

CPSC 314, Written Homework 2 solution

Out: Fri 05 Feb 2016

Due: Fri 12 Feb 2016 2pm (hand in at start of lecture)

Value: 4% of final grade

Total Points: 100

Marking Issue

1. Problem 1 to 7:

- For solutions who have intermediate steps, 2 points deduction for each incorrect derivation and 2 points for correctness of the final result. An incorrect derivation will not effect later ones, always assuming your previous steps are all correct.

- For solutions who have no deriving steps, just give marks by judging how the results match the expected ones. Basically 2 points deduction for each mismatching.

2. Problem 8:

- 10 points for each question. 6 points for sketch and 4 points for reasoning. No need to give/match the exact numerical result, but size of the sketch should be close enough to the solution key. Full credit is given as long as the shape and size is correct based on intuitions and proper reasoning. Partial credits might be given based on individual cases.

1. (10 pts) Give the camera/viewing transformation matrix for an eye position $(-5,3,-2)$, a lookat point $(-5, 0, -2)$ and an up vector $(1,0,0)$.

Solution

$$\vec{g} = \vec{l} - \vec{e} = (-5, 0, -2) - (-5, 3, -2) = (0, -3, 0)$$

$$\vec{w} = -\frac{\vec{g}}{\|\vec{g}\|} = -\frac{(0, -3, 0)}{3} = (0, 1, 0)$$

$$\vec{u} = \frac{\vec{l} \times \vec{w}}{\|\vec{l} \times \vec{w}\|} = \frac{(1, 0, 0) \times (0, 1, 0)}{1} = (0, 0, 1)$$

$$\vec{v} = \vec{w} \times \vec{u} = (0, 1, 0) \times (0, 0, 1) = (1, 0, 0)$$

$$\mathbf{R}^{-1} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{1}$$

$$\mathbf{T}^{-1} = \begin{pmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{aligned} \mathbf{M}_{W2V} &= \mathbf{R}^{-1}\mathbf{T}^{-1} \\ &= \begin{pmatrix} 0 & 0 & 1 & 2 \\ 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 0 & 1 \end{pmatrix} \end{aligned}$$

2. (10 pts) Give the perspective projection matrix for a view volume with a near plane $z = -2$, far plane $z = -16$, a left plane $x = -2$, a right plane $x = 2$, a top plane $y = 2$, and a bottom plane $y = -2$.

Solution

$$\begin{aligned}
 x = -2 &= \text{left} : \text{left} = -2 \\
 x = 2 &= \text{right} : \text{right} = 2 \\
 y = -2 &= \text{bot} : \text{bot} = -2 \\
 z = 2 &= \text{top} : \text{top} = 2 \\
 z = -2 &= -\text{near} : \text{near} = 2 \\
 z = -16 &= -\text{far} : \text{far} = 16
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{M}_P &= \begin{pmatrix} \frac{2n}{r-l} & 0 & \frac{r+l}{r-l} & 0 \\ 0 & \frac{2n}{t-b} & \frac{t+b}{t-b} & 0 \\ 0 & 0 & -\frac{f+n}{f-n} & -\frac{2fn}{f-n} \\ 0 & 0 & -1 & 0 \end{pmatrix} \\
 &= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -\frac{9}{7} & -\frac{32}{7} \\ 0 & 0 & -1 & 0 \end{pmatrix} \\
 &= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1.286 & -4.571 \\ 0 & 0 & -1 & 0 \end{pmatrix}
 \end{aligned} \tag{2}$$

3. (10 pts) Give the NDC-to-display transformation matrix for a viewport 800 pixels wide and 600 pixels high, with the origin in the upper left of the display.

Solution

General formulation of N2D transformation includes: reflect in y, scale by width, height and depth, and translate by width/2, height/2 and depth/2.

$$\mathbf{M}_{N2D} = \begin{pmatrix} 1 & 0 & 0 & \frac{(\text{width}-1)}{2} \\ 0 & 1 & 0 & \frac{(\text{height}-1)}{2} \\ 0 & 0 & 1 & \frac{\text{depth}}{2} \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{\text{width}}{2} & 0 & 0 & 0 \\ 0 & \frac{\text{height}}{2} & 0 & 0 \\ 0 & 0 & \frac{\text{depth}}{2} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{3}$$

In this question, width = 800, height = 600, depth = 1. Substitute numbers into those matrices, we get:

$$\begin{aligned}
 \mathbf{M}_{N2D} &= \begin{pmatrix} 1 & 0 & 0 & \frac{799}{2} \\ 0 & 1 & 0 & \frac{599}{2} \\ 0 & 0 & 1 & \frac{1}{2} \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 400 & 0 & 0 & 0 \\ 0 & 300 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \\
 &= \begin{pmatrix} 400 & 0 & 0 & \frac{799}{2} \\ 0 & -300 & 0 & \frac{599}{2} \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 1 \end{pmatrix} \\
 &= \begin{pmatrix} 400 & 0 & 0 & 399.5 \\ 0 & -300 & 0 & 299.5 \\ 0 & 0 & 0.5 & 0.5 \\ 0 & 0 & 0 & 1 \end{pmatrix}
 \end{aligned} \tag{4}$$

4. (10 pts) In world coordinate system, given a point (-3, 5, 1), what are its coordinates in the camera coordinate system, after the viewing transformation from problem 1 above has been applied to it. **Update Feb 05: point changed from (3, 3, 1) to (-3, 5, 1)**

Solution

$$\vec{P}_{VCS} = \mathbf{M}_{W2V} \vec{P}_{WCS} = \begin{pmatrix} 0 & 0 & 1 & 2 \\ 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -3 \\ 5 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \\ 2 \\ 1 \end{pmatrix} \tag{5}$$

5. (10 pts) Now calculate its coordinates in the clipping coordinate system, by applying the perspective warp for the view frustum specified in problem 2 to the point in camera coordinates (that is, the answer from problem 4).

Solution

$$\vec{P}_{CCS} = \mathbf{M}_P \vec{P}_{VCS} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -\frac{9}{7} & -\frac{32}{7} \\ 0 & 0 & -1 & 0 \end{pmatrix} \begin{pmatrix} 3 \\ 2 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \\ -\frac{50}{7} \\ -2 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \\ -7.1429 \\ -2 \end{pmatrix} \quad (6)$$

6. (10 pts) Calculate its coordinates in the normalized device coordinate system, by applying the perspective divide to your answer from problem 5.

Solution

$$\vec{P}_{NDCS} = \vec{P}_{CCS} / \vec{P}_{CCS}[4] = \vec{P}_{CCS} / -2 = \begin{pmatrix} -\frac{3}{2} \\ -1 \\ \frac{25}{7} \\ 1 \end{pmatrix} = \begin{pmatrix} -1.5 \\ -1 \\ 3.5714 \\ 1 \end{pmatrix} \quad (7)$$

7. (10 pts) Finally, calculate its coordinates in the display coordinate system, by applying the viewport transformation matrix from problem 3 to your answer from problem 6.

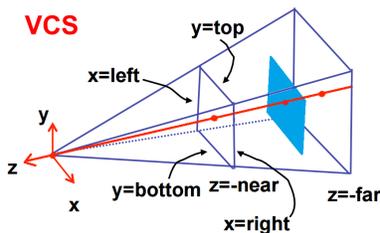
Solution

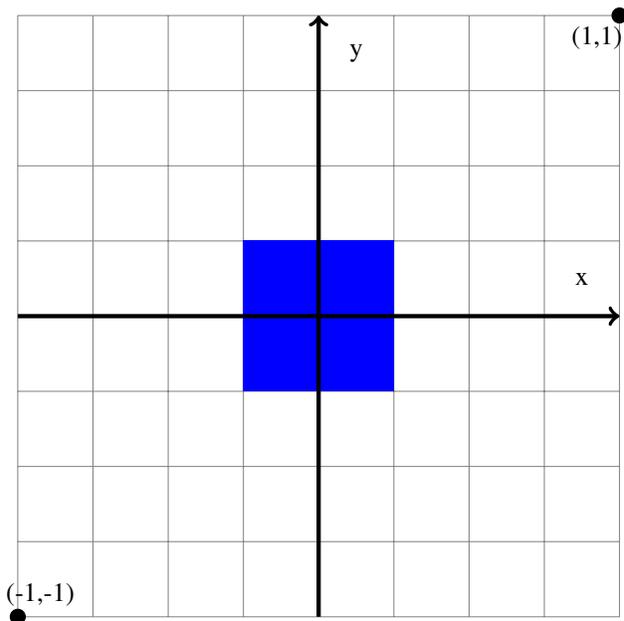
$$\vec{P}_{DCS} = \mathbf{M}_{N2D} \vec{P}_3 = \begin{pmatrix} -200.5 \\ 599.5 \\ 2.2857 \\ 1 \end{pmatrix} \quad (8)$$

8. (30 pts) In the camera coordinate system of problem 2, using perspective projection, we put a planar square object on $z = -8$, perpendicular to z -axis, so that the negative z -axis pierces the centre of the object. Given the edge of the square to be 4, we would have an image in NDCS of the square object as shown below.

- If the near value is doubled, what would the observed effect be on the resulting image of the object?
- if we change top plane to be $y = 4$ and bottom plane $y = -4$, what would the observed effect be on the resulting image of the object?
- considering tilting the top of the square towards the near plane, by rotating it about a vector parallel to the x axis in camera coordinate and passing through the centre of the square, by an angle of 45 degree, so that the top of square gets nearer and the bottom is farther to the near plane. What is the shape of the projection of the square (a square, rectangular, parallelogram, trapezoid)?

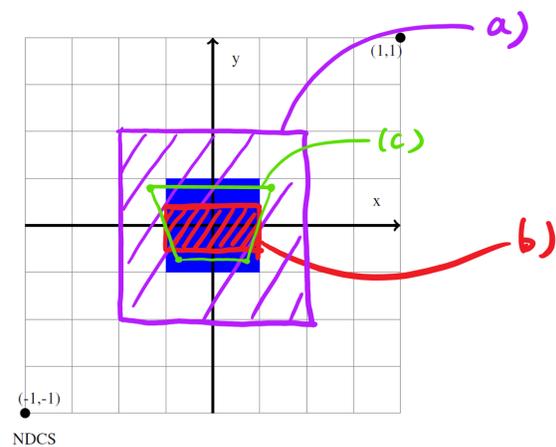
For each question sketch your result in the graph and **provide reasons** to get full credits.





NDCS

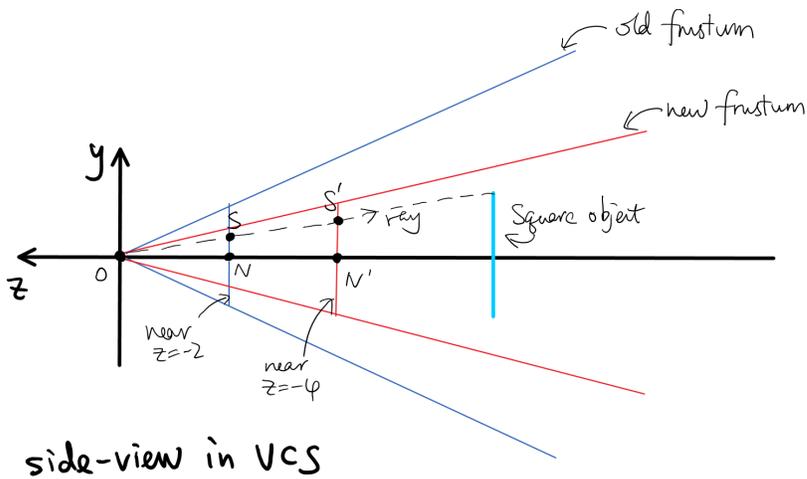
Solution



NDCS

a) Given near plane is doubled while all other planes remain the same, both the vertical and horizontal FOV (field of view) will become smaller. The projection of the square will become larger due to a smaller FOV. As a result, there will be a zoom-in effect on the image.

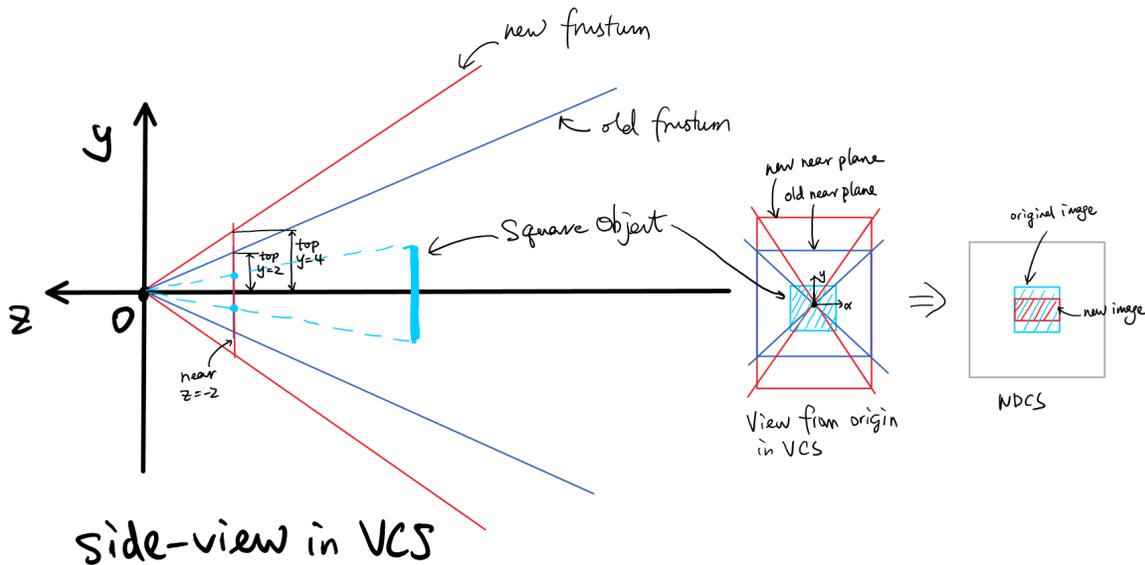
If you want to calculate the exact position of each vertex of the square in NDCS (though not necessarily required in this question), you could use the projection matrix and the perspective divide to get its exact coordinates in NDCS. A better way would be just use the geometry relationship in top or side-view of VCS.



Imagine we have a ray starting from origin of VCS and arriving at the corner of the square. The ray will intersect with the near plane at point S. After moving near plane to $z = -4$, the new intersection point S' will appear to be higher along x or y-axis (depends on top-view or side-view). According to the similarity between triangle OSN and $OS'N'$, we could say $SN/S'N' = ON/ON'$, while $ON = \text{old_near} = 2$ and $ON' = \text{new_near} = 4$. So $S'N' = 2*SN$. Since we still have the same size of the near plane (same top/bot and left/right), the result image would be twice as large as it was.

b) Given top and bottom plane is doubled while all other planes remain the same, the vertical FOV becomes larger and horizontal FOV remains the same. The projection of the square will become smaller but only along y axis due to a larger vertical FOV. As a result, there will be a squish-like effect along y on the image.

Similar to the previous question, you could use the projection matrix and the perspective divide if you want to get the coordinates for each vertex in NDCS. A better way would be just use the geometry relationship in side-view of VCS since only vertical FOV has been changed.

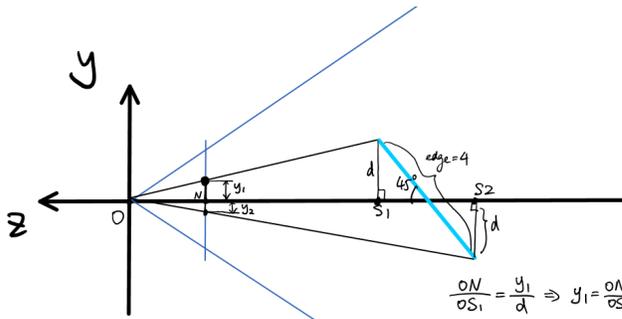


Though the intersection point caused by the square vertex ray and the near plane will not change in VCS, the size of the near plane has changed since we double the top/bot. As a result, when the intersection point goes from VCS to NDCS, it would be squeezed to half of the y coordinate compared to the original square.

c) The image of the square will be a trapezoid, where the top side is the long one, while the bottom the smaller. Objects appear smaller when they are further away.

Similar to the previous two, you could just compute the new coordinates in VCS for each vertex on the square, then apply the matrix to get the result image in NDCS.

Or you could also use the geometry and similarity to calculate the coordinates.



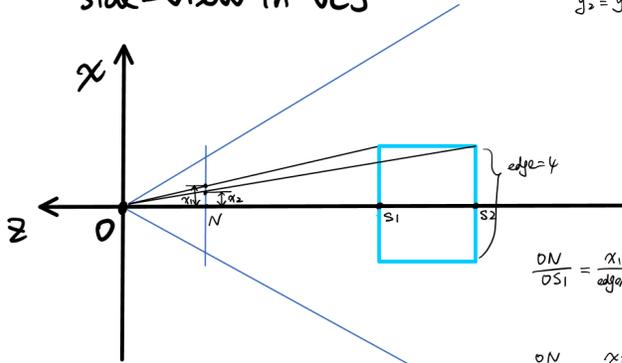
$$\frac{ON}{OS_1} = \frac{y_1}{d} \Rightarrow y_1 = \frac{ON}{OS_1} \cdot d = \frac{-2}{-8+2\cos 45^\circ} \cdot 2\sin 45^\circ = \frac{2\sqrt{2}}{8-\sqrt{2}} = 0.4295$$

$$y'_1 = y_1 / \text{top} = 0.2148$$

$$\frac{ON}{OS_2} = \frac{y_2}{d} \Rightarrow y_2 = \frac{ON}{OS_2} \cdot d = \frac{-2}{-8-2\cos 45^\circ} \cdot 2\sin 45^\circ = \frac{2\sqrt{2}}{8+\sqrt{2}} = 0.2006$$

$$y'_2 = y_2 / \text{bot} = -0.15$$

side-view in VCS



$$\frac{ON}{OS_1} = \frac{x_1}{\text{edge}/2} \Rightarrow x_1 = \frac{ON}{OS_1} \cdot \frac{\text{edge}}{2} = \frac{4}{8-\sqrt{2}} = 0.6074$$

$$x'_1 = x_1 / \text{right} = 0.3037$$

$$\frac{ON}{OS_2} = \frac{x_2}{\text{edge}/2} \Rightarrow x_2 = \frac{ON}{OS_2} \cdot \frac{\text{edge}}{2} = \frac{4}{8+\sqrt{2}} = 0.4248$$

$$x'_2 = x_2 / \text{right} = 0.2124$$

top-view in VCS

