CPSC 314 - Term 2 2010
HW3 Solutions.

1. Two ways to calculate $\vec{N}_B$; one is more "correct" than the other, but both will receive credit for the assignment.

Method 1 (more "correct")

Calculate normals on faces AB & BC, and interpolate

$$\vec{AB} = B - A = \langle 5, 1 \rangle$$

$$\Rightarrow \vec{N}_{AB} = \frac{\langle 1, -5 \rangle}{\| \langle 1, -5 \rangle \|} = \frac{1}{\sqrt{26}} \langle 1, -5 \rangle$$

$$\approx \langle .1961\ -.9806 \rangle$$

$$\vec{BC} = C - B = \langle 2, 2 \rangle$$

$$\Rightarrow \vec{N}_{BC} = \frac{\langle 2, -2 \rangle}{\| \cdot \|} = \frac{1}{\sqrt{2}} \langle 1, -1 \rangle$$

$$\approx \langle .7071, -.7071 \rangle$$

Interpolate

$$\boxed{\vec{N}_B = \frac{\vec{N}_{AB} + \vec{N}_{BC}}{\| \cdot \|} \approx \langle .4719, -.8817 \rangle}$$

Method 2.

Interpolate between normals at vertices. A & C

$$\vec{N}_A = -\frac{1}{\sqrt{2}} \langle 1, 1 \rangle$$

$$\vec{N}_C = \langle 1, 0 \rangle$$

$$\boxed{\vec{N}_B = \frac{\vec{N}_A + \vec{N}_C}{\| \cdot \|} \approx \langle .3827, -.9239 \rangle}$$

2 pts for correct $\vec{N}_B$

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1 b)

Blinn-Phong Lighting Model

$$I = \underbrace{k_A I_A}_{\text{ambient}} + \underbrace{k_D I_D(\vec{N}\cdot\vec{L})}_{\text{diffuse}} + \underbrace{k_S I_S(\vec{H}\cdot\vec{V})^{\alpha}}_{\text{specular}}$$

$k_A = \langle .1\ .1\ .1\rangle$
$k_D = \langle .3\ .8\ .9\rangle$
$k_S = \langle 1\ 1\ 1\rangle$
$I_A = \langle .1\ .2\ .1\rangle$
$I_D = I_S = I_L = \langle 1\ 1\ .9\rangle$
$\alpha = 20$

$k_A I_A = \langle .01\ .02\ .01\rangle$
$k_D I_L = \langle .3\ .8\ .81\rangle$
$k_S I_L = \langle 1\ 1\ .9\rangle$

Point B

Diffuse $\vec{N}_B = \langle .4719, -.8817\rangle$ or $\langle .3827\ -.9239\rangle$
$\vec{L}_B = \frac{1}{\sqrt{2}}\langle 1\ -1\rangle$

$\vec{N}_B\cdot\vec{L}_B \approx .9571$ or $.9239$
$k_D I_D(\vec{N}_B\cdot\vec{L}_B) \approx \langle .2871\ .7657\ .7752\rangle$ or
$\langle .2772\ .7391\ .7483\rangle$

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Specular

$$\vec{V}_B = \frac{E - B}{\|\cdot\|} \approx \langle .4472 \quad -.8944 \rangle$$

$$\vec{H}_B = \frac{\vec{V}_B + \vec{L}_B}{\|\cdot\|} \approx \langle .5847 \quad -.8112 \rangle$$

$$(\vec{H}_B \cdot \vec{N}_B)^\alpha \approx .8372 \text{ or } .5814$$

$$k_S I_S (\vec{H}_B \cdot \vec{N}_B) \approx \langle .8372 \ .8372 \ .7534 \rangle \text{ or } \langle .5814 \ .5814 \ .5233 \rangle$$

Total

$$I_B \approx \langle .01 \quad .02 \quad .01 \rangle +$$
$$\langle .2871 \quad .7657 \quad .7752 \rangle \text{ or } \langle .2772 \quad .7391 \quad .7483 \rangle +$$
$$\langle .8372 \quad .8372 \quad .7534 \rangle \text{ or } \langle .5814 \quad .5814 \quad .5233 \rangle$$
$$= \langle 1.134 \quad 1.623 \quad 1.539 \rangle \text{ or}$$
$$\langle .8686 \quad 1.3405 \quad 1.2816 \rangle$$

Don't forget to clamp!

$$\boxed{I_B = \langle 1 \quad 1 \quad 1 \rangle \text{ or } \langle .8686 \quad 1 \quad 1 \rangle}$$

6 points
(-2 if answer differs)

Point C

Diffuse

$$\vec{N}_C = \langle 1 \quad 0 \rangle$$
$$\vec{L}_C = \langle 0 \quad -1 \rangle$$

$$\vec{N}_C \perp \vec{L}_C \rightarrow \vec{N}_C \cdot \vec{L}_C = \emptyset$$

$$\rightarrow k_D I_D (\vec{N}_C \cdot \vec{L}_C) = \langle 0 \quad 0 \quad 0 \rangle$$

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Specular

$\vec{V}_c = \langle 0, -1 \rangle$
$\vec{H}_c = \langle 0, -1 \rangle$

$\rightarrow (\vec{H}_c \cdot \vec{N}_c) = 0$
$k_s I_s (\vec{H}_c \cdot \vec{N}_c)^\alpha = \langle 0\ 0\ 0 \rangle$

Total

$$\boxed{\begin{aligned} I_c &= k_A I_A \\ &= \langle .01\ \ .02\ \ .01 \rangle \end{aligned}}$$

6 pts

Point D

$\boxed{\text{Using Flat shading model so } I_D = I_B \text{ or } I_C}$

4 pts.

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1c)

Point B - see 1b 2 pts
Point C - see 1b 2 pts

Point D - linearly interpolate between B & C

$$I_D = \frac{1}{2} I_B + \frac{1}{2} I_C$$

$$= k_A I_A + \frac{1}{2} k_D I_D (\vec{N}_B \cdot \vec{L}_B) + \frac{1}{2} k_s I_s (\vec{H}_B \cdot \vec{N}_B)^k$$

$I_D \approx \langle .01 \ .02 \ .01 \rangle +$

$\langle .1436 \ .3828 \ .3876 \rangle$ or $\langle .1386 \ .3696 \ .3742 \rangle$

$\langle .4186 \ .4186 \ .3767 \rangle$ or $\langle .2907 \ .2907 \ .2616 \rangle$

$$\boxed{I_D = \langle .5721 \ .8214 \ .7743 \rangle \text{ or } \langle .4393 \ .6803 \ .6458 \rangle}$$ 12 pts.

1d)

Points B & C - see 1c 4 pk.

Point D - linearly interpolate $\vec{N}_B$ & $\vec{N}_C \rightarrow \vec{N}_D$

$$\vec{N}_D = \frac{\vec{N}_B + \vec{N}_C}{\| \cdot \|} \approx \langle .8579 \ -.5139 \rangle \text{ or } \langle .8315 \ -.5556 \rangle$$

$\vec{V}_D \approx \langle .1961 \ -.9806 \rangle$

$\vec{L}_D \approx \langle .3162 \ -.9487 \rangle$

$$\vec{H}_D = \frac{\vec{L}_D + \vec{V}_D}{\| \cdot \|} \approx \langle .2567 \ -.9665 \rangle$$

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1d)

Diffuse

$k_D I_D (\vec{N}_0 \cdot \vec{L}_D) \approx \langle .2276 \;\; .6070 \;\; .6146 \rangle$ or
$\langle .2370 \;\; .6320 \;\; .6399 \rangle$

Specular

$k_S I_S (\vec{H}_0 \cdot \vec{L}_D)^2 \approx \langle .0013 \;\; .0013 \;\; .0012 \rangle$ or
$\langle .0032 \;\; .0032 \;\; .0029 \rangle$

Total

$I_D \approx \langle .2389 \;\; .6283 \;\; .6258 \rangle$ or
$\langle .2502 \;\; .6552 \;\; .6528 \rangle$

12 pts.

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2.

$\langle R\ G\ B \rangle = \langle .3\ .5\ .2 \rangle$

YIQ

$$\begin{bmatrix} Y \\ I \\ Q \end{bmatrix} = \begin{bmatrix} .3 & .59 & .11 \\ .6 & -.28 & -.32 \\ .21 & -.52 & .31 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

$$\boxed{\begin{bmatrix} Y \\ I \\ Q \end{bmatrix} \approx \begin{bmatrix} .4070 \\ -.02400 \\ -.1350 \end{bmatrix}}$$

4 pts

HSV

Since $B < G$

$$H = \cos^{-1}\left[\frac{\frac{1}{2}(2R - G - B)}{\sqrt{(R-G)^2 + (R-B)(G-B)}}\right]$$

$$\approx 100.9 \ [\text{deg}]$$

$$V = \max(R, G, B) = .5$$

$$S = 1 - \frac{\min(R, G, B)}{V} = 1 - \frac{.2}{.5} = .6$$

$$\boxed{HSV = \langle 100.9 \quad .6 \quad .5 \rangle}$$

4 pts.

CMY

$$\boxed{\begin{aligned} C &= 1 - R = .7 \\ M &= 1 - G = .5 \\ Y &= 1 - B = .8 \end{aligned}}$$

2 pt.

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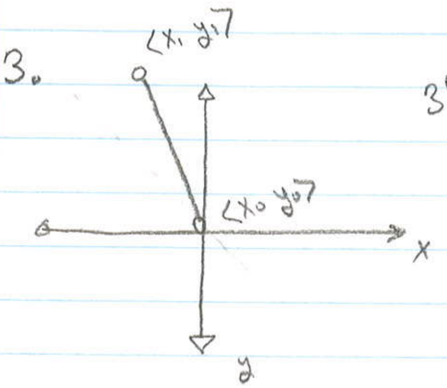
3.



3rd Octant

<u>Pseudocode</u>

```
dx = x1 - x0
dy = y1 - y0
slope = dx/dy
x = x0
for ( y = round(y0) ; y < y1 ; y++)
{
    plot(round(x), y)
    x = x + slope
}
```

8 pts. for logic.

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3 cont.

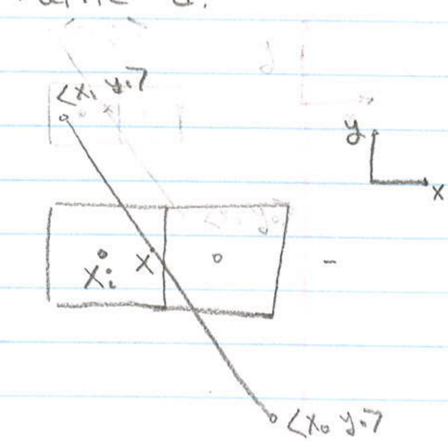
Full version of Bresenham – using integer arithmetic. 7 pts

```
int x = x0 - 1/2

d = 2(x1 - x0)(y0+1) + (y0 - y1)(2x0 - 1) +
    2x0y1 - 2x1y0

for (int y = y0; y < y1; y++)
{
    Plot(x, y)
    if (d ≤ 0) {
        x = x - 1
        d = d + 2(x1 - x0) - 2(y0 - y1)
    } else {
        d = d + 2(x1 - x0)
    }
}
```

Derivation of error metric d.



Want

$$x - x_i \le \frac{1}{2} \qquad (1)$$

$x = my + b$

$m = \dfrac{x_1 - x_0}{y_1 - y_0} \qquad b = \dfrac{x_0(y_1 - y_0) - y_0(x_1 - x_0)}{y_1 - y_0}$

Substitute in (1):

$$\frac{y(x_1 - x_0) + x_0(y_1 - y_0) - y_0(x_1 - x_0)}{y_1 - y_0} - \frac{x_i(y_1 - y_0)}{y_1 - y_0} \le \frac{1}{2}$$

$$\to y(x_1 - x_0) + x_0(y_1 - y_0) - y_0(x_1 - x_0) - x_i(y_1 - y_0) \le \frac{y_1 - y_0}{2}$$

$$\to 2(x_1 - x_0)y + 2(y_0 - y_1)x_i + 2x_0y_1 - 2\cancel{x_0y_0} - 2x_1y_0 + 2\cancel{x_0y_0} \le y_1 - y_0$$

$$\to \underbrace{2(x_1 - x_0)y + (y_0 - y_1)(2x_i + 1) + 2x_0y_1 - 2x_1y_0 \le 0.}_{d \le 0}$$

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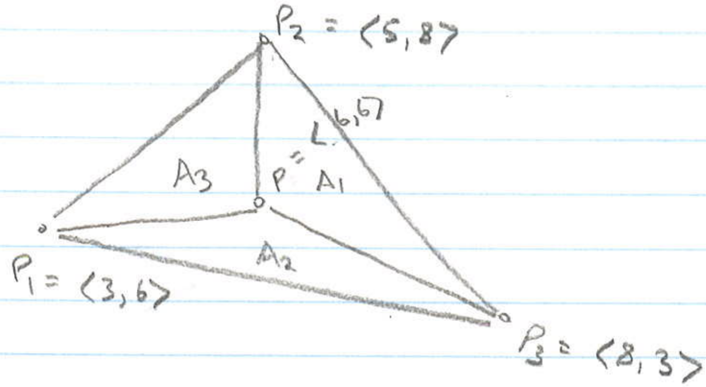
4.



$$\vec{PP_1} = P_1 - P = \langle -3, 0 \rangle$$
$$\vec{PP_2} = P_2 - P = \langle -1, 2 \rangle$$
$$\vec{PP_3} = P_3 - P = \langle 2, -3 \rangle$$

$$A_1 = \frac{1}{2}\|\vec{PP_3} \times \vec{PP_2}\| = \frac{1}{2}\|\langle 0,0,1 \rangle\| = \frac{1}{2}$$
$$A_2 = \frac{1}{2}\|\vec{PP_1} \times \vec{PP_3}\| = \frac{1}{2}\|\langle 0,0,9 \rangle\| = 4.5$$
$$A_3 = \frac{1}{2}\|\vec{PP_2} \times \vec{PP_1}\| = \frac{1}{2}\|\langle 0,0,6 \rangle\| = 3$$

$$A = A_1 + A_2 + A_3 = 8$$

$$\boxed{\begin{aligned} \alpha &= \frac{A_1}{A} = \frac{1}{16} = .0625 \\ \beta &= \frac{A_2}{A} = \frac{9}{16} = .5625 \\ \gamma &= \frac{A_3}{A} = \frac{3}{8} = .375 \end{aligned}}$$

21 pts.

$$\langle r\ g\ b \rangle \text{ @ } P = \alpha\langle .3\ .2\ .9 \rangle \text{ or } \alpha\langle .3\ .9\ .2 \rangle + \beta\langle .5\ .5\ .9 \rangle + \gamma\langle .7\ .2\ .7 \rangle$$

$$\boxed{\begin{aligned} &= \langle .5625\ .3688\ .8250 \rangle \text{ or} \\ &\langle .5625\ .4125\ .7812 \rangle \end{aligned}}$$

4 pts