University of British Columbia
CPSC 314 Computer Graphics
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Transformations V, Viewing I

Week 3, Fri Jan 22

http://www.ugrad.cs.ubc.ca/~cs314/Vjan2010

Review: Display Lists

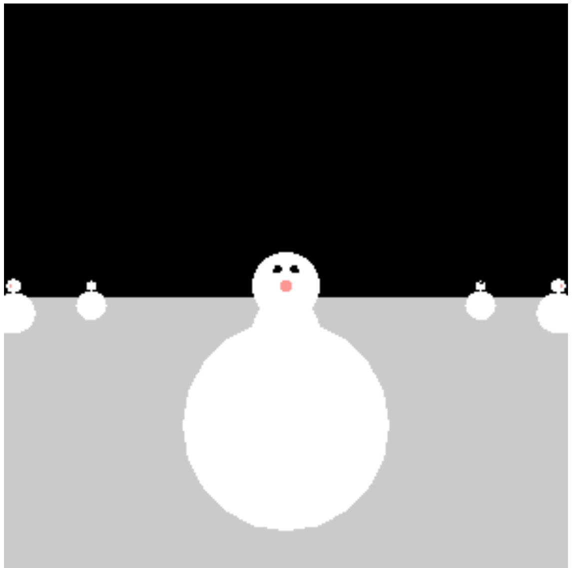
- precompile/cache block of OpenGL code for reuse
  - usually more efficient than immediate mode
    - exact optimizations depend on driver
  - good for multiple instances of same object
    - but cannot change contents, not parametrizable
  - good for static objects redrawn often
    - display lists persist across multiple frames
    - interactive graphics: objects redrawn every frame from new viewpoint from moving camera
  - can be nested hierarchically
- snowman example

  http://www.lighthouse3d.com/opengl/displaylists

One Snowman

```
void drawSnowMan() {

glColor3f(1.0f, 1.0f, 1.0f);

// Draw Body
glTranslatef(0.0f ,0.75f, 0.0f);
glutSolidSphere(0.75f,20,20);

// Draw Head
glTranslatef(0.0f, 1.0f, 0.0f);
glutSolidSphere(0.25f,20,20);

// Draw Eyes
glPushMatrix();
glColor3f(0.0f,0.0f,0.0f);
glTranslatef(0.05f, 0.10f, 0.18f);
glutSolidSphere(0.05f,10,10);
glTranslatef(-0.1f, 0.0f, 0.0f);
glutSolidSphere(0.05f,10,10);
glPopMatrix();

// Draw Nose
glColor3f(1.0f, 0.5f , 0.5f);
glRotatef(0.0f,1.0f, 0.0f, 0.0f);
glutSolidCone(0.08f,0.5f,10,2);
}
```

Instantiate Many Snowmen



```
// Draw 36 Snowmen
for(int i = -3; i < 3; i++)
   for(int j=-3; j < 3; j++) {
   glPushMatrix();
   glTranslatef(i*10.0, 0, j * 10.0);
   // Call the function to draw a snowman
   drawSnowMan();
   glPopMatrix();
}
```

36K polygons, 55 FPS

Making Display Lists

```
GLuint createDL() {
GLuint snowManDL;
// Create the id for the list
snowManDL = glGenLists(1);
glNewList(snowManDL,GL_COMPILE);
drawSnowMan();
glEndList();
return(snowManDL); }

snowmanDL = createDL();
for(int i = -3; i < 3; i++)
  for(int j=-3; j < 3; j++) {
    glPushMatrix();
    glTranslatef(i*10.0, 0, j * 10.0);
    glCallList(Dlid);
    glPopMatrix(); }
```

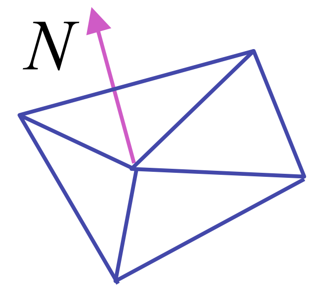
36K polygons, 153 FPS

Transforming Normals

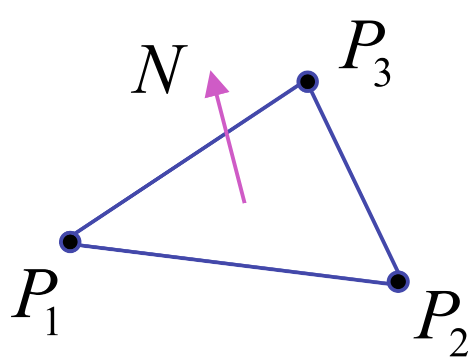
Transforming Geometric Objects

- lines, polygons made up of vertices
  - transform the vertices
  - interpolate between
- does this work for everything? no!
  - normals are trickier

Computing Normals



- normal
  - direction specifying orientation of polygon
    - w=0 means direction with homogeneous coords
    - vs. w=1 for points/vectors of object vertices
  - used for lighting
    - must be normalized to unit length
  - can compute if not supplied with object



$$N = (P_2 - P_1) \times (P_3 - P_1)$$

Transforming Normals

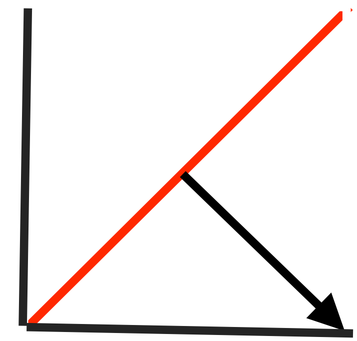
$$\begin{bmatrix} x' \\ y' \\ z' \\ 0 \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} & m_{13} & T_x \\ m_{21} & m_{22} & m_{23} & T_y \\ m_{31} & m_{32} & m_{33} & T_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 0 \end{bmatrix}$$

- so if points transformed by matrix **M**, can we just transform normal vector by **M** too?
  - translations OK: w=0 means unaffected
  - rotations OK
  - uniform scaling OK

- these all maintain direction

Transforming Normals

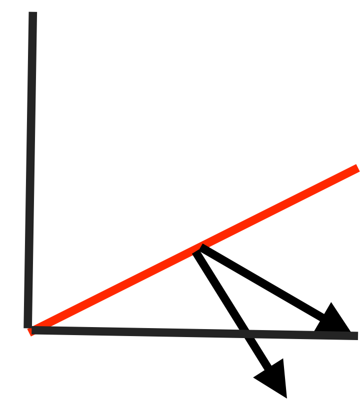
- nonuniform scaling does not work
- x-y=0 plane
  - line x=y
  - normal: [1,-1,0]
    - direction of line x=-y
    - (ignore normalization for now)



Transforming Normals

- apply nonuniform scale: stretch along x by 2
  - new plane x = 2y
- transformed normal: [2,-1,0]

$$\begin{bmatrix} 2 \\ -1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}$$



  - normal is direction of line x = -2y or x+2y=0
  - not perpendicular to plane!
  - should be direction of 2x = -y

Planes and Normals

- plane is all points perpendicular to normal
  - $N \bullet P = 0$ (with dot product)
  - $N^T \bullet P = 0$ (matrix multiply requires transpose)

$$N = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}, P = \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$

- explicit form: plane = $ax + by + cz + d$

Finding Correct Normal Transform

- transform a plane

$$P \longrightarrow P' = MP$$
$$N \longrightarrow N' = QN$$

given M,
what should Q be?

$$N'^T P' = 0$$

stay perpendicular

$$(QN)^T (MP) = 0$$

substitute from above

$$N^T Q^T M P = 0$$

$$(\mathbf{AB})^T = \mathbf{B}^T \mathbf{A}^T$$

$$Q^T M = I$$

$$\mathbf{N}^T \mathbf{P} = 0 \text{ if } \mathbf{Q}^T \mathbf{M} = \mathbf{I}$$

$$\mathbf{Q} = \left(\mathbf{M}^{-1}\right)^{\mathbf{T}}$$

thus the normal to any surface can be transformed by the inverse transpose of the modelling transformation

Reading for This and Next 2 Lectures

- FCG Chapter 7 Viewing
- FCG Section 6.3.1 Windowing Transforms

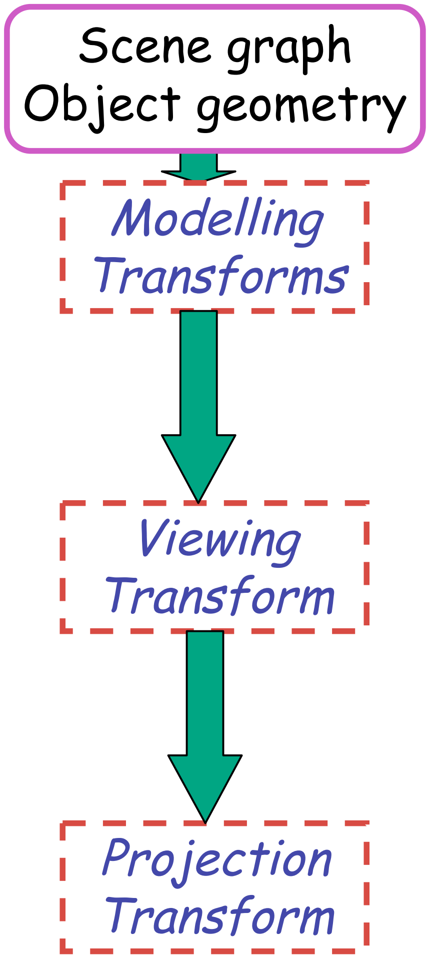
- RB rest of Chap Viewing
- RB rest of App Homogeneous Coords

Viewing

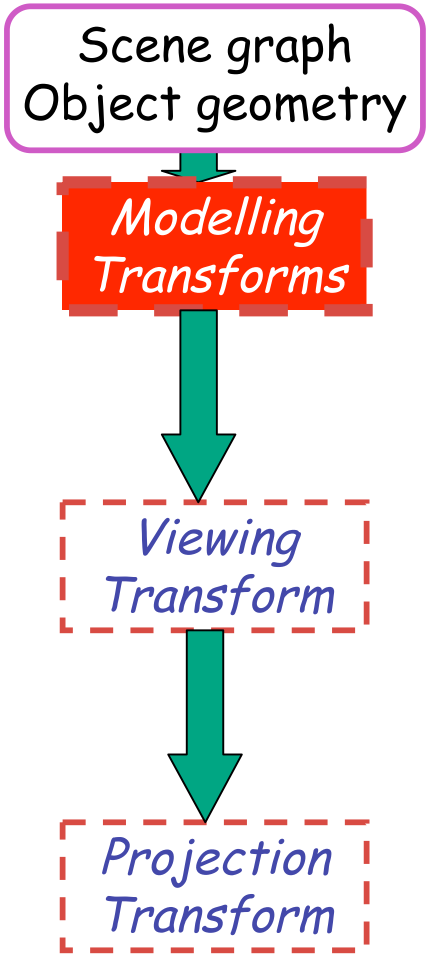
Using Transformations

- three ways
  - modelling transforms
    - place objects within scene (shared world)
    - affine transformations
  - viewing transforms
    - place camera
    - rigid body transformations: rotate, translate
  - projection transforms
    - change type of camera
    - projective transformation

Rendering Pipeline

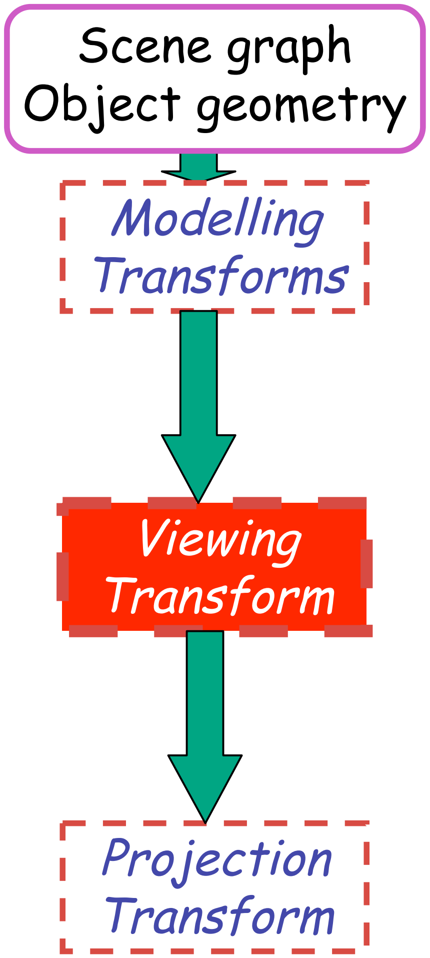


Rendering Pipeline

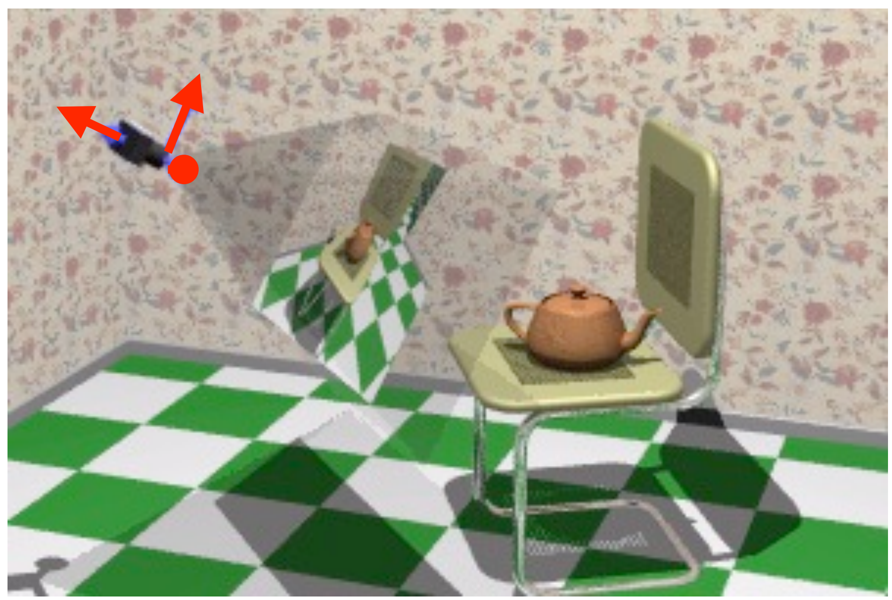


- result
  - all vertices of scene in shared 3D world coordinate system

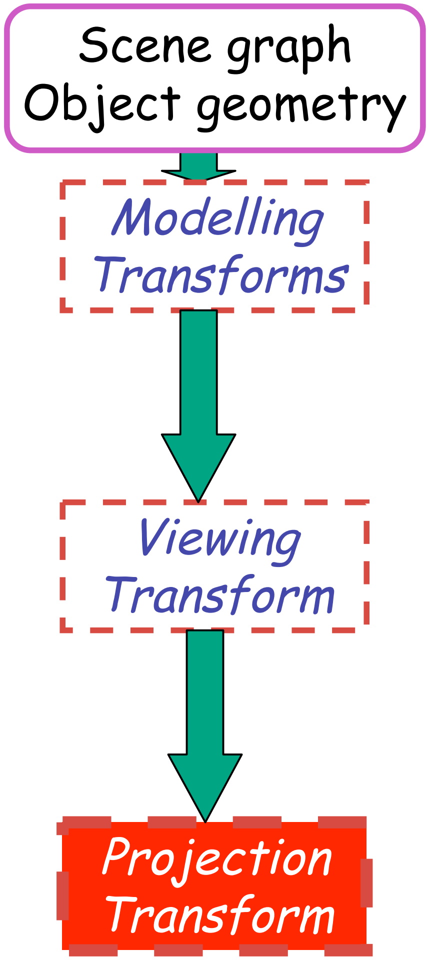
Rendering Pipeline



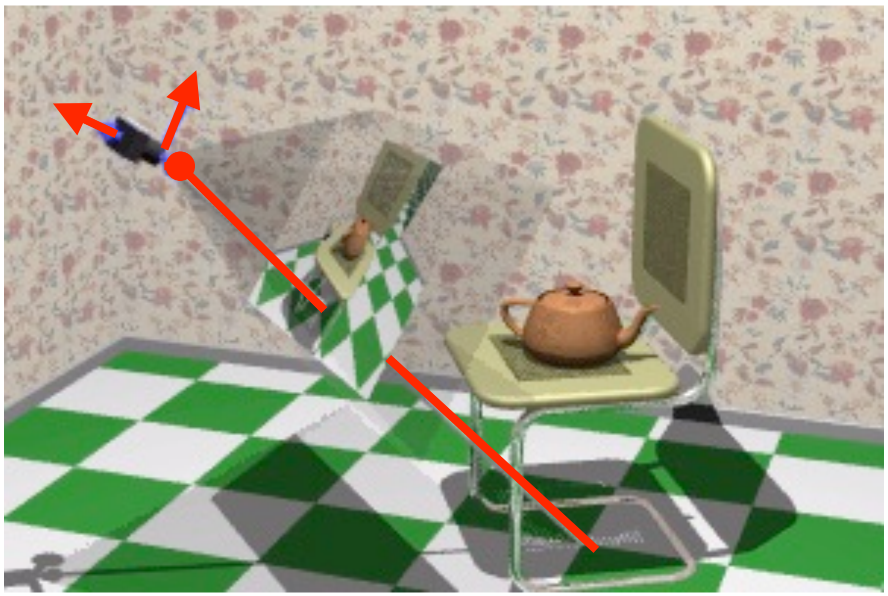
- result
  - scene vertices in 3D view (camera) coordinate system



Rendering Pipeline



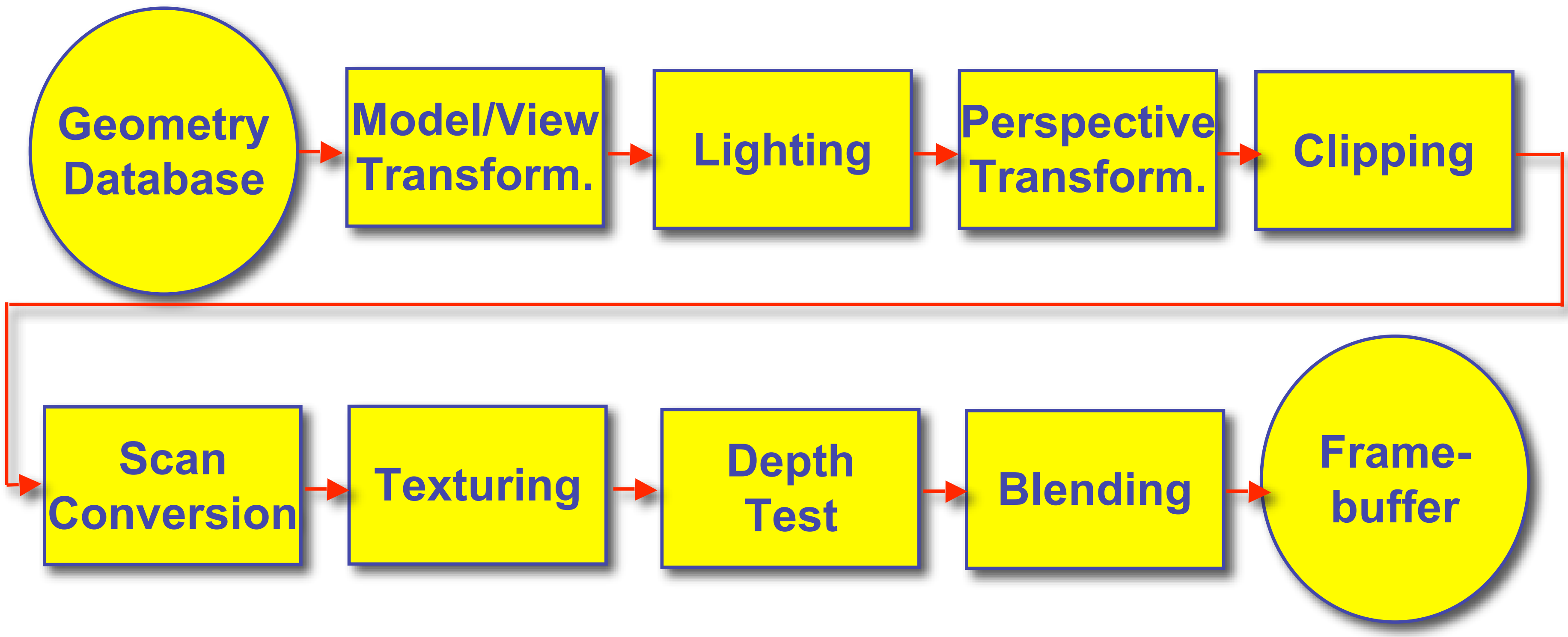
- result
  - 2D screen coordinates of clipped vertices



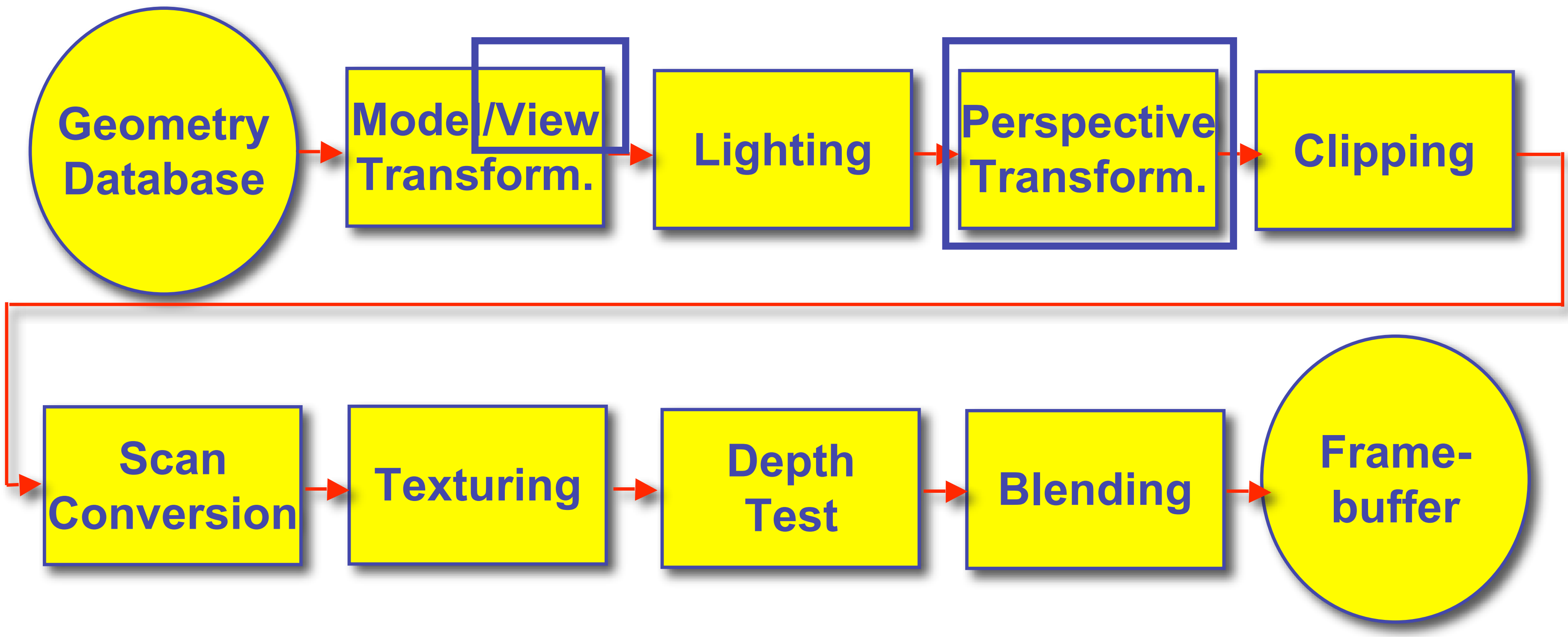
Viewing and Projection

- need to get from 3D world to 2D image
- projection: geometric abstraction
  - what eyes or cameras do
- two pieces
  - viewing transform:
    - where is the camera, what is it pointing at?
  - perspective transform: 3D to 2D
    - flatten to image

Rendering Pipeline



Rendering Pipeline



OpenGL Transformation Storage

- modeling and viewing stored together
  - possible because no intervening operations
- perspective stored in separate matrix

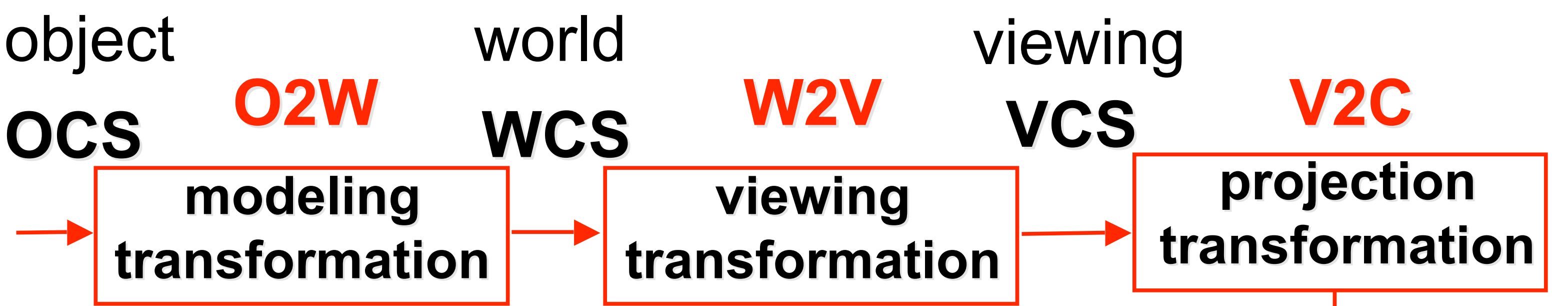
- specify which matrix is target of operations
  - common practice: return to default modelview mode after doing projection operations

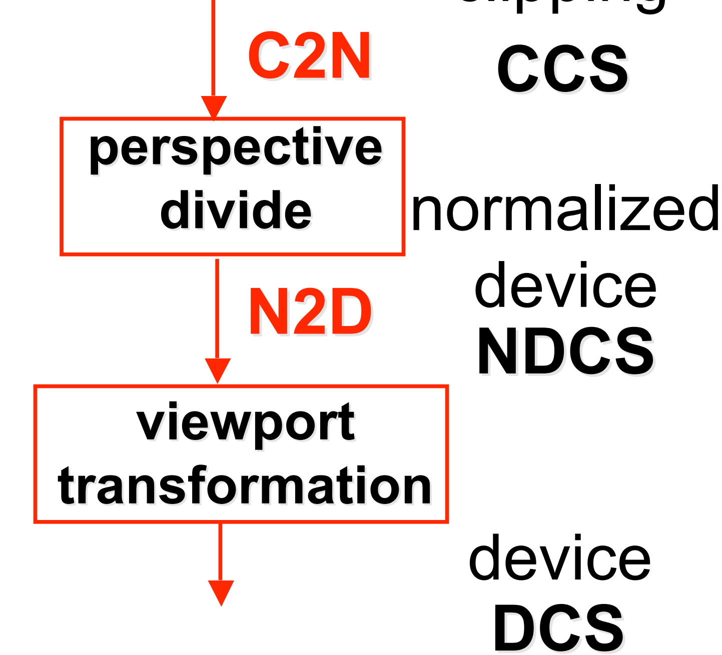
```
glMatrixMode(GL_MODELVIEW);
glMatrixMode(GL_PROJECTION);
```

Coordinate Systems

- result of a transformation
- names
  - convenience
    - mouse: leg, head, tail
  - standard conventions in graphics pipeline
    - object/modelling
    - world
    - camera/viewing/eye
    - screen/window
    - raster/device

Projective Rendering Pipeline





OCS - object/model coordinate system

WCS - world coordinate system

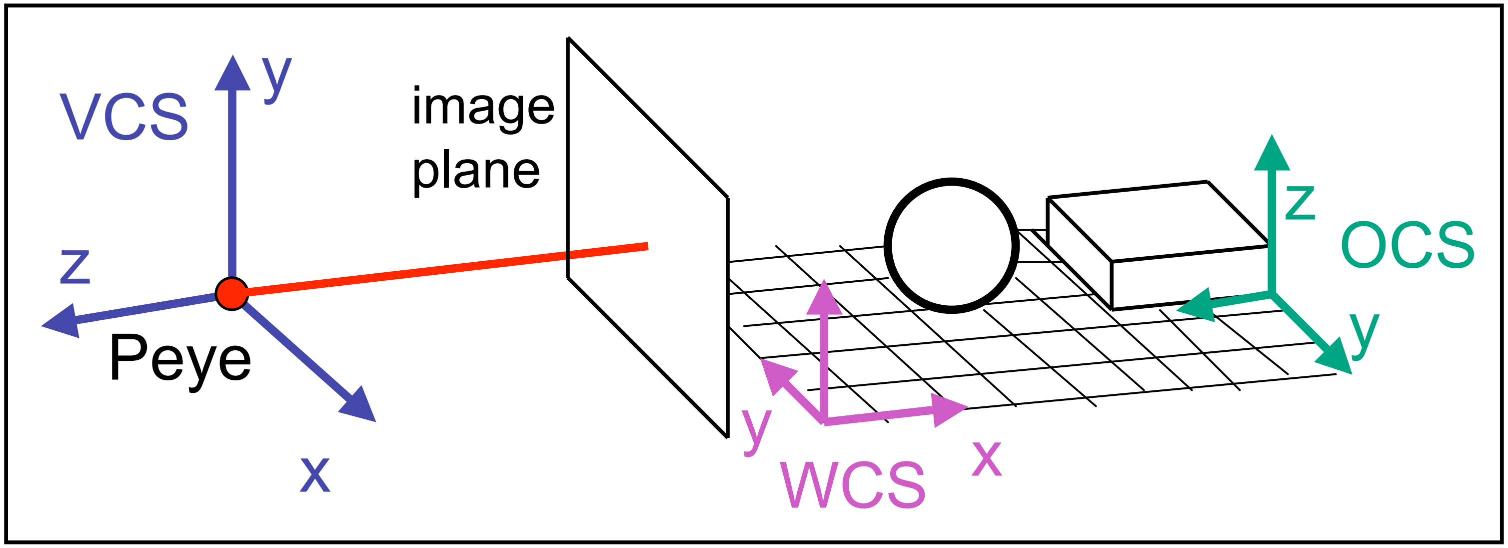
VCS - viewing/camera/eye coordinate system

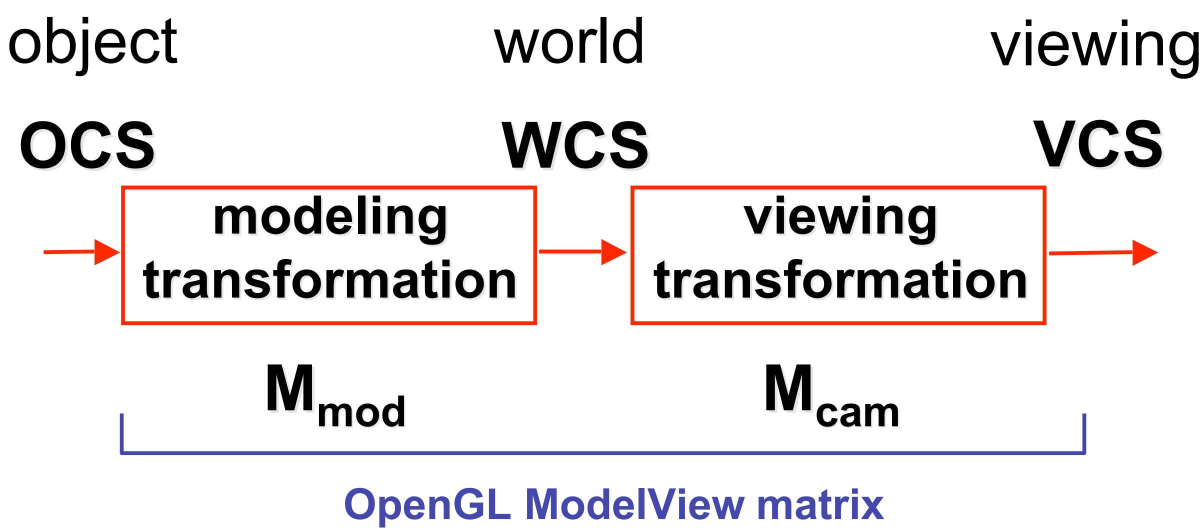
CCS - clipping coordinate system

NDCS - normalized device coordinate system

DCS - device/display/screen coordinate system

Viewing Transformation





Basic Viewing

- starting spot - OpenGL
  - camera at world origin
    - probably inside an object
  - y axis is up
  - looking down negative z axis
    - why? RHS with x horizontal, y vertical, z out of screen
- translate backward so scene is visible
  - move distance d = focal length

- where is camera in P1 template code?
  - 5 units back, looking down -z axis

Convenient Camera Motion

- rotate/translate/scale versus
  - eye point, gaze/lookat direction, up vector
  - demo: Robins transformation, projection

OpenGL Viewing Transformation

```
gluLookAt(ex,ey,ez,lx,ly,lz,ux,uy,uz)
```

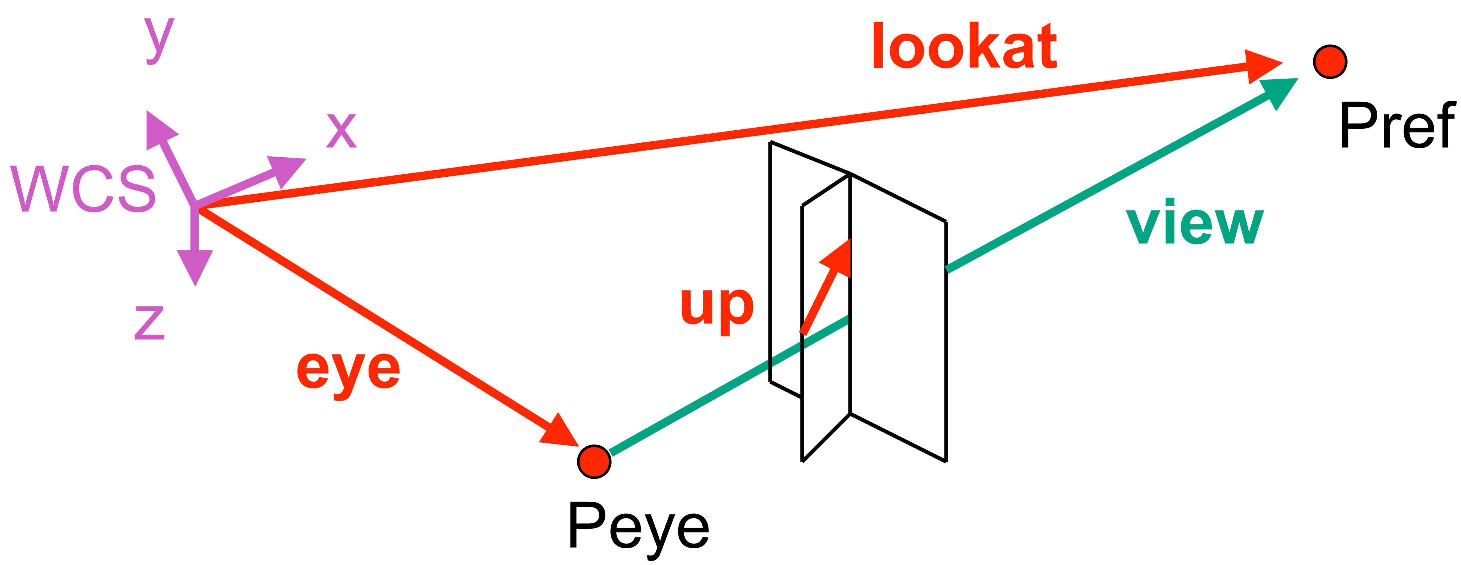
- postmultiplies current matrix, so to be safe:

```
glMatrixMode(GL_MODELVIEW);
glLoadIdentity();
gluLookAt(ex,ey,ez,lx,ly,lz,ux,uy,uz)
// now ok to do model transformations
```

- demo: Nate Robins tutorial *projection*

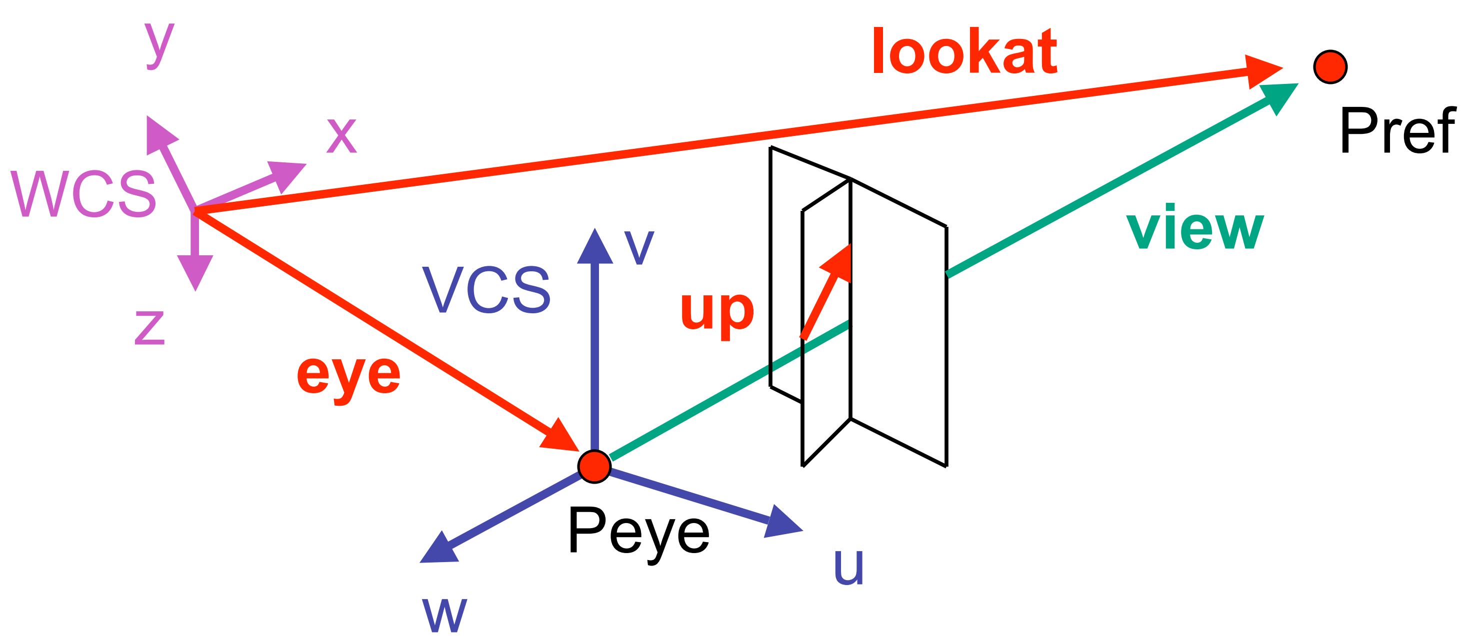
Convenient Camera Motion

- rotate/translate/scale versus
  - eye point, gaze/lookat direction, up vector



From World to View Coordinates: W2V

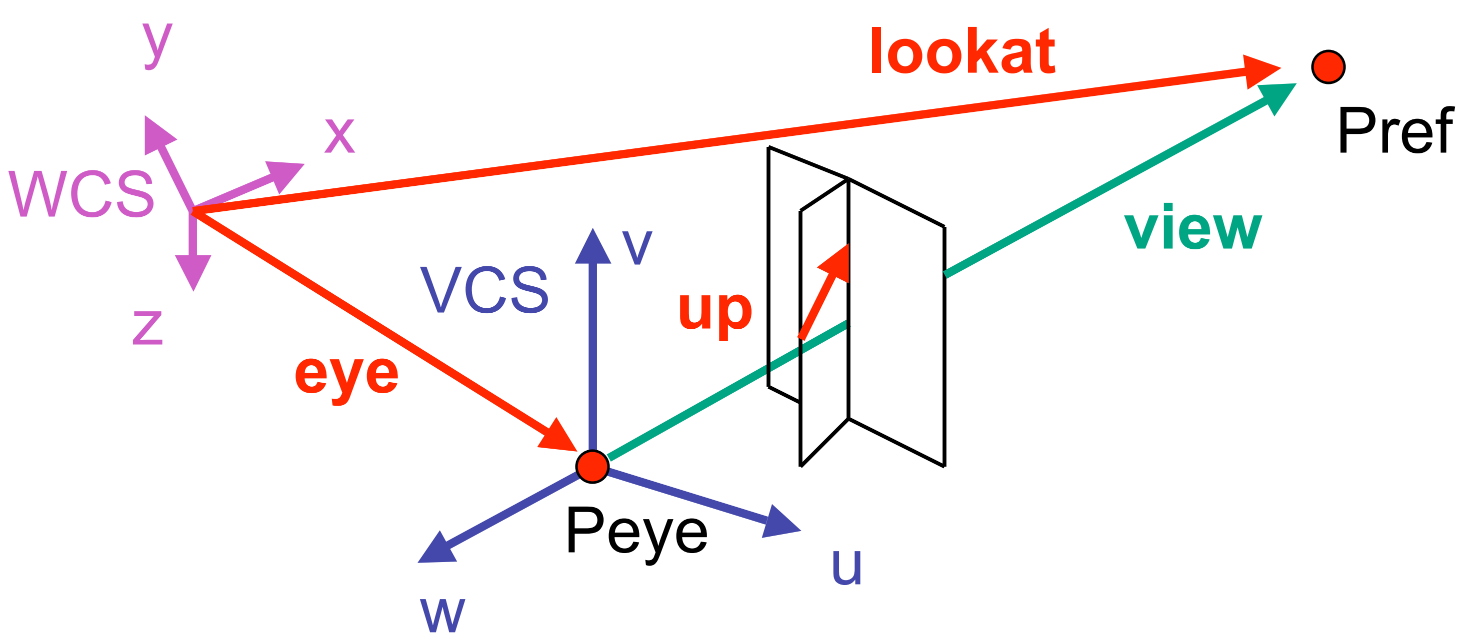
- translate **eye** to origin
- rotate **view** vector (**lookat** – **eye**) to **w** axis
- rotate around **w** to bring **up** into **vw**-plane



Deriving W2V Transformation

- translate **eye** to origin

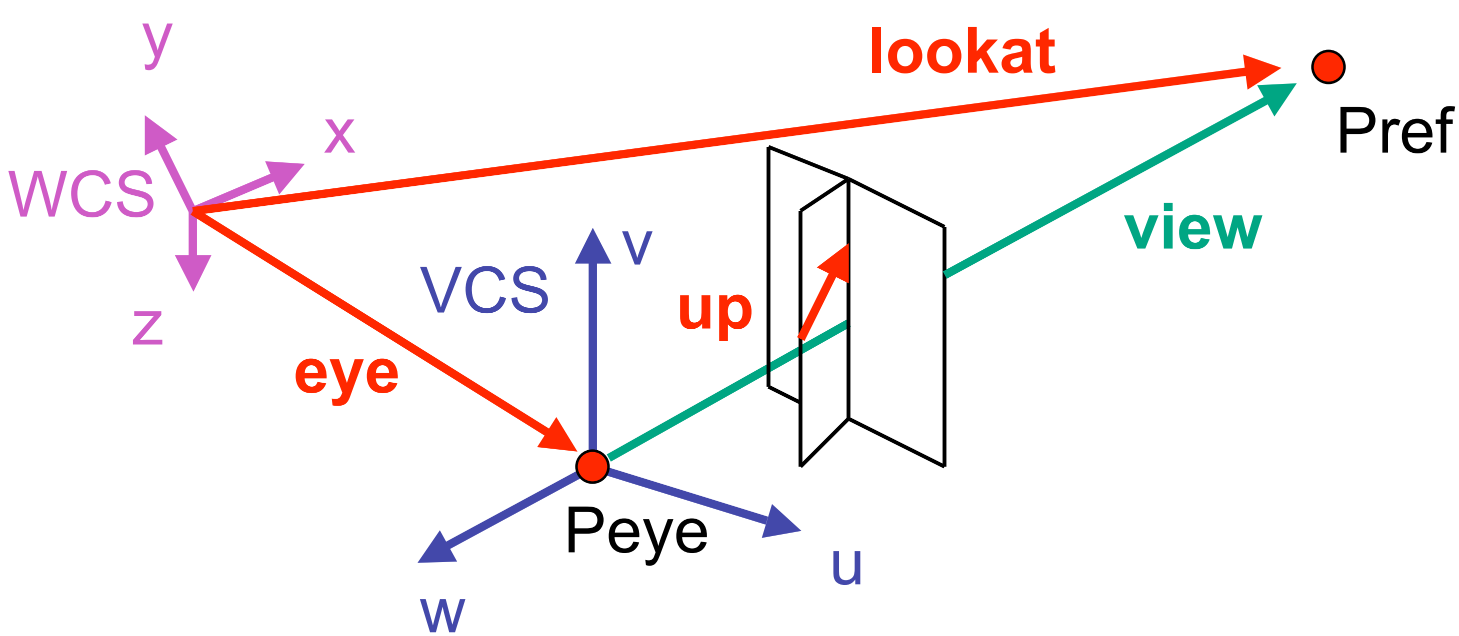
$$\mathbf{T} = \begin{bmatrix} 1 & 0 & 0 & e_x \\ 0 & 1 & 0 & e_y \\ 0 & 0 & 1 & e_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



Deriving W2V Transformation

- rotate **view** vector (**lookat** – **eye**) to **w** axis
  - **w**: normalized opposite of **view/gaze** vector **g**

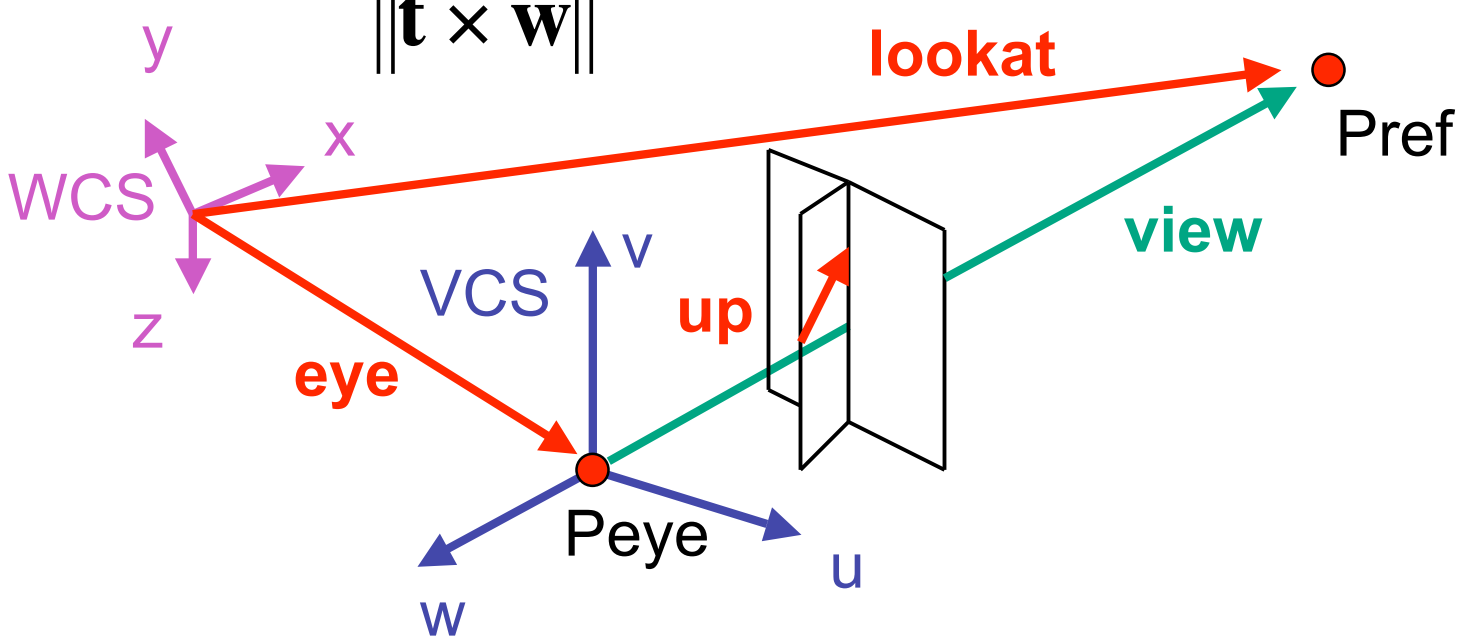
$$\mathbf{w} = -\hat{\mathbf{g}} = -\frac{\mathbf{g}}{\|\mathbf{g}\|}$$



Deriving W2V Transformation

- rotate around **w** to bring up into **vw**-plane
  - **u** should be perpendicular to **vw**-plane, thus perpendicular to **w** and up vector **t**
  - **v** should be perpendicular to **u** and **w**

$$\mathbf{u} = \frac{\mathbf{t} \times \mathbf{w}}{\|\mathbf{t} \times \mathbf{w}\|} \qquad \mathbf{v} = \mathbf{w} \times \mathbf{u}$$



Deriving W2V Transformation

- rotate from WCS **xyz** into **uvw** coordinate system with matrix that has columns **u**, **v**, **w**

$$\mathbf{u} = \frac{\mathbf{t} \times \mathbf{w}}{\|\mathbf{t} \times \mathbf{w}\|} \qquad \mathbf{v} = \mathbf{w} \times \mathbf{u} \qquad \mathbf{w} = -\hat{\mathbf{g}} = -\frac{\mathbf{g}}{\|\mathbf{g}\|}$$

$$\mathbf{R} = \begin{bmatrix} u_x & v_x & w_x & 0 \\ u_y & v_y & w_y & 0 \\ u_z & v_z & w_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \mathbf{T} = \begin{bmatrix} 1 & 0 & 0 & e_x \\ 0 & 1 & 0 & e_y \\ 0 & 0 & 1 & e_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad M_{W2V} = TR$$

- reminder: rotate from **uvw** to **xyz** coord sys with matrix **M** that has columns **u,v,w**

W2V vs. V2W

- M_{W2V}=TR

$$\mathbf{T} = \begin{bmatrix} 1 & 0 & 0 & e_x \\ 0 & 1 & 0 & e_y \\ 0 & 0 & 1 & e_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \mathbf{R} = \begin{bmatrix} u_x & v_x & w_x & 0 \\ u_y & v_y & w_y & 0 \\ u_z & v_z & w_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- we derived position of camera in world
  - invert for world with respect to camera
- $M_{V2W}=(M_{W2V})^{-1}=R^{-1}T^{-1}$

$$\mathbf{R}^{-1} = \begin{bmatrix} u_x & u_y & u_z & 0 \\ v_x & v_y & v_z & 0 \\ w_x & w_y & w_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \mathbf{T}^{-1} = \begin{bmatrix} 1 & 0 & 0 & -e_x \\ 0 & 1 & 0 & -e_y \\ 0 & 0 & 1 & -e_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

  - inverse is transpose for orthonormal matrices
  - inverse is negative for translations

W2V vs. V2W

- M_{W2V}=TR

$$\mathbf{T} = \begin{bmatrix} 1 & 0 & 0 & e_x \\ 0 & 1 & 0 & e_y \\ 0 & 0 & 1 & e_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \mathbf{R} = \begin{bmatrix} u_x & v_x & w_x & 0 \\ u_y & v_y & w_y & 0 \\ u_z & v_z & w_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

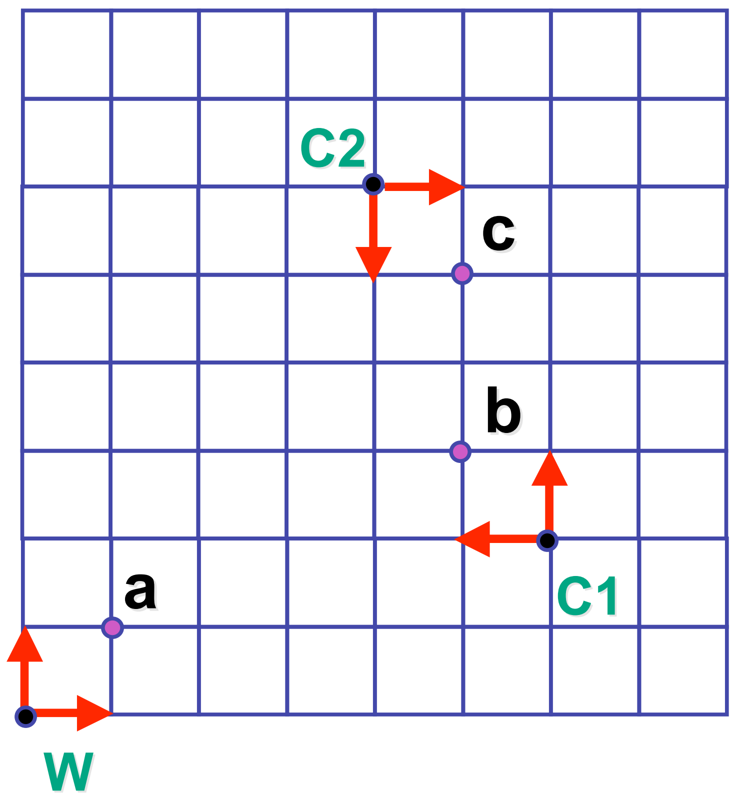
- we derived position of camera in world
  - invert for world with respect to camera
- $M_{V2W}=(M_{W2V})^{-1}=R^{-1}T^{-1}$

$$\mathbf{M}_{view2world} = \begin{bmatrix} u_x & u_y & u_z & 0 \\ v_x & v_y & v_z & 0 \\ w_x & w_y & w_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -e_x \\ 0 & 1 & 0 & -e_y \\ 0 & 0 & 1 & -e_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} u_x & u_y & u_z & -e_x \\ v_x & v_y & v_z & -e_y \\ w_x & w_y & w_z & -e_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Moving the Camera or the World?

- two equivalent operations
  - move camera one way vs. move world other way
- example
  - initial OpenGL camera: at origin, looking along -z axis
  - create a unit square parallel to camera at z = -10
  - translate in z by 3 possible in two ways
    - camera moves to z = -3
      - Note OpenGL models viewing in left-hand coordinates
    - camera stays put, but world moves to -7
  - resulting image same either way
    - possible difference: are lights specified in world or view coordinates?

World vs. Camera Coordinates Example



$a = (1,1)_W$

$b = (1,1)_{C1} = (5,3)_W$

$c = (1,1)_{C2} = (1,3)_{C1} = (5,5)_W$