University of British Columbia
CPSC 314 Computer Graphics
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Viewing II

Week 4, Mon Jan 25

http://www.ugrad.cs.ubc.ca/~cs314/Vjan2010

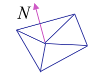
Reading for This and Next 2 Lectures

- FCG Chapter 7 Viewing
- FCG Section 6.3.1 Windowing Transforms
- RB rest of Chap Viewing
- RB rest of App Homogeneous Coords

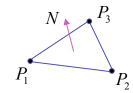
Review: Display Lists

- precompile/cache block of OpenGL code for reuse
  - usually more efficient than immediate mode
    - exact optimizations depend on driver
  - good for multiple instances of same object
    - but cannot change contents, not parametrizable
  - good for static objects redrawn often
    - display lists persist across multiple frames
    - interactive graphics: objects redrawn every frame from new viewpoint from moving camera
  - can be nested hierarchically
- snowman example: 3x performance improvement, 36K polys

Review: Computing Normals



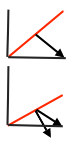
- normal
  - direction specifying orientation of polygon
    - w=0 means direction with homogeneous coords
    - vs. w=1 for points/vectors of object vertices
  - used for lighting
    - must be normalized to unit length
  - can compute if not supplied with object



$$N = (P_2 - P_1) \times (P_3 - P_1)$$

Review: Transforming Normals

- cannot transform normals using same matrix as points
  - nonuniform scaling would cause to be not perpendicular to desired plane!



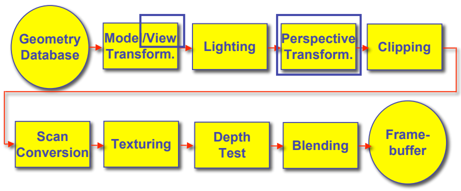
$$P \rightarrow P' = MP$$
$$N \rightarrow N' = QN$$

given M,
what should Q be?

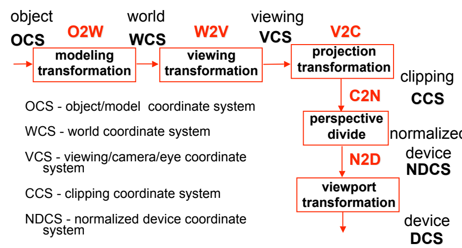
$$\mathbf{Q} = \left(\mathbf{M}^{-1}\right)^T$$

inverse transpose of the modelling transformation

Review: Rendering Pipeline



Review: Projective Rendering Pipeline



OCS - object/model coordinate system

WCS - world coordinate system

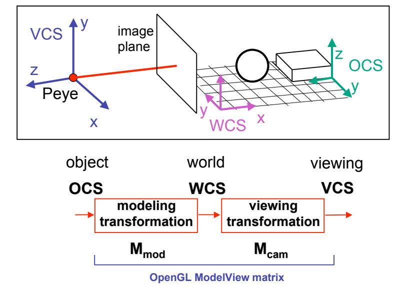
VCS - viewing/camera/eye coordinate system

CCS - clipping coordinate system

NDCS - normalized device coordinate system

DCS - device/display/screen coordinate system

Review: Viewing Transformation



Review: Basic Viewing

- starting spot - OpenGL
  - camera at world origin
    - probably inside an object
  - y axis is up
  - looking down negative z axis
    - why? RHS with x horizontal, y vertical, z out of screen
- translate backward so scene is visible
  - move distance d = focal length
- where is camera in P1 template code?
  - 5 units back, looking down -z axis

Convenient Camera Motion

- rotate/translate/scale versus
  - eye point, gaze/lookat direction, up vector
  - demo: Robins transformation, projection

OpenGL Viewing Transformation

```
gluLookAt(ex,ey,ez,lx,ly,lz,ux,uy,uz)
```

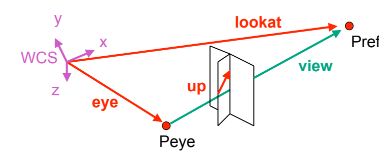
- postmultiplies current matrix, so to be safe:

```
glMatrixMode(GL_MODELVIEW);
glLoadIdentity();
gluLookAt(ex,ey,ez,lx,ly,lz,ux,uy,uz)
// now ok to do model transformations
```

- demo: Nate Robins tutorial *projection*

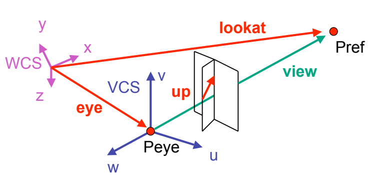
Convenient Camera Motion

- rotate/translate/scale versus
  - eye point, gaze/lookat direction, up vector



From World to View Coordinates: W2V

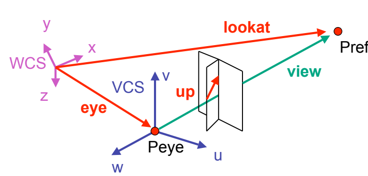
- translate **eye** to origin
- rotate **view** vector (**lookat** – **eye**) to **w** axis
- rotate around **w** to bring **up** into **vw**-plane



Deriving W2V Transformation

- translate **eye** to origin

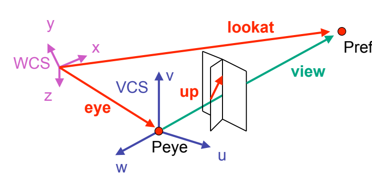
$$\mathbf{T} = \begin{bmatrix} 1 & 0 & 0 & e_x \\ 0 & 1 & 0 & e_y \\ 0 & 0 & 1 & e_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



Deriving W2V Transformation

- rotate **view** vector (**lookat** – **eye**) to **w** axis
  - **w**: normalized opposite of **view/gaze** vector **g**

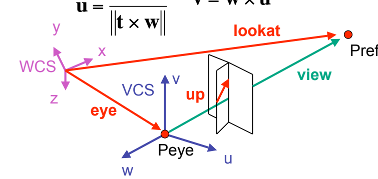
$$\mathbf{w} = -\hat{\mathbf{g}} = -\frac{\mathbf{g}}{\|\mathbf{g}\|}$$



Deriving W2V Transformation

- rotate around **w** to bring **up** into **vw**-plane
  - **u** should be perpendicular to **vw**-plane, thus perpendicular to **w** and **up** vector **t**
  - **v** should be perpendicular to **u** and **w**

$$\mathbf{u} = \frac{\mathbf{t} \times \mathbf{w}}{\|\mathbf{t} \times \mathbf{w}\|} \qquad \mathbf{v} = \mathbf{w} \times \mathbf{u}$$



Deriving W2V Transformation

- rotate from WCS **xyz** into **uvw** coordinate system with matrix that has columns **u**, **v**, **w**

$$\mathbf{u} = \frac{\mathbf{t} \times \mathbf{w}}{\|\mathbf{t} \times \mathbf{w}\|} \qquad \mathbf{v} = \mathbf{w} \times \mathbf{u} \qquad \mathbf{w} = -\hat{\mathbf{g}} = -\frac{\mathbf{g}}{\|\mathbf{g}\|}$$

$$\mathbf{R} = \begin{bmatrix} u_x & v_x & w_x & 0 \\ u_y & v_y & w_y & 0 \\ u_z & v_z & w_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \mathbf{T} = \begin{bmatrix} 1 & 0 & 0 & e_x \\ 0 & 1 & 0 & e_y \\ 0 & 0 & 1 & e_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad M_{W2V}=TR$$

- reminder: rotate from **uvw** to **xyz** coord sys with matrix **M** that has columns **u,v,w**

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W2V vs. V2W

- M_{W2V}=TR

$$\mathbf{T} = \begin{bmatrix} 1 & 0 & 0 & e_x \\ 0 & 1 & 0 & e_y \\ 0 & 0 & 1 & e_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \mathbf{R} = \begin{bmatrix} u_x & v_x & w_x & 0 \\ u_y & v_y & w_y & 0 \\ u_z & v_z & w_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- we derived position of camera in world
  - invert for world with respect to camera
- $M_{V2W}=(M_{W2V})^{-1}=R^{-1}T^{-1}$

$$\mathbf{R}^{-1} = \begin{bmatrix} u_x & u_y & u_z & 0 \\ v_x & v_y & v_z & 0 \\ w_x & w_y & w_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \mathbf{T}^{-1} = \begin{bmatrix} 1 & 0 & 0 & -e_x \\ 0 & 1 & 0 & -e_y \\ 0 & 0 & 1 & -e_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

  - inverse is transpose for orthonormal matrices
  - inverse is negative for translations

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W2V vs. V2W

- M_{W2V}=TR

$$\mathbf{T} = \begin{bmatrix} 1 & 0 & 0 & e_x \\ 0 & 1 & 0 & e_y \\ 0 & 0 & 1 & e_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \mathbf{R} = \begin{bmatrix} u_x & v_x & w_x & 0 \\ u_y & v_y & w_y & 0 \\ u_z & v_z & w_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- we derived position of camera in world
  - invert for world with respect to camera
- $M_{V2W}=(M_{W2V})^{-1}=R^{-1}T^{-1}$

$$\mathbf{M}_{view2world} = \begin{bmatrix} u_x & u_y & u_z & 0 \\ v_x & v_y & v_z & 0 \\ w_x & w_y & w_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -e_x \\ 0 & 1 & 0 & -e_y \\ 0 & 0 & 1 & -e_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} u_x & u_y & u_z & -e_x \\ v_x & v_y & v_z & -e_y \\ w_x & w_y & w_z & -e_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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Moving the Camera or the World?

- two equivalent operations
  - move camera one way vs. move world other way
- example
  - initial OpenGL camera: at origin, looking along -z axis
  - create a unit square parallel to camera at z = -10
  - translate in z by 3 possible in two ways
    - camera moves to z = -3
      - Note OpenGL models viewing in left-hand coordinates
    - camera stays put, but world moves to -7
  - resulting image same either way
    - possible difference: are lights specified in world or view coordinates?
- third operation: scaling the world
  - smaller vs farther away

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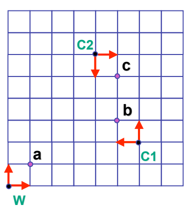
World vs. Camera Coordinates Example



$a = (1,1)_W$

$b = (1,1)_{C1} = (5,3)_W$

$c = (1,1)_{C2} = (1,3)_{C1} = (5,5)_W$

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Projections I

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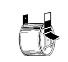
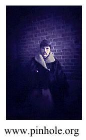
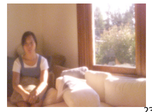
Pinhole Camera

- ingredients
  - box, film, hole punch
- result
  - picture







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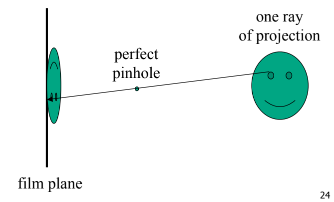
Pinhole Camera

- theoretical perfect pinhole
- light shining through tiny hole into dark space yields upside-down picture



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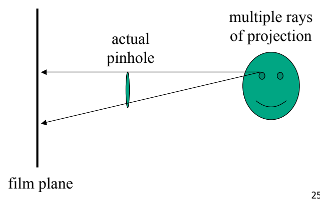
Pinhole Camera

- non-zero sized hole
- blur: rays hit multiple points on film plane



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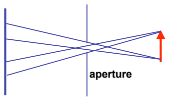
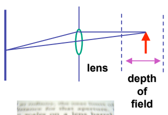
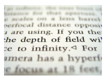
Real Cameras

- pinhole camera has small aperture (lens opening)
  - minimize blur
- problem: hard to get enough light to expose the film
- solution: lens
  - permits larger apertures
  - permits changing distance to film plane without actually moving it
    - cost: limited depth of field where image is in focus







http://en.wikipedia.org/wiki/Image:DOF-ShallowDepthofField.jpg

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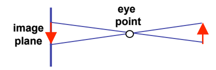
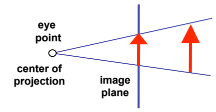
Graphics Cameras

- real pinhole camera: image inverted



- computer graphics camera: convenient equivalent



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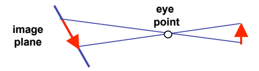
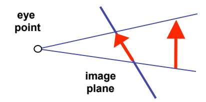
General Projection

- image plane need not be perpendicular to view plane





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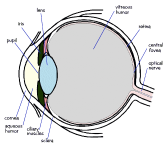
Perspective Projection

- our camera must model perspective



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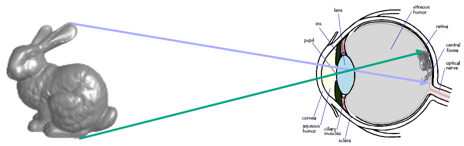
Perspective Projection

- our camera must model perspective



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Projective Transformations

- planar geometric projections
  - planar: onto a plane
  - geometric: using straight lines
  - projections: 3D -> 2D
- aka projective mappings
- counterexamples?

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Projective Transformations

- properties
  - lines mapped to lines and triangles to triangles
  - parallel lines do NOT remain parallel
    - e.g. rails vanishing at infinity

  - affine combinations are NOT preserved
    - e.g. center of a line does not map to center of projected line (perspective foreshortening)

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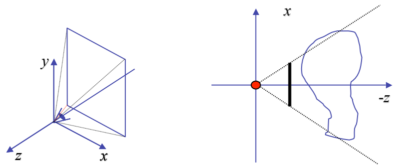
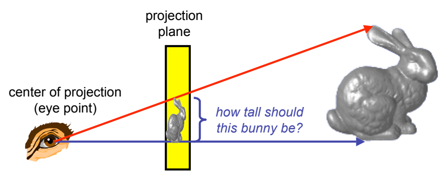
Perspective Projection

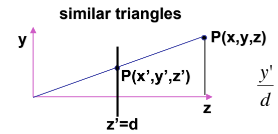
- project all geometry
  - through common center of projection (eye point)
  - onto an image plane



Perspective Projection



Basic Perspective Projection



$$\frac{y'}{d} = \frac{y}{z} \rightarrow y' = \frac{y \cdot d}{z}$$

$$\frac{x'}{d} = \frac{x}{z} \rightarrow x' = \frac{x \cdot d}{z} \qquad \text{but} \qquad z' = d$$

- nonuniform foreshortening
  - not affine

Perspective Projection

- desired result for a point $[x, y, z, 1]^T$ projected onto the view plane:

$$\frac{x'}{d} = \frac{x}{z}, \quad \frac{y'}{d} = \frac{y}{z}$$

$$x' = \frac{x \cdot d}{z} = \frac{x}{z/d}, \quad y' = \frac{y \cdot d}{z} = \frac{y}{z/d}, \quad z' = d$$

- what could a matrix look like to do this?

Simple Perspective Projection Matrix

$$\begin{bmatrix} \frac{x}{z/d} \\ \frac{y}{z/d} \\ d \end{bmatrix}$$

Simple Perspective Projection Matrix

$$\begin{bmatrix} \frac{x}{z/d} \\ \frac{y}{z/d} \\ d \end{bmatrix} \text{ is homogenized version of } \begin{bmatrix} x \\ y \\ z \\ z/d \end{bmatrix} \text{ where w = z/d}$$

Simple Perspective Projection Matrix

$$\begin{bmatrix} \frac{x}{z/d} \\ \frac{y}{z/d} \\ d \end{bmatrix} \text{ is homogenized version of } \begin{bmatrix} x \\ y \\ z \\ z/d \end{bmatrix} \text{ where w = z/d}$$

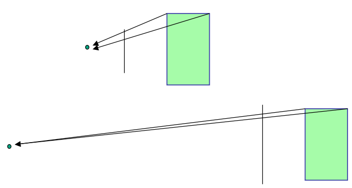
$$\begin{bmatrix} x \\ y \\ z \\ z/d \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Perspective Projection

- expressible with 4x4 homogeneous matrix
  - use previously untouched bottom row
- perspective projection is irreversible
  - many 3D points can be mapped to same (x, y, d) on the projection plane
  - no way to retrieve the unique z values

Moving COP to Infinity

- as COP moves away, lines approach parallel
  - when COP at infinity, orthographic view



Orthographic Camera Projection

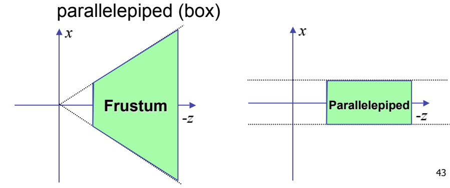
- camera's back plane parallel to lens
- infinite focal length
- no perspective convergence
- just throw away z values

$$\begin{bmatrix} x_p \\ y_p \\ z_p \end{bmatrix} = \begin{bmatrix} x \\ y \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x_p \\ y_p \\ z_p \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

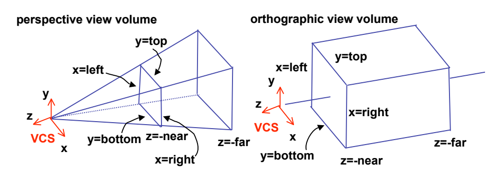
Perspective to Orthographic

- transformation of space
  - center of projection moves to infinity
  - view volume transformed
    - from frustum (truncated pyramid) to parallelepiped (box)



View Volumes

- specifies field-of-view, used for clipping
- restricts domain of ***z*** stored for visibility test

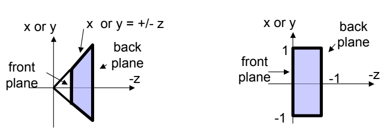


Canonical View Volumes

- standardized viewing volume representation

perspective

orthographic
orthogonal
parallel



Why Canonical View Volumes?

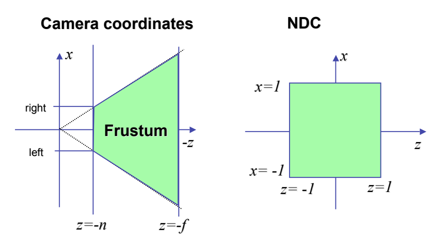
- permits standardization
  - clipping
    - easier to determine if an arbitrary point is enclosed in volume with canonical view volume vs. clipping to six arbitrary planes
  - rendering
    - projection and rasterization algorithms can be reused

Normalized Device Coordinates

- convention
  - viewing frustum mapped to specific parallelepiped
    - Normalized Device Coordinates (NDC)
    - same as clipping coords
  - only objects inside the parallelepiped get rendered
  - which parallelepiped?
    - depends on rendering system

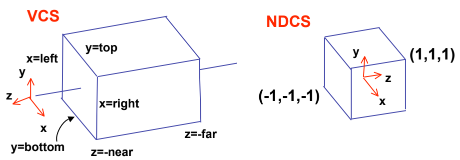
Normalized Device Coordinates

left/right x =+/- 1, top/bottom y =+/- 1, near/far z =+/- 1



Understanding Z

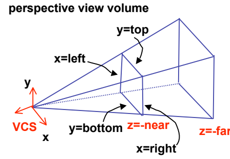
- z axis flip changes coord system handedness
  - RHS before projection (eye/view coords)
  - LHS after projection (clip, norm device coords)

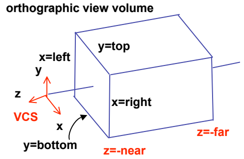


Understanding Z

near, far always positive in OpenGL calls

```
glOrtho(left,right,bot,top,near,far);
glFrustum(left,right,bot,top,near,far);
glPerspective(fovy,aspect,near,far);
```



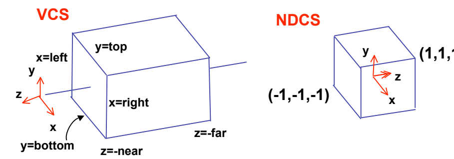


Understanding Z

- why near and far plane?
  - near plane:
    - avoid singularity (division by zero, or very small numbers)
  - far plane:
    - store depth in fixed-point representation (integer), thus have to have fixed range of values (0…1)
    - avoid/reduce numerical precision artifacts for distant objects

Orthographic Derivation

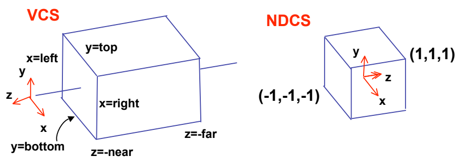
- scale, translate, reflect for new coord sys



Orthographic Derivation

- scale, translate, reflect for new coord sys

$$y' = a \cdot y + b \qquad \begin{aligned} y = top &\rightarrow y' = 1 \\ y = bot &\rightarrow y' = -1 \end{aligned}$$



Orthographic Derivation

- scale, translate, reflect for new coord sys

$$y' = a \cdot y + b \qquad \begin{aligned} y = top &\rightarrow y' = 1 \\ y = bot &\rightarrow y' = -1 \end{aligned} \qquad \begin{aligned} 1 &= a \cdot top + b \\ -1 &= a \cdot bot + b \end{aligned}$$

$$\begin{aligned} & b = 1 - a \cdot top, b = -1 - a \cdot bot \\ & 1 - a \cdot top = -1 - a \cdot bot \\ & 1 - (-1) = -a \cdot bot - (-a \cdot top) \\ & 2 = a(-bot + top) \\ & a = \frac{2}{top - bot} \end{aligned}$$

$$\begin{aligned} & 1 = \frac{2}{top - bot} top + b \\ & b = 1 - \frac{2 \cdot top}{top - bot} \\ & b = \frac{(top - bot) - 2 \cdot top}{top - bot} \\ & b = \frac{-top - bot}{top - bot} \end{aligned}$$

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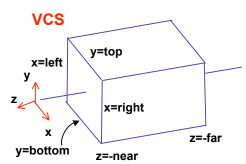
$$y' = a \cdot y + b \qquad \begin{aligned} y = top &\rightarrow y' = 1 \\ y = bot &\rightarrow y' = -1 \end{aligned}$$



$$a = \frac{2}{top - bot}$$

$$b = -\frac{top + bot}{top - bot}$$

same idea for right/left, far/near

Orthographic Derivation

- scale, translate, reflect for new coord sys

$$P' = \begin{bmatrix} \frac{2}{right - left} & 0 & 0 & -\frac{right + left}{right - left} \\ 0 & \frac{2}{top - bot} & 0 & -\frac{top + bot}{top - bot} \\ 0 & 0 & \frac{-2}{far - near} & -\frac{far + near}{far - near} \\ 0 & 0 & 0 & 1 \end{bmatrix} P$$

Orthographic Derivation

- scale, translate, reflect for new coord sys

$$P' = \begin{bmatrix} \frac{2}{right - left} & 0 & 0 & -\frac{right + left}{right - left} \\ 0 & \frac{2}{top - bot} & 0 & -\frac{top + bot}{top - bot} \\ 0 & 0 & \frac{-2}{far - near} & -\frac{far + near}{far - near} \\ 0 & 0 & 0 & 1 \end{bmatrix} P$$

Orthographic Derivation

- scale, translate, reflect for new coord sys

$$P' = \begin{bmatrix} \frac{2}{right - left} & 0 & 0 & -\frac{right + left}{right - left} \\ 0 & \frac{2}{top - bot} & 0 & -\frac{top + bot}{top - bot} \\ 0 & 0 & \frac{-2}{far - near} & -\frac{far + near}{far - near} \\ 0 & 0 & 0 & 1 \end{bmatrix} P$$

Orthographic Derivation

- scale, translate, reflect for new coord sys

$$P' = \begin{bmatrix} \frac{2}{right - left} & 0 & 0 & -\frac{right + left}{right - left} \\ 0 & \frac{2}{top - bot} & 0 & -\frac{top + bot}{top - bot} \\ 0 & 0 & \frac{-2}{far - near} & -\frac{far + near}{far - near} \\ 0 & 0 & 0 & 1 \end{bmatrix} P$$

Orthographic OpenGL

```
glMatrixMode(GL_PROJECTION);
glLoadIdentity();
glOrtho(left,right,bot,top,near,far);
```