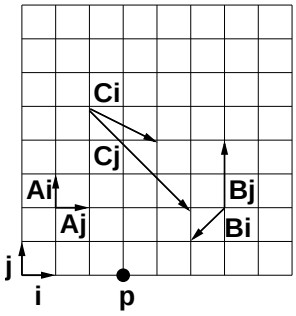
CPSC 314, Written Homework 1: Transformations. Solutions

Out: Mon 21 Jan 2008
Due: Wed 6 Feb 2008 1pm sharp
Value: 4% of final grade
Total Points: 100

1. (21 pts) The point coordinate P can be expressed as P = 3*i + 0*j, where i and j are basis vectors of unit length along the x and y axes, respectively. Describe the point P in terms of the 3 other coordinate systems given below.



   a) P = (-2,2) b) P = (3,.5) c) P = (2,-1)

2. (3 pts) Write down the 4x4 matrix for scaling an object by 2 in y and 3 in Z.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3. (10 pts) Give the OpenGL commands required to encode M. You may assume the matrix stack has been initialized with `glIdentity()`.

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

   glTranslatef(1.0, 1.0, 1.0); glScalef(1.0, 1.0, 2.0);

4. (6 pts) Homogenize the point (2,10,8,4).

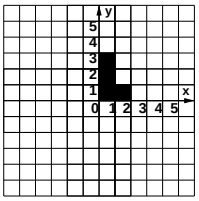
   (.5, 2.5, 2, 1)

5. (20 pts) Give the 4x4 OpenGL modelview matrix at the four lines A, B, C, and D below.

```
glLoadIdentity();
glTranslate(2,3,0);
A
glRotate(90, 0,1,0);
B
glPushMatrix();
glScale(1,.5,1);
glTranslate(1,1,0);
C
glPopMatrix();
glScale(2,1,1);
D
```

```
A
   | 1 0 0 2 |
   | 0 1 0 3 |
   | 0 0 1 0 |
   | 0 0 0 1 |

B
   | 1 0 0 2 |       |  0 0 1 0 |     |  0 0 1 2 |
   | 0 1 0 3 |   *   |  0 1 0 0 | =   |  0 1 0 3 |
   | 0 0 1 0 |       | -1 0 0 0 |     | -1 0 0 0 |
   | 0 0 0 1 |       |  0 0 0 1 |     |  0 0 0 1 |

C
   | 0 0 1 2 |       |  1 0   0 0 |     |  1 0 0 1 |   |  0 0   1  2   |
   | 0 1 0 3 |   *   |  0 0.5 0 0 | *   |  0 1 0 1 | = |  0 0.5 0  3.5 |
   |-1 0 0 0 |       |  0 0   1 0 |     |  0 0 1 0 |   | -1 0   0 -1   |
   | 0 0 0 1 |       |  0 0   0 1 |     |  0 0 0 1 |   |  0 0   0  1   |

D (multiply by matrix from line B)

   |  0 0 1 2 |      | 2 0 0 0 |      |  0 0 1 2 |
   |  0 1 0 3 | *    | 0 1 0 0 |  =   |  0 1 0 3 |
   | -1 0 0 0 |      | 0 0 1 0 |      | -2 0 0 0 |
   |  0 0 0 1 |      | 0 0 0 1 |      |  0 0 0 1 |
```

6. (40 pts) For each equation below, sketch the new location L' of the L shape on the grid and provide the OpenGL sequence needed to carry out those operations. Use the function `drawL()`, which draws an L shape with the lower left corner at the current origin as shown below. You may assume the matrix mode is `GL_MODELVIEW` and that the stack has been initialized with `glLoadIdentity()`. For reference, the OpenGL command syntax is `glRotatef(angle,x,y,z)`, `glTranslatef(x,y,z)`, `glScalef(x,y,z)`.




```
drawL();
```

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \mathbf{C} = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \mathbf{D} = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

```
Observe that...
A -> translate by 1 along X axis -> glTranslate[d|f]( 1, 0, 0 )
B -> rotate by 3/2 pi about Z axis - > glRotate[d|f]( 3 / 2 * PI, 0, 0, 1 )
C -> translate by -1 along X axis -> glTranslate[d|f]( -1, 0, 0 )
D -> scale by 2 along X axis -> glScale[d|f]( 2, 1, 1 )
```

a)

```
glTranslate[d|f]( 1, 0, 0 );
glRotate[d|f]( 3 / 2 * PI, 0, 0, 1 );
glTranslate[d|f]( -1, 0, 0 );
drawL();
```

b)

```
glTranslate[d|f]( 1, 0, 0 );
glTranslate[d|f]( -1, 0, 0 );
glScale[d|f]( 2, 1, 1 );
drawL();
```

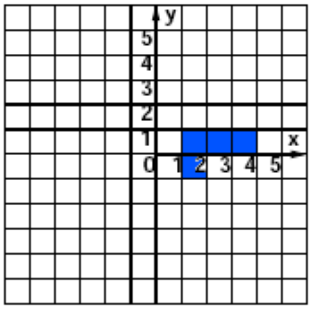
c)

```
glRotate[d|f]( 3 / 2 * PI, 0, 0, 1 );
glTranslate[d|f]( 1, 0, 0 );
glRotate[d|f]( 3 / 2 * PI, 0, 0, 1 );
drawL();
```

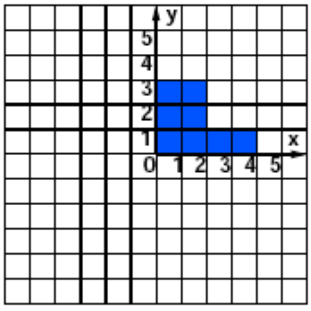
d)

```
glTranslate[d|f]( -1, 0, 0 );
glRotate[d|f]( 3 / 2 * PI, 0, 0, 1 );
glTranslate[d|f]( 1, 0, 0 );
glScale[d|f]( 2, 1, 1 );
drawL();
```

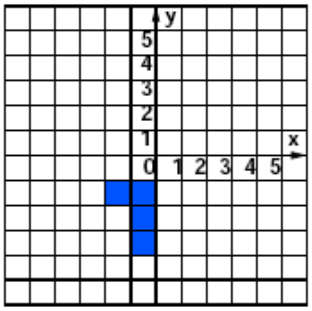
a) L' = ABC L



b) L' = ACD L



c) L' = BAB L



d) L' = CBAD L