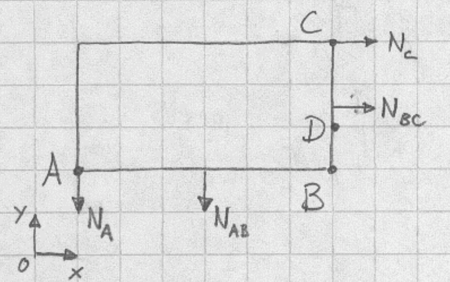
CPSC 314 HW3 Key

1) $I_a = [\,0.1 \;\; 0.1 \;\; 0.2\,]$ $k_d = [\,0.9 \;\; 0.2 \;\; 0.9\,]$

$I_L = [\,1.0 \;\; 0.9 \;\; 0.9\,]$ $k_a = [\,0.2 \;\; 0.2 \;\; 0.2\,]$

$k_s = [\,1.0 \;\; 1.0 \;\; 0.\,]$

n-shiny = 30



N_A = vertex normal @ A = $[\,0 \;\; -1 \;\; 0\,]$
N_C = vertex normal @ C = $[\,1 \;\; 0 \;\; 0\,]$

$A = [\,1 \;\; 2 \;\; 0\,]$ L E
$B = [\,7 \;\; 2 \;\; 0\,]$
$C = [\,7 \;\; 5 \;\; 0\,]$
$D = [\,7 \;\; 3 \;\; 0\,]$

N_{AB} = face normal of AB = $[\,0 \;\; -1 \;\; 0\,]$
N_{BC} = face normal of BC = $[\,1 \;\; 0 \;\; 0\,]$

1a) $$N_B = \frac{N_{AB} + N_{BC}}{\|N_{AB} + N_{BC}\|} = \left[\,\frac{\sqrt{2}}{2} \;\; -\frac{\sqrt{2}}{2} \;\; 0\,\right]$$

1b) Point B

B_{Amb} = ambient @ point B = $I_a k_a = [\,0.02 \;\; 0.02 \;\; 0.04\,]$

B_{Diff} = diffuse @ point B = $I_L k_d (N_B \cdot \ell_B)$

$$\ell_B = \frac{(L - B)}{\|L - B\|}$$ ← (normalized light direction vector)

$\hookrightarrow B_{Diff} = [\,0.888 \;\; 0.178 \;\; 0.799\,]$

B_{Spec} = specular @ point B = $I_L k_s (v_B \cdot r_{BL})^{n\text{-shiny}}$

$$v_B = \frac{(E - B)}{\|E - B\|} \qquad r_{BL} = 2 * N_B * (N_B \cdot \ell_B) - \ell_B$$

(view vector) (reflected ray from light L)

$\hookrightarrow B_{Spec} = [\,0.971 \;\; 0.874 \;\; 0\,]$

clamped

B_{Tot} = total @ point B = $B_{Amb} + B_{Diff} + B_{Spec} = [\,1.0 \;\; 1.0 \;\; 0.839\,]$

Point C)

For Flat Shading you need only need point B <u>or</u> point C, but we will need both for part 1c)

$$C_{Amb} = [\,0.02 \quad 0.02 \quad 0.04\,]$$

$$C_{Diff} = [\,0.285 \quad 0.057 \quad 0.256\,]$$

$$C_{spec} = [\,0.4349 \quad 0.3914 \quad 0\,] \times 10^{-8}$$

$$C_{Tot} = [\,0.3046 \quad 0.0769 \quad 0.2961\,]$$

Note: Shading model does not effect lighting at vertizes. lighting values at B, C do not change for parts 1c & 1d.

Point D)

For Flat Shading:

Points on a face take the illumination values of one of that face's vertizes.

$\therefore$ Illumination @ point D = Illumination @ point B or @ Point C

1c) Gouraud Shading

Illumination values are linearly interpolated across faces.

$$I_D = \frac{2}{3} I_B + \frac{1}{3} I_C$$

$$D_{Amb} = \frac{2}{3} B_{Amb} + \frac{1}{3} C_{Amb} = [\,0.02 \quad 0.02 \quad 0.04\,]$$

$$D_{Diff} = \frac{2}{3} B_{Diff} + \frac{1}{3} C_{Diff} = [\,0.687 \quad 0.1377 \quad 0.618\,]$$

$$D_{spec} = \frac{2}{3} B_{spec} + \frac{1}{3} C_{spec} = [\,0.6444 \quad 0.5800 \quad 0\,]$$

$$D_{Total} = D_{Amb} + D_{Diff} + D_{spec} = \frac{2}{3} B_{Total} + \frac{1}{3} C_{Total}$$

$$= [\,0.7682 \quad 0.6923 \quad 0.6580\,]$$

Computations in Detail

$*$ → componentwise mult, regular mult.
$\times$ → cross product
$\cdot$ → dot product

$$B_{Amb} = \underbrace{\begin{bmatrix} 0.1 \\ 0.1 \\ 0.2 \end{bmatrix}}_{I_\alpha} * \underbrace{\begin{bmatrix} 0.2 \\ 0.2 \\ 0.2 \end{bmatrix}}_{k_\alpha} = \begin{bmatrix} 0.02 \\ 0.02 \\ 0.04 \end{bmatrix}$$

$$l_B = \frac{\begin{bmatrix} 9 \\ -1 \\ 0 \end{bmatrix} - \begin{bmatrix} 7 \\ 2 \\ 0 \end{bmatrix}}{\left\| \begin{bmatrix} 9 \\ -1 \\ 0 \end{bmatrix} - \begin{bmatrix} 7 \\ 2 \\ 0 \end{bmatrix} \right\|} = \frac{\begin{bmatrix} 2 \\ -3 \\ 0 \end{bmatrix}}{\sqrt{2^2+3^2}} = \begin{bmatrix} 0.5547 \\ -0.8321 \\ 0 \end{bmatrix}$$

$$B_{Diff} = \underbrace{\begin{bmatrix} 1.0 \\ 0.9 \\ 0.9 \end{bmatrix}}_{I_L} * \underbrace{\begin{bmatrix} 0.9 \\ 0.2 \\ 0.9 \end{bmatrix}}_{k_d} * \left(\underbrace{\begin{bmatrix} \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} \\ 0 \end{bmatrix}}_{N_B} \cdot \underbrace{\begin{bmatrix} 0.5547 \\ -0.8321 \\ 0 \end{bmatrix}}_{l_B} \right)$$

$$= \begin{bmatrix} 0.9 \\ 0.18 \\ 0.81 \end{bmatrix} * \left(\frac{\sqrt{2}}{2} \cdot 0.5547 + \frac{-\sqrt{2}}{2} \cdot (-0.83221) \right) = \begin{bmatrix} 0.9 \\ 0.18 \\ 0.81 \end{bmatrix} \cdot 0.9867 = \begin{bmatrix} 0.88803 \\ 0.177606 \\ 0.7992 \end{bmatrix}$$

$$B_{Spec} = \underbrace{\begin{bmatrix} 1.0 \\ 0.9 \\ 0.9 \end{bmatrix}}_{I_L} * \underbrace{\begin{bmatrix} 1.0 \\ 1.0 \\ 0 \end{bmatrix}}_{k_s} * \left(\underbrace{\begin{bmatrix} 0.8575 \\ -0.5145 \\ 0 \end{bmatrix}}_{V_B} \cdot \underbrace{\begin{bmatrix} 0.8322 \\ -0.5547 \\ 0 \end{bmatrix}}_{r_{BL}} \right)^{30}$$

$$= \begin{bmatrix} 1.0 \\ 0.9 \\ 0 \end{bmatrix} * 0.9706$$

$$= \begin{bmatrix} 0.9706 \\ 0.8735 \\ 0 \end{bmatrix}$$

$$V_B = \frac{\begin{bmatrix} 12 \\ -1 \\ 0 \end{bmatrix} - \begin{bmatrix} 7 \\ 2 \\ 0 \end{bmatrix}}{\left\| \begin{bmatrix} 12 \\ -1 \\ 0 \end{bmatrix} - \begin{bmatrix} 7 \\ 2 \\ 0 \end{bmatrix} \right\|} = \frac{\begin{bmatrix} 5 \\ -3 \\ 0 \end{bmatrix}}{\sqrt{5^2+3^2}} = \begin{bmatrix} 0.857 \\ -0.514 \\ 0 \end{bmatrix}$$

$$r_{BL} = 2 * \underbrace{\begin{bmatrix} \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} \\ 0 \end{bmatrix}}_{N_B} * \left(\underbrace{\begin{bmatrix} \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} \\ 0 \end{bmatrix}}_{N_B} \cdot \underbrace{\begin{bmatrix} 0.5547 \\ -0.8321 \\ 0 \end{bmatrix}}_{l_B} \right) - l_B$$

$$r_{BL} = \begin{bmatrix} \sqrt{2} \\ -\sqrt{2} \\ 0 \end{bmatrix} * (0.9806) - \begin{bmatrix} 0.5547 \\ -0.8321 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.83 \\ -0.554 \\ 0 \end{bmatrix}$$

$$l_c = \text{normalise}\left(\begin{bmatrix}9\\-1\\0\end{bmatrix} - \begin{bmatrix}7\\5\\0\end{bmatrix}\right) = \frac{\begin{bmatrix}2\\-6\\0\end{bmatrix}}{\sqrt{4+36}} = \begin{bmatrix}0.3162\\-0.9487\\0\end{bmatrix}$$

$$C_{Amb} = B_{Amb}$$

$$C_{Diff} = \underset{I_L\,k_{Diff}}{\begin{bmatrix}0.9\\0.18\\0.81\end{bmatrix}} * \left(\underset{N_c}{\begin{bmatrix}1\\0\\0\end{bmatrix}} \cdot \underset{l_c}{\begin{bmatrix}0.3162\\-0.9487\\0\end{bmatrix}}\right) = \begin{bmatrix}0.9\\0.18\\0.81\end{bmatrix} \cdot (0.3162) = \begin{bmatrix}0.285\\0.057\\0.256\end{bmatrix}$$

$$C_{spec} = \underset{I_L\,k_S}{\begin{bmatrix}1.0\\0.9\\0\end{bmatrix}} * \left(\underset{V_c}{\begin{bmatrix}0.6402\\-0.7682\\0\end{bmatrix}} \cdot \underset{r_{CL}}{\begin{bmatrix}0.3162\\0.9487\\0\end{bmatrix}}\right)^{30} = \begin{bmatrix}1.0\\0.9\\0\end{bmatrix} * 4.349\times10^{-9} = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

effectively zero

$$V_c = \frac{\begin{bmatrix}12\\-1\\0\end{bmatrix} - \begin{bmatrix}7\\5\\0\end{bmatrix}}{\sqrt{5^2+6^2}} = \begin{bmatrix}0.6402\\-0.7682\\0\end{bmatrix}$$

$$r_{CL} = 2 * \underset{N_c}{\begin{bmatrix}1\\0\\0\end{bmatrix}} * \left(\underset{N_c}{\begin{bmatrix}1\\0\\0\end{bmatrix}} \cdot \underset{l_c}{\begin{bmatrix}0.3162\\-0.9487\\0\end{bmatrix}}\right) - l_c$$

$$= \begin{bmatrix}2\\0\\0\end{bmatrix} * 0.3162 - \begin{bmatrix}0.3162\\-0.9487\\0\end{bmatrix}$$

$$= \begin{bmatrix}0.3162\\0.9487\\0\end{bmatrix}$$

$$C_{Tot} = \begin{bmatrix}0.305\\0.077\\0.296\end{bmatrix}$$

FLAT SHADING

$$D_{Amb} = B_{Amb}$$

$$l_D = \text{normalise}\left(\underset{L}{\begin{bmatrix}9\\-1\\0\end{bmatrix}} - \underset{D}{\begin{bmatrix}7\\3\\0\end{bmatrix}}\right) = \frac{\begin{bmatrix}2\\-4\\0\end{bmatrix}}{\sqrt{4+16}}$$

$$l_D = \begin{bmatrix}0.4472\\-0.8944\\0\end{bmatrix}$$

$$D_{Diff} = \underset{I_L\,k_D}{\begin{bmatrix}0.9\\0.18\\0.81\end{bmatrix}} * \left(\underset{N_D}{\begin{bmatrix}1\\0\\0\end{bmatrix}} \cdot \underset{l_D}{\begin{bmatrix}0.4472\\-0.8944\\0\end{bmatrix}}\right) = \begin{bmatrix}0.9\\0.18\\0.81\end{bmatrix} * 0.4472$$

$$= \begin{bmatrix}0.403\\0.080\\0.362\end{bmatrix}$$

$$D_{spec} = \underset{I_L\,k_S}{\begin{bmatrix}1.0\\0.9\\0\end{bmatrix}} * \left(\underbrace{\underset{V_D}{\begin{bmatrix}0.7808\\-0.6247\\0\end{bmatrix}} \cdot \underset{r_{DL}}{\begin{bmatrix}0.4472\\0.8944\\0\end{bmatrix}}}_{\text{see next page}}\right)^{30} = \begin{bmatrix}1.0\\0.9\\0\end{bmatrix} * 4.334\times10^{-21} = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

(effectively)

1d) Normals are linearly interpolated across face.

$$N_D = \frac{2}{3}N_B + \frac{1}{3}N_C = [\,0.8047 \quad -0.4714 \quad 0\,]$$

normalize $\rightarrow [\,0.8629 \quad -0.5054 \quad 0\,]$

Now recompute lighting at point D with new normal

$$D_{Amb} = I_A \cdot k_A = [\,0.02 \quad 0.02 \quad 0.04\,]$$

by now you've noticed this never changes.

$$D_{Diff} = I_L k_d \,(N_D \cdot \ell_D) = [\,0.7541 \quad 0.1508 \quad 0.6785\,]$$

$$\ell_D = \frac{(L-D)}{\|L-D\|}$$

$$D_{spec} = I_L k_s \,(v_D \cdot r_{DL})^{30} = [\,0. \quad 0 \quad 0\,]$$

$$v_D = \frac{(E-D)}{\|E-D\|} \qquad r_{DL} = 2 * N_D * (N_D \bullet \ell_D) - \ell_D$$

$$D_{Total} = D_{Amb} + D_{Diff} + D_{spec} = [\,0.7741 \quad 0.1708 \quad 0.7185\,]$$

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2) Colour $\quad RGB = [\,0.5000 \quad 0.2000 \quad 0.8000\,]$

$$CMY = [\,1 \quad 1 \quad 1\,] - RGB = [\,0.500 \quad 0.800 \quad 0.200\,]$$

$$YIQ = \begin{bmatrix} 0.3 & 0.59 & 0.11 \\ 0.6 & -0.28 & -0.32 \\ 0.21 & -0.52 & 0.31 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} 0.3560 \\ -0.0120 \\ 0.2490 \end{bmatrix}$$

HSV:

$$V = \frac{R+G+B}{3} = 0.500 \qquad \text{value (brightness)}$$

$$S = 1 - \frac{\min(R,G,B)}{V} = 0.600 \qquad \text{saturation}$$

$$H' = \arccos\left[\frac{\frac{1}{2}[(R-G)+(R-B)]}{\sqrt{(R-G)^2+(R-B)(G-B)}}\right] \qquad \text{hue angle}$$

note: if $B > G \quad H = 360 - H'$
else $\quad H = H'$

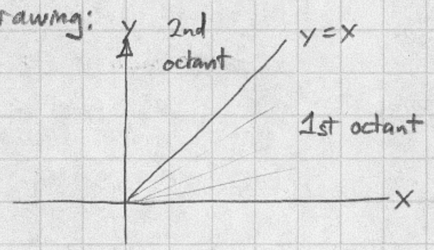
$$H = 360 - H' = 360 - 90$$

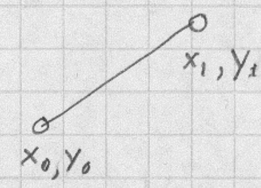
$$H = 270°$$

3) Rasterization

(swap x, y in algorithm

line Drawing:





- instead of iterating along x, computing y-values, iterate along y, computing x-values

Basic Algorithm:

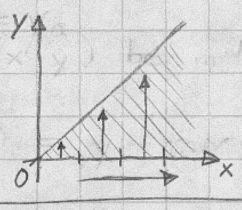
```
float dx, dy, x, y, slope

    dx = x1 - x0
    dy = y1 - y0

    slope = dx/dy      <- note inversion!  run/rise

    x = x0

    for (int y = round(y0); y < y1 ; y++)
    {
        PlotPixel (round(x), y);

        x = x + slope;
    }
```

3 con't)

line $y = mx$

Full Bresenham Algorithm

1st octant $m \in [0, 1]$



$y = y_0$

error func →

$$2d = 2*(y_0 - y_1)(x_0 + 1) + (x_1 - x_0)(2y_0 + 1) + 2x_0 y_1 - 2x_1 y_0$$

```
for (x = x0 ; x < x1 ; x++)
{
   PlotPixel (x, y);
   if (d < 0)
   {  y = y+1
      d = d+2(x1-x0) + 2(y0-y1)
   }
   else
   {
      d = d+2(y0 - y1)
   }
}
```

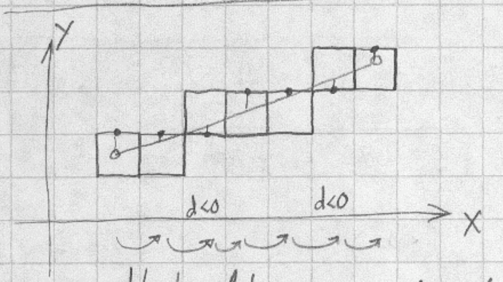
if line is above midpoint → if (d<0)

step up → y = y+1

add error → d = d+2(x1-x0) + 2(y0-y1)

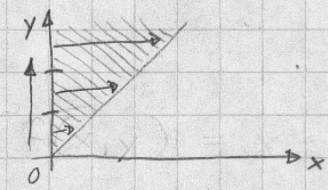
don't step up → d = d+2(y0 - y1)

add error → }



black dots are midpoints

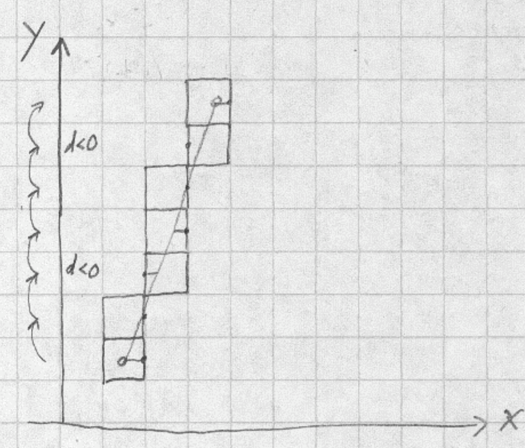
2nd Octant $m \in (1, \infty)$



$x = x_0$

$$2d = 2*(x_0 - x_1)(y_0 + 1) + (y_1 - y_0)(2x_0 + 1) + 2y_0 x_1 - 2y_1 x_0$$

```
for (y = y0 ; y < y1 ; y++)
{
   PlotPixel (x, y);
   if (d < 0)
   {  x = x +1
      d = d + 2 (y1 - y0) + 2(x0 - x1)
   }
   else
   {
      d = d + 2 (x0 - x1)
   }
}
```



4)

$P1 = [2 \;\; 5 \;\; 0]$

$P2 = [5 \;\; 12 \;\; 0]$

$P3 = [10 \;\; 0 \;\; 0]$

colour

$C_1 = [0.5 \;\; 0.7 \;\; 0.1]$

$C_2 = [0.8 \;\; 0.8 \;\; 0.2]$

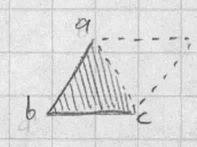
$C_3 = [0.9 \;\; 0.1 \;\; 0.6]$

$P = [4 \;\; 7 \;\; 0]$

$$\alpha = \frac{Area(P, P2, P3)}{Area(P1, P2, P3)} \qquad \beta = \frac{Area(P, P1, P3)}{Area(P1, P2, P3)} \qquad \gamma = \frac{Area(P, P1, P2)}{Area(P1, P2, P3)}$$

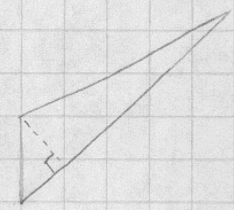
$\alpha = 0.5211$ $\qquad \beta = 0.3662$ $\qquad \gamma = 0.1127$

$$Area(a, b, c) = \frac{1}{2} \| \underbrace{(a-b) \times (b-c)}_{\text{cross product}} \|$$



or:

$$Area(a, b, c) = \frac{1}{2} \sqrt{\|a-b\|^2 \|b-c\|^2 - (\underbrace{(a-b) \cdot (b-c)}_{\text{dot product}})^2}$$



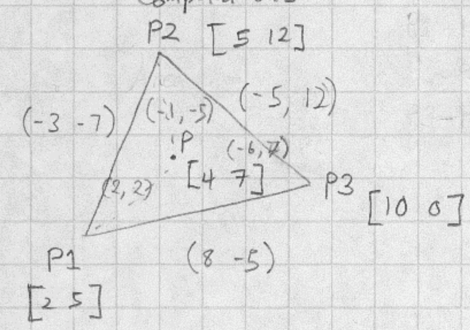
colour @ p

$$C_p = \alpha C_1 + \beta C_2 + \gamma C_3$$

$$C_p = 0.5211 \cdot \begin{bmatrix} 0.5 \\ 0.7 \\ 0.1 \end{bmatrix}^T + 0.3662 \cdot \begin{bmatrix} 0.8 \\ 0.8 \\ 0.2 \end{bmatrix}^T + 0.1127 \cdot \begin{bmatrix} 0.9 \\ 0.1 \\ 0.6 \end{bmatrix}^T$$

$$C_p = [0.6549 \;\; 0.6690 \;\; 0.1930]$$

Computations



$$Area(P1, P2, P3) = \frac{1}{2} \sqrt{\|P_1 - P_2\|^2 \|P_2 - P_3\|^2 - ((P_1 - P_2)(P_2 - P_3))^2}$$

$$= \frac{1}{2} \sqrt{58 \cdot 169 - 4761} = 35.5$$

$$Area(P, P2, P3) = \frac{1}{2} \sqrt{\|P - P_2\|^2 \|P_2 - P_3\|^2 - ((P - P_2)(P_2 - P_3))^2}$$

$$= \frac{1}{2} \sqrt{26 \cdot 169 - 3025} = 18.5$$

$$Area(P, P1, P3) = \frac{1}{2} \sqrt{\|P - P_1\|^2 \|P_1 - P_3\|^2 - ((P - P_1) \cdot (P_1 - P_3))^2}$$

$$= \frac{1}{2} \sqrt{8 \cdot 89 - 36} = 13$$

$$Area(P, P1, P2) = \frac{1}{2} \sqrt{\|P - P_1\|^2 \|P_1 - P_2\|^2 - ((P - P_1) \cdot (P_1 - P_2))^2} = \frac{1}{2} \sqrt{8 \cdot 58 - 400} = 4$$

$$\left. \begin{aligned} \alpha &= \frac{18.5}{35.5} = 0.5211 \\ \beta &= \frac{13}{35.5} = 0.3662 \\ \gamma &= \frac{4}{35.5} = 0.1127 \end{aligned} \right\} \text{sum to } 1 \; \boxed{\checkmark}$$