CS314 Jan07 HW#1 Solns

1) $P_A = (-1, -2)$

$P_B = (-1, 3)$

$P_C = (-.5, -2)$

2) rotation matrix:

$$\begin{bmatrix} \cos(\theta) & 0 & \sin(\theta) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\theta = 270^\circ$

$\cos(\theta) = 0$, $\sin(\theta) = -1$

answer:

$$\begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3) Restated question:
Write down the 4x4 matrix for shearing an object along x by 2 in the y direction, and along x by 3 in the z direction.

shear along x in y dir:

$$\begin{bmatrix} 1 & hyx & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad hyx = 2$$

shear along x in z dir:

$$\begin{bmatrix} 1 & 0 & hzx & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad hzx = 3$$

$$\begin{bmatrix} 1 & hyx & hzx & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

answer

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

4) $M = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 2 & 0 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

either option 1

$A = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ translate by (3,2,1)

$B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ scale by (1,2,1)

$M = AB$

or

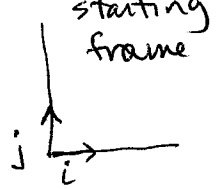
option 2

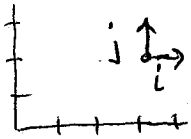
$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ scale by (1,2,1)

$B = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

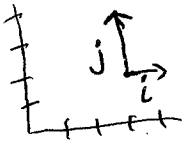
5) local/frame. read $M = AB$ from left to right

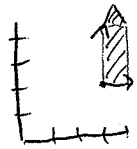
option 1: first, move frame by (3,2,1) with respect to starting frame



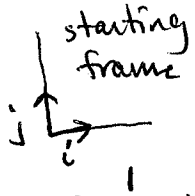


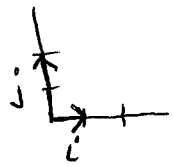
next, scale frame by 2 in y with respect to current frame





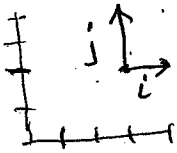
option 2: first, scale frame by 2 in y with respect to starting frame





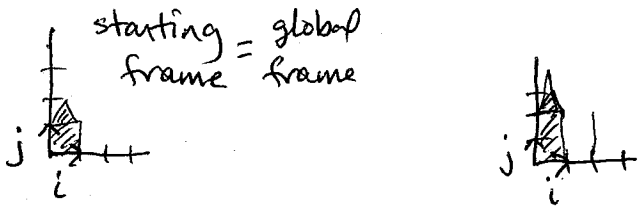
next, move frame by (3,1,1) with respect to current (scaled) frame

(so moves by 2 units wrt original y axis)

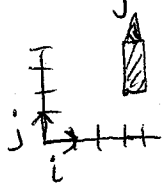


6) global/object. read $M = AB$ from right to left

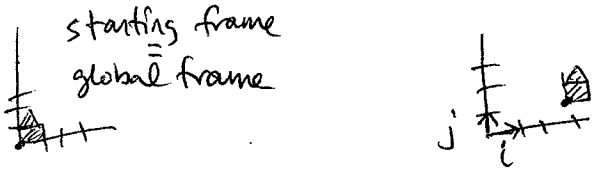
<u>option 1:</u> first, scale the object by 2 in y with respect to the global frame (using house as example object in pictures)



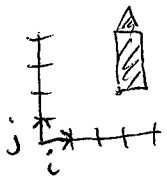
next, translate the object by (3,2,1) in the global frame



<u>option 2:</u> first, translate the object by (3,1,1) in the global frame



next, scale the object by 2 in y with respect to the global frame



7) <u>option 1:</u>
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glTranslate(3,2,1);
glScale(1,2,1);
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<u>option 2:</u>
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glScale(1,2,1);
glTranslate(3,1,1);
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8) $\left(\frac{8}{5}, \frac{15}{5}, \frac{9}{5}, \frac{5}{5}\right) = \left(\frac{8}{5}, 3, \frac{9}{5}, 1\right)$

9)

$$a' = \begin{bmatrix} 2.828 & 0 & .707 & 1 \\ 0 & 5 & 0 & 0 \\ -.707 & 1 & .707 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 4.35 \\ 5 \\ 1 \\ 1 \end{bmatrix}$$

$$= Ma$$

$$b' = Mb = \begin{bmatrix} 7.363 \\ 10 \\ 1.293 \\ 1 \end{bmatrix}$$

$$c' = Mc = \begin{bmatrix} .293 \\ 0 \\ -.707 \\ 1 \end{bmatrix}$$

10) before transform: $n = (b-a) \times (c-a) = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} -1 \\ -1 \\ -2 \\ 1 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \\ 0 \\ 1 \end{bmatrix}$

normalize n: length$(n) = \sqrt{8} = 2.83$

$$\text{normal} = \begin{bmatrix} \frac{-2}{2.83} \\ \frac{-2}{2.83} \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} .665 \\ .338 \\ .665 \\ 1 \end{bmatrix} \begin{bmatrix} -.707 \\ .707 \\ 0 \\ 1 \end{bmatrix}$$

after transform: $n' = (b'-a') \times (c'-a') = \begin{bmatrix} 2.828 \\ 5 \\ .293 \\ 1 \end{bmatrix} \times \begin{bmatrix} -4.242 \\ -5 \\ -1.707 \\ 1 \end{bmatrix} = \begin{bmatrix} -.707 \\ 3.585 \\ 7.07 \\ 1 \end{bmatrix}$

normalize n': length$(n') = \sqrt{112.82} = 10.62$

$$\text{normal} = \begin{bmatrix} -.665 \\ .338 \\ .665 \\ 1 \end{bmatrix}$$

11) $A = rot(z, 0, 0, 1) = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ $F = trans(2,3,0) = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$G = trans(1,1,0) = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ $H = scale(1, .5, 1) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & .5 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$I = scale(2,1,1) = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$B = AF = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & -3 \\ 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$J = GH = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & .5 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & .5 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$C = BJ = \begin{bmatrix} 0 & -1 & 0 & -3 \\ 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & .5 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -.5 & 0 & -4 \\ 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$D = BI = \begin{bmatrix} 0 & -1 & 0 & -3 \\ 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & -3 \\ 2 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$