1. We have to translate and then rotate the coordinate system

$$V = TR = \begin{bmatrix} u_x & u_y & u_z & 0 \\ v_x & v_y & v_z & 0 \\ w_x & w_y & w_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & -l_x \\ 0 & 1 & 0 & -l_y \\ 0 & 0 & 1 & -l_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$= M_{W2V}$ (from point's perspe

$$= \begin{bmatrix} -\frac{7}{\sqrt{50}} & 0 & -\frac{1}{\sqrt{50}} & \frac{6}{\sqrt{50}} \\ 0 & -1 & 0 & 1 \\ -\frac{1}{\sqrt{50}} & 0 & \frac{7}{\sqrt{50}} & -\frac{42}{\sqrt{50}} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$= M_{V2W}$ (from coordinat frame's perspect

$$= \begin{bmatrix} -0.9899 & 0 & -0.1414 & 0.8485 \\ 0 & -1 & 0 & 1 \\ -0.1414 & 0 & 0.9899 & -5.9397 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The above matrix has values substituted from these three vectors below

$$\boxed{u = \frac{t \times w}{\|t \times w\|} \qquad v = \frac{w \times u}{\|w \times u\|} \qquad w = -\hat{g}}$$

$$g = (1,1,-1) - (0,1,6) = (1,0,-7)$$

$$w = -\hat{g} = \frac{-g}{\|g\|} = \frac{-1}{\sqrt{1^2+0^2+7^2}}(1,0,-7) = \left(\frac{-1}{\sqrt{50}}, 0, \frac{7}{\sqrt{50}}\right)$$
(normalized)

$$t = (0,-1,0)$$

$$u = \frac{t \times w}{\|t \times w\|} = \frac{1}{\|t \times w\|}\begin{bmatrix} i & j & k \\ 0 & -1 & 0 \\ \frac{-1}{\sqrt{50}} & 0 & \frac{7}{\sqrt{50}} \end{bmatrix} = \left(\frac{-7}{\sqrt{50}}, 0, \frac{-1}{\sqrt{50}}\right)$$
(normalized)

$$v = \frac{w \times u}{\|w \times u\|} = \frac{1}{\|w \times u\|}\begin{bmatrix} i & j & k \\ -\frac{1}{\sqrt{50}} & 0 & \frac{7}{\sqrt{50}} \\ -\frac{7}{\sqrt{30}} & 0 & \frac{-1}{\sqrt{50}} \end{bmatrix} = \left(0, \frac{-49}{\sqrt{50}}, 0\right) \times \frac{1}{\|w \times v\|}$$
(normalized)

$$= \frac{1}{\frac{49}{\sqrt{50}}}\left(0, \frac{-49}{\sqrt{50}}, 0\right)$$

$$= (0,-1,0)$$

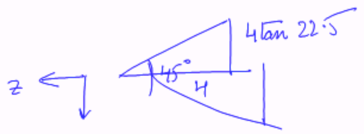
2. Perspective Projection

$n = 4$

$f = 20$

$fov_h = 45^\circ$

$aspect = 4:3$



$\frac{w}{h} = \frac{4}{3}$

$\frac{h}{2} = \frac{3}{4} \times w = 3 \tan 22.5$

$l = -4 \tan 22.5$

$r = 4 \tan 22.5$

$t = 3 \tan 22.5$

$b = -3 \tan 22.5$

$n = 4 \quad f = 20$

$$\begin{bmatrix} \frac{\cancel{2 \times 4}}{\cancel{8} \tan 22.5} & 0 & 0 & 0 \\ 0 & \frac{8}{6 \tan 22.5} & 0 & 0 \\ 0 & 0 & \frac{-\cancel{24}\,6}{\cancel{16}\,4} & -\frac{\cancel{2 \times 4 \times 20}\,10}{\cancel{16}\,4} \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2.414 & 0 & 0 & 0 \\ 0 & 3.219 & 0 & 0 \\ 0 & 0 & -1.5 & -10 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

3. $W = 800$ pixels
$H = 500$ pixels

NDC - to - display transformation matrix

$$= \begin{bmatrix} 1 & 0 & 0 & 399.5 \\ 0 & 1 & 0 & 249.5 \\ 0 & 0 & 1 & 0.5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 400 & 0 & 0 & 0 \\ 0 & 250 & 0 & 0 \\ 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

translate scale reflect

$$= \begin{bmatrix} 400 & 0 & 0 & 399.5 \\ 0 & -250 & 0 & 249.5 \\ 0 & 0 & 0.5 & 0.5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

4. $M\ P^T$ will give us the required coordinates in the camera coordinate system

All the transformation matrices used below are from point's perspective

(0.8485, 1, -5.9397, 1)
(-5.0912, 1, -6.7882, 1)
(-1.6971, 1, -9.3338, 1)
(-1.9799, -4, -7.3539, 1)

One of them is shown as example below

$$M_{W2V} \times P^T = \begin{bmatrix} -0.9899 & 0 & -0.1414 & 0.8485 \\ 0 & -1 & 0 & 1 \\ -0.1414 & 0 & 0.9899 & -5.9397 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.8485 \\ 1 \\ -5.9397 \\ 1 \end{bmatrix}$$

5.

(2.0483, 3.2190, -1.0905, 5.9397)
(-12.2901, 3.2190, 0.1823, 6.7882)
(-4.0967, 3.2190, 4.0007, 9.3338)
(-4.7795, -12.8760, 1.0309, 7.3539)

Explained below

P_T is in camera coordinate system

P_T' is now clipping coordinates

$$M_{V2CC} \times P_T = P_T'$$

$$\begin{bmatrix} 2.414 & 0 & 0 & 0 \\ 0 & 3.219 & 0 & 0 \\ 0 & 0 & -1.5 & -10 \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 0.8485 \\ 1 \\ -5.9397 \\ 1 \end{bmatrix} = \begin{bmatrix} 2.0483 \\ 3.2190 \\ -1.0905 \\ 5.9397 \end{bmatrix}$$

6. (0.3449, 0.5419, -0.1836, 1.0000)
(-1.8105, 0.4742, 0.0269, 1.0000)
(-0.4389, 0.3449, 0.4286, 1.0000)
(-0.6499, -1.7509, 0.1402, 1.0000)

These points are now in NDC

Since NDC extends from (-1,-1,-1) to (1,1
Notice that 2nd and 4th point are out of this range. This should end up outsi
the viewport finally

Explained below

P_T is in CC

P_T' is in NDC

$$\underbrace{\begin{bmatrix} 2.0483 \\ 3.2190 \\ -1.0905 \\ 5.9397 \end{bmatrix}}_{P_T} \qquad \underbrace{\begin{bmatrix} 2.0483/5.9397 \\ 3.2190/5.9397 \\ -1.0905/5.9397 \\ 5.9397/5.9397 \end{bmatrix}}_{P_T'} = \begin{bmatrix} 0.3449 \\ 0.5419 \\ -0.1836 \\ 1 \end{bmatrix}$$

7.

(537.4429, 114.0133, 0.4082, 1.0000)
(-324.7000, 130.9491, 0.5134, 1.0000)
(223.9364, 163.2812, 0.7143, 1.0000)
(139.5308, 687.2263, 0.5701, 1.0000)

These points are in display coordinate system

Notice that the 2nd and 4th point are outside the viewpo as expected

Explained below

$$\begin{bmatrix} .400 & 0 & 0 & 399.5 \\ 0 & -250 & 0 & 249.5 \\ 0 & 0 & 0.5 & 0.5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.3449 \\ 0.5419 \\ -0.1836 \\ 1 \end{bmatrix} = \begin{bmatrix} 537.4429 \\ 114.0133 \\ 0.4082 \\ 1 \end{bmatrix}$$

M_{NDC2DC} $\quad$ P_T (in NDC) $\quad$ P_T' (in display coordinates)

8.

$$\begin{bmatrix} x' \\ y' \\ z' \\ w' \end{bmatrix} = \begin{bmatrix} E & 0 & A & 0 \\ 0 & F & B & 0 \\ 0 & 0 & C & D \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$\frac{Ex + Az}{w'} = \frac{x'}{w'} \qquad (1)$$

$$w' = -z \qquad (2)$$

Eq (a)

$$\frac{x'}{w'} = 1 = \frac{E\,x}{-z} - A$$

$$\Rightarrow E\,\frac{\text{right}}{\text{near}} - A = 1 \qquad (a)$$

Eq (b)

$$\frac{x'}{w'} = -1 = \frac{E\,x}{-z} - A$$

$$\Rightarrow E\,\frac{\text{left}}{\text{near}} - A = -1 \qquad (b)$$

(a) − (b)

$$E\,\frac{(\text{right} - \text{left})}{\text{near}} = 2$$

$$\boxed{E = \frac{2\,\text{near}}{\text{right} - \text{left}}}$$

Put in (a)

$$-A = 1 - \frac{2\,\text{right}}{\text{right} - \text{left}}$$

$$-A = -\frac{\text{right} - \text{left}}{\text{right} - \text{left}}$$

$$A = \frac{\text{right} + \text{left}}{\text{right} - \text{left}}$$

9\. CCD imaging chip = 4 X 3 mm

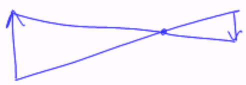
Painting = 50 cm tall

↳ completely fills vertical fov

focal length = 20 mm = 2 cm

This is how a virtual camera's ray diagram would look like.

But a real world camera's ray diagram looks like this



Both result in the same similar triangle configurations

Using similar triangles

$$\frac{50}{d} = \frac{.3}{2} \Rightarrow d = \frac{1000}{3}$$

$$= 333.33 \text{ cm}$$