CS 314 Homework 2

① $\text{gluLookAt}(2, 10, 3,\ -2, 2, 0,\ \overbrace{-1, -1, 0}^{t});$

-1 pt each for sign errors

1. compute normalized gaze vector

$$g = \begin{bmatrix} -4 \\ -8 \\ -3 \end{bmatrix} \ (\text{lookAt} - \text{eye}) \qquad \|g\| = \sqrt{16 + 64 + 9} = \sqrt{89}$$

$$w = -\frac{g}{\|g\|} = -\frac{1}{\sqrt{89}}\begin{bmatrix} -4 \\ -8 \\ -3 \end{bmatrix} = \underline{\frac{1}{\sqrt{89}}\begin{bmatrix} 4 \\ 8 \\ 3 \end{bmatrix} = \begin{bmatrix} 0.424 \\ 0.848 \\ 0.318 \end{bmatrix}} \qquad \textbf{2 pt}$$

2. compute normalized vector that's perpendicular to gaze & up vector

$$u = \frac{1}{\|t \times w\|}\, t \times w \Rightarrow \qquad t \times w = \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix} \times \frac{1}{\sqrt{89}}\begin{bmatrix} 4 \\ 8 \\ 3 \end{bmatrix} = \frac{1}{\sqrt{89}}\begin{bmatrix} -3 \\ 3 \\ -4 \end{bmatrix}$$

$$\|t \times w\| = \frac{1}{\sqrt{89}}\sqrt{9 + 9 + 16} = \frac{\sqrt{34}}{\sqrt{89}}$$

$$\underline{u = \frac{1}{\sqrt{34}}\begin{bmatrix} -3 \\ 3 \\ -4 \end{bmatrix} = \begin{bmatrix} -0.5145 \\ 0.5145 \\ -0.686 \end{bmatrix}} \qquad 2 \text{ pt}$$

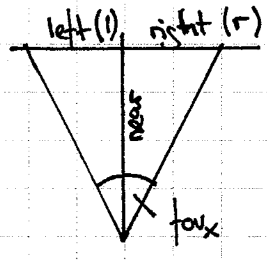
3. compute normalized vector that's perpendicular to gaze and z.

$$v = w \times u = \frac{1}{\sqrt{89}}\begin{bmatrix} 4 \\ 8 \\ 3 \end{bmatrix} \times \frac{1}{\sqrt{34}}\begin{bmatrix} -3 \\ 3 \\ -4 \end{bmatrix} = \underline{\frac{1}{\sqrt{89}\sqrt{34}}\begin{bmatrix} -41 \\ 7 \\ 36 \end{bmatrix} = \begin{bmatrix} -0.745 \\ 0.127 \\ 0.654 \end{bmatrix}} \qquad 2\text{pt}$$

4. compute final matrix

$$M = R \cdot T = \begin{bmatrix} u_x & u_y & u_z & 0 \\ v_x & v_y & v_z & 0 \\ w_x & w_y & w_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -eye_x \\ 0 & 1 & 0 & -eye_y \\ 0 & 0 & 1 & -eye_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} -3/\sqrt{34} & 3/\sqrt{34} & -4/\sqrt{34} & -12/\sqrt{34} \\ -41/\sqrt{3026} & 7/\sqrt{3026} & 36/\sqrt{3026} & -96/\sqrt{3026} \\ 4/\sqrt{89} & 8/\sqrt{89} & 3/\sqrt{89} & -97/\sqrt{89} \\ 0 & 0 & 0 & 1 \end{bmatrix} = \underline{\begin{bmatrix} -0.51 & 0.51 & -0.69 & -2.1 \\ -0.75 & 0.13 & 0.65 & -1.75 \\ 0.42 & 0.85 & 0.32 & -10.28 \\ 0 & 0 & 0 & 1 \end{bmatrix}} \qquad 2\text{pt}$$

② 

near $= 1$, far $= 100$, aspect $= \frac{1}{2}$

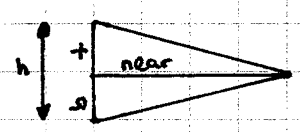
$fov_x = 30°$

$\tan\left(\frac{fov_x}{2}\right) = \frac{r}{near}$ $\underline{r = \tan(15°) = 0.268}$ 1pt

$\underline{l = -r = -0.268}$ 1 pt

- width of image plane at near $w = 2 \cdot r = 0.5356$
- get height of image plane at near : $\frac{w}{h} = \text{aspect} = \frac{1}{2}$

$h = 2 \cdot 0.5356$



$\underline{t = \frac{h}{2} = 0.5356}$ 1 pt

$\underline{b = -t = -0.5356}$ 1 pt

$$P = \begin{bmatrix} \frac{2 \cdot near}{r-l} & 0 & \frac{r+l}{r-l} & 0 \\ 0 & \frac{2 \cdot near}{t-b} & \frac{t+b}{t-b} & 0 \\ 0 & 0 & -\frac{far+near}{far-near} & -\frac{2\, far\, near}{far-near} \\ 0 & 0 & -1 & 0 \end{bmatrix} = \underline{\begin{bmatrix} 3.734 & 0 & 0 & 0 \\ 0 & 1.867 & 0 & 0 \\ 0 & 0 & -1.02 & -2.02 \\ 0 & 0 & -1 & 0 \end{bmatrix}} \quad 4pt$$

OR

calculate fov_y and use simpler gluPerspective (fov_y, aspect, near, far) matrix.

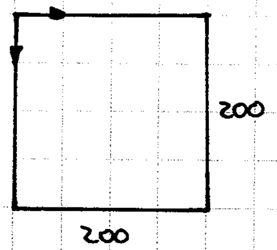
① set width of image plane and t as described above

$\tan\left(\frac{fov_y}{2}\right) = t$ $\underline{fov_y = 2 \cdot \tan^{-1}(t) = 56.3768°}$ 4 pt

$$P = \begin{bmatrix} f/\text{aspect} & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & -\frac{far+near}{far-near} & -\frac{2 \cdot far \cdot near}{far-near} \\ 0 & 0 & -1 & 0 \end{bmatrix} = \underline{\begin{bmatrix} 3.734 & 0 & 0 & 0 \\ 0 & 1.867 & 0 & 0 \\ 0 & 0 & -1.02 & -2.02 \\ 0 & 0 & -1 & 0 \end{bmatrix}} \quad 4\ pt$$

(2pt for these 2 values)

③



$$V = \begin{bmatrix} 1 & 0 & 0 & 99.5 \\ 0 & 1 & 0 & 99.5 \\ 0 & 0 & 1 & 0.5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 100 & 0 & 0 & 0 \\ 0 & 100 & 0 & 0 \\ 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \underline{\begin{bmatrix} 100 & 0 & 0 & 99.5 \\ 0 & -100 & 0 & 99.5 \\ 0 & 0 & 0.5 & 0.5 \\ 0 & 0 & 0 & 1 \end{bmatrix}}$$

translate — 2pt; scale — 2pt; reflect — 2pt; 2pt

full marks for correct matrix multiplication with incorrect matrices for ④ - ⑦

④

$$p'_{world} = \begin{pmatrix} 3 \\ 2 \\ 1 \\ 1 \end{pmatrix} \quad p''_{world} = \begin{pmatrix} 0 \\ 0 \\ -3 \\ 1 \end{pmatrix} \quad p'''_{world} = \begin{pmatrix} -2 \\ -1 \\ 2 \\ 1 \end{pmatrix} \quad p^{IV}_{world} = \begin{pmatrix} 1 \\ 5 \\ -1 \\ 1 \end{pmatrix}$$

cascading errors within ④ - ⑦ no marks!

multiply world coordinates by view matrix

$$p'_{camera} = M \cdot p'_{world} = \begin{pmatrix} -3.26 \\ -3.07 \\ -6.7 \\ 1 \end{pmatrix} \text{ 2pt} \qquad p''_{camera} = M \cdot p''_{world} = \begin{pmatrix} 0 \\ -3.71 \\ -11.24 \\ 1 \end{pmatrix} \text{ 2pt}$$

$$p'''_{camera} = M \cdot p'''_{world} = \begin{pmatrix} -2.92 \\ 0.93 \\ -11.34 \\ 1 \end{pmatrix} \text{ 2pt} \qquad p^{IV}_{camera} = M \cdot p^{IV}_{world} = \begin{pmatrix} 0.69 \\ -2.51 \\ -5.94 \\ 1 \end{pmatrix} \text{ 2pt}$$

with
$$M = \begin{pmatrix} -0.51 & 0.51 & -0.69 & -2.1 \\ -0.75 & 0.13 & 0.65 & -1.75 \\ 0.42 & 0.85 & 0.32 & -10.28 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

⑤ multiply points in camera coordinates by projection matrix

$$p'_{clip} = P \cdot p'_{camera} = \begin{pmatrix} -12.17 \\ -5.74 \\ 5.12 \\ 6.7 \end{pmatrix} \text{ 2pt} \qquad p''_{clip} = P \cdot p''_{camera} = \begin{pmatrix} 0 \\ -6.92 \\ 9.44 \\ 11.24 \end{pmatrix} \text{ 2pt} \qquad p'''_{clip} = P \cdot p'''_{camera} = \begin{pmatrix} -10.89 \\ 1.73 \\ 9.55 \\ 11.34 \end{pmatrix} \text{ 2pt}$$

$$p^{IV}_{clip} = P \cdot p^{IV}_{camera} = \begin{pmatrix} 2.56 \\ -4.68 \\ 4.03 \\ 5.94 \end{pmatrix} \text{ 2pt}$$

with
$$P = \begin{pmatrix} 3.734 & 0 & 0 & 0 \\ 0 & 1.867 & 0 & 0 \\ 0 & 0 & -1.02 & -2.02 \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

⑥ $$p'_{NDC} = \begin{bmatrix} p'_{clip}[0] / p'_{clip}[3] \\ p'_{clip}[1] / p'_{clip}[3] \\ p'_{clip}[2] / p'_{clip}[3] \\ 1.0 \end{bmatrix} = \begin{bmatrix} -1.74 \\ -0.82 \\ 0.73 \\ 1.0 \end{bmatrix} \text{ 2pt} \qquad \text{with} \quad p'_{clip} = \begin{bmatrix} p'_{clip}[0] = -12.17 \\ p'_{clip}[1] = -5.74 \\ p'_{clip}[2] = 5.12 \\ p'_{clip}[3] = 6.7 \end{bmatrix}$$

↳ divide each element of the vector by it's homogeneous coordinate w

similarly:

$$p''_{NDC} = \begin{bmatrix} 0 \\ -0.62 \\ 0.84 \\ 1.0 \end{bmatrix} \qquad p'''_{NDC} = \begin{bmatrix} -0.96 \\ 0.15 \\ 0.84 \\ 1.0 \end{bmatrix} \qquad p^{iv}_{NDC} = \begin{bmatrix} 0.43 \\ -0.79 \\ 0.68 \\ 1.0 \end{bmatrix}$$

2 pt 2pt 2pt

⑦ multiply points in normalized device coordinates by viewport matrix

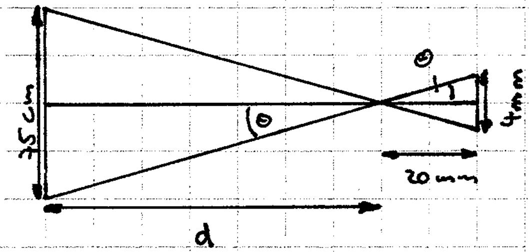
$$p_{2D} = V \cdot p_{NDC} = \begin{bmatrix} 100 & 0 & 0 & 99.5 \\ 0 & -100 & 0 & 99.5 \\ 0 & 0 & 0.5 & 0.5 \\ 0 & 0 & 0 & 1 \end{bmatrix} p_{NDC} \qquad p_{2D} = \begin{bmatrix} \text{pixel coord } x \\ \text{pixel coord } y \\ \text{depth} \in [0.0, 1.0] \\ 1.0 \end{bmatrix}$$

$$p'_{2D} = \begin{bmatrix} -74.42 \\ 181.49 \\ 0.87 \\ 1.0 \end{bmatrix} \qquad p''_{2D} = \begin{bmatrix} 99.5 \\ 161.12 \\ 0.92 \\ 1.0 \end{bmatrix} \qquad p'''_{2D} = \begin{bmatrix} 3.52 \\ 84.24 \\ 0.92 \\ 1.0 \end{bmatrix} \qquad p^{iv}_{2D} = \begin{bmatrix} 142.65 \\ 178.40 \\ 0.84 \\ 1.0 \end{bmatrix}$$

2pt 2pt 2pt 2pt

correct depth and pixelshift (-0.5) optional

⑧



distance to image

4 pt for image

$$\frac{75/2 \text{ cm}}{d \text{ cm}} = \frac{4/2 \text{ mm}}{20 \text{ mm}}$$

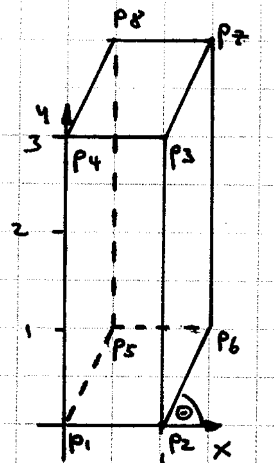
2 pt for equation

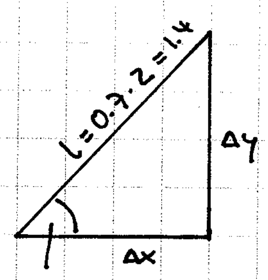
$\underline{d = 375 \text{ cm}}$

2 pt for final value

(9)

a)





$\Theta = 70^\circ$

$\sin\Theta = \frac{\Delta y}{1.4}$ $\qquad \underline{\Delta y = 1.32}$

$\cos\Theta = \frac{\Delta x}{1.4}$ $\qquad \underline{\Delta x = 0.48}$

1pt for the drawing

$$p_1 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \quad p_2 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix} \quad p_3 = \begin{pmatrix} 1 \\ 3 \\ 0 \\ 1 \end{pmatrix} \quad p_4 = \begin{pmatrix} 0 \\ 3 \\ 0 \\ 1 \end{pmatrix} \quad p_5 = \begin{pmatrix} 0.48 \\ 1.32 \\ 0 \\ 1 \end{pmatrix} \quad p_6 = \begin{pmatrix} 1.48 \\ 1.32 \\ 0 \\ 1 \end{pmatrix} \quad p_7 = \begin{pmatrix} 1.48 \\ 4.32 \\ 0 \\ 1 \end{pmatrix} \quad p_8 = \begin{pmatrix} 0.48 \\ 4.32 \\ 0 \\ 1 \end{pmatrix}$$

1pt for each correct point

9pts total for a)

b) two different ways to solve for the Matrix

① Solve equation system with the given points in 3D and their corresponding points in 2D.

$$M = \begin{pmatrix} m_{00} & m_{01} & m_{02} & m_{03} \\ m_{10} & m_{11} & m_{12} & m_{13} \\ m_{20} & m_{21} & m_{22} & m_{23} \\ m_{30} & m_{31} & m_{32} & m_{33} \end{pmatrix}$$

I use p_1
$$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} m_{00} & m_{01} & m_{02} & m_{03} \\ m_{10} & m_{11} & m_{12} & m_{13} \\ m_{20} & m_{21} & m_{22} & m_{23} \\ m_{30} & m_{31} & m_{32} & m_{33} \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

$\to m_{03} = 0,\ m_{13} = 0,\ m_{23} = 0,\ m_{33} = 1$

0.5pt for each matrix entry only when solving with this approach

II use p_5
$$\begin{pmatrix} 0.48 \\ 1.32 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} m_{00} & m_{01} & m_{02} & 0 \\ m_{10} & m_{11} & m_{12} & 0 \\ m_{20} & m_{21} & m_{22} & 0 \\ m_{30} & m_{31} & m_{32} & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ -2 \\ 1 \end{pmatrix}$$

$\to m_{02} = -0.24,\ m_{12} = -0.66,\ m_{22} = 0,\ m_{32} = 0$

III use p_2
$$\begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} m_{00} & m_{01} & -0.24 & 0 \\ m_{10} & m_{11} & -0.66 & 0 \\ m_{20} & m_{21} & 0 & 0 \\ m_{30} & m_{31} & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

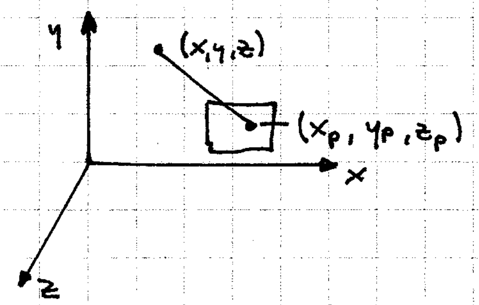
$\to m_{00} = 1,\ m_{10} = 0,\ m_{20} = 0,\ m_{30} = 0$

IV use p_8
$$\begin{pmatrix} 0.48 \\ 4.32 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & m_{01} & -0.24 & 0 \\ 0 & m_{11} & -0.66 & 0 \\ 0 & m_{21} & 0 & 0 \\ 0 & m_{31} & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 3 \\ -2 \\ 1 \end{pmatrix}$$

$m_{01} = 0$
$m_{11} = 1$
$m_{21} = 0$
$m_{31} = 0$

, thus
$$\boxed{M = \begin{pmatrix} 1 & 0 & -0.24 & 0 \\ 0 & 1 & -0.66 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}}$$

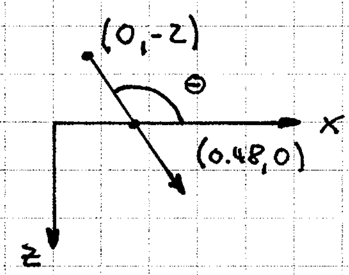
9) b) ② second possible solution (with the same answer)



we need $(x, y, z) \rightarrow (x_p, y_p, z_p)$

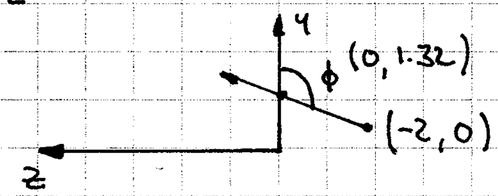
use p_5 $(0, 0, -2) \rightarrow (0.48, 1.32, 0)$

$$M = \begin{pmatrix} 1 & 0 & -\cot\theta & 0 \\ 0 & 1 & -\cot\phi & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$ where



1 pt for drawing

and



1 pt for drawing

Ⅰ
$x_p = x - z\cot\theta$

$0.48 = 2\cot\theta \rightarrow \underline{\cot\theta = 0.24}$ 2pt

Ⅱ
$y_p = y - z\cot\phi$

$1.32 = 2\cot\phi \rightarrow \underline{\cot\phi = 0.66}$ 2pt

thus $M = \begin{pmatrix} 1 & 0 & -0.24 & 0 \\ 0 & 1 & -0.66 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ 2pt

confirm that points in the xy plane are not changed by projection:

$$\begin{pmatrix} 1 & 0 & -0.24 & 0 \\ 0 & 1 & -0.66 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} x \\ y \\ 0 \\ 1 \end{pmatrix}$$ ↯ 1 pt

9 pt total for b)

(10) to convert from world coordinates x_{world}, y_{world} to pixel coordinates x_{pixel}, y_{pixel} we need to

a) flip the y axis

b) shift by $-x_c$, $-y_c$ (a point at (x_c, y_c) would be translated to $(0,0)$)

c) convert from feet to inches (divide by S)

d) convert from inches to pixels (for width multiply by $\frac{800}{W}$, for height by $\frac{600}{H}$)

e) shift to center of window (by $(400, 300)$)

$$P_{pixel} := \begin{pmatrix} x_{pixel} \\ y_{pixel} \\ 1 \end{pmatrix} = \underbrace{\overset{2pt}{\begin{pmatrix} 1 & 0 & 400 \\ 0 & 1 & 300 \\ 0 & 0 & 1 \end{pmatrix}}}_{\text{shift to center e)}} \cdot \underbrace{\overset{2pt}{\begin{pmatrix} \frac{800}{W} & 0 & 0 \\ 0 & \frac{600}{H} & 0 \\ 0 & 0 & 1 \end{pmatrix}}}_{\text{convert from } \cancel{\text{feet to inches c)}} \text{ inches to pixels d)}} \cdot \underbrace{\overset{2pt}{\begin{pmatrix} \frac{1}{S} & 0 & 0 \\ 0 & \frac{1}{S} & 0 \\ 0 & 0 & 1 \end{pmatrix}}}_{\text{convert from feet to inches c)}} \cdot \underbrace{\overset{2pt}{\begin{pmatrix} 1 & 0 & -x_c \\ 0 & 1 & -y_c \\ 0 & 0 & 1 \end{pmatrix}}}_{\text{shift to origin b)}} \cdot \underbrace{\overset{2pt}{\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}}}_{\text{flip y a)}} \cdot \begin{pmatrix} x_{world} \\ y_{world} \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} x_{pixel} \\ y_{pixel} \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{800}{WS} & 0 & -\frac{800}{WS} x_c + 400 \\ 0 & -\frac{600}{HS} & -\frac{600}{HS} y_c + 300 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_{world} \\ y_{world} \\ 1 \end{pmatrix}$$

2pt

pseudo-code : (assume W & H are defined)

```
function mapWorldToScreen(x_world, y_world, x_c, y_c, S)
    x_pixel = ((x_world - x_c) · 800 / (WS) + 400);
    y_pixel = ((y_world - y_c) · (-600) / (HS) + 300);
```

4pt