CPSC 314
Midterm

March 7, 2008

Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page.

Name: _____

Student Number: _____

Question 1	/ 12
Question 2	/ 22
Question 3	/ 12
Question 4	/ 14
Question 5	/ 40
extra	/ 3
TOTAL	/ 100

This exam has 6 questions, for a total of 100 points.

1. Short Quesions

   (a) (2 points) Express the point $P(x, y, z, h) = P(1, 2, 3, 0.5)$ described in homogeneous coordinates using cartesian coordinates.

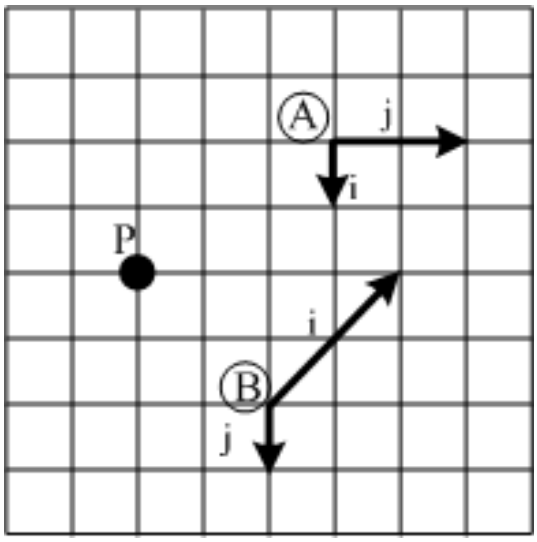
   (b) (2 points) True or false: straight lines remain straight lines after a perspective transformation? Explain.

   (c) (4 points) Give the inverse of the following transformation matrix:
   $$\begin{bmatrix} 2 & 0 & 0 & 4 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

   (d) (2 points) True or false: Implicit rasterization algorithm works for *all* polygons? Explain.

   (e) (2 points) True or false: In 3D is R * T = T * R where T is a translational and R a rotation matrix? Explain

2. Coordinate Frames



(a) (6 points) Express point P in each of the two coordinate frames.

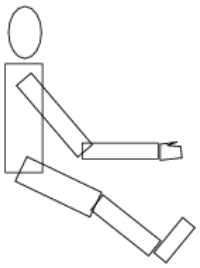
(b) (8 points) Derive the 3×3 homogeneous transformation matrix which takes a point from frame A coordinates and expresses it in terms of frame B coordinates. Verify your solution using your answer to part (a).

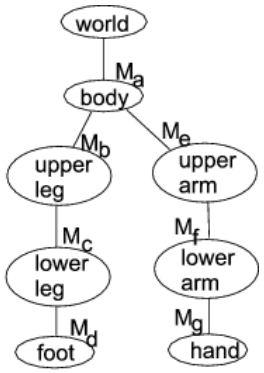
(c) (8 points) Give the sequence of OpenGL transformations that would produce the following transformation matrix:

$$\begin{bmatrix} 0 & -1 & 0 & 4 \\ 1 & 0 & 0 & -1 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3. Scene Graphs, Viewing Transformation

The transformation matrices in the following scene graph define the relative transformations of each body part with respect to their parent.





(a) (6 points) Determine an expression for the composite transformation matrix that would be used to draw the hand, i.e., that takes a point from the hand coordinate frame to the world coordinate frame.

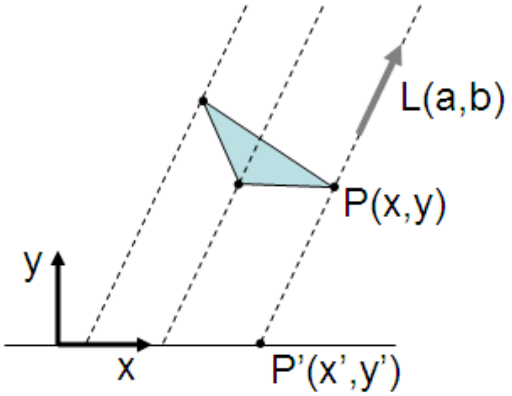
(b) (6 points) Determine an expression for the transformation matrix that takes a point from the hand coordinate frame to the foot coordinate frame.

4. Projections

(a) (4 points) Sketch a two-point perspective view of a cube. How would one produce such a photograph or image?

(b) (10 points) Projections can be used to compute shadows on the ground plane. In the figure below, the vector $L(a, b)$ points in the towards the light source. For the given triangle, we wish to compute its ground-plane shadow by projecting its three vertices onto the ground plane along the direction of the lighting, as indicated by the dashed lines.

Using similar triangles and geometry, give expressions for x' and y', given a known point location $P(x, y)$ and a known lighting direction $L(a, b)$. Using these expressions, give the transformation matrix M that would perform the same projection, i.e., $P' = MP$.



5. Scan Conversion

A circle is defined by a center $C = (C_x, C_y)$ and a radius R (assume that C_x, C_y and R are integer).

(a) (4 points) Write an implicit equation of the circle.

(b) (4 points) Write a parametric equation of the circle.

(c) (12 points) Write the pseudocode for rasterizing an arc on the circle starting at $(C_x - R, 0)$ and ending at $(C_x - R/2, C_y - R/2)$. Make your code as efficient as possible. (Hint 1: start with a floating point algorithm first and convert it to integer if you can, Hint 2: note the slope of the arc).

(d) (10 points) Write the pseudocode for scan converting the circle (including its interior) using a variation of the implicit rasterization algorthm.

(e) (10 points) Write the pseudocode for scan converting the cricle (including its interior) using a variation of the scanline algorithm.

6. For those who are bored

   How many pieces can a perfect bagel (torus) be sliced into with 3 straight cuts, without rearranging the pieces between cuts?