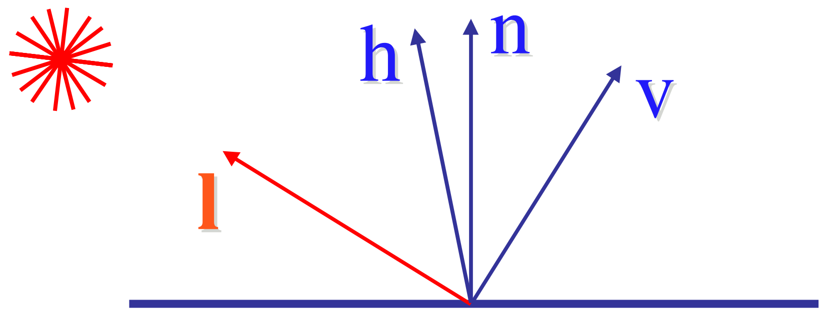




$$2\,(\mathbf{N}\,(\mathbf{N} \cdot \mathbf{L})) - \mathbf{L} = \mathbf{R}$$

Review: Reflection Equations 2

- Blinn improvement



$$\mathbf{I}_{\mathbf{specular}} = \mathbf{k}_{\mathbf{s}} \mathbf{I}_{\mathbf{light}} (\mathbf{h} \bullet \mathbf{n})^{n_{shiny}}$$

$$\mathbf{h} = (\mathbf{l} + \mathbf{v})/2$$

- full Phong lighting model
  - combine ambient, diffuse, specular components

$$\mathbf{I}_{\mathbf{total}} = \mathbf{k}_{\mathbf{s}} \mathbf{I}_{\mathbf{ambient}} + \sum_{i=1}^{\#lights} \mathbf{I}_{\mathbf{i}} (\mathbf{k}_{\mathbf{d}} (\mathbf{n} \bullet \mathbf{l}_{\mathbf{i}}) + \mathbf{k}_{\mathbf{s}} (\mathbf{v} \bullet \mathbf{r}_{\mathbf{i}})^{n_{shiny}})$$

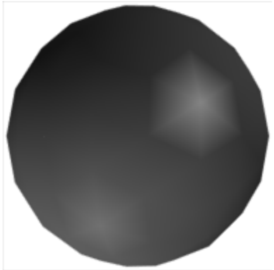
  - don't forget to normalize all vectors: n,l,r,v,h

Review: Lighting

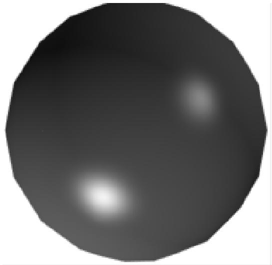
- lighting models
  - ambient
    - normals don't matter
  - Lambert/diffuse
    - angle between surface normal and light
  - Phong/specular
    - surface normal, light, and viewpoint

Review: Shading Models

- flat shading
  - compute Phong lighting once for entire polygon



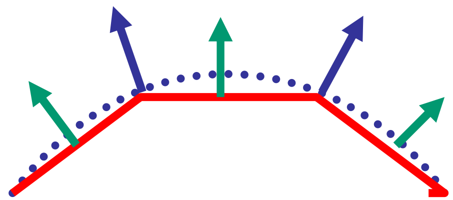
- Gouraud shading
  - compute Phong lighting at the vertices and interpolate lighting values across polygon
- Phong shading



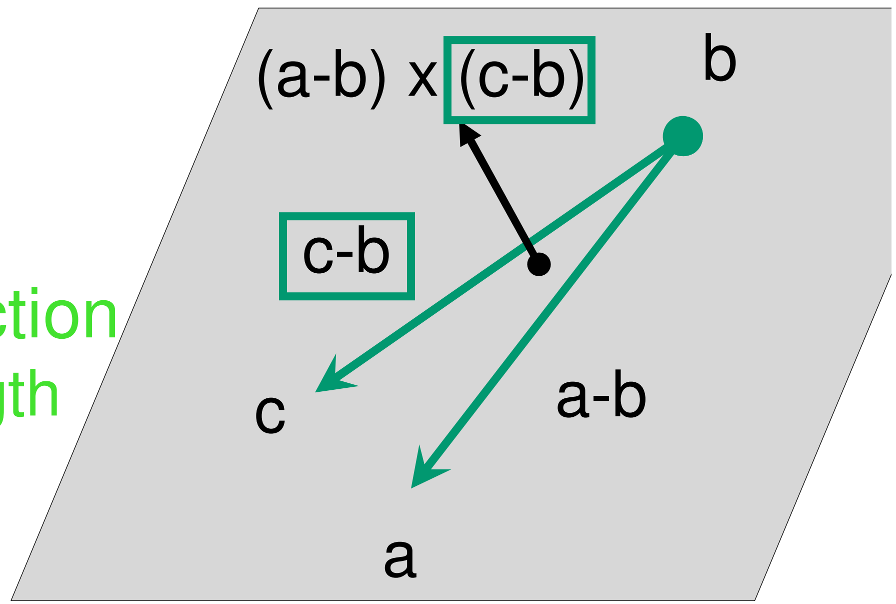
  - compute averaged vertex normals
  - interpolate normals across polygon and perform Phong lighting across polygon

Correction/Review: Computing Normals

- per-vertex normals by interpolating per-facet normals
  - OpenGL supports both

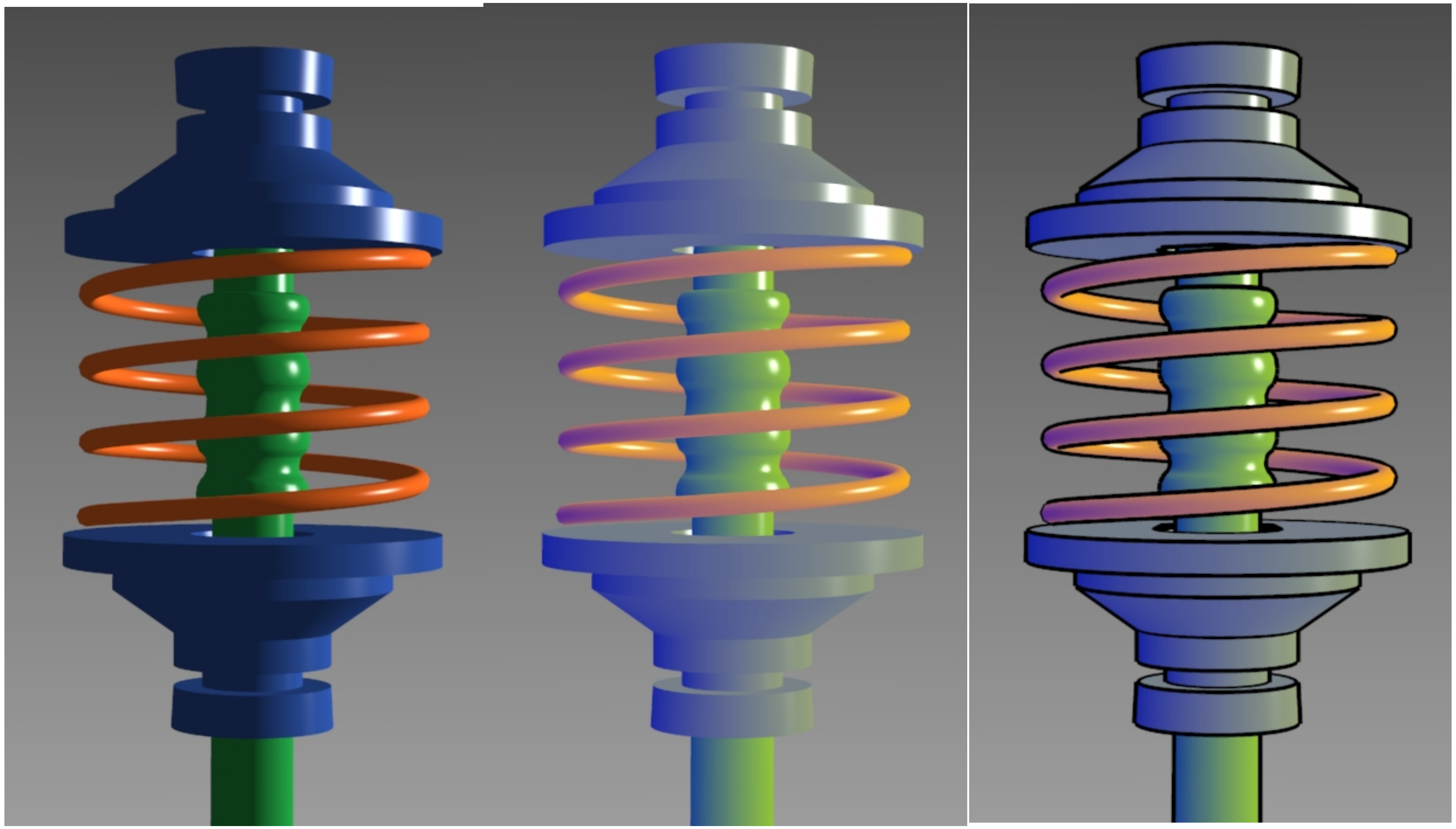


- computing normal for a polygon
  - three points form two vectors
    - pick a point
    - vectors from
      - A: point to previous
      - B: point to next
  - AxB: normal of plane direction
    - normalize: make unit length
  - which side of plane is up?
    - counterclockwise
      point order convention



Review: Non-Photorealistic Shading

- cool-to-warm shading $k_w = \frac{1 + \mathbf{n} \cdot \mathbf{l}}{2}, c = k_w c_w + (1 - k_w) c_c$
- draw silhouettes: if $(\mathbf{e} \cdot \mathbf{n_0})(\mathbf{e} \cdot \mathbf{n_1}) \leq 0$, **e**=edge-eye vector
- draw creases: if $(\mathbf{n_0} \cdot \mathbf{n_1}) \leq threshold$



http://www.cs.utah.edu/~gooch/SIG98/paper/drawing.html

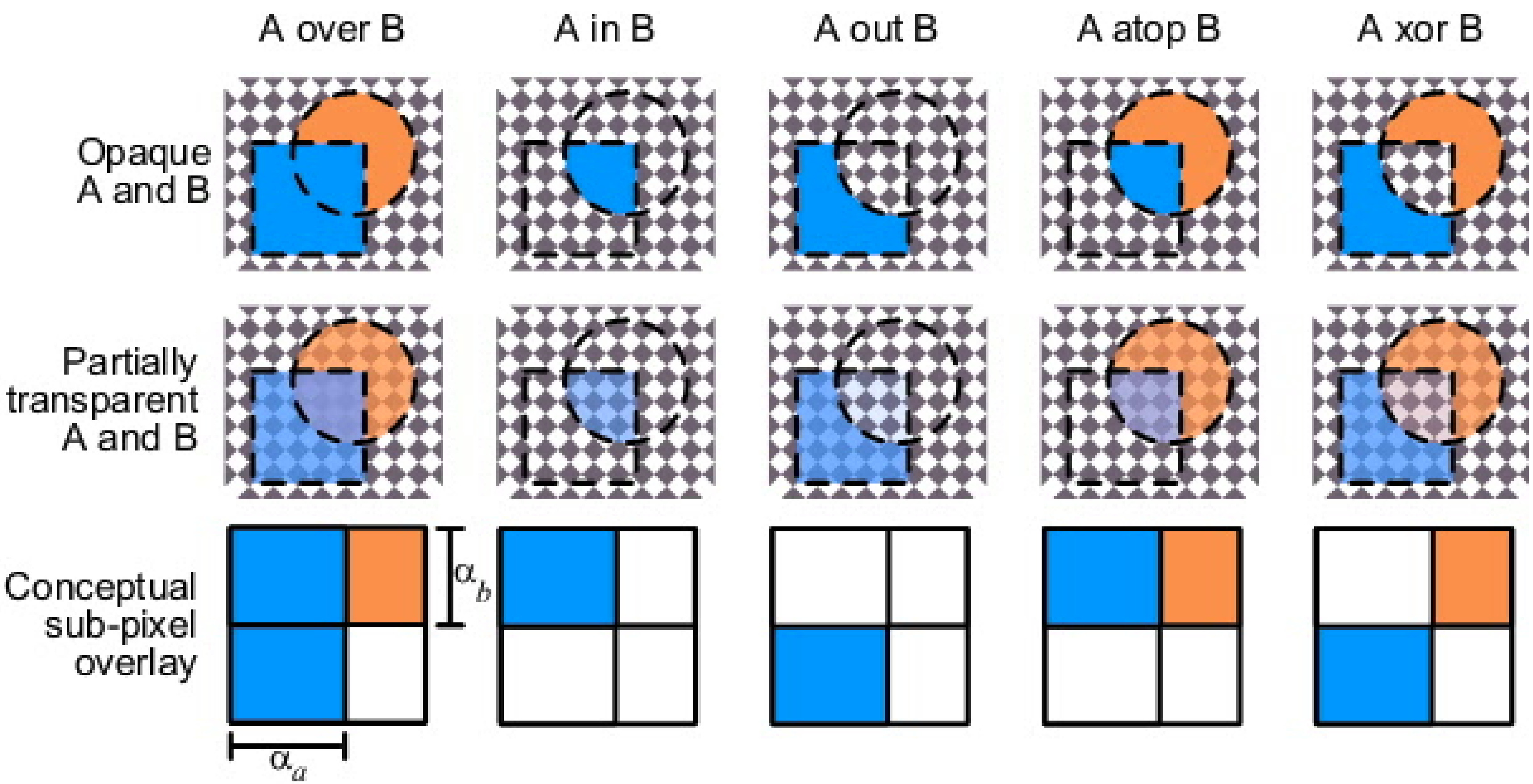
End of Class Last Time

- use version control for your projects!
  - CVS, RCS
- partially work through problem with lighting

Compositing

Compositing

- how might you combine multiple elements?
- foreground color **A**, background color **B**



Premultiplying Colors

- specify opacity with alpha channel: (r,g,b,α)
  - $\alpha=1$: opaque, $\alpha=.5$: translucent, $\alpha=0$: transparent

- **A** *over* **B**
  - $\mathbf{C} = \alpha\mathbf{A} + (1-\alpha)\mathbf{B}$

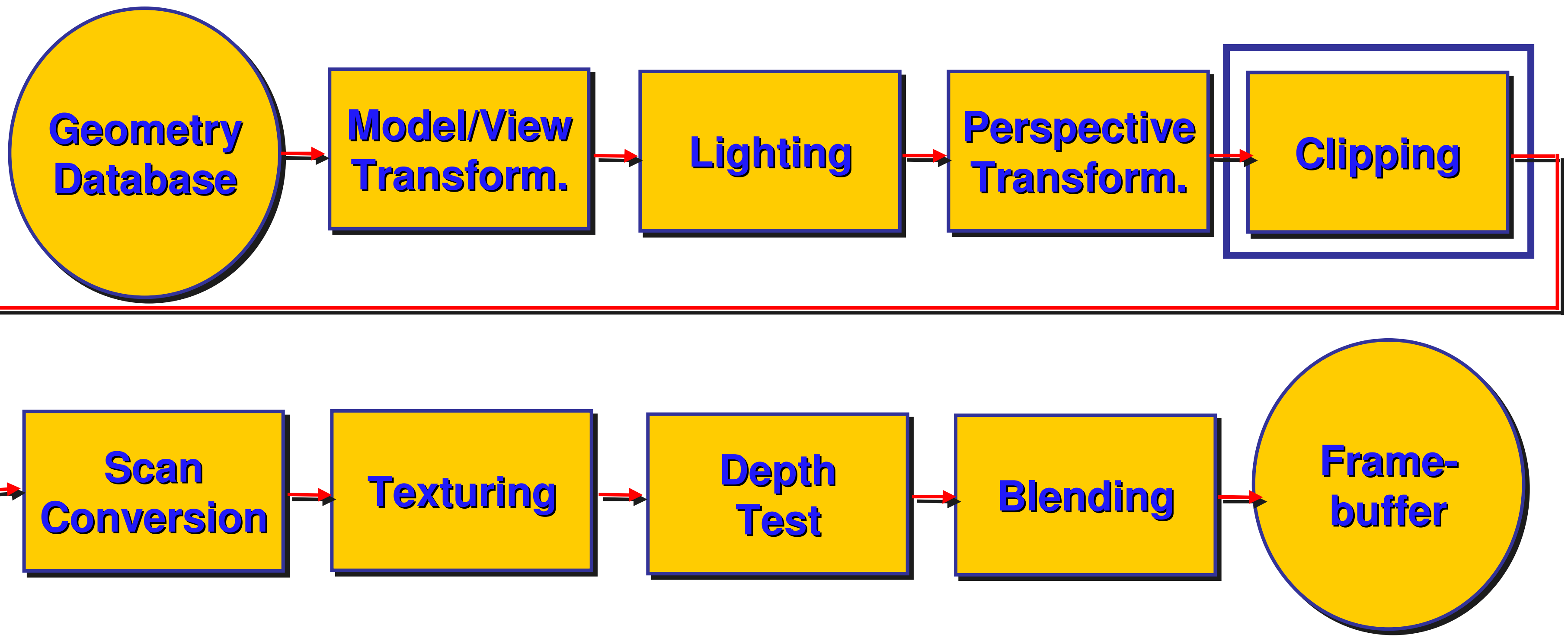
- but what if **B** is also partially transparent**?**
  - $\mathbf{C} = \alpha\mathbf{A} + (1-\alpha)\ \beta\mathbf{B} = \ \beta\mathbf{B} + \alpha\mathbf{A} + \beta\mathbf{B} - \alpha\ \beta\mathbf{B}$
  - $\gamma = \beta + (1-\beta)\alpha = \beta + \alpha - \alpha\beta$
    - 3 multiplies, different equations for alpha vs. RGB

- premultiplying by alpha
  - $\mathbf{C'} = \gamma\,\mathbf{C},\ \mathbf{B'} = \beta\mathbf{B},\ \mathbf{A'} = \alpha\mathbf{A}$

  - $\mathbf{C'} = \mathbf{B'} + \mathbf{A'} - \alpha\mathbf{B'}$
  - $\gamma = \beta + \alpha - \alpha\beta$
    - 1 multiply to find C, same equations for alpha and RGB

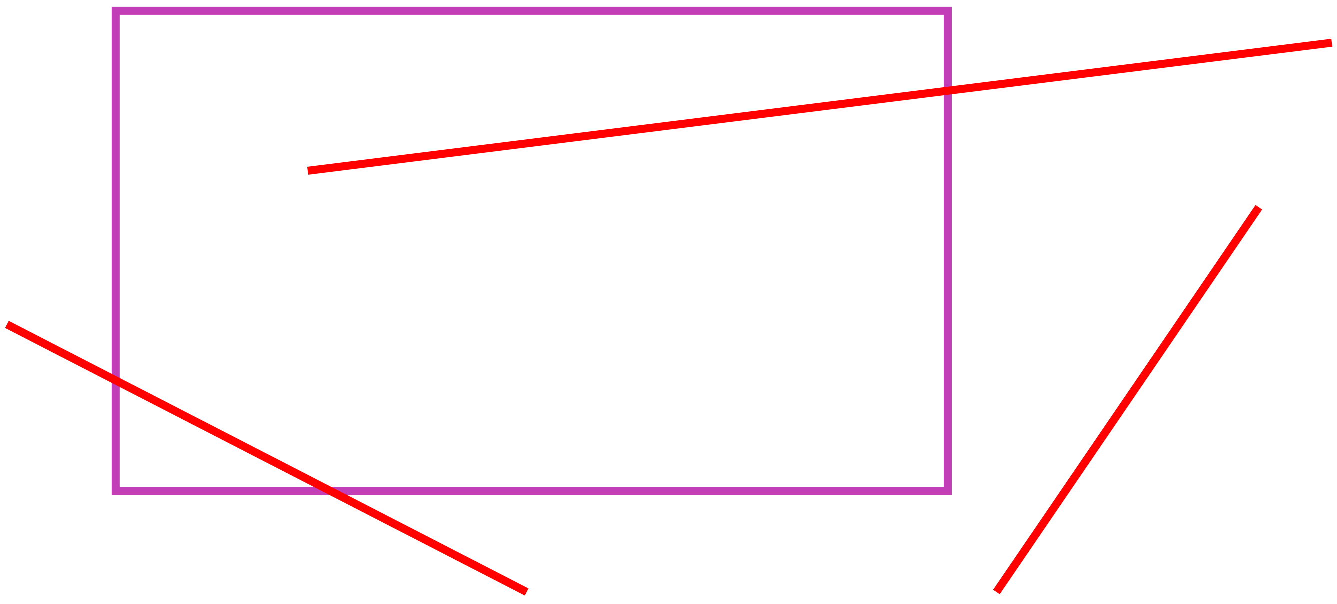
Clipping

Rendering Pipeline



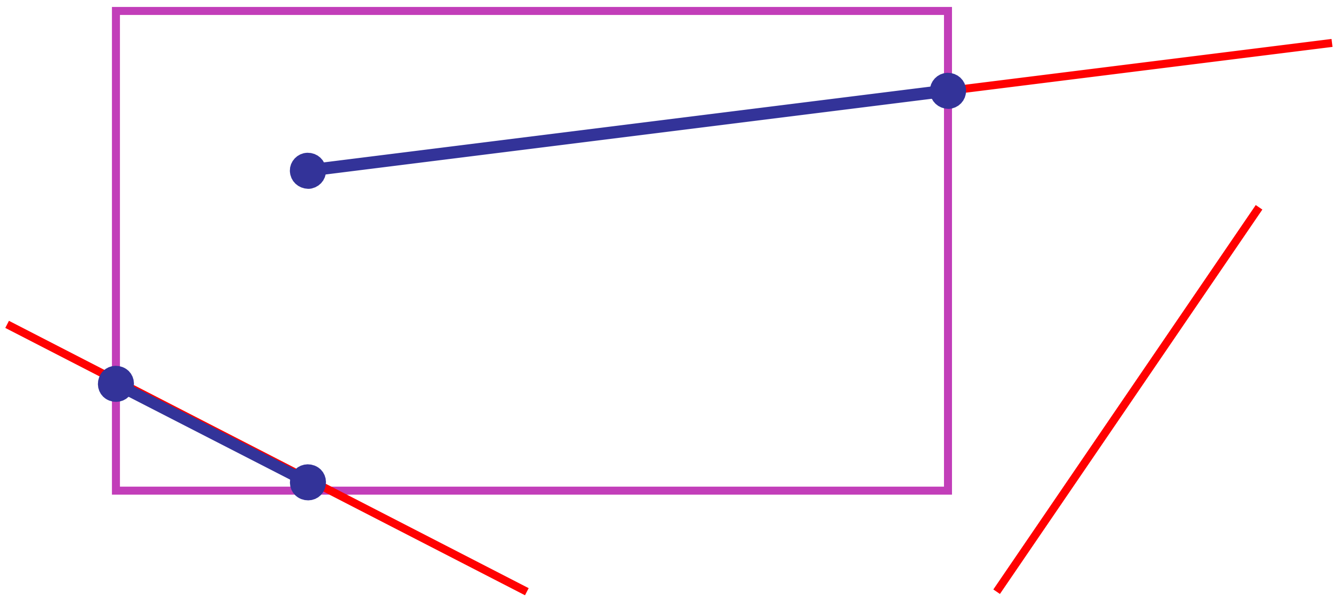
Next Topic: Clipping

- we've been assuming that all primitives (lines, triangles, polygons) lie entirely within the *viewport*
  - in general, this assumption will not hold:



Clipping

- analytically calculating the portions of primitives within the viewport



Why Clip?

- bad idea to rasterize outside of framebuffer bounds
- also, don’t waste time scan converting pixels outside window
  - could be billions of pixels for very close objects!

Line Clipping

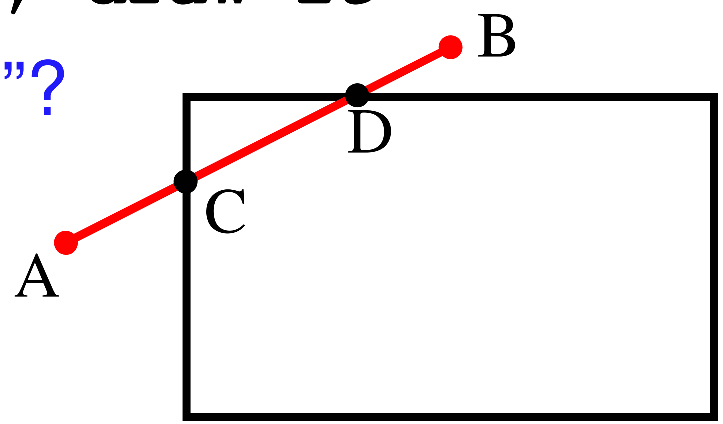
- 2D
  - determine portion of line inside an axis-aligned rectangle (screen or window)
- 3D
  - determine portion of line inside axis-aligned parallelpiped (viewing frustum in NDC)
  - simple extension to 2D algorithms

Clipping

- naïve approach to clipping lines:

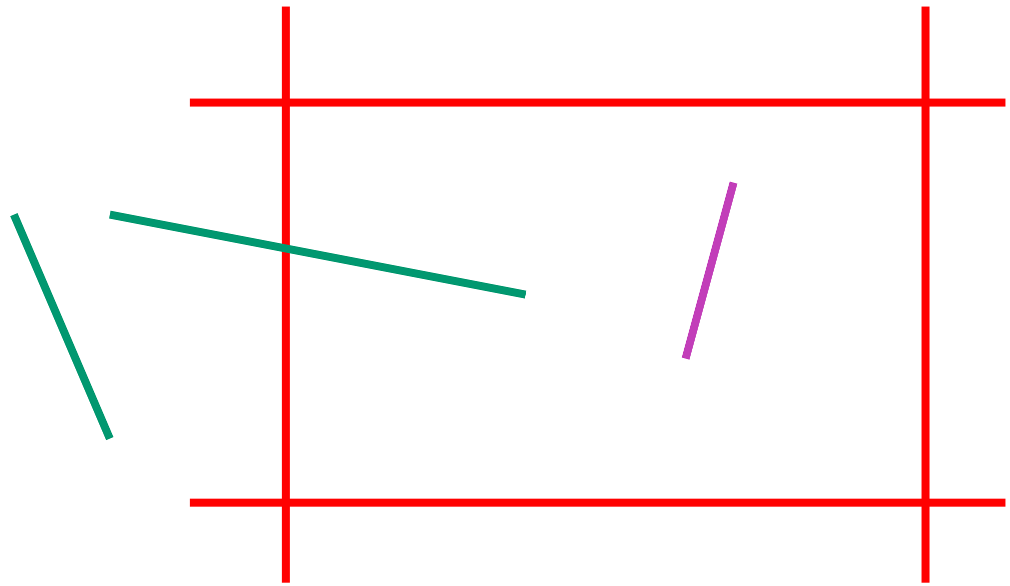
```
for each line segment
    for each edge of viewport
        find intersection point
        pick "nearest" point
    if anything is left, draw it
```

- what do we mean by "nearest"?
- how can we optimize this?



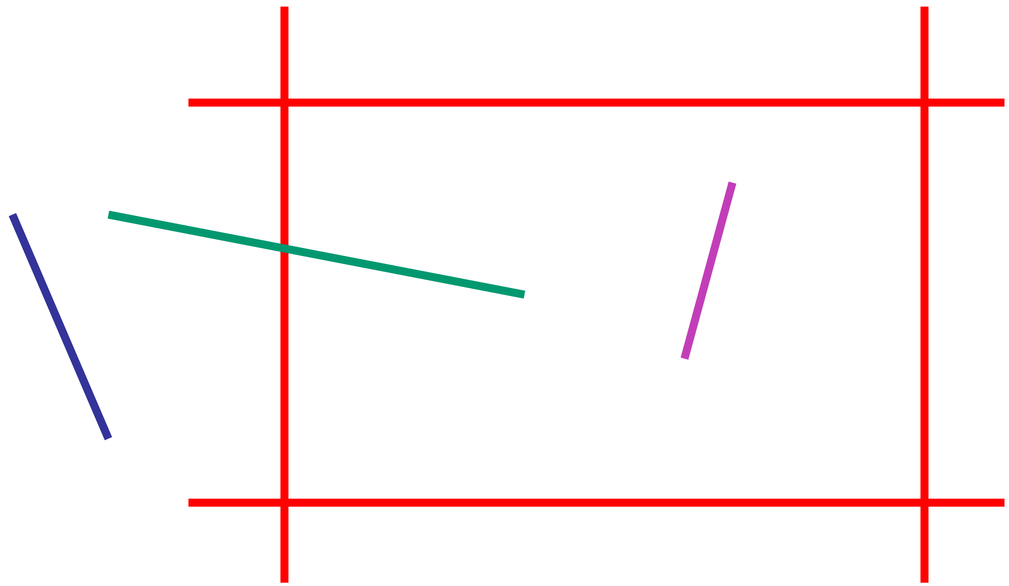
Trivial Accepts

- big optimization: trivial accept/rejects
  - Q: how can we quickly determine whether a line segment is entirely inside the viewport?
  - A: test both endpoints



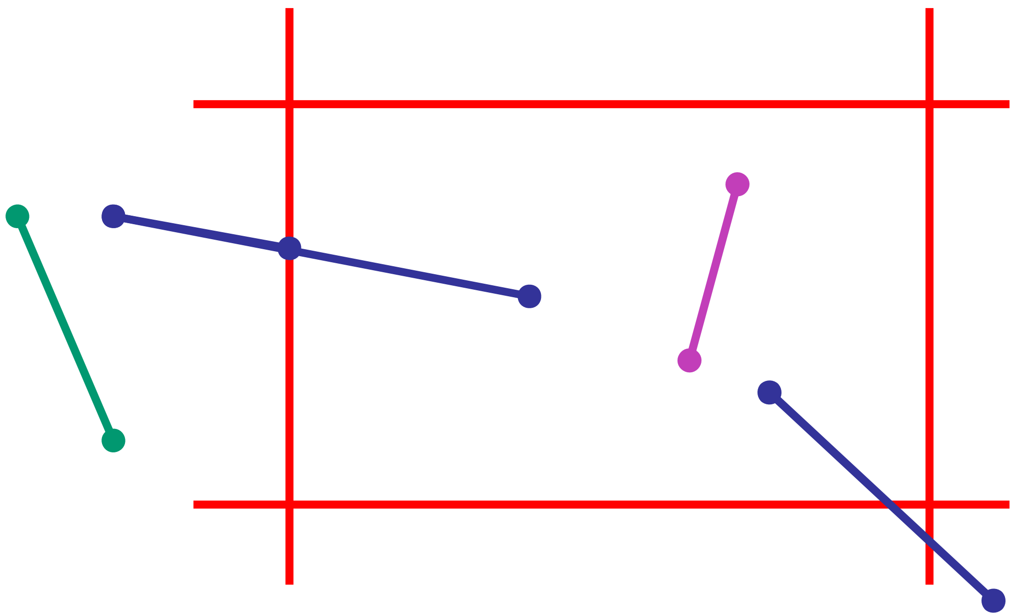
Trivial Rejects

- Q: how can we know a line is outside viewport?
- A: if both endpoints on wrong side of same edge, can trivially reject line



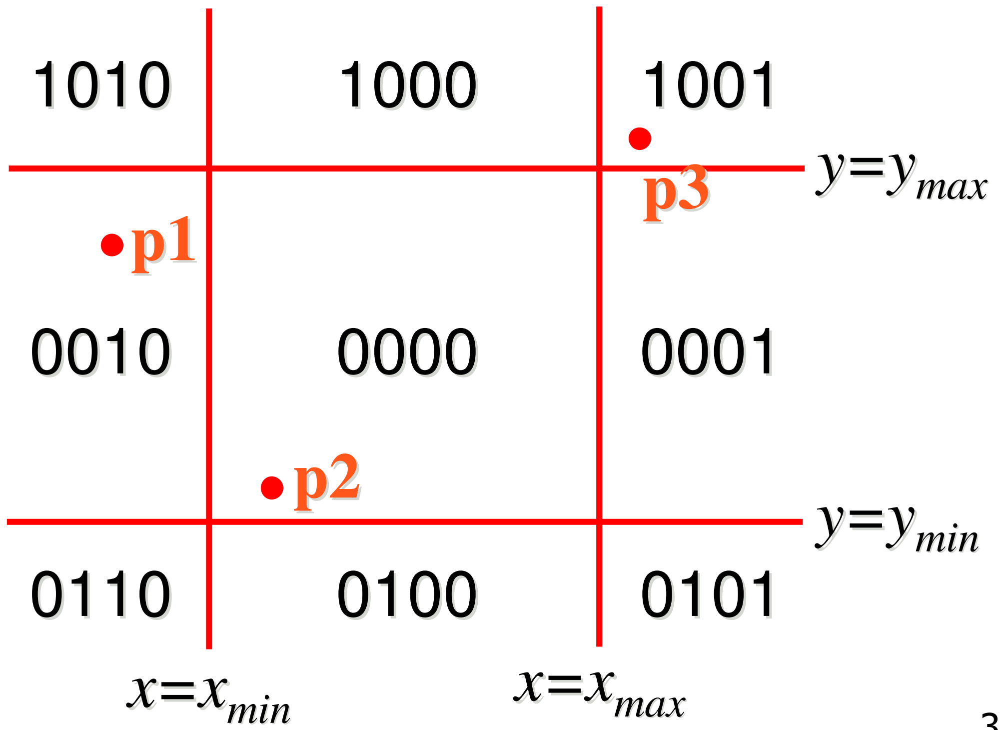
Clipping Lines To Viewport

- combining trivial accepts/rejects
  - trivially accept lines with both endpoints inside all edges of the viewport
  - trivially reject lines with both endpoints outside the same edge of the viewport
  - otherwise, reduce to trivial cases by splitting into two segments



Cohen-Sutherland Line Clipping

- outcodes
  - 4 flags encoding position of a point relative to top, bottom, left, and right boundary
    - $OC(p1)=0010$
    - $OC(p2)=0000$
    - $OC(p3)=1001$



Cohen-Sutherland Line Clipping

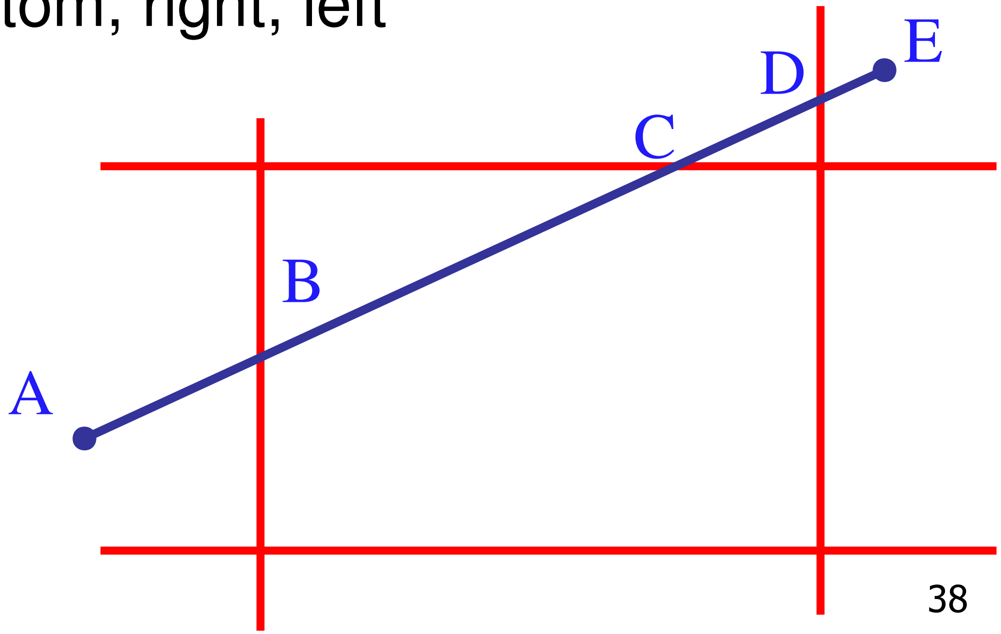
- assign outcode to each vertex of line to test
  - line segment: (**p1**,**p2**)
- trivial cases
  - OC(**p1**)== 0 && OC(**p2**)==0
    - both points inside window, thus line segment completely visible (trivial accept)
  - (OC(**p1**) & OC(**p2**))!= 0
    - there is (at least) one boundary for which both points are outside (same flag set in both outcodes)
    - thus line segment completely outside window (trivial reject)

Cohen-Sutherland Line Clipping

- if line cannot be trivially accepted or rejected, subdivide so that one or both segments can be discarded
- pick an edge that the line crosses (*how?*)
- intersect line with edge (*how?*)
- discard portion on wrong side of edge and assign outcode to new vertex
- apply trivial accept/reject tests; repeat if necessary

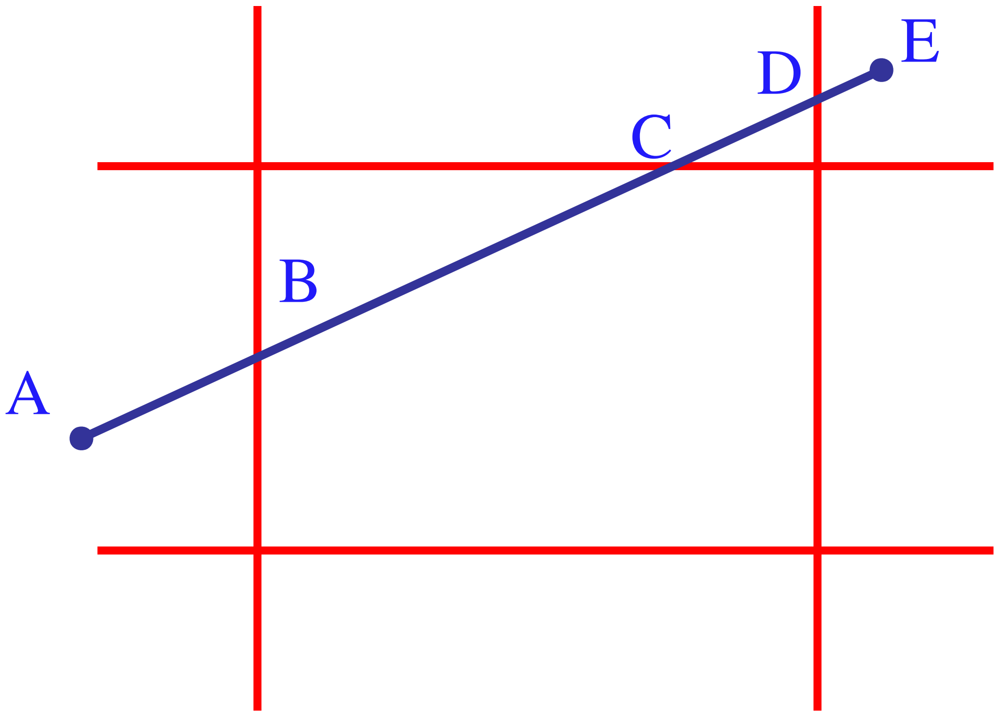
Cohen-Sutherland Line Clipping

- if line cannot be trivially accepted or rejected, subdivide so that one or both segments can be discarded
- pick an edge that the line crosses
  - check against edges in same order each time
    - for example: top, bottom, right, left



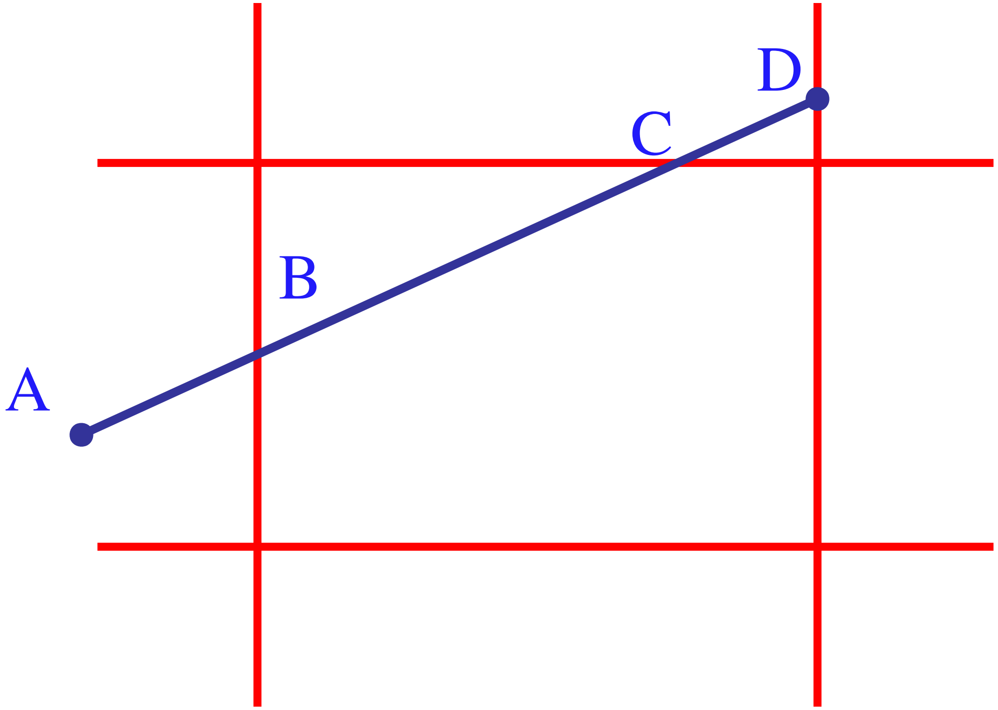
Cohen-Sutherland Line Clipping

- intersect line with edge (how?)



Cohen-Sutherland Line Clipping

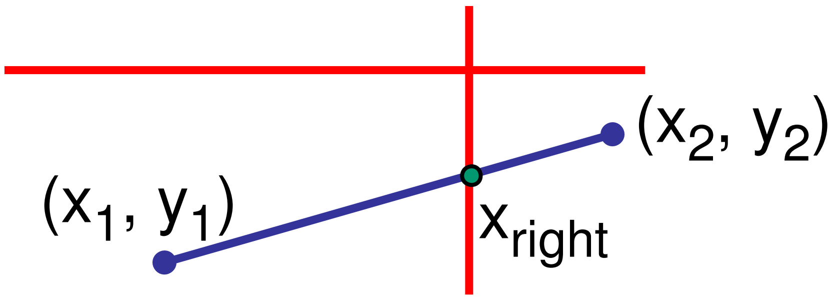
- discard portion on wrong side of edge and assign outcode to new vertex



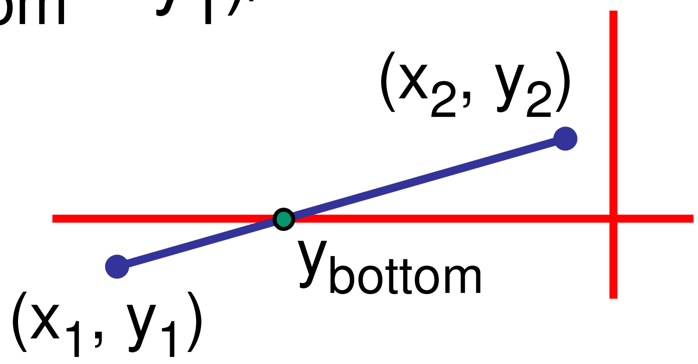
- apply trivial accept/reject tests and repeat if necessary

Viewport Intersection Code

- $(x_1, y_1), (x_2, y_2)$ intersect vertical edge at x_{right}
  - $y_{intersect} = y_1 + m(x_{right} - x_1)$
  - $m=(y_2-y_1)/(x_2-x_1)$



- $(x_1, y_1), (x_2, y_2)$ intersect horiz edge at y_{bottom}
  - $x_{intersect} = x_1 + (y_{bottom} - y_1)/m$
  - $m=(y_2-y_1)/(x_2-x_1)$



Cohen-Sutherland Discussion

- use opcodes to quickly eliminate/include lines
  - best algorithm when trivial accepts/rejects are common
- must compute viewport clipping of remaining lines
  - non-trivial clipping cost
  - redundant clipping of some lines
- more efficient algorithms exist

Line Clipping in 3D

- approach
  - clip against parallelpiped in NDC
    - after perspective transform
  - means that clipping volume always the same
    - xmin=ymin= -1, xmax=ymax= 1 in OpenGL

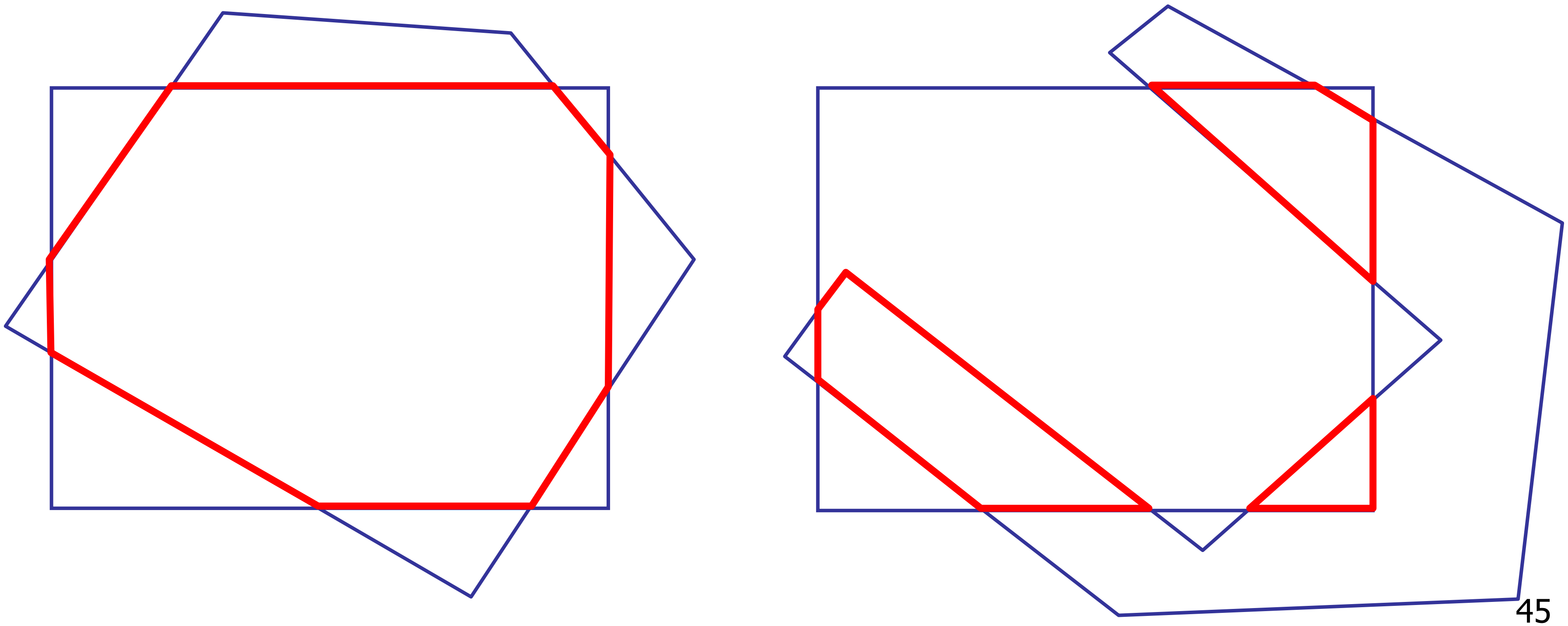
  - boundary lines become boundary planes
    - but outcodes still work the same way
    - additional front and back clipping plane
      - zmin = -1, zmax = 1 in OpenGL

Polygon Clipping

- objective
  - 2D: clip polygon against rectangular window
    - or general convex polygons
    - extensions for non-convex or general polygons
  - 3D: clip polygon against parallelpiped

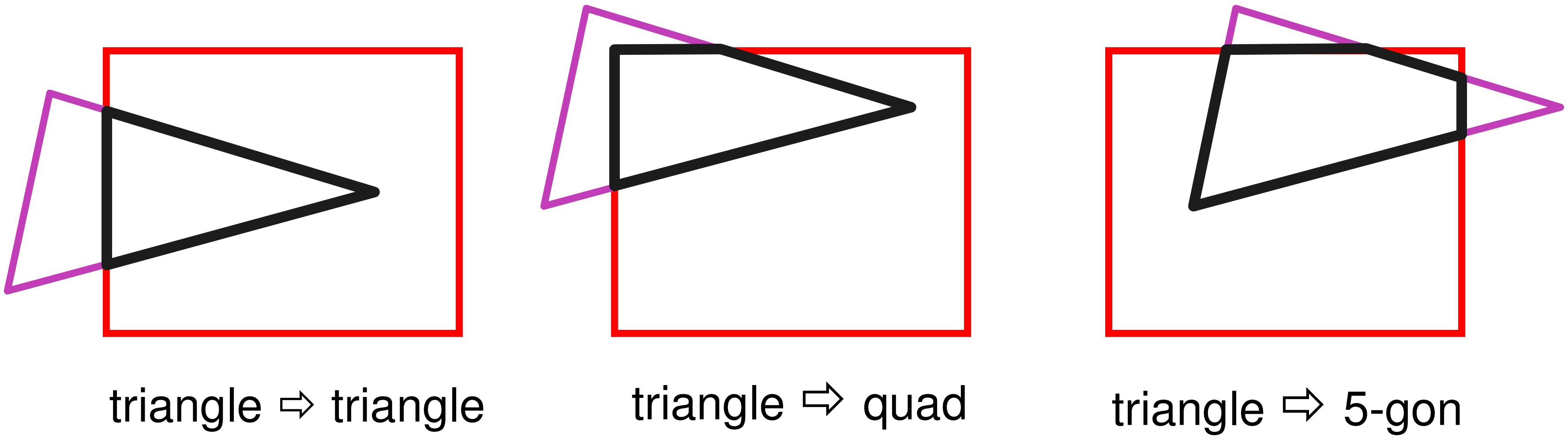
Polygon Clipping

- not just clipping all boundary lines
  - may have to introduce new line segments



Why Is Clipping Hard?

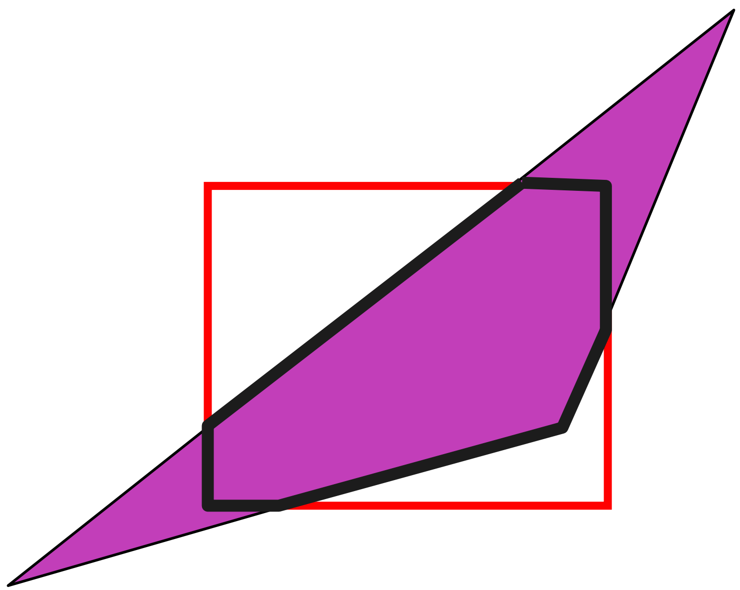
- what happens to a triangle during clipping?
- possible outcomes:



- how many sides can a clipped triangle have?

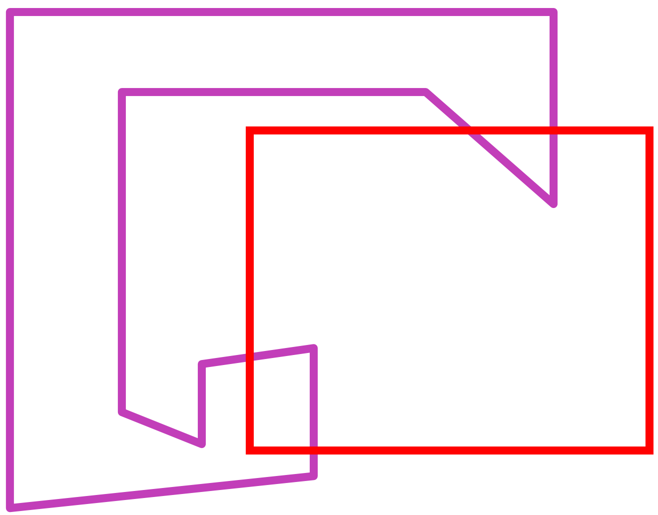
How Many Sides?

- seven…



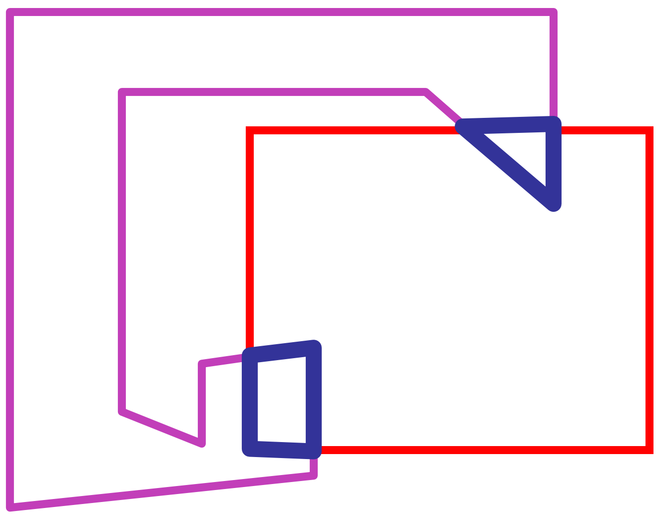
Why Is Clipping Hard?

- a really tough case:



Why Is Clipping Hard?

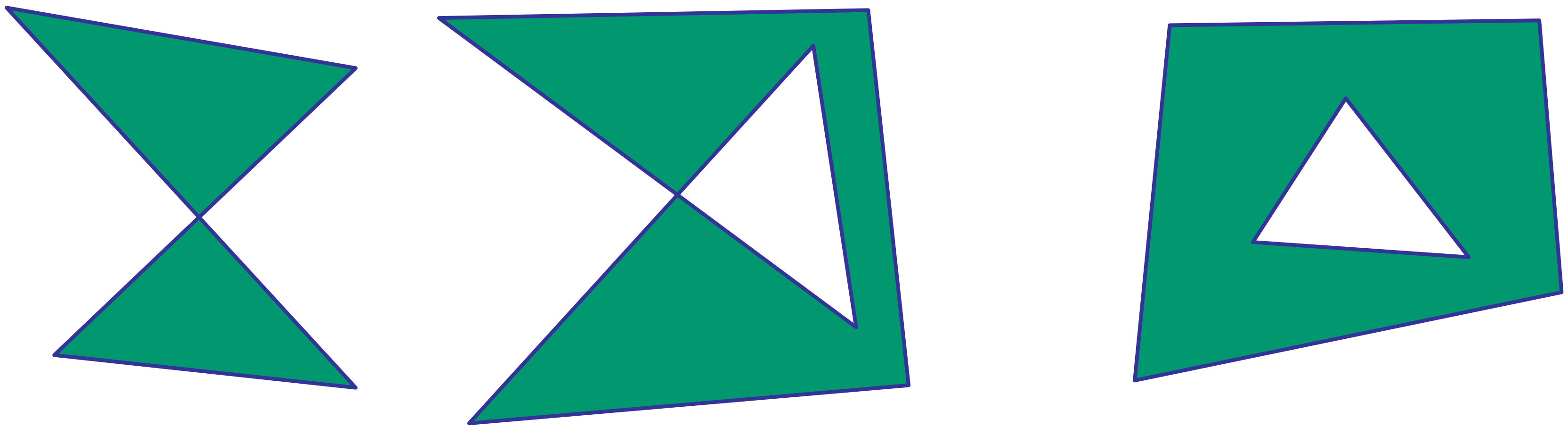
- a really tough case:



concave polygon ⇨ multiple polygons

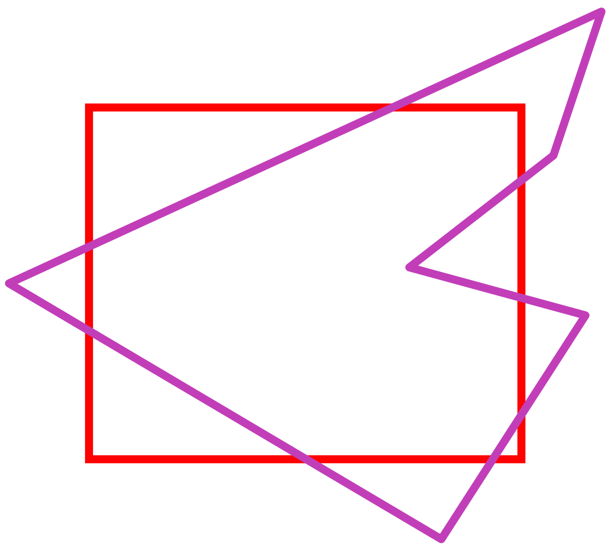
Polygon Clipping

- classes of polygons
  - triangles
  - convex
  - concave
  - holes and self-intersection



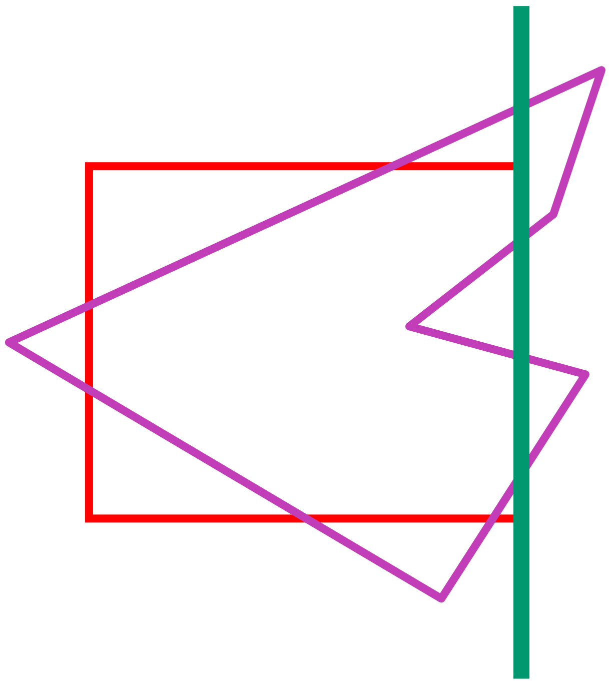
Sutherland-Hodgeman Clipping

- basic idea:
  - consider each edge of the viewport individually
  - clip the polygon against the edge equation
  - after doing all edges, the polygon is fully clipped



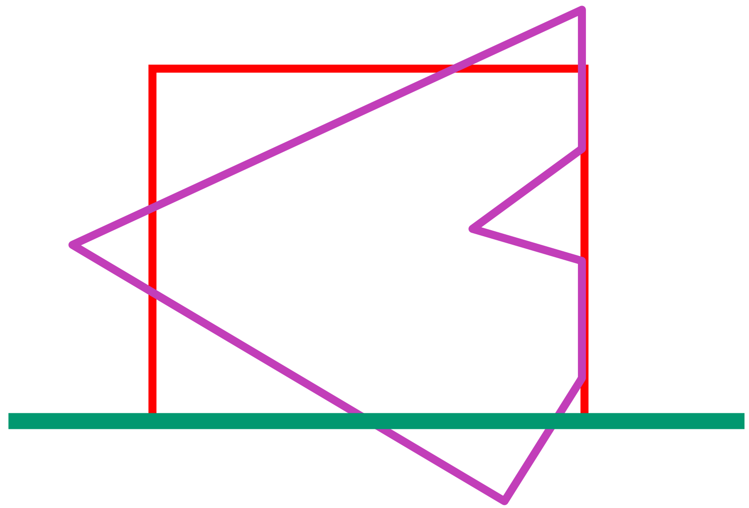
Sutherland-Hodgeman Clipping

- basic idea:
  - consider each edge of the viewport individually
  - clip the polygon against the edge equation
  - after doing all edges, the polygon is fully clipped



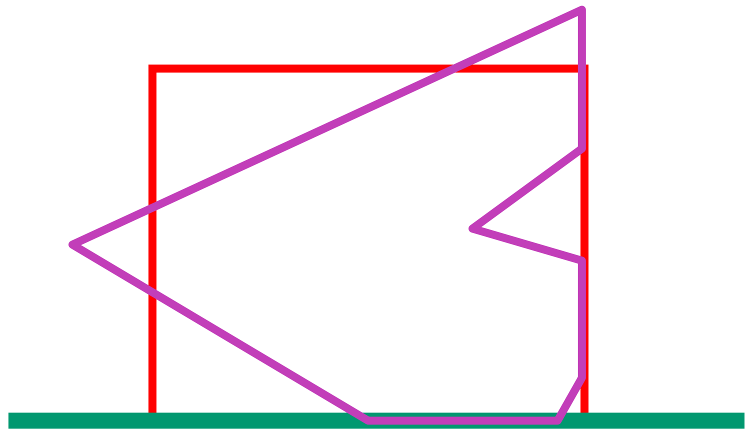
Sutherland-Hodgeman Clipping

- basic idea:
  - consider each edge of the viewport individually
  - clip the polygon against the edge equation
  - after doing all edges, the polygon is fully clipped



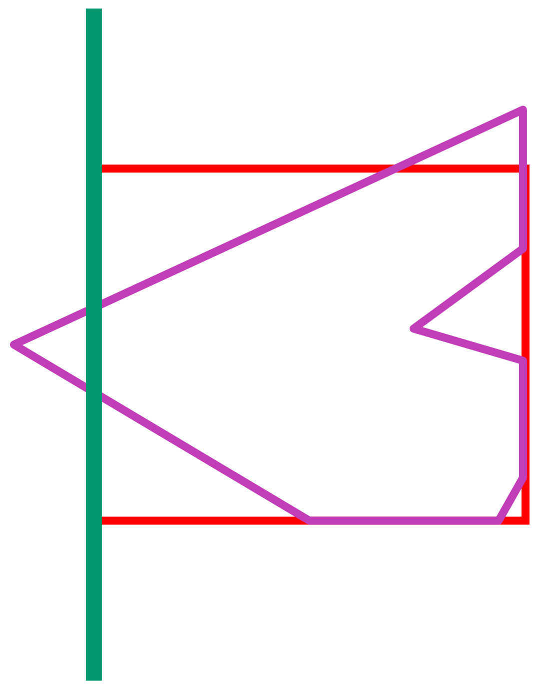
Sutherland-Hodgeman Clipping

- basic idea:
  - consider each edge of the viewport individually
  - clip the polygon against the edge equation
  - after doing all edges, the polygon is fully clipped



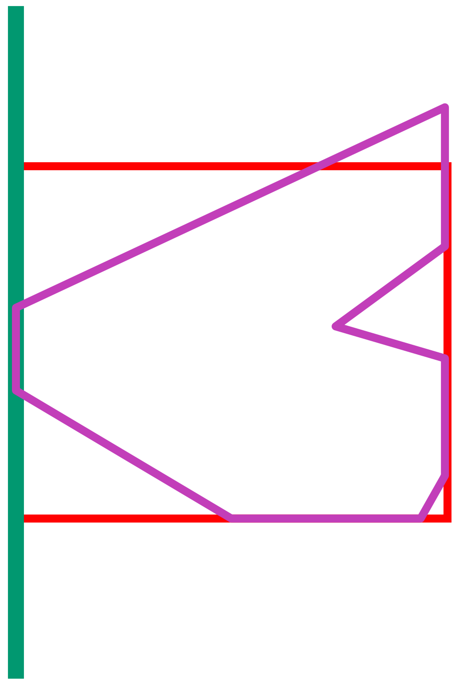
Sutherland-Hodgeman Clipping

- basic idea:
  - consider each edge of the viewport individually
  - clip the polygon against the edge equation
  - after doing all edges, the polygon is fully clipped



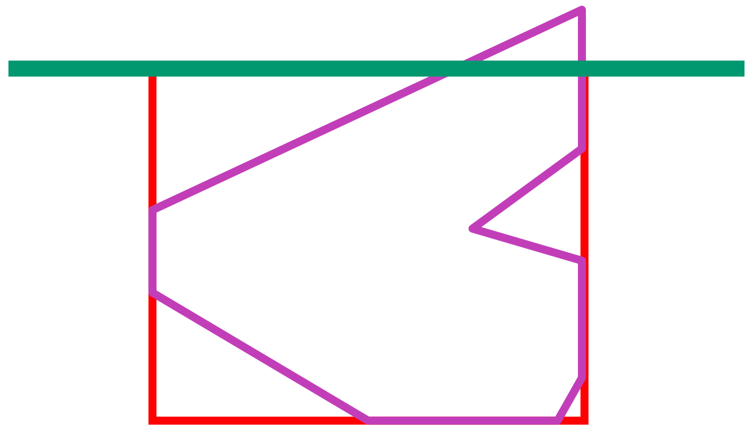
Sutherland-Hodgeman Clipping

- basic idea:
  - consider each edge of the viewport individually
  - clip the polygon against the edge equation
  - after doing all edges, the polygon is fully clipped



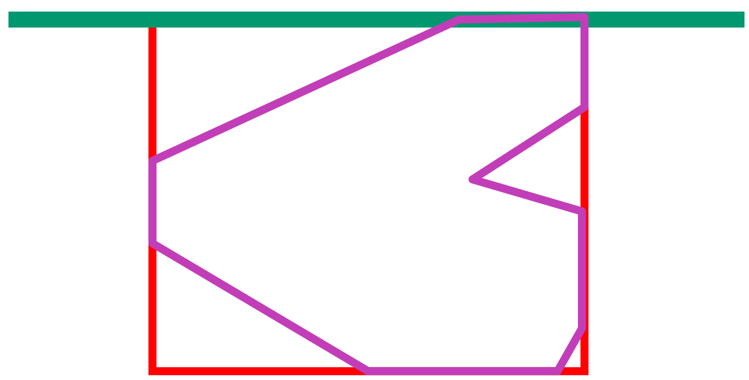
Sutherland-Hodgeman Clipping

- basic idea:
  - consider each edge of the viewport individually
  - clip the polygon against the edge equation
  - after doing all edges, the polygon is fully clipped



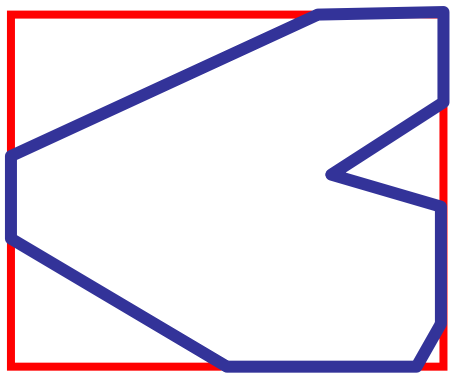
Sutherland-Hodgeman Clipping

- basic idea:
  - consider each edge of the viewport individually
  - clip the polygon against the edge equation
  - after doing all edges, the polygon is fully clipped



Sutherland-Hodgeman Clipping

- basic idea:
  - consider each edge of the viewport individually
  - clip the polygon against the edge equation
  - after doing all edges, the polygon is fully clipped

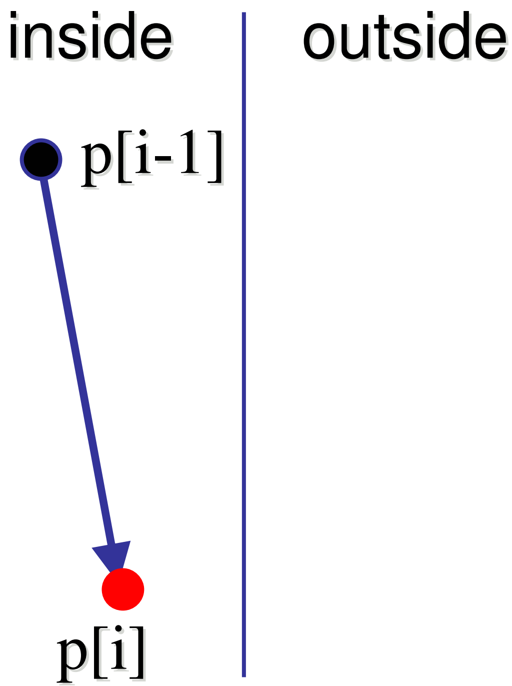


Sutherland-Hodgeman Algorithm

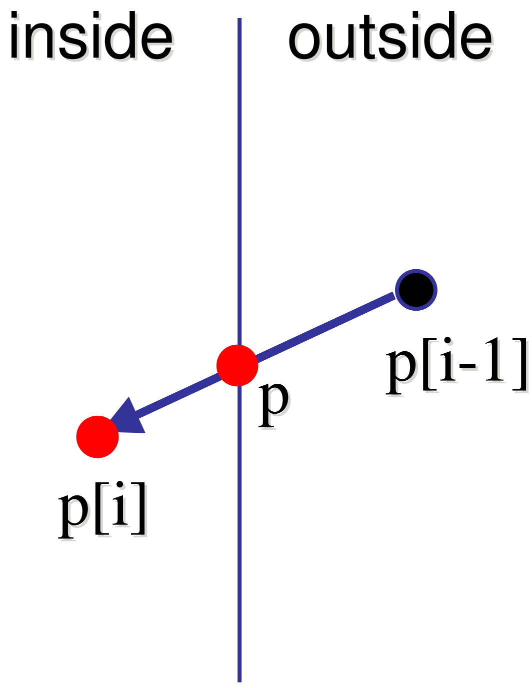
- input/output for algorithm
  - input: list of polygon vertices in order
  - output: list of clipped poygon vertices consisting of old vertices (maybe) and new vertices (maybe)
- basic routine
  - go around polygon one vertex at a time
  - decide what to do based on 4 possibilities
    - is vertex inside or outside?
    - is previous vertex inside or outside?

Clipping Against One Edge

- p[i] inside: 2 cases



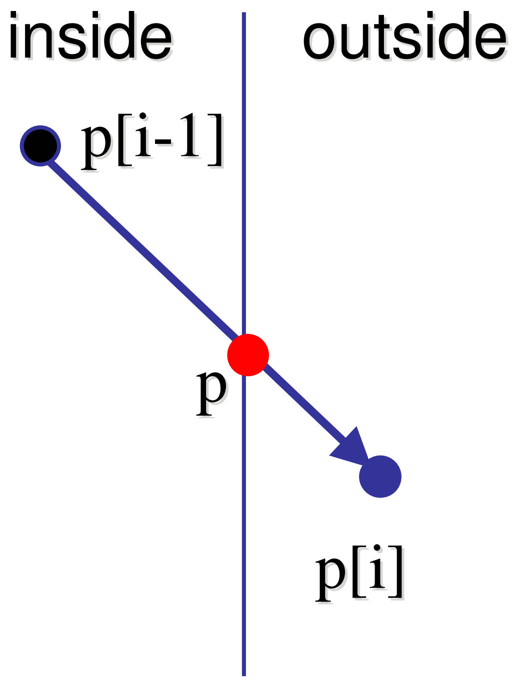
output: p[i]



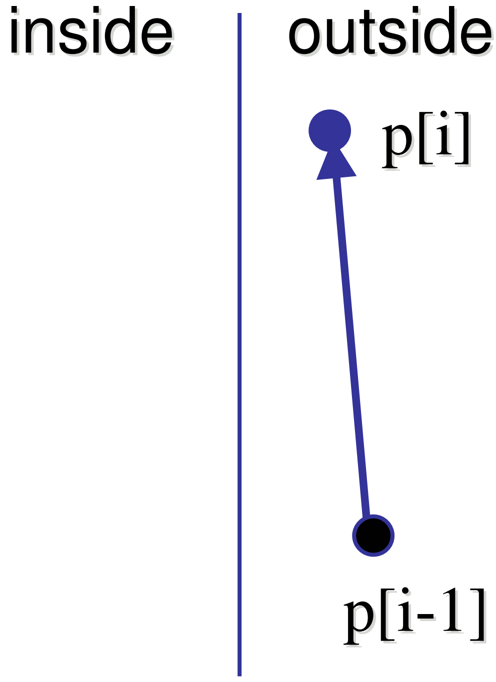
output: p, p[i]

Clipping Against One Edge

- p[i] outside: 2 cases



output: p

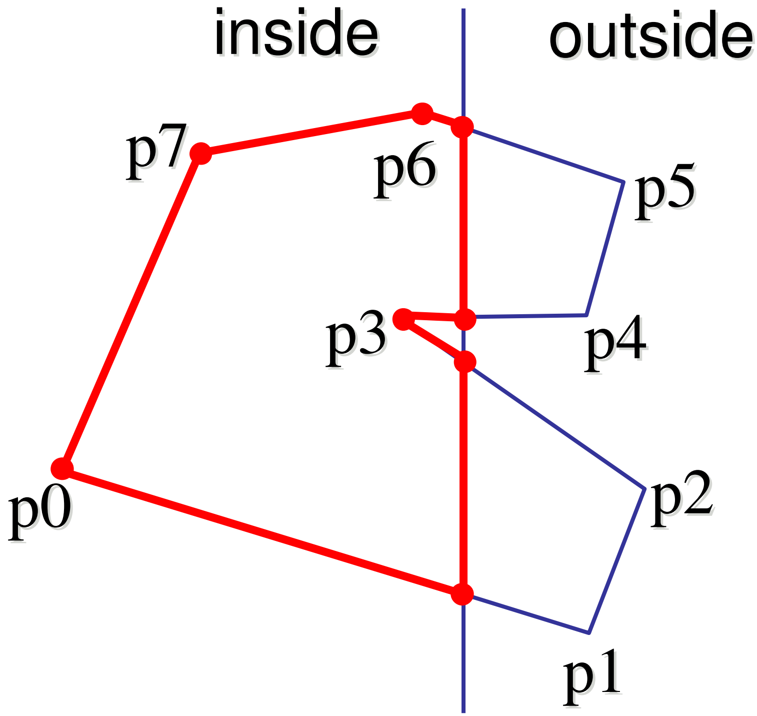


output: nothing

Clipping Against One Edge

```
clipPolygonToEdge( p[n], edge ) {
    for( i= 0 ; i< n ; i++ ) {
        if( p[i] inside edge ) {
          if( p[i-1] inside edge ) output p[i];     // p[-1]= p[n-1]
          else {
            p= intersect( p[i-1], p[i], edge ); output p, p[i];
          }
        } else {                                    // p[i] is outside edge
        if( p[i-1] inside edge ) {
          p= intersect(p[i-1], p[I], edge ); output p;
        }
    }
}
```

Sutherland-Hodgeman Example

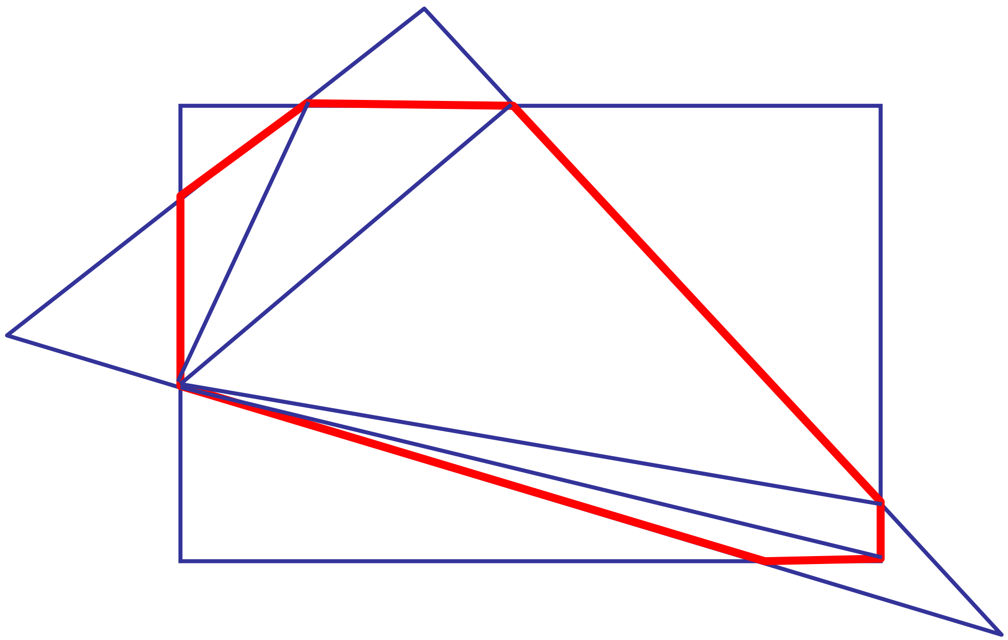


Sutherland-Hodgeman Discussion

- similar to Cohen/Sutherland line clipping
  - inside/outside tests: outcodes
  - intersection of line segment with edge: window-edge coordinates
- clipping against individual edges independent
  - great for hardware (pipelining)
  - all vertices required in memory at same time
    - not so good, but unavoidable
    - another reason for using triangles only in hardware rendering

Sutherland/Hodgeman Discussion

- for rendering pipeline:
  - re-triangulate resulting polygon (can be done for every individual clipping edge)



Curves

Parametric Curves

- parametric form for a line:

$$x = x_0 t + (1-t)x_1$$
$$y = y_0 t + (1-t)y_1$$
$$z = z_0 t + (1-t)z_1$$

- x, y and z are each given by an equation that involves:
  - parameter *t*
  - some user specified control points, x_0 and x_1
- this is an example of a parametric curve

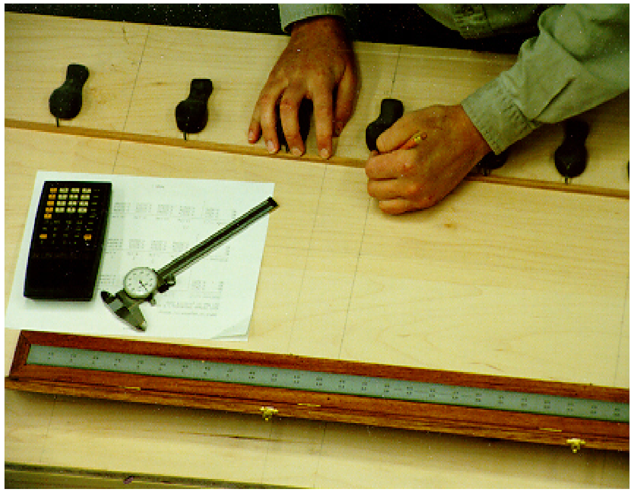
Splines

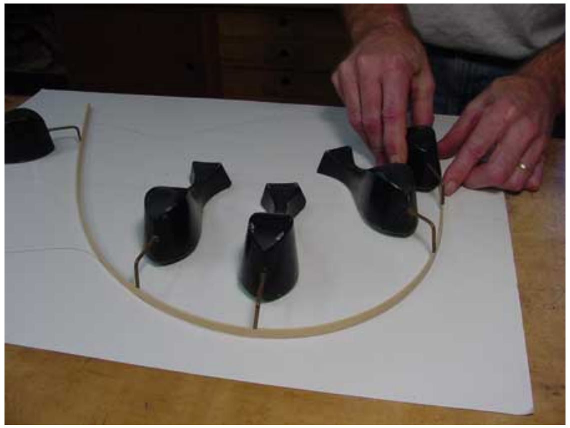
- a *spline* is a parametric curve defined by *control points*
  - term “spline” dates from engineering drawing, where a spline was a piece of flexible wood used to draw smooth curves
  - control points are *adjusted by the user* to control shape of curve

Splines - History

- draftsman used ‘ducks’ and strips of wood (splines) to draw curves
- wood splines have second-order continuity, pass through the control points

a duck (weight)





ducks trace out curve

Hermite Spline

- *hermite spline* is curve for which user provides:
  - endpoints of curve
  - parametric derivatives of curve at endpoints
    - parametric derivatives are *dx/dt, dy/dt, dz/dt*
  - more derivatives would be required for higher order curves

Hermite Cubic Splines

- example of knot and continuity constraints

Hermite Specification

Hermite Spline (2)

- say user provides x_0, x_1, x'_0, x'_1
- cubic spline has degree 3, is of the form:

$$x = at^3 + bt^2 + ct + d$$

  - for some constants a, b, c and d derived from the control points, but how?
- we have constraints:
  - curve must pass through x_0 when $t=0$
  - derivative must be x'_0 when $t=0$
  - curve must pass through x_1 when $t=1$
  - derivative must be x'_1 when $t=1$

Hermite Spline (3)

- solving for the unknowns gives

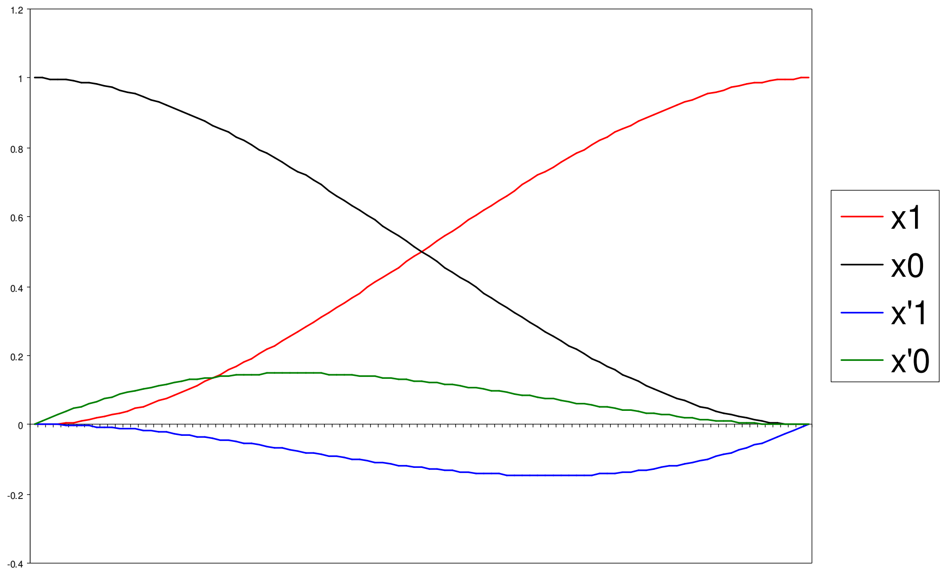
$$
\begin{aligned}
a &= -2x_1 + 2x_0 + x_1' + x_0' \\
b &= 3x_1 - 3x_0 - x_1' - 2x_0' \\
c &= x_0' \\
d &= x_0
\end{aligned}
$$

- rearranging gives

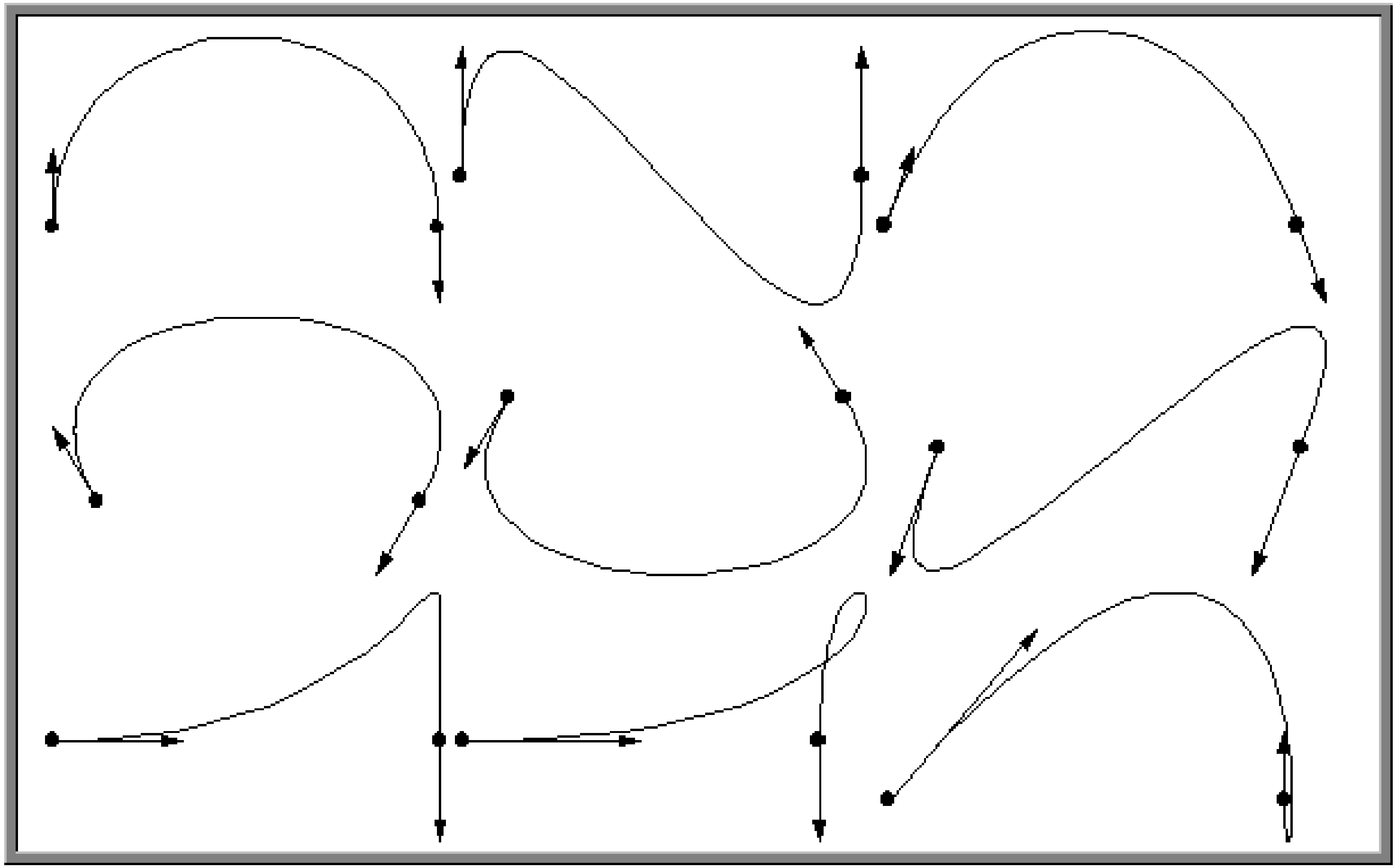
$$
\begin{aligned}
x = x_1(-2t^3 + 3t^2) \\
+ x_0(2t^3 - 3t^2 + 1) \\
+ \mathrm{x}_1'(t^3 - t^2) \\
+ x_0'(t^3 - 2t^2 + t)
\end{aligned}
\quad \text{or} \quad
x = \begin{bmatrix} x_1 & x_0 & x_1' & x_0' \end{bmatrix}
\begin{bmatrix} -2 & 3 & 0 & 0 \\ 2 & -3 & 0 & 1 \\ 1 & -1 & 0 & 0 \\ 1 & -2 & 1 & 0 \end{bmatrix}
\begin{bmatrix} t^3 \\ t^2 \\ t \\ 1 \end{bmatrix}
$$

Basis Functions

- a point on a Hermite curve is obtained by multiplying each control point by some function and summing
- functions are called *basis functions*



Sample Hermite Curves



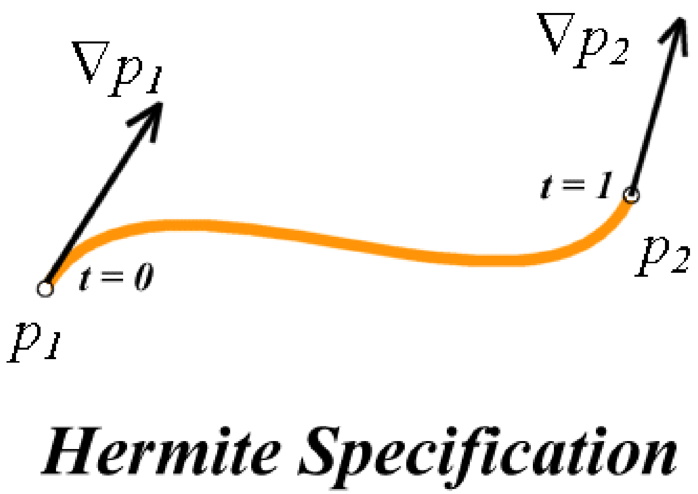
Splines in 2D and 3D

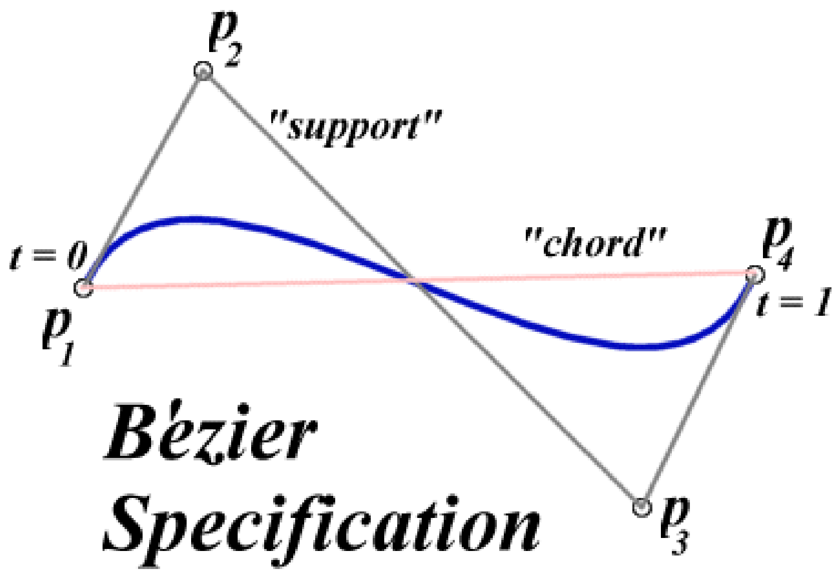
- so far, defined only 1D splines: $x = f(t : x_0, x_1, x'_0, x'_1)$
- for higher dimensions, define control points in higher dimensions (that is, as vectors)

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x_1 & x_0 & x'_1 & x'_0 \\ y_1 & y_0 & y'_1 & y'_0 \\ z_1 & z_0 & z'_1 & z'_0 \end{bmatrix} \begin{bmatrix} -2 & 3 & 0 & 0 \\ 2 & -3 & 0 & 1 \\ 1 & -1 & 0 & 0 \\ 1 & -2 & 1 & 0 \end{bmatrix} \begin{bmatrix} t^3 \\ t^2 \\ t \\ 1 \end{bmatrix}$$

Bézier Curves

- similar to Hermite, but more intuitive definition of endpoint derivatives
- four control points, two of which are knots





Bézier Curves

- derivative values of Bezier curve at knots dependent on adjacent points

$$\nabla p_1 = 3(p_2 - p_1)$$
$$\nabla p_4 = 3(p_4 - p_3)$$

Bézier vs. Hermite

- can write Bezier in terms of Hermite
  - note: just matrix form of previous

$$\underbrace{\begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \\ \frac{dx_1}{dt} & \frac{dy_1}{dt} \\ \frac{dx_2}{dt} & \frac{dy_2}{dt} \end{bmatrix}}_{\mathbf{G}_{Hermite}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ -3 & 3 & 0 & 0 \\ 0 & 0 & -3 & 3 \end{bmatrix} \underbrace{\begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_4 & y_4 \end{bmatrix}}_{\mathbf{G}_{Bezier}}$$

Bézier vs. Hermite

- Now substitute this in for previous Hermite

$$\begin{bmatrix} a_x & a_y \\ b_x & b_y \\ c_x & c_y \\ d_x & d_y \end{bmatrix} = \underbrace{\begin{bmatrix} 2 & -2 & 1 & 1 \\ -3 & 3 & -2 & -1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}}_{\mathbf{M}_{Hermite}} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ -3 & 3 & 0 & 0 \\ 0 & 0 & -3 & 3 \end{bmatrix} \underbrace{\begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_4 & y_4 \end{bmatrix}}_{\mathbf{G}_{Bezier}}$$

Bézier Basis, Geometry Matrices

$$\begin{bmatrix} a_x & a_y \\ b_x & b_y \\ c_x & c_y \\ d_x & d_y \end{bmatrix} = \underbrace{\begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}}_{\mathbf{M}_{Bezier}} \underbrace{\begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_4 & y_4 \end{bmatrix}}_{\mathbf{G}_{Bezier}}$$

- but why is M_{Bezier} a good basis matrix?

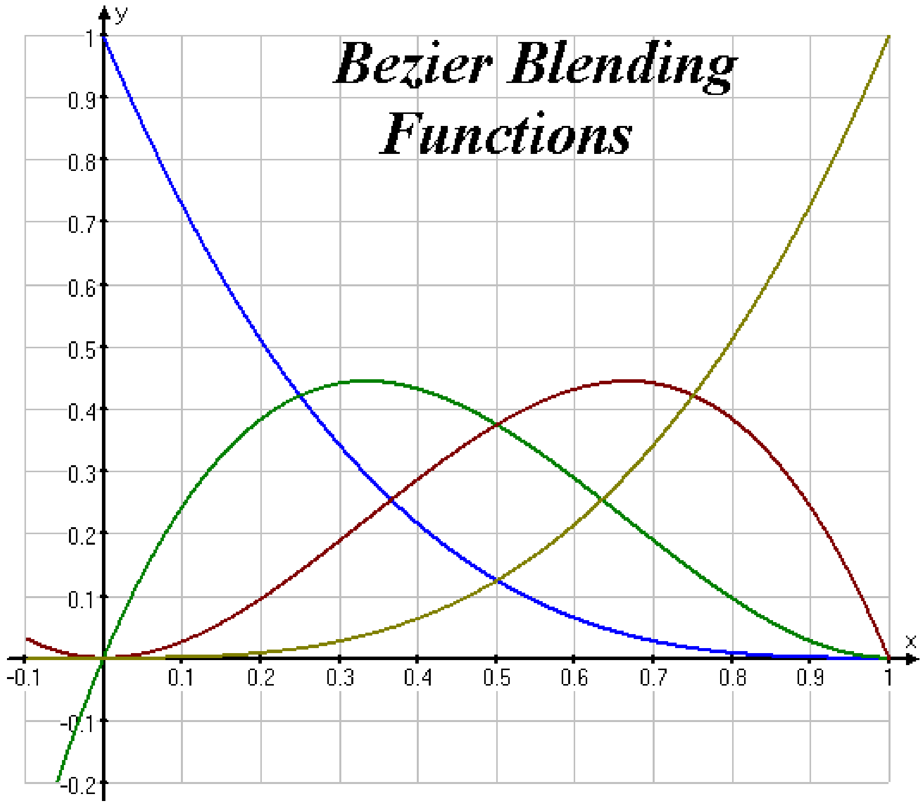
Bézier Blending Functions

- look at blending functions
- family of polynomials called order-3 Bernstein polynomials
  - C(3, k) t^k $(1\text{-}t)^{3\text{-}k}$; 0<= k <= 3
  - all positive in interval [0,1]
  - sum is equal to 1

$$p(t) = \begin{bmatrix} (1-t)^3 \\ 3t(1-t)^2 \\ 3t^2(1-t) \\ t^3 \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{bmatrix}$$

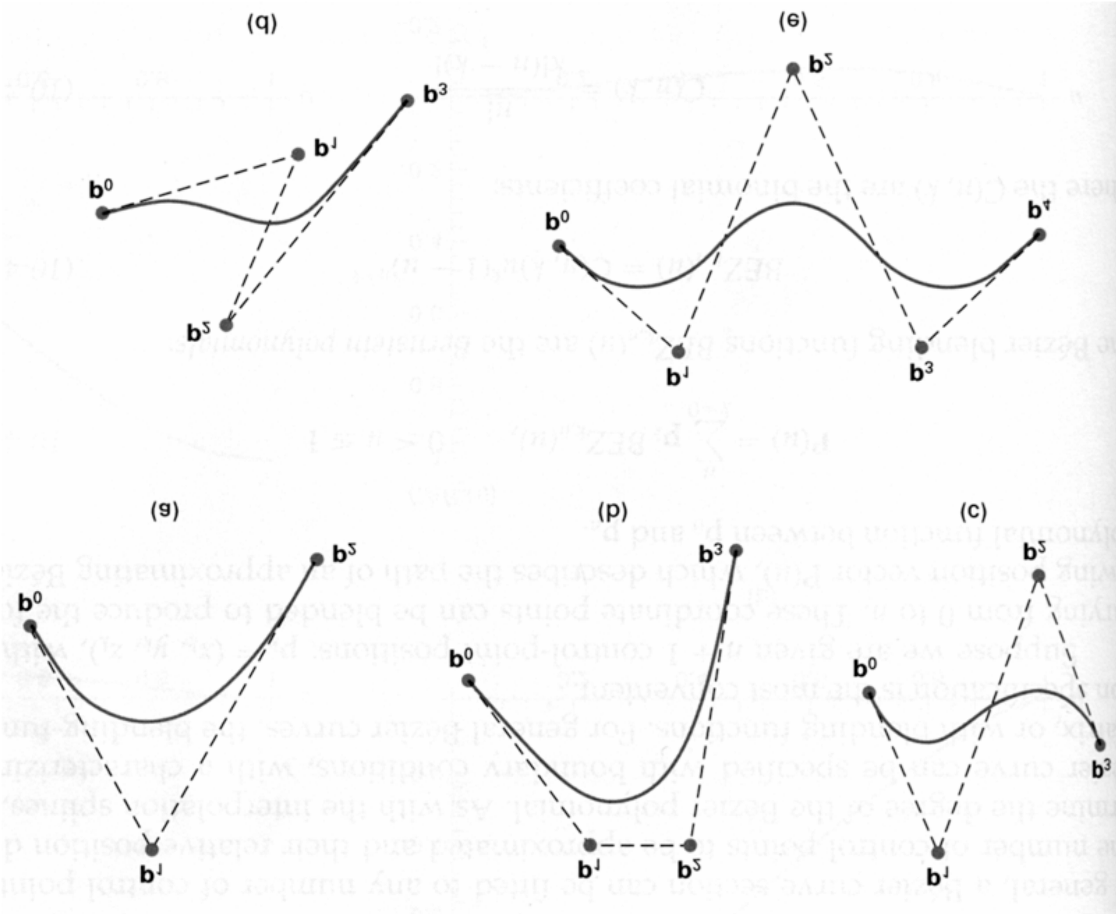
Bézier Blending Functions

- every point on curve is linear combination of control points
- weights of combination are all positive
- sum of weights is 1
- therefore, curve is a convex combination of the control points



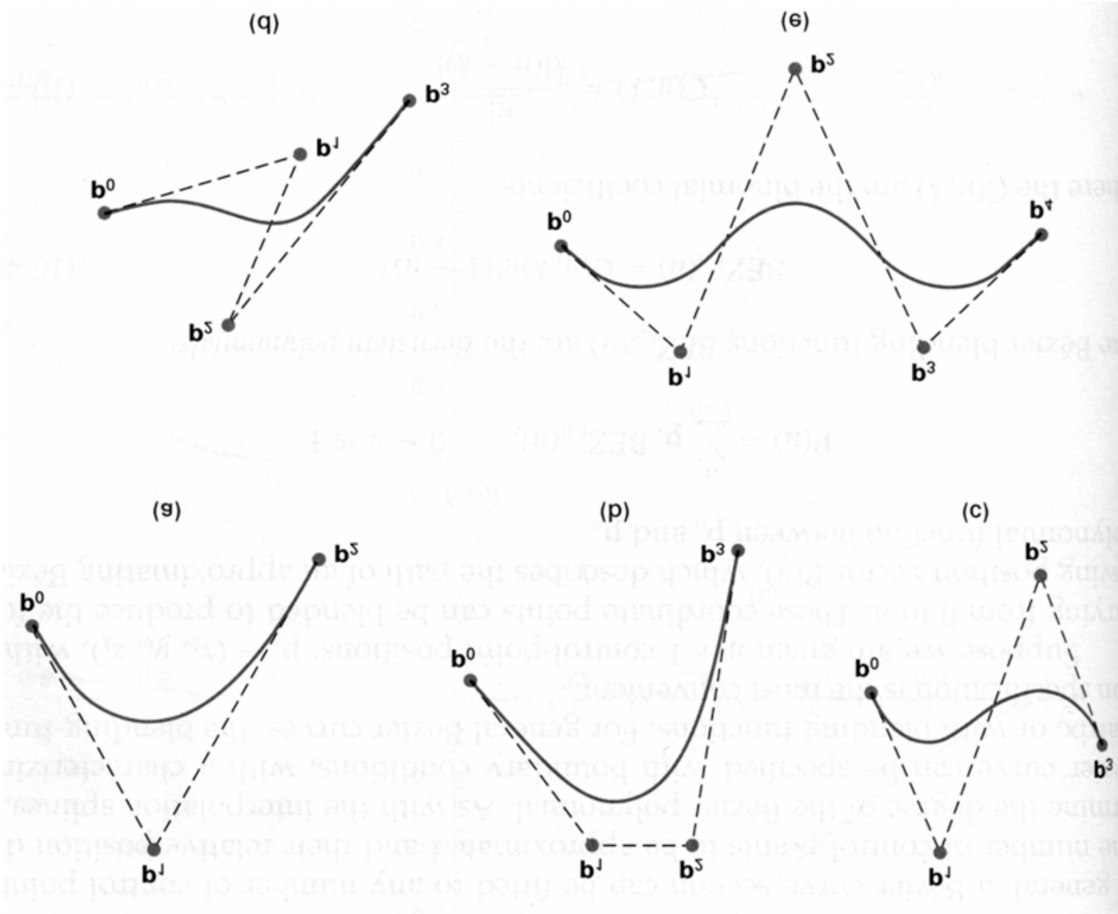
Bézier Curves

- curve will always remain within convex hull (bounding region) defined by control points



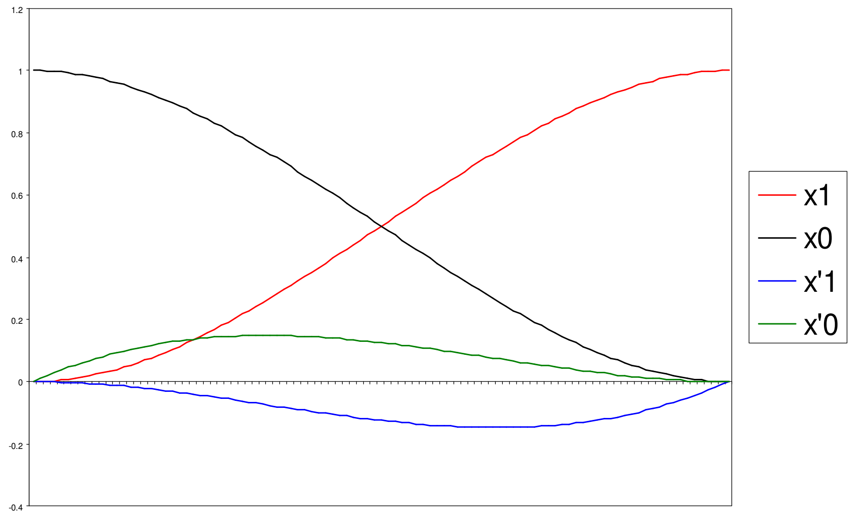
Bézier Curves

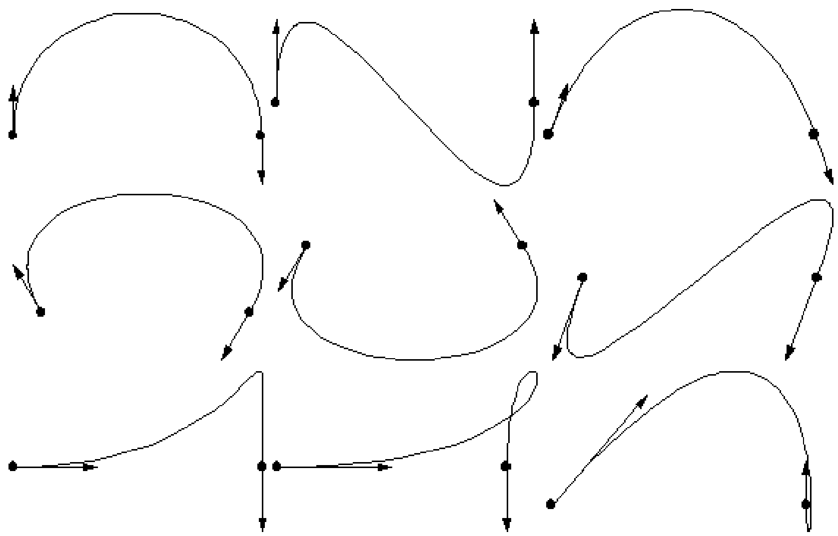
- interpolate between first, last control points
- 1st point's tangent along line joining 1st, 2nd pts
- 4th point's tangent along line joining 3rd, 4th pts



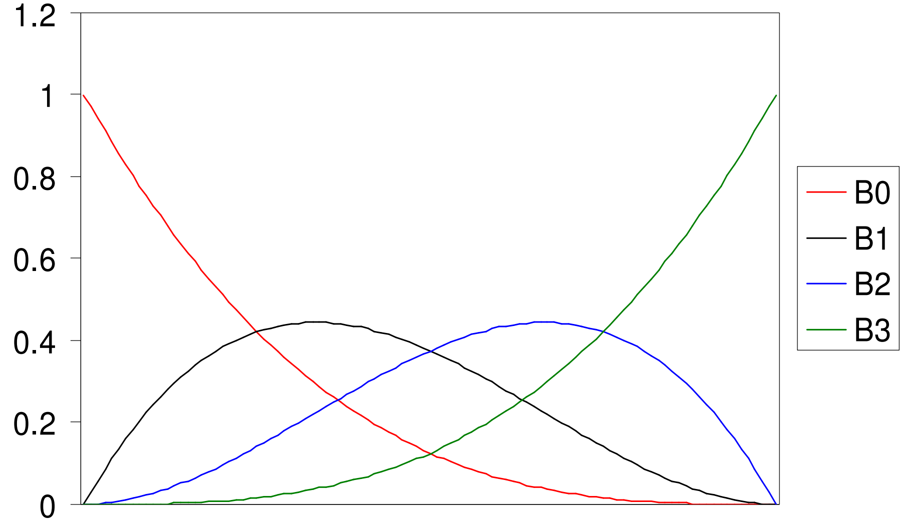
Comparing Hermite and Bézier

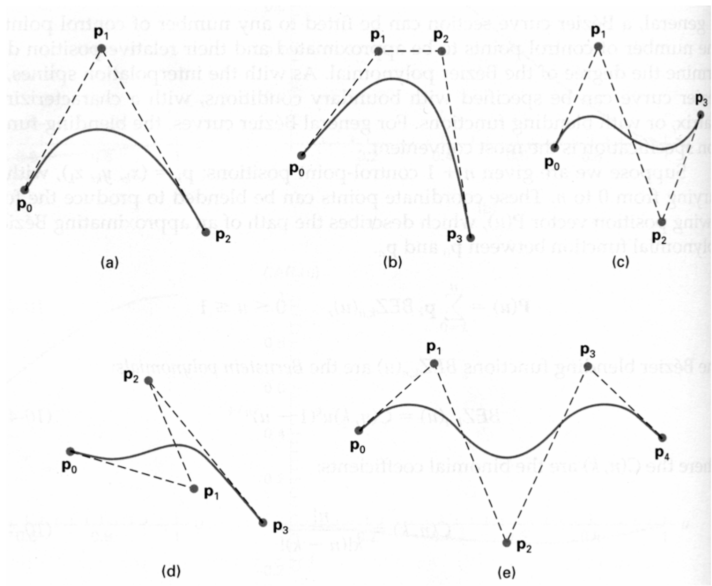
Hermite





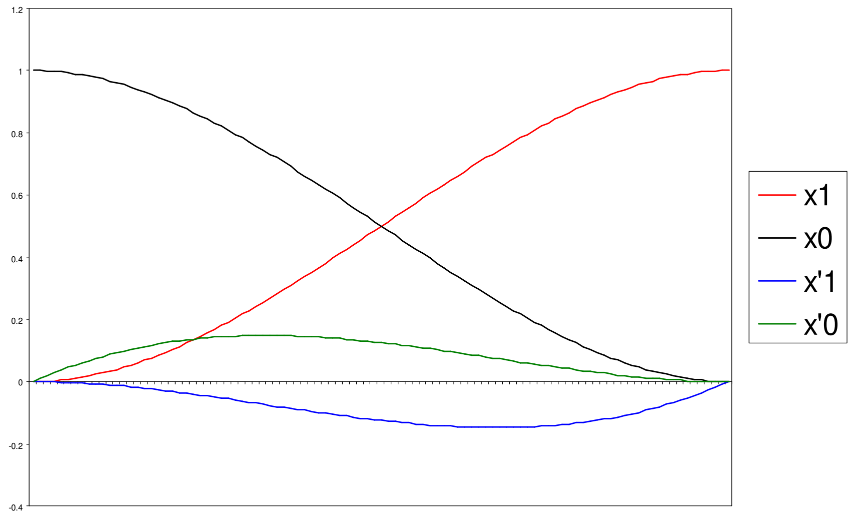
Bézier

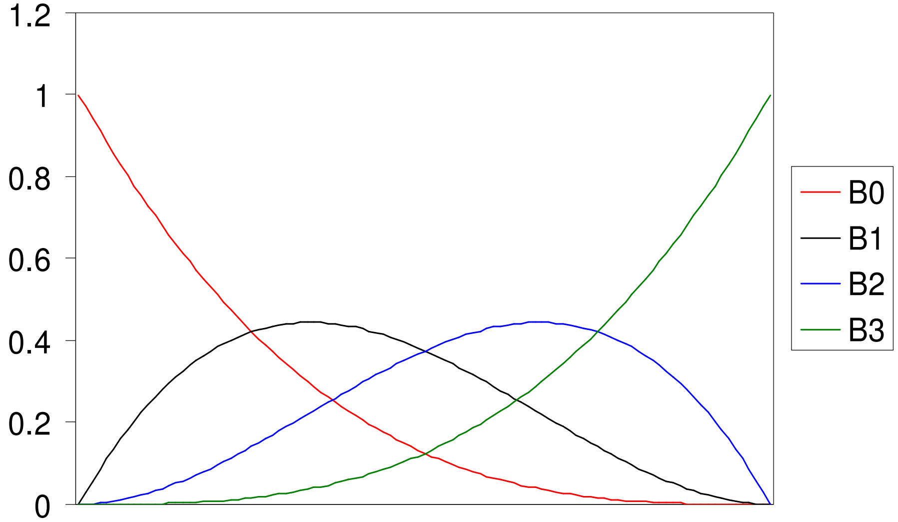


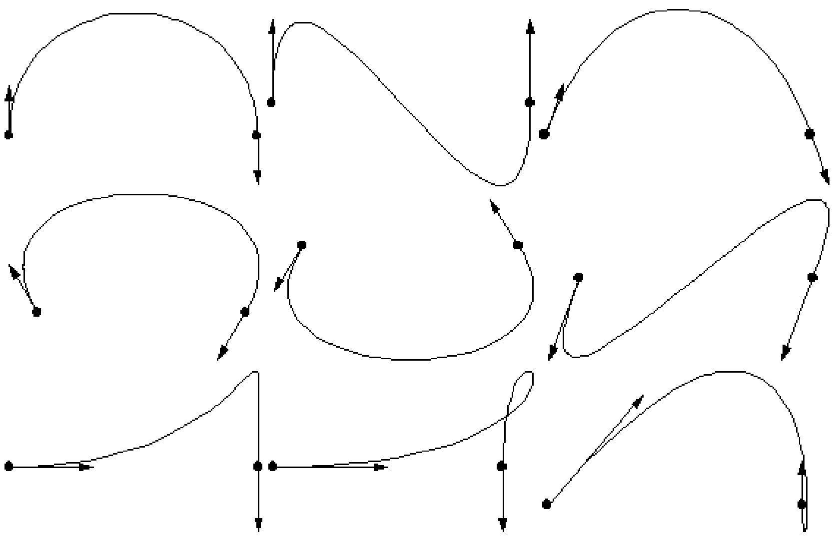


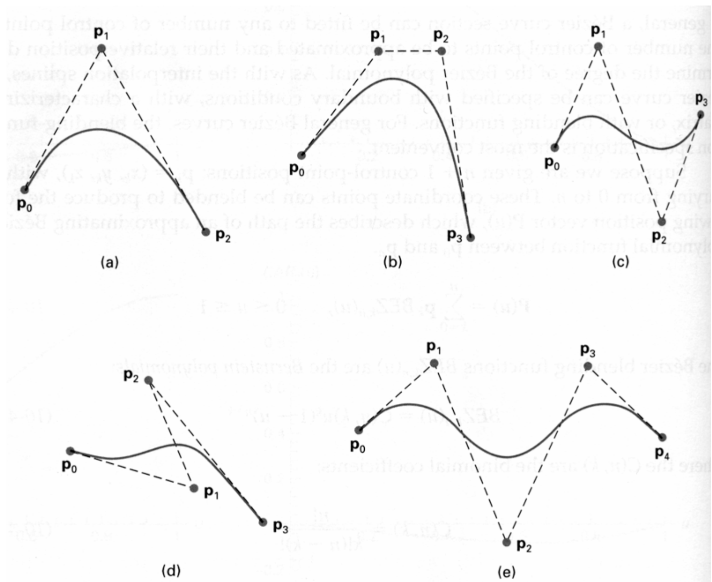
Comparing Hermite and Bezier

demo: www.siggraph.org/education/materials/HyperGraph/modeling/splines/demoprog/curve.html



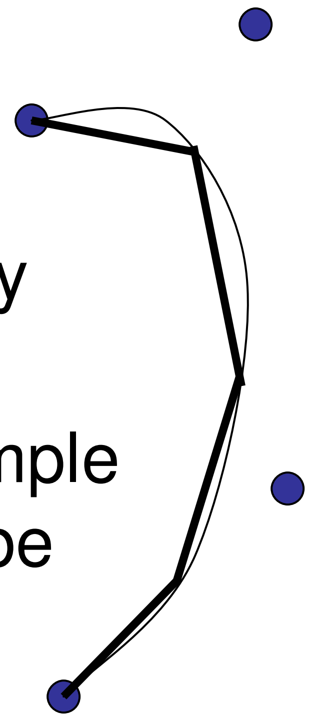






Rendering Bezier Curves: Simple

- evaluate curve at fixed set of parameter values, join points with straight lines
- advantage: very simple
- disadvantages:
  - expensive to evaluate the curve at many points
  - no easy way of knowing how fine to sample points, and maybe sampling rate must be different along curve
  - no easy way to adapt: hard to measure deviation of line segment from exact curve

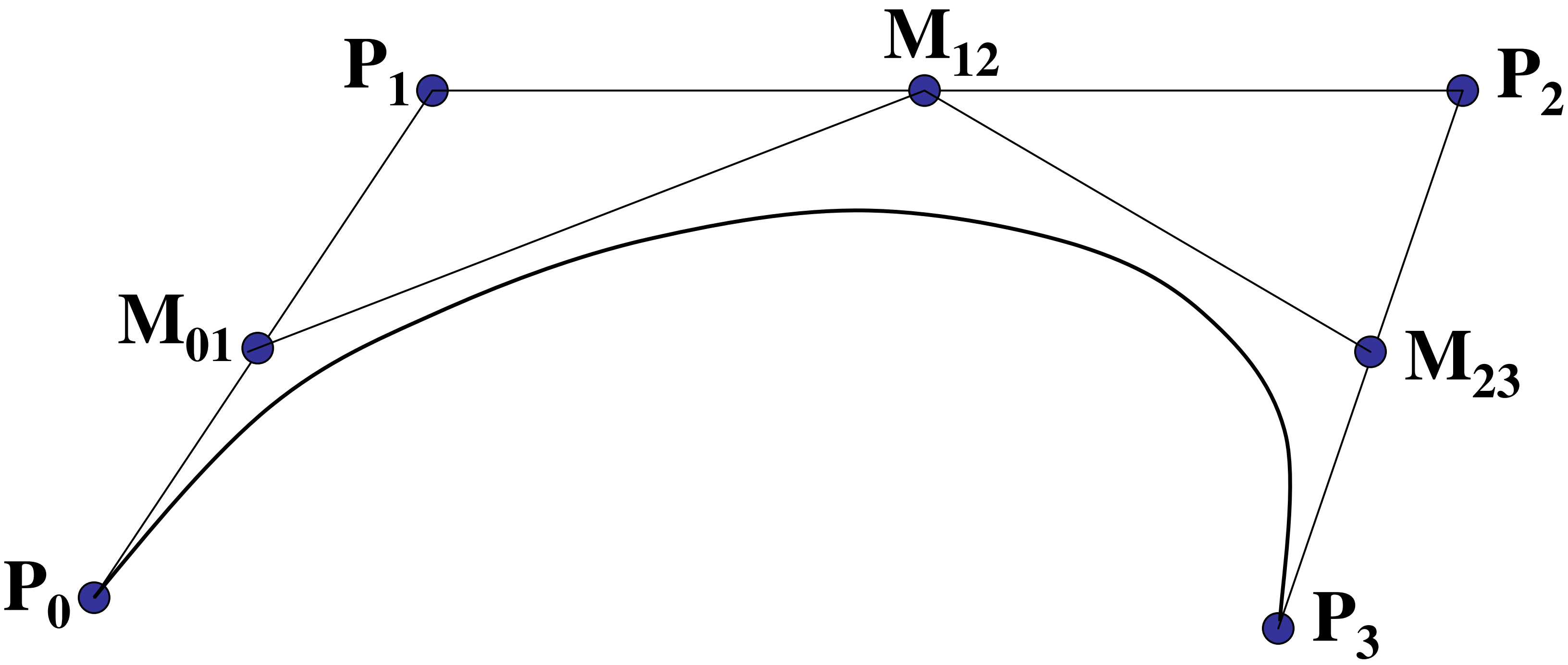


Rendering Beziers: Subdivision

- a cubic Bezier curve can be broken into two shorter cubic Bezier curves that exactly cover original curve
- suggests a rendering algorithm:
  - keep breaking curve into sub-curves
  - stop when control points of each sub-curve are nearly collinear
  - draw the control polygon: polygon formed by control points

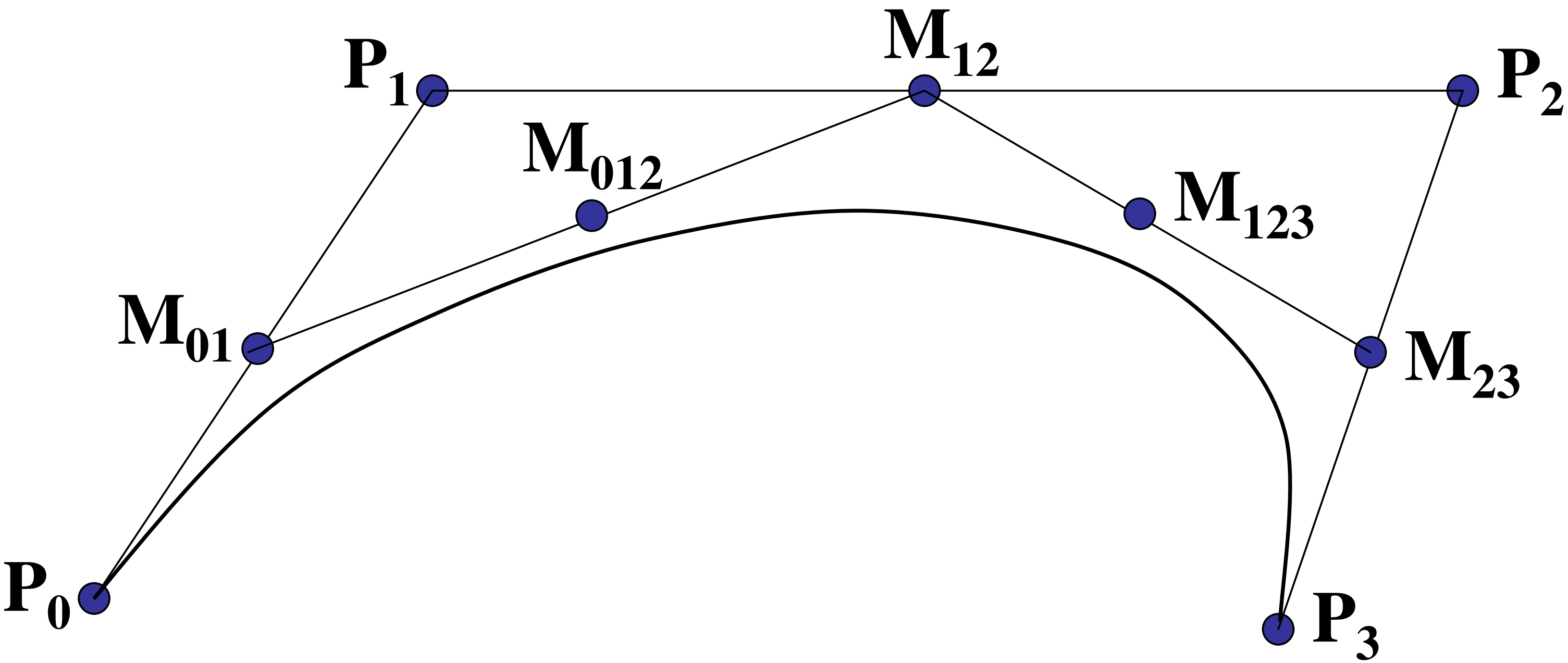
Sub-Dividing Bezier Curves

- step 1: find the midpoints of the lines joining the original control vertices. call them M_{01}, M_{12}, M_{23}



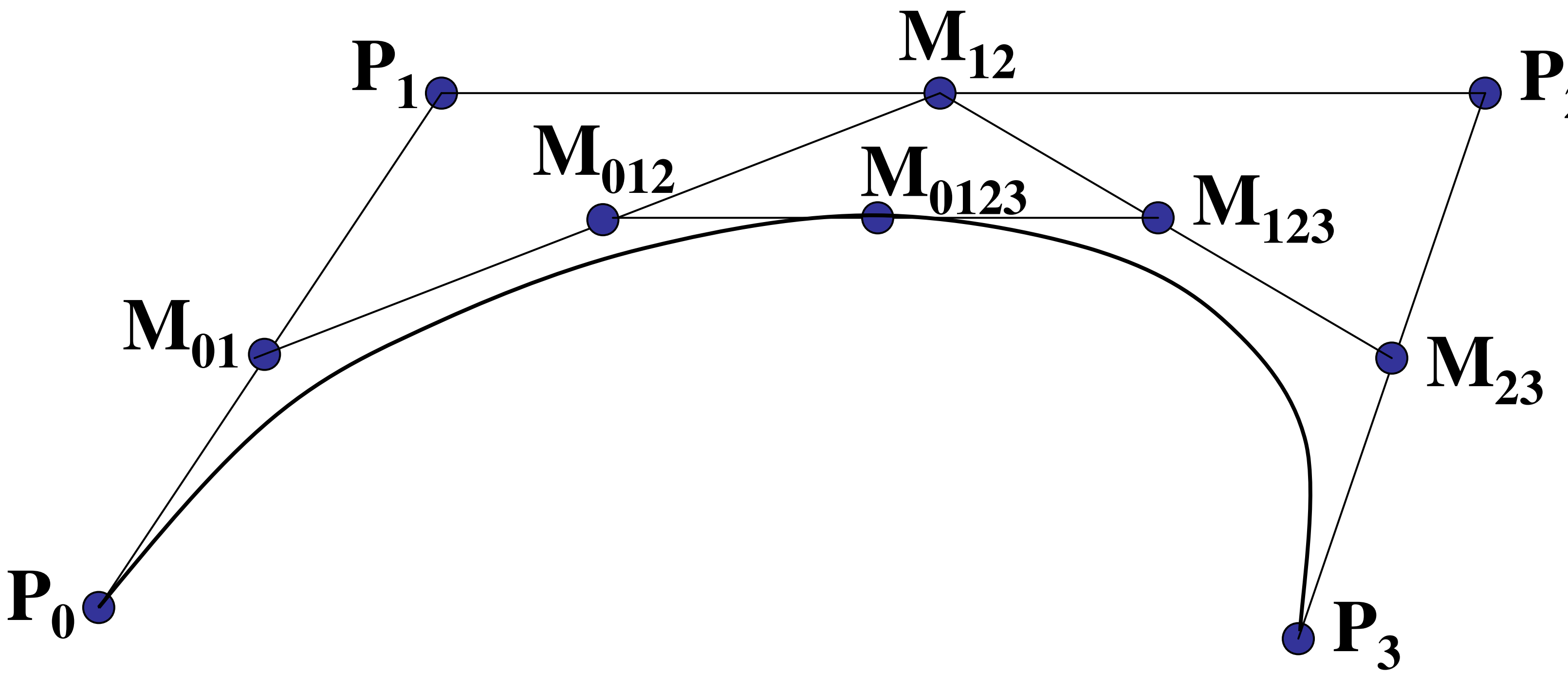
Sub-Dividing Bezier Curves

- step 2: find the midpoints of the lines joining M_{01}, M_{12} and M_{12}, M_{23}. call them M_{012}, M_{123}



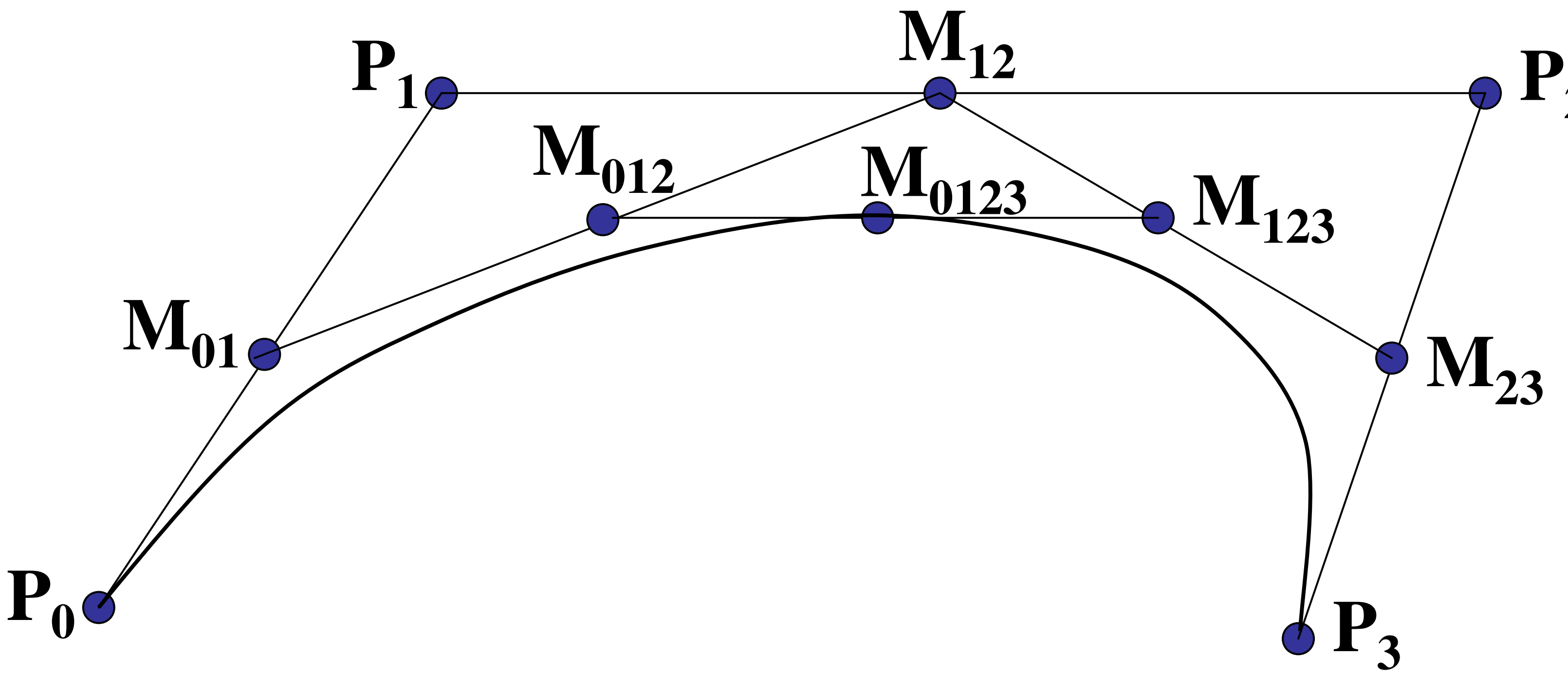
Sub-Dividing Bezier Curves

- step 3: find the midpoint of the line joining M_{012}, M_{123}. call it M_{0123}



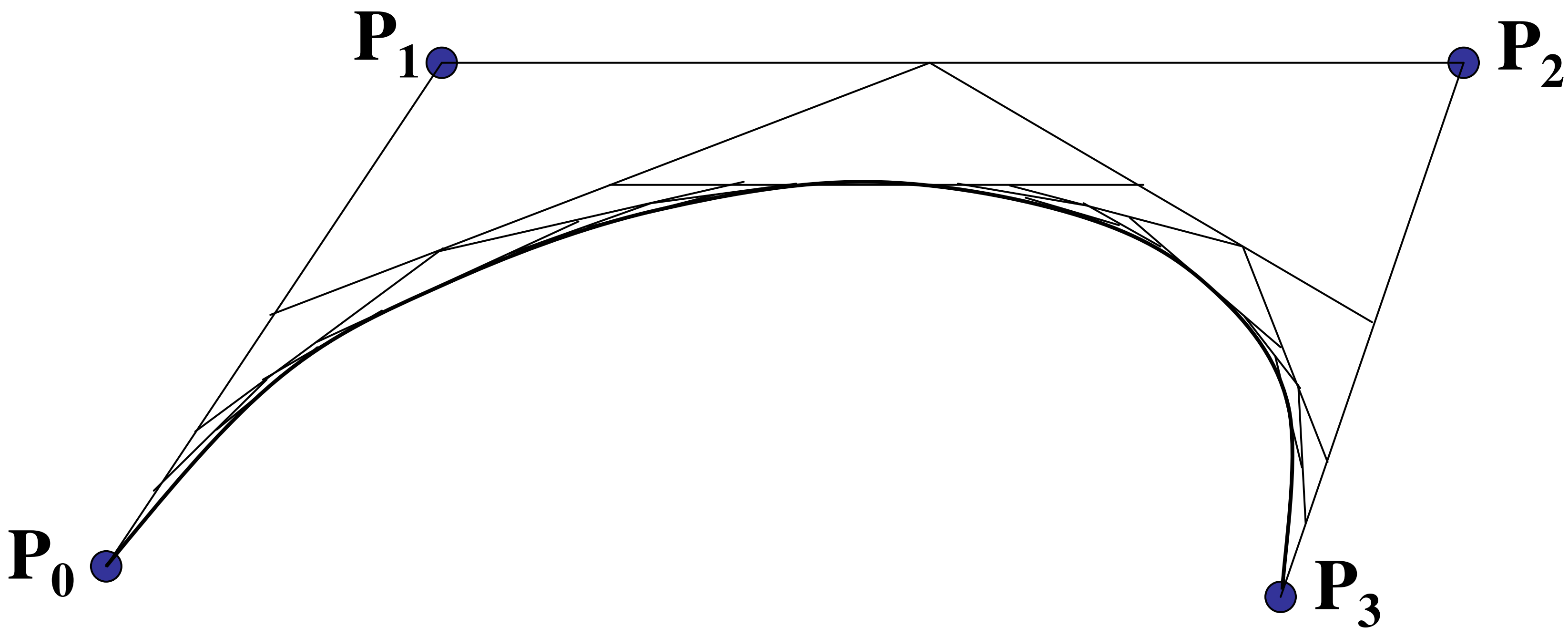
Sub-Dividing Bezier Curves

- curve P_0, M_{01}, M_{012}, M_{0123} exactly follows original from t=0 to t=0.5
- curve M_{0123} , M_{123} , M_{23}, P_3 exactly follows original from t=0.5 to t=1



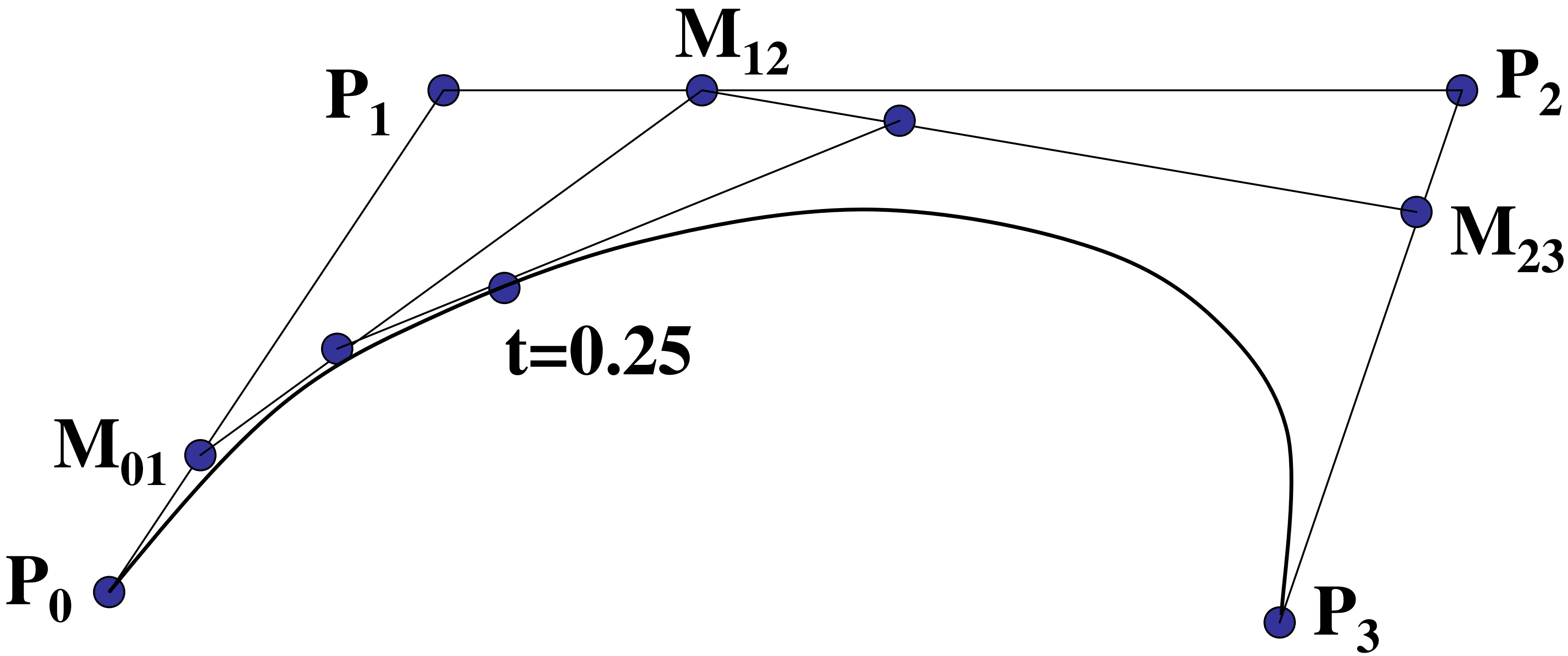
Sub-Dividing Bezier Curves

- continue process to create smooth curve



de Casteljau's Algorithm

- can find the point on a Bezier curve for any parameter value *t* with similar algorithm
  - for *t=0.25*, instead of taking midpoints take points 0.25 of the way

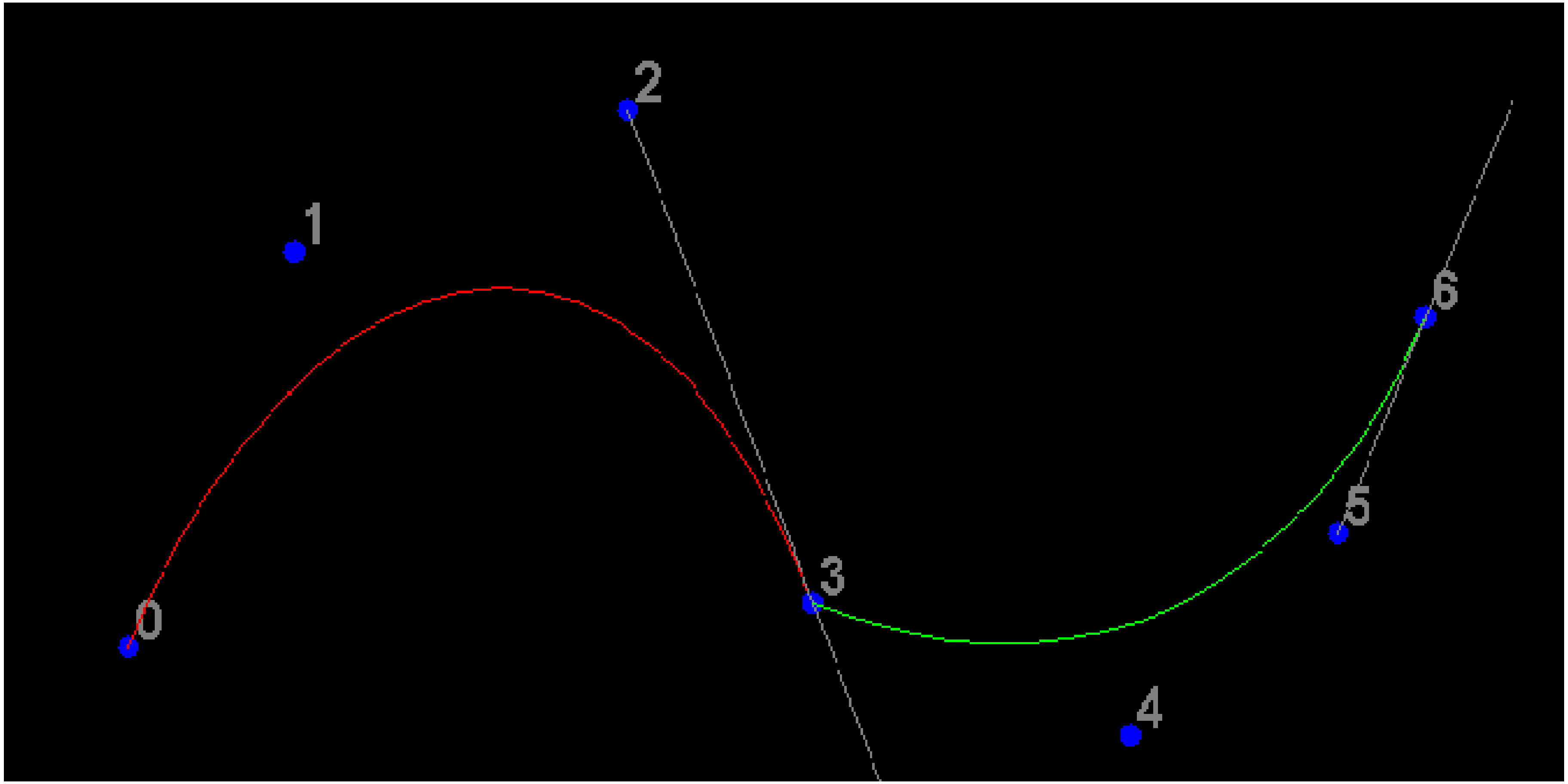


demo: www.saltire.com/applets/advanced_geometry/spline/spline.htm

Longer Curves

- a single cubic Bezier or Hermite curve can only capture a small class of curves
  - at most 2 inflection points
- one solution is to raise the degree
  - allows more control, at the expense of more control points and higher degree polynomials
  - control is not local, one control point influences entire curve
- better solution is to join pieces of cubic curve together into piecewise cubic curves
  - total curve can be broken into pieces, each of which is cubic
  - local control: each control point only influences a limited part of the curve
  - interaction and design is much easier

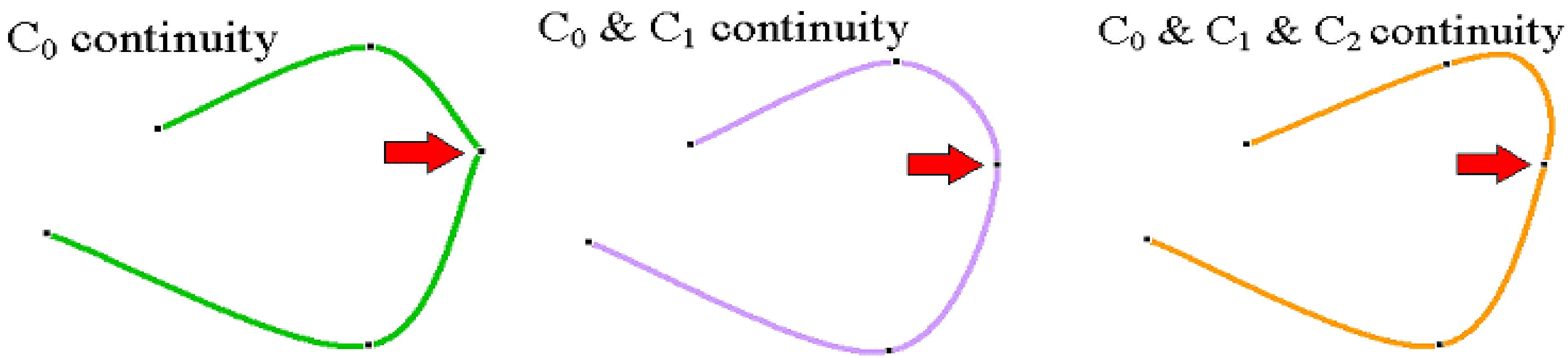
Piecewise Bezier: Continuity Problems



demo: www.cs.princeton.edu/~min/cs426/jar/bezier.html

Continuity

- when two curves joined, typically want some degree of continuity across knot boundary
  - C0, “C-zero”, point-wise continuous, curves share same point where they join
  - C1, “C-one”, continuous derivatives
  - C2, “C-two”, continuous second derivatives



Geometric Continuity

- derivative continuity is important for animation
  - if object moves along curve with constant parametric speed, should be no sudden jump at knots
- for other applications, *tangent continuity* suffices
  - requires that the tangents point in the same direction
  - referred to as G^1 *geometric continuity*
  - curves could be made C^1 with a re-parameterization
  - geometric version of C^2 is G^2, based on curves having the same radius of curvature across the knot

Achieving Continuity

- Hermite curves
  - user specifies derivatives, so C^1 by sharing points and derivatives across knot
- Bezier curves
  - they interpolate endpoints, so C^0 by sharing control pts
  - introduce additional constraints to get C^1
    - parametric derivative is a constant multiple of vector joining first/last 2 control points
    - so C^1 achieved by setting $P_{0,3}=P_{1,0}=J$, and making $P_{0,2}$ and J and $P_{1,1}$ collinear, with $J-P_{0,2}=P_{1,1}-J$
    - C^2 comes from further constraints on $P_{0,1}$ and $P_{1,2}$

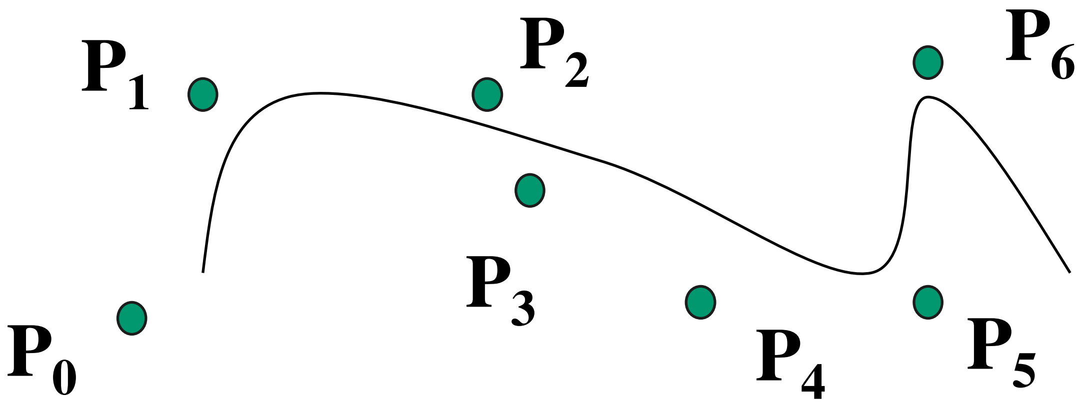
  - leads to...

B-Spline Curve

- start with a sequence of control points
- select four from middle of sequence

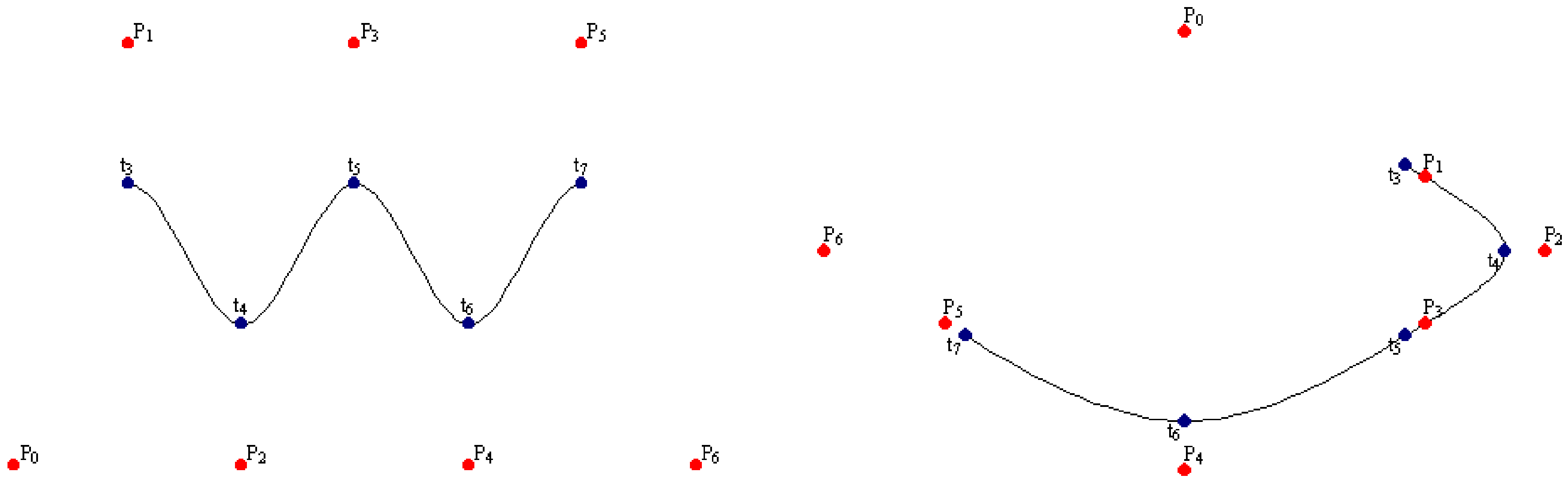
  (p_{i-2}, p_{i-1}, p_i, p_{i+1})

  - Bezier and Hermite goes between p_{i-2} and p_{i+1}
  - B-Spline doesn't interpolate (touch) any of them but approximates the going through p_{i-1} and p_i



B-Spline

- by far the most popular spline used
- C_0, C_1, and C_2 continuous



demo: www.siggraph.org/education/materials/HyperGraph/modeling/splines/demoprog/curve.html

B-Spline

- locality of points

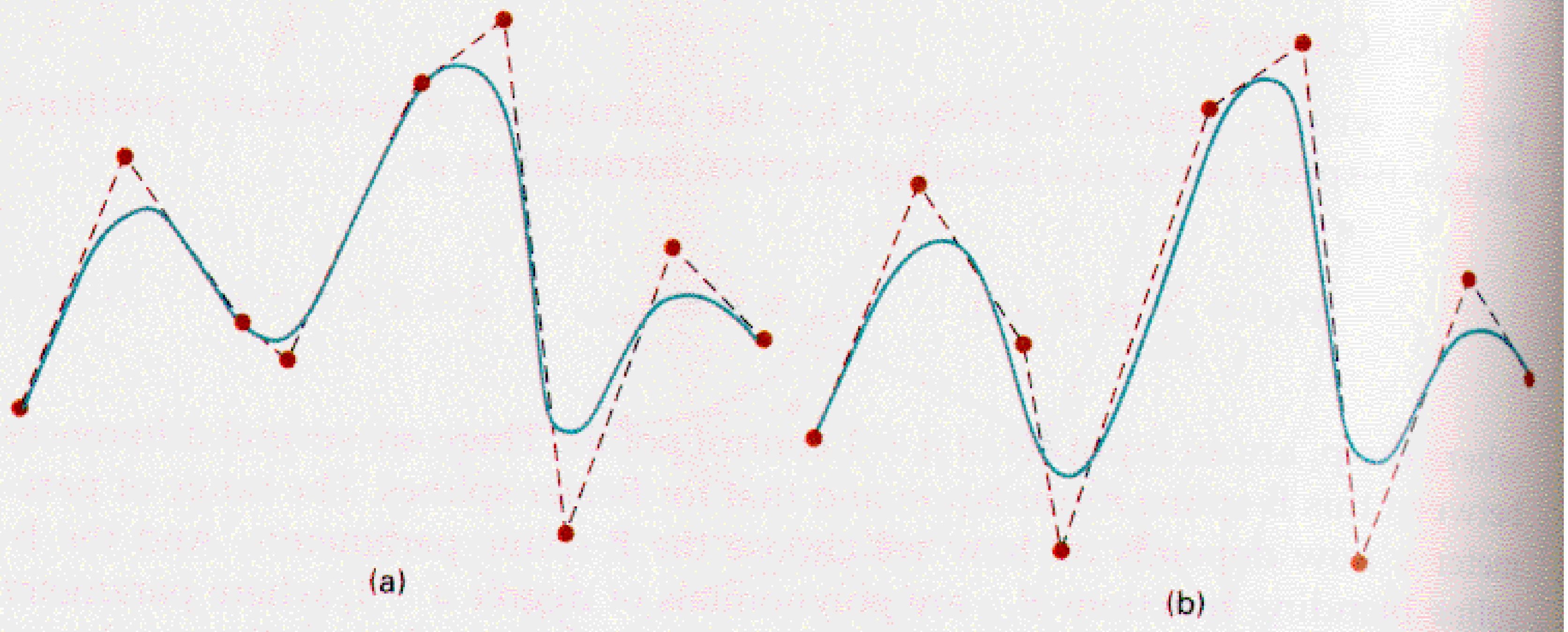
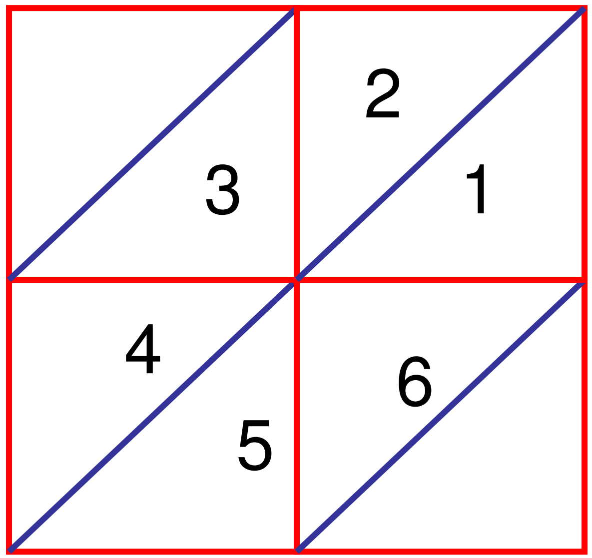


Figure 10-41
Local modification of a B-spline curve. Changing one of the control points in (a) produces curve (b), which is modified only in the neighborhood of the altered control point.

Project 3

- bumpy plane
  - vertex height varies randomly by 20% of face width
  - world coordinate light, camera coord light
  - regenerate terrain
  - toggle colors
- six triangles around a vertex
- [demo]



Project 3: Normals

- calculate once (per terrain)
  - per-face normals
  - then interpolate for per-vertex
- use when drawing
  - specify interleaved with vertices
- explicitly drawing normals
  - bristles at vertices
  - visual debugging

Project 3: Data Structures

- suggestion: 100x100x4 array for vertex coords
- colors?
- normals? per-face, per-vertex

Project 4

- create your own graphics game or tutorial
- required functionality
  - 3D, interactive, lighting/shading
  - texturing, picking, HUD
- advanced functionality pieces
  - two for 1-person team
  - four for 2-person team
  - six for 3-person eam

P4: Advanced Functionality

- (new) navigation
- procedural modelling/textures
  - particle systems
- collision detection
- simulated dynamics
- level of detail control
- advanced rendering effects
- whatever else you want to do
  - proposal is a check with me

P4 Proposal

- due Wed 1 Jun 4pm
  - either electronic handin, or box handin for hardcopy
  - short (< 1 page) description
    - how game works
    - how it will fulfill required functionality
    - advanced functionality
  - must include at least one annotated screenshot mockup sketch
    - hand-drawn scanned or using computer tools

P4 Writeup

- **what:** a high level description of what you've created, including an explicit list of the advanced functionality items
- **how:** mid-level description of the algorithms and data structures that you've used
- **howto:** detailed instructions of the low-level mechanics of how to actually play (keyboard controls, etc)
- **sources:** sources of inspiration and ideas (especially any source code you looked at for inspiration on the Internet)
- include screen shots with handin for HOF eligibility

P4 Grading

- final project due 11:59pm Fri Jun 17
  - face to face demos again
  - I will be grading
- grading
  - 50% base: required functions, gameplay, etc
  - 50% advanced functionality
  - buckets, tentative mapping
    - zero = 0
    - minus = 40
    - check-minus = 60
    - check = 80
    - check-plus = 100
    - plus 105