University of British Columbia
CPSC 314 Computer Graphics
Jan-Apr 2013

Tamara Munzner

Viewing

http://www.ugrad.cs.ubc.ca/~cs314/Vjan2013

Reading for This Module

- FCG Chapter 7 Viewing
- FCG Section 6.3.1 Windowing Transforms

- RB rest of Chap Viewing
- RB rest of App Homogeneous Coords

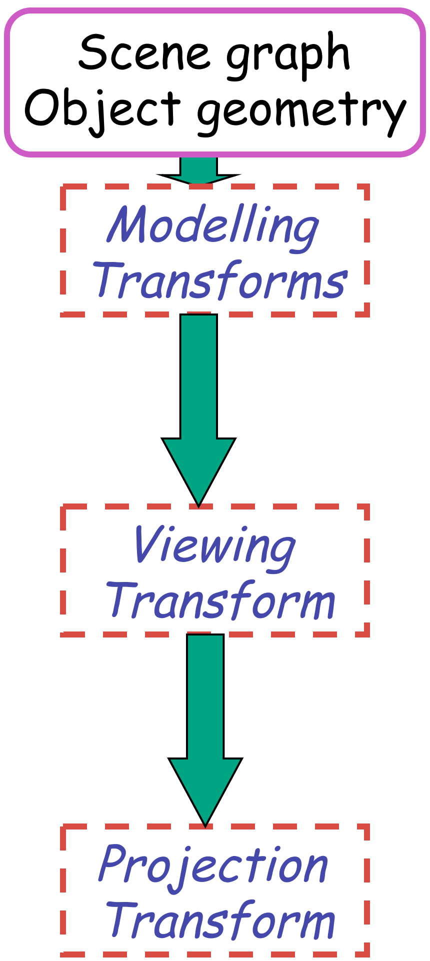
- RB Chap Selection and Feedback
- RB Sec Object Selection Using the Back Buffer
  - (in Chap Now That You Now)

Viewing

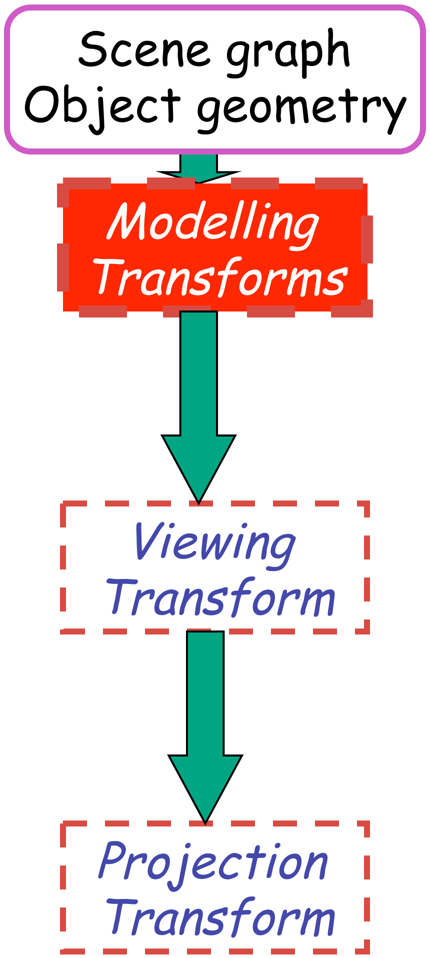
Using Transformations

- three ways
  - modelling transforms
    - place objects within scene (shared world)
    - affine transformations
  - viewing transforms
    - place camera
    - rigid body transformations: rotate, translate
  - projection transforms
    - change type of camera
    - projective transformation

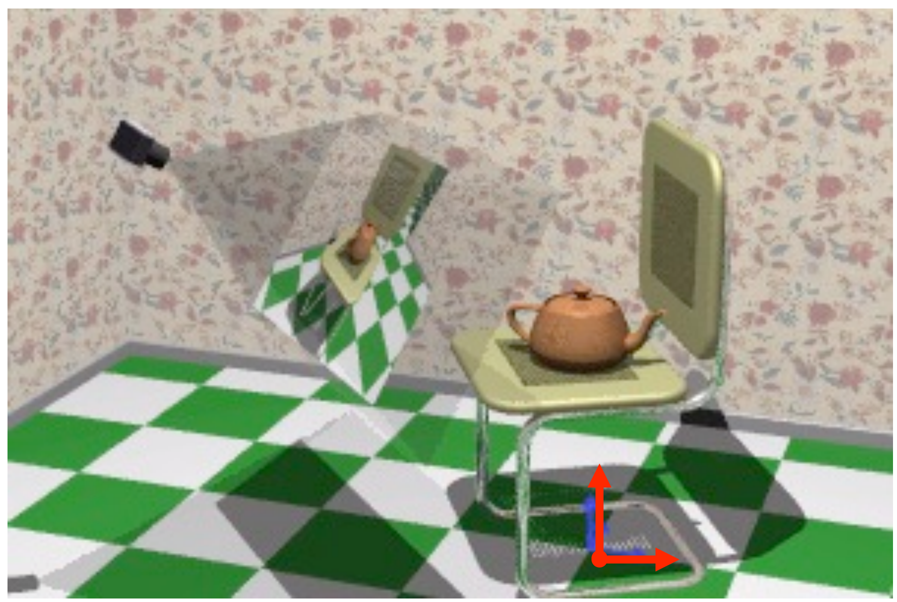
Rendering Pipeline



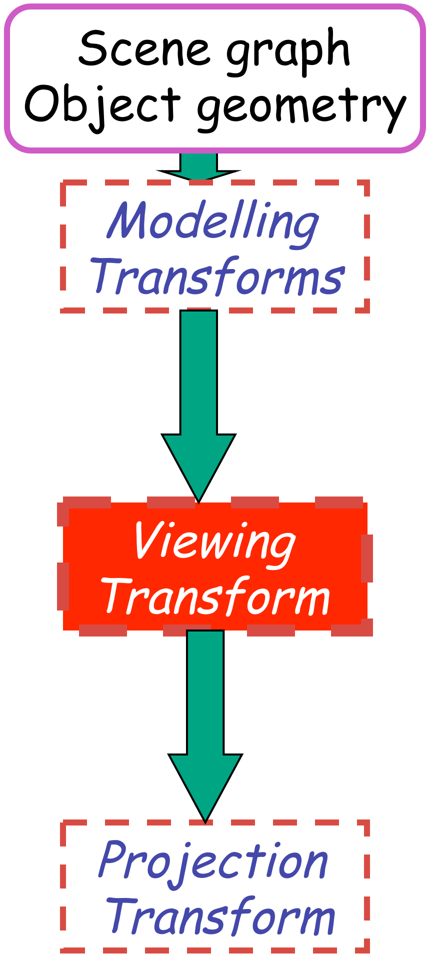
Rendering Pipeline



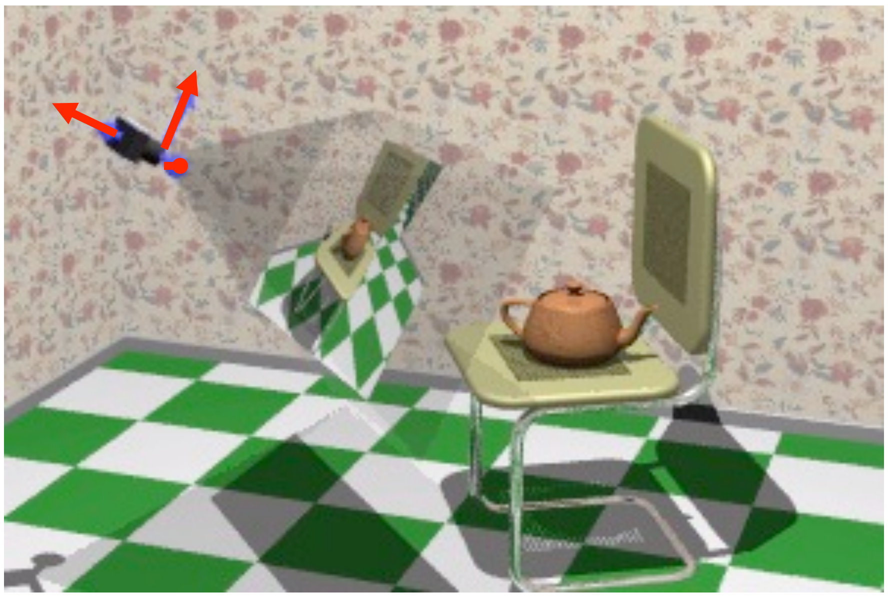
- result
  - all vertices of scene in shared 3D world coordinate system



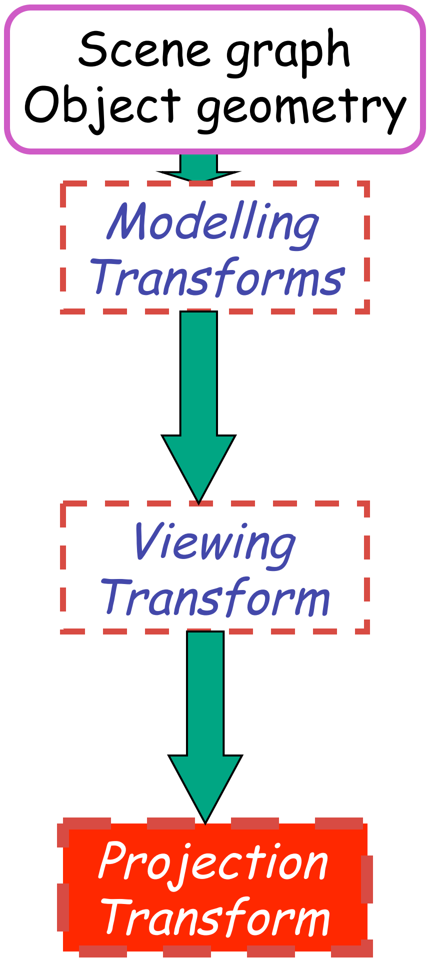
Rendering Pipeline



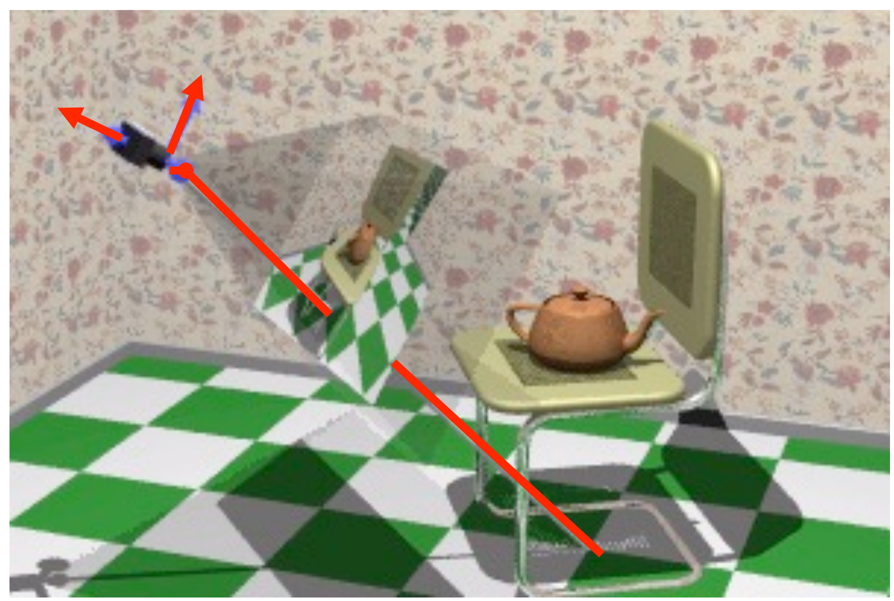
- result
  - scene vertices in 3D view (camera) coordinate system



Rendering Pipeline



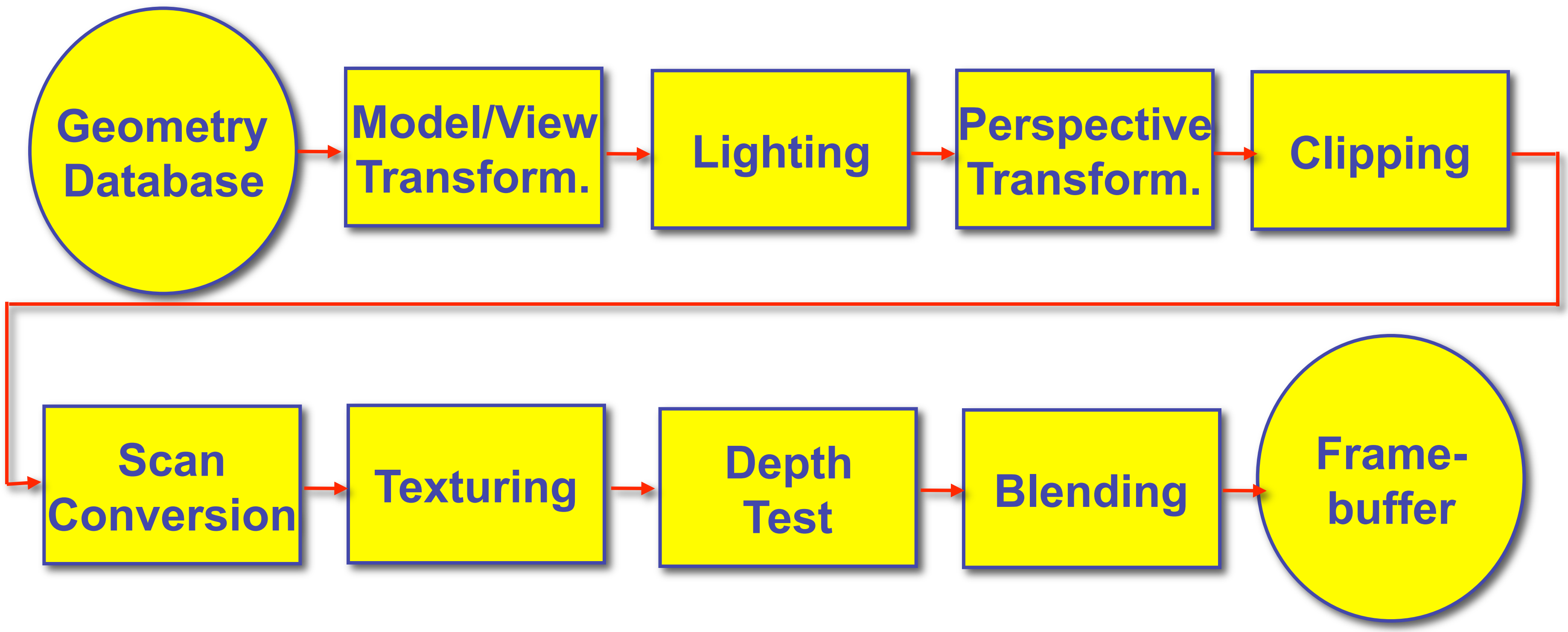
- result
  - 2D screen coordinates of clipped vertices



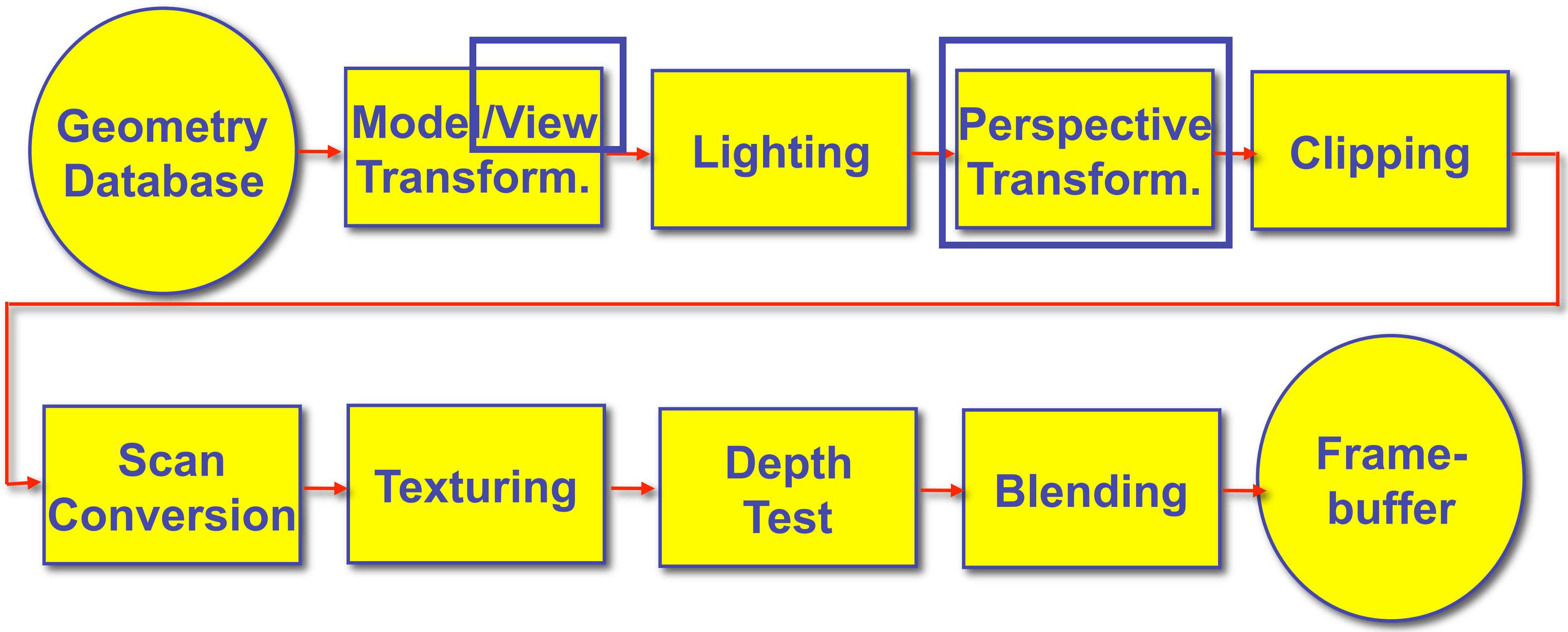
Viewing and Projection

- need to get from 3D world to 2D image
- projection: geometric abstraction
  - what eyes or cameras do
- two pieces
  - viewing transform:
    - where is the camera, what is it pointing at?
  - perspective transform: 3D to 2D
    - flatten to image

Rendering Pipeline



Rendering Pipeline



OpenGL Transformation Storage

- modeling and viewing stored together
  - possible because no intervening operations
- perspective stored in separate matrix

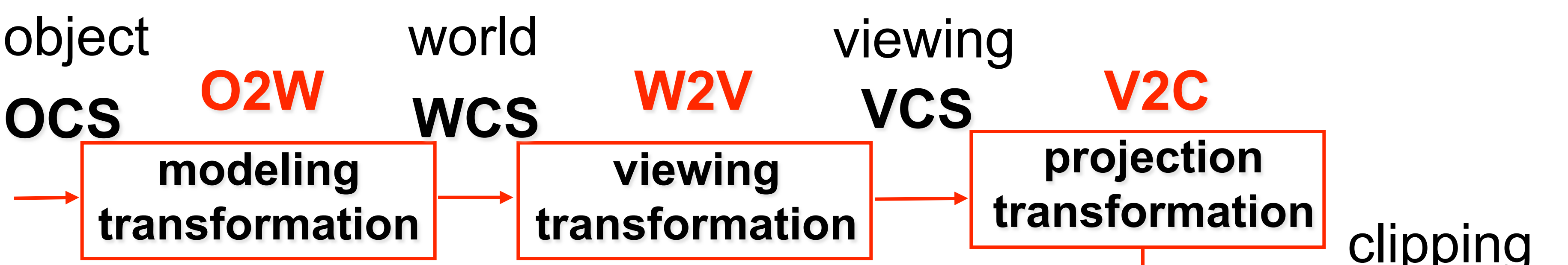
- specify which matrix is target of operations
  - common practice: return to default modelview mode after doing projection operations

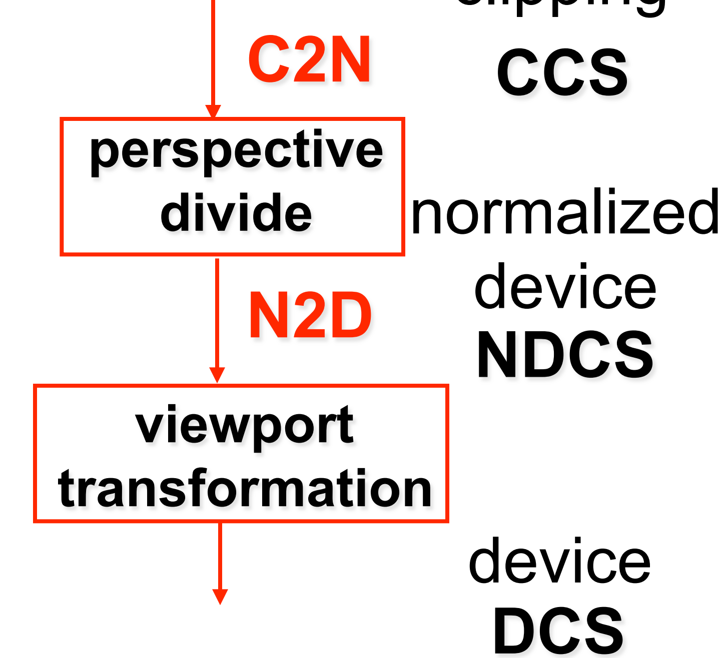
```
glMatrixMode(GL_MODELVIEW);
glMatrixMode(GL_PROJECTION);
```

Coordinate Systems

- result of a transformation
- names
  - convenience
    - animal: leg, head, tail
  - standard conventions in graphics pipeline
    - object/modelling
    - world
    - camera/viewing/eye
    - screen/window
    - raster/device

Projective Rendering Pipeline





OCS - object/model coordinate system

WCS - world coordinate system

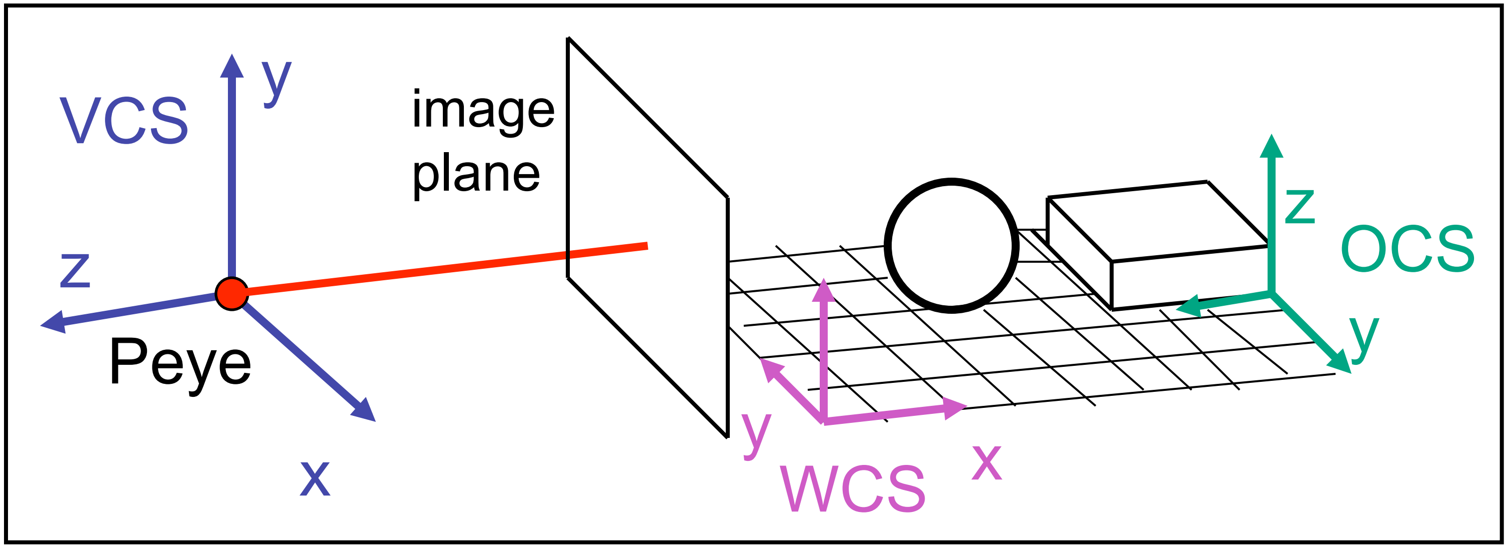
VCS - viewing/camera/eye coordinate system

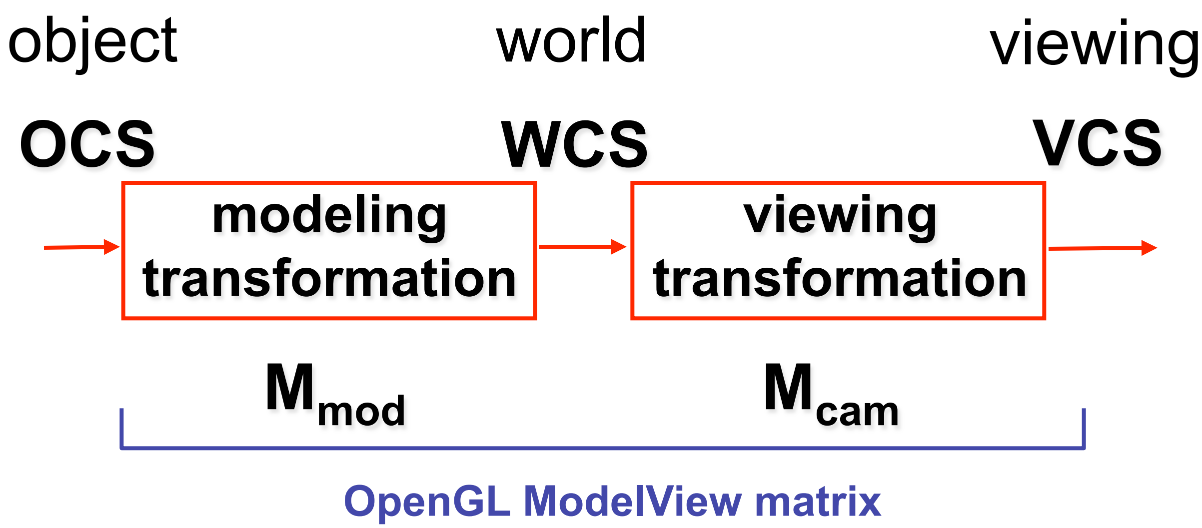
CCS - clipping coordinate system

NDCS - normalized device coordinate system

DCS - device/display/screen coordinate system

Viewing Transformation





Basic Viewing

- starting spot - OpenGL
  - camera at world origin
    - probably inside an object
  - y axis is up
  - looking down negative z axis
    - why? RHS with x horizontal, y vertical, z out of screen
- translate backward so scene is visible
  - move distance d = focal length

- where is camera in P1 template code?
  - 5 units back, looking down -z axis

Convenient Camera Motion

- rotate/translate/scale versus
  - eye point, gaze/lookat direction, up vector
  - demo: Robins transformation, projection

OpenGL Viewing Transformation

```
gluLookAt(ex,ey,ez,lx,ly,lz,ux,uy,uz)
```

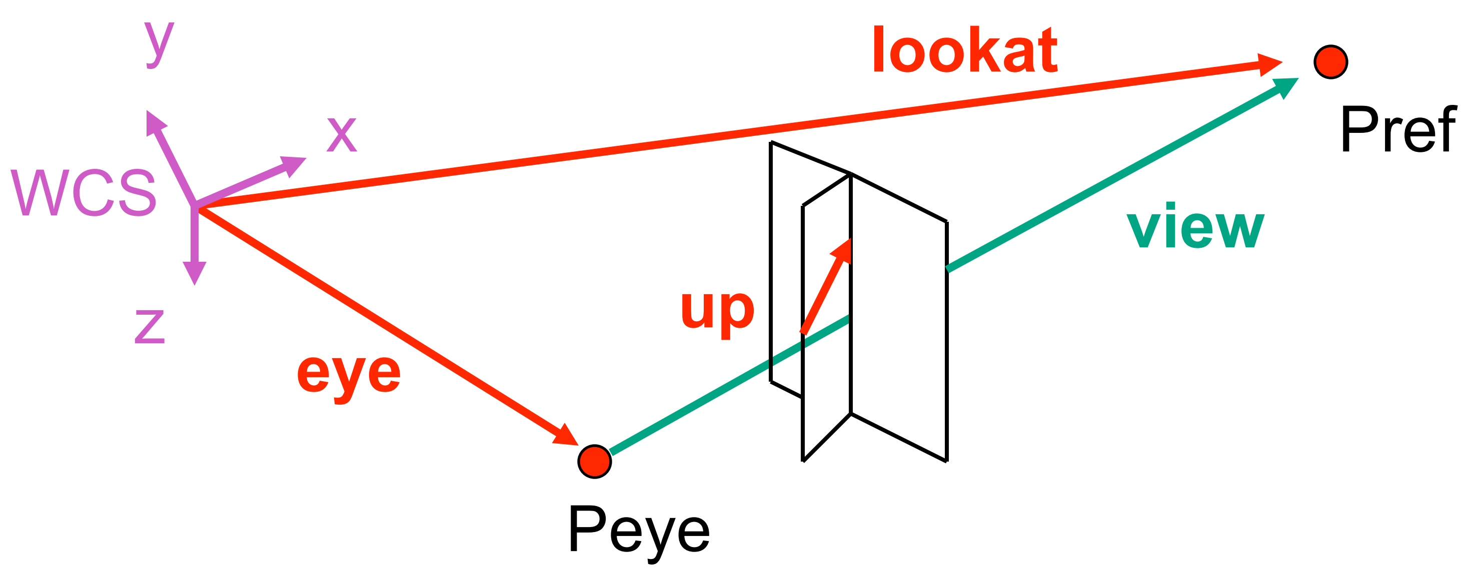
- postmultiplies current matrix, so to be safe:

```
glMatrixMode(GL_MODELVIEW);
glLoadIdentity();
gluLookAt(ex,ey,ez,lx,ly,lz,ux,uy,uz)
// now ok to do model transformations
```

- demo: Nate Robins tutorial *projection*

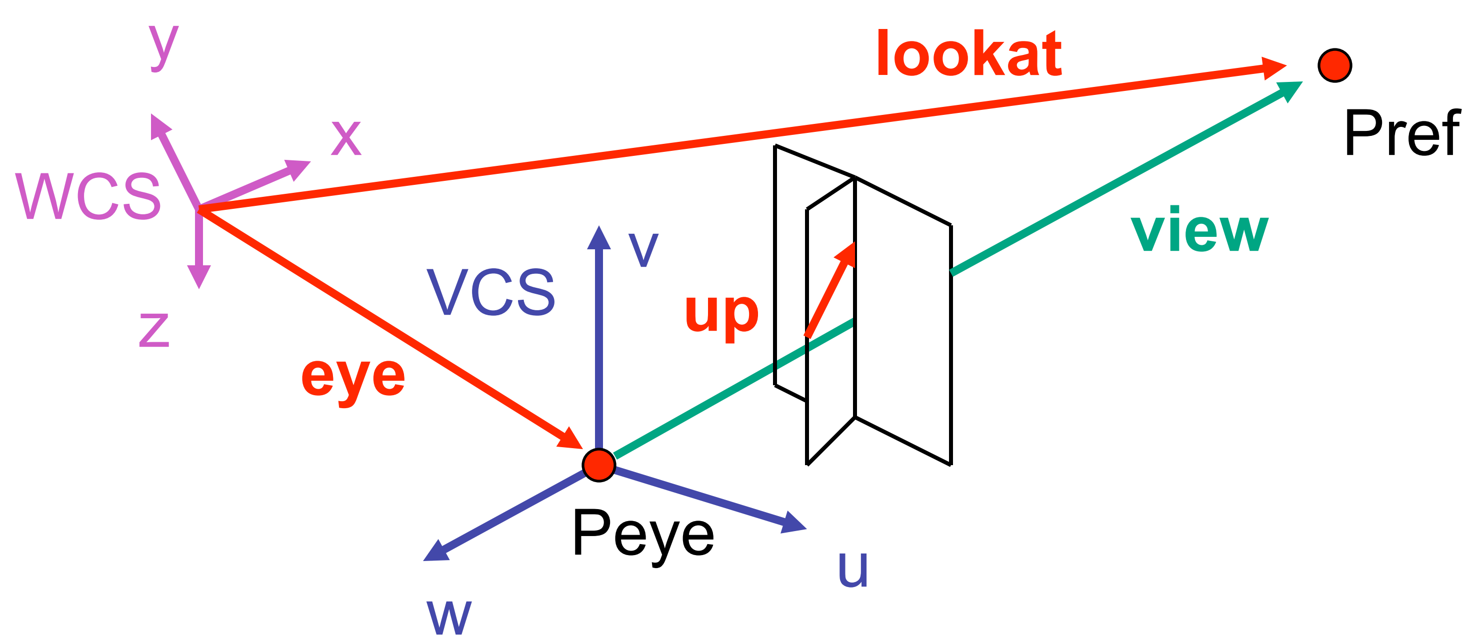
Convenient Camera Motion

- rotate/translate/scale versus
  - eye point, gaze/lookat direction, up vector



Placing Camera in World Coords: V2W

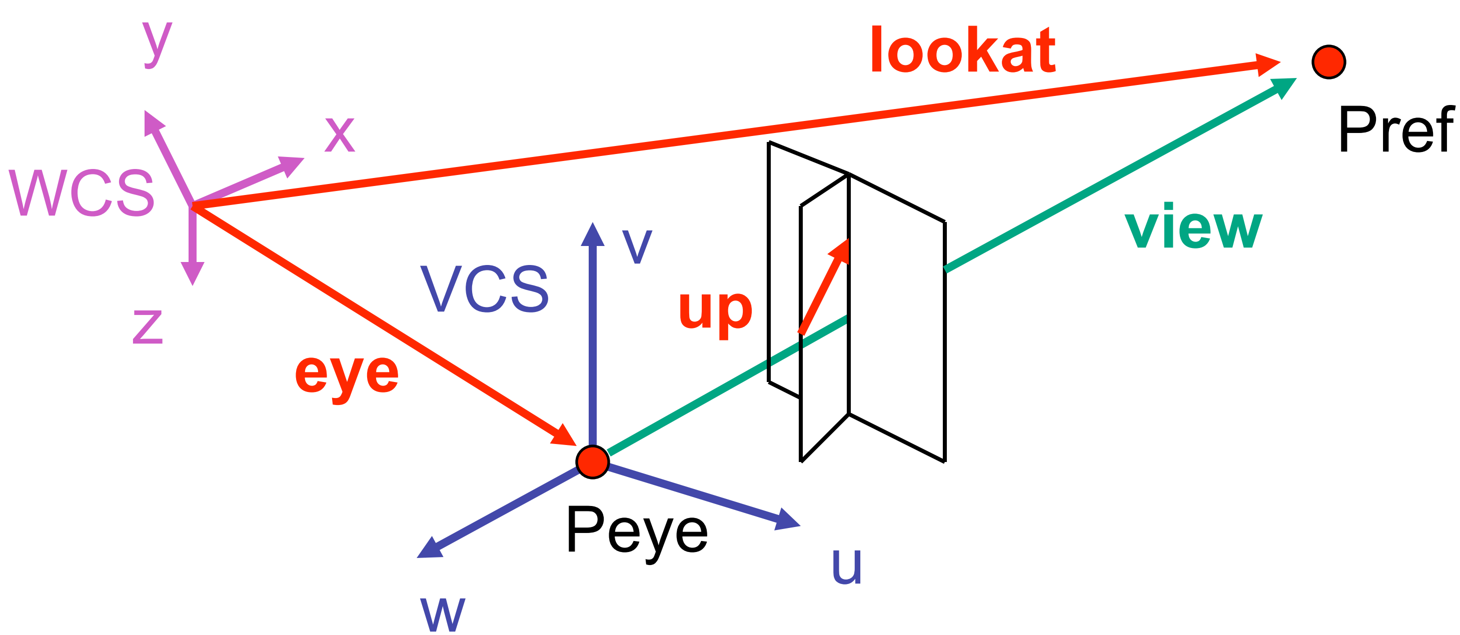
- treat camera as if it's just an object
  - translate from origin to **eye**
  - rotate **view** vector (**lookat** – **eye**) to **w** axis
  - rotate around **w** to bring **up** into **vw**-plane



Deriving V2W Transformation

- translate origin to **eye**

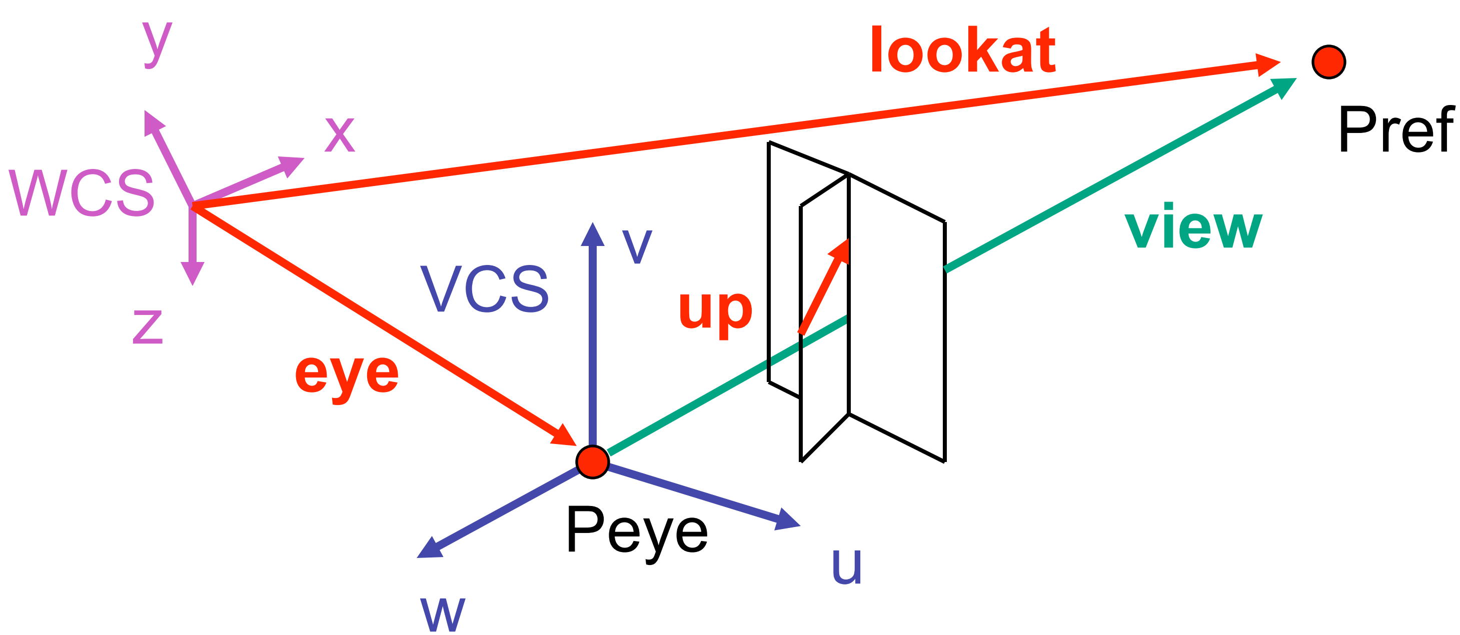
$$\mathbf{T} = \begin{bmatrix} 1 & 0 & 0 & e_x \\ 0 & 1 & 0 & e_y \\ 0 & 0 & 1 & e_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



Deriving V2W Transformation

- rotate **view** vector (**lookat** – **eye**) to **w** axis
  - **w**: normalized opposite of **view/gaze** vector **g**

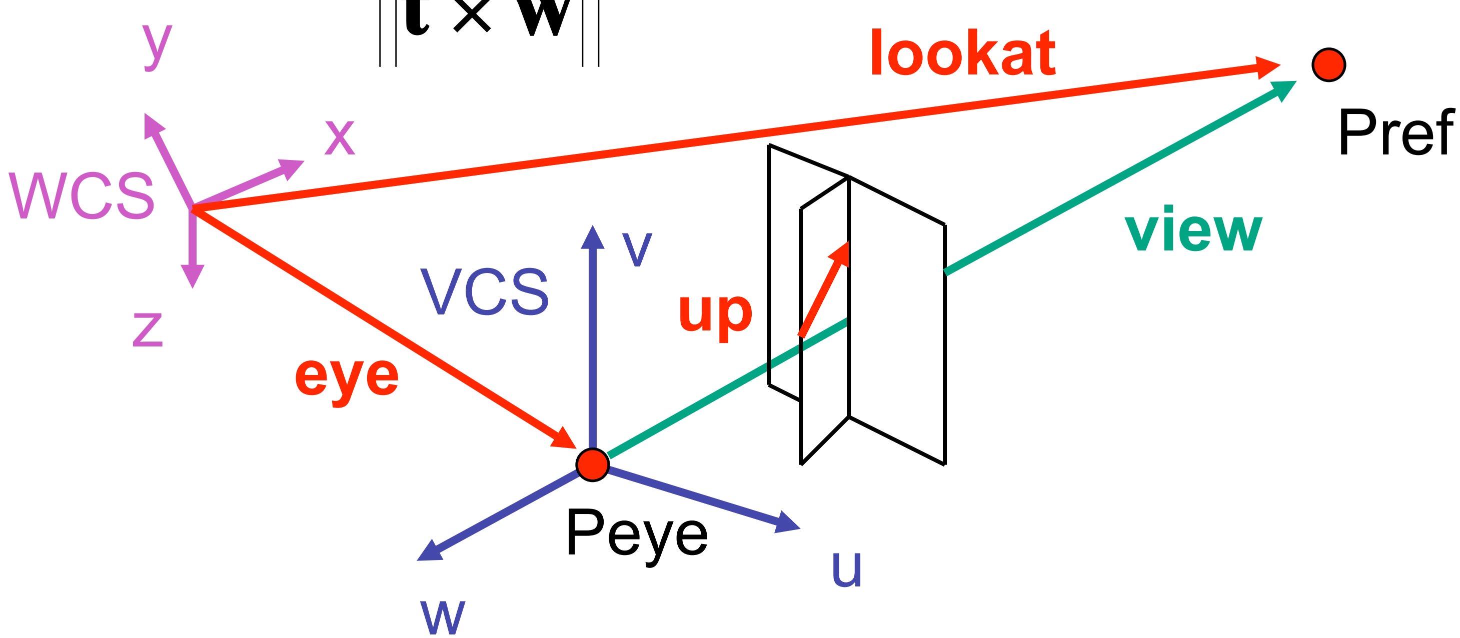
$$\mathbf{w} = -\hat{\mathbf{g}} = -\frac{\mathbf{g}}{\left\|\mathbf{g}\right\|}$$



Deriving V2W Transformation

- rotate around **w** to bring **up** into **vw**-plane
  - **u** should be perpendicular to **vw**-plane, thus perpendicular to **w** and **up** vector **t**
  - **v** should be perpendicular to **u** and **w**

$$\mathbf{u} = \frac{\mathbf{t} \times \mathbf{w}}{\left\|\mathbf{t} \times \mathbf{w}\right\|} \qquad \mathbf{v} = \mathbf{w} \times \mathbf{u}$$



Deriving V2W Transformation

- rotate from WCS **xyz** into **uvw** coordinate system with matrix that has columns **u**, **v**, **w**

$$\mathbf{u} = \frac{\mathbf{t} \times \mathbf{w}}{\left\|\mathbf{t} \times \mathbf{w}\right\|} \qquad \mathbf{v} = \mathbf{w} \times \mathbf{u} \qquad \mathbf{w} = -\hat{\mathbf{g}} = -\frac{\mathbf{g}}{\left\|\mathbf{g}\right\|}$$

$$\mathbf{T} = \begin{bmatrix} 1 & 0 & 0 & e_x \\ 0 & 1 & 0 & e_y \\ 0 & 0 & 1 & e_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \mathbf{R} = \begin{bmatrix} u_x & v_x & w_x & 0 \\ u_y & v_y & w_y & 0 \\ u_z & v_z & w_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad M_{V2W} = TR$$

- reminder: rotate from **uvw** to **xyz** coord sys with matrix **M** that has columns **u,v,w**

V2W vs. W2V

- M_{V2W}=TR

$$\mathbf{T} = \begin{bmatrix} 1 & 0 & 0 & e_x \\ 0 & 1 & 0 & e_y \\ 0 & 0 & 1 & e_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \mathbf{R} = \begin{bmatrix} u_x & v_x & w_x & 0 \\ u_y & v_y & w_y & 0 \\ u_z & v_z & w_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- we derived position of camera as object in world
  - invert for gluLookAt: go from world to camera!
- $M_{W2V}=(M_{V2W})^{-1}=R^{-1}T^{-1}$

$$\mathbf{R}^{-1} = \begin{bmatrix} u_x & u_y & u_z & 0 \\ v_x & v_y & v_z & 0 \\ w_x & w_y & w_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \mathbf{T}^{-1} = \begin{bmatrix} 1 & 0 & 0 & -e_x \\ 0 & 1 & 0 & -e_y \\ 0 & 0 & 1 & -e_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

  - inverse is transpose for orthonormal matrices
  - inverse is negative for translations

V2W vs. W2V

- $M_{W2V}=(M_{V2W})^{-1}=R^{-1}T^{-1}$

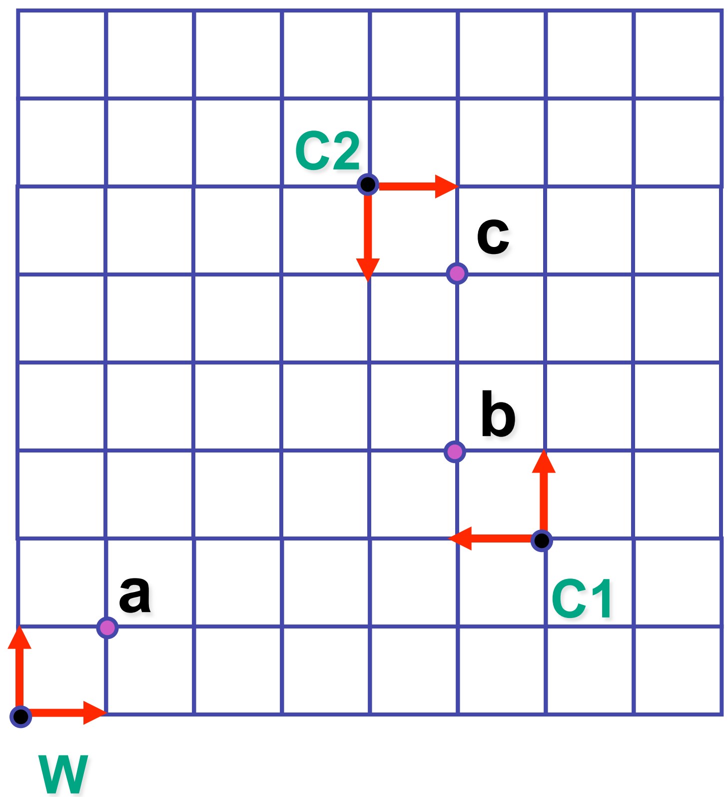
$$\mathbf{M}_{world2view} = \begin{bmatrix} u_x & u_y & u_z & 0 \\ v_x & v_y & v_z & 0 \\ w_x & w_y & w_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -e_x \\ 0 & 1 & 0 & -e_y \\ 0 & 0 & 1 & -e_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} u_x & u_y & u_z & -\mathbf{e}\bullet\mathbf{u} \\ v_x & v_y & v_z & -\mathbf{e}\bullet\mathbf{v} \\ w_x & w_y & w_z & -\mathbf{e}\bullet\mathbf{w} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{M}_{W2V} = \begin{bmatrix} u_x & u_y & u_z & -e_x * u_x + -e_y * u_y + -e_z * u_z \\ v_x & v_y & v_z & -e_x * v_x + -e_y * v_y + -e_z * v_z \\ w_x & w_y & w_z & -e_x * w_x + -e_y * w_y + -e_z * w_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Moving the Camera or the World?

- two equivalent operations
  - move camera one way vs. move world other way
- example
  - initial OpenGL camera: at origin, looking along -z axis
  - create a unit square parallel to camera at z = -10
  - translate in z by 3 possible in two ways
    - camera moves to z = -3
      - Note OpenGL models viewing in left-hand coordinates
    - camera stays put, but world moves to -7
  - resulting image same either way
    - possible difference: are lights specified in world or view coordinates?

World vs. Camera Coordinates Example



$a = (1,1)_W$

$b = (1,1)_{C1} = (5,3)_W$

$c = (1,1)_{C2} = (1,3)_{C1} = (5,5)_W$

Projections I

Pinhole Camera

- ingredients
  - box, film, hole punch
- result
  - picture

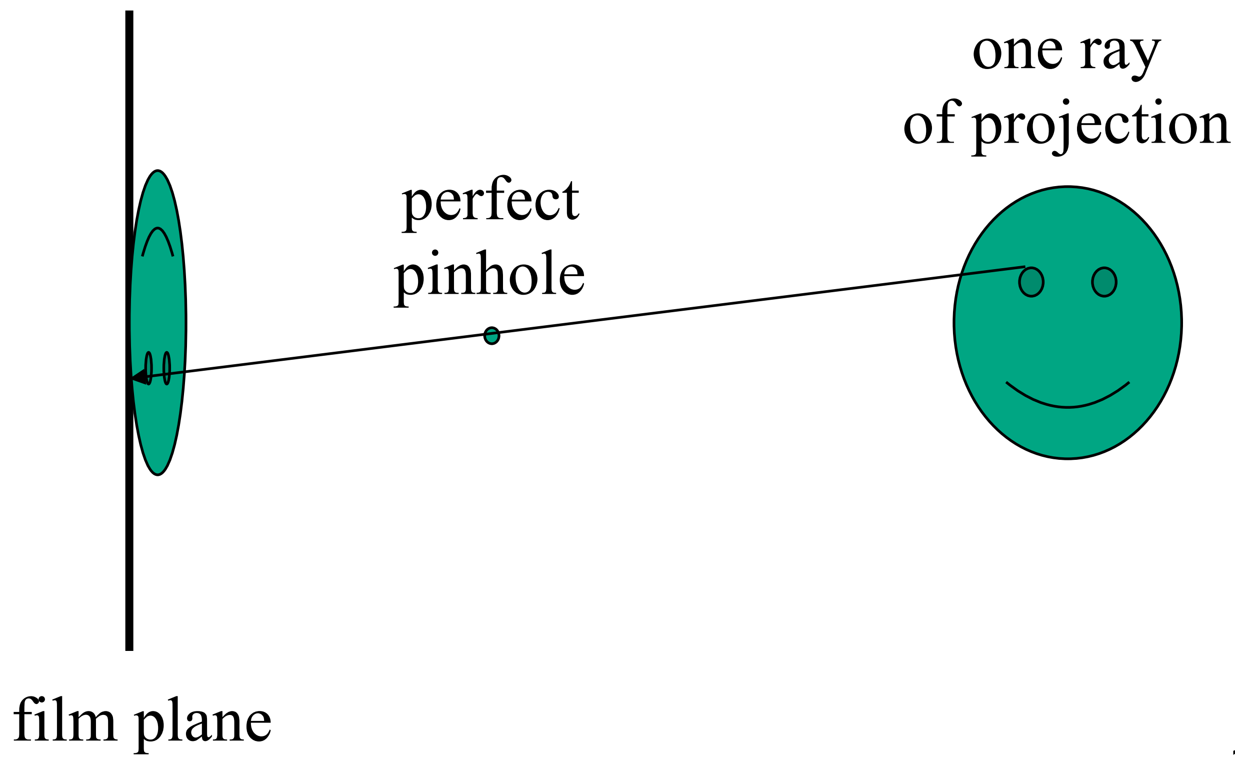
www.kodak.com

www.pinhole.org

www.debevec.org/Pinhole

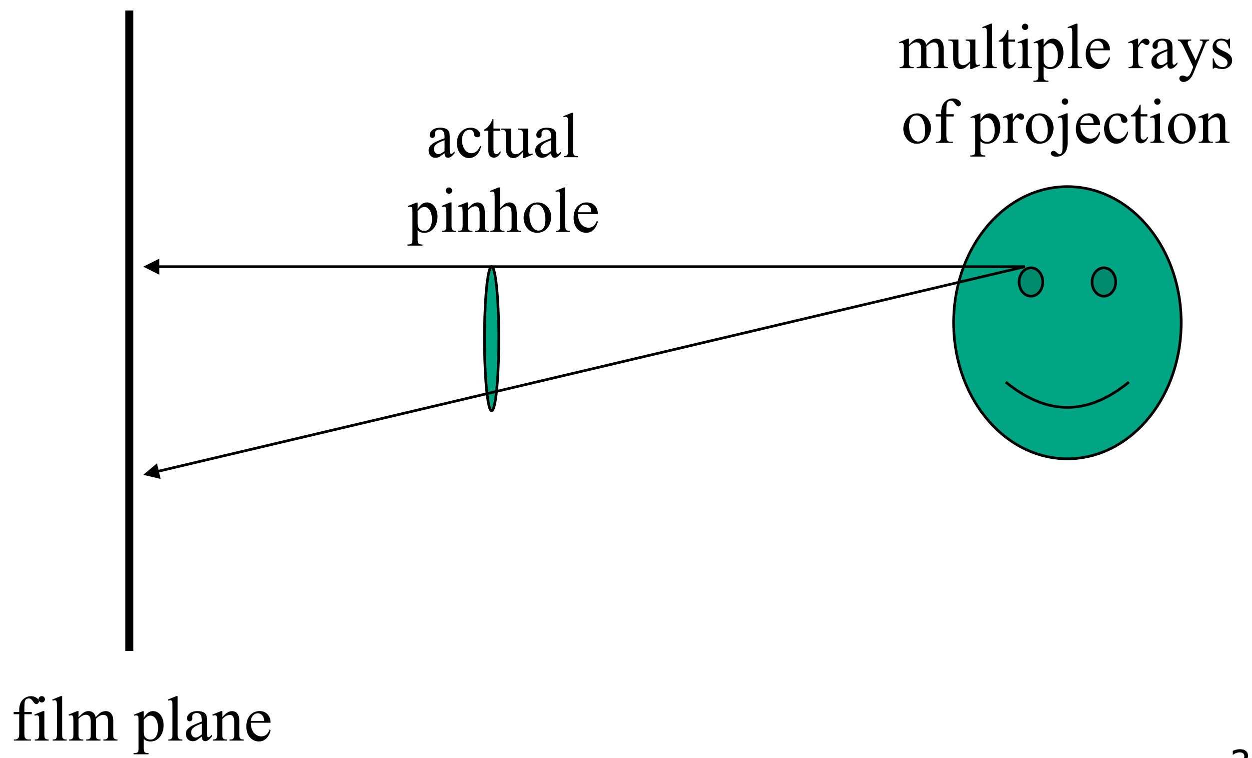
Pinhole Camera

- theoretical perfect pinhole
  - light shining through tiny hole into dark space yields upside-down picture



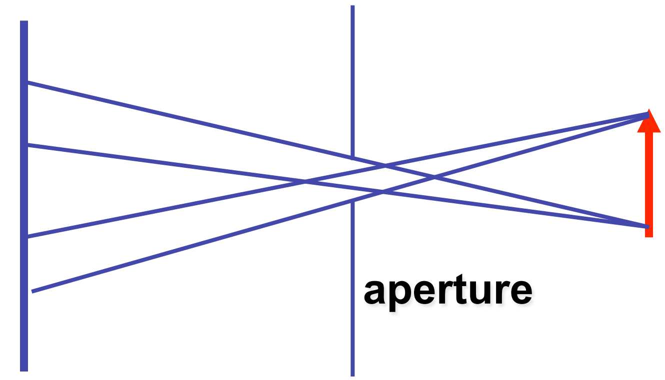
Pinhole Camera

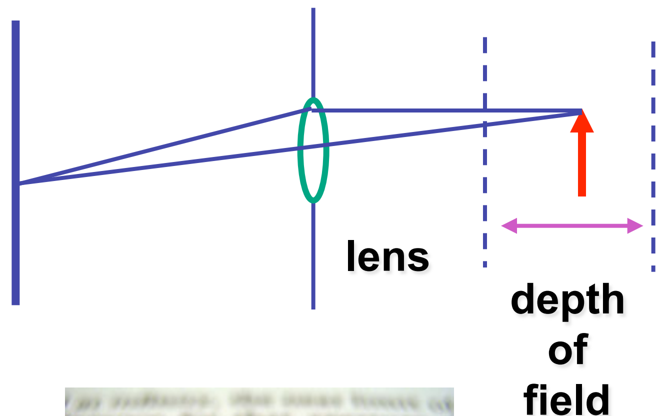
- non-zero sized hole
  - blur: rays hit multiple points on film plane

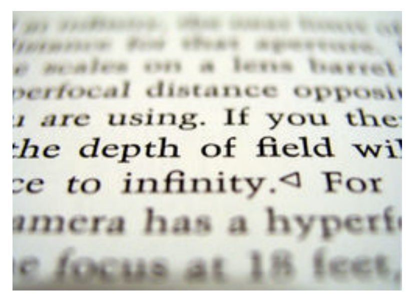


Real Cameras

- pinhole camera has small aperture (lens opening)
  - minimize blur
- problem: hard to get enough light to expose the film
- solution: lens
  - permits larger apertures
  - permits changing distance to film plane without actually moving it
    - cost: limited depth of field where image is in focus



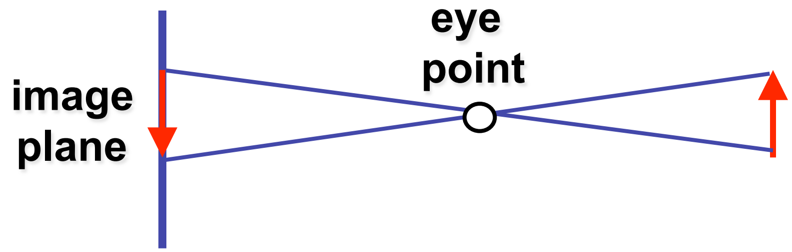




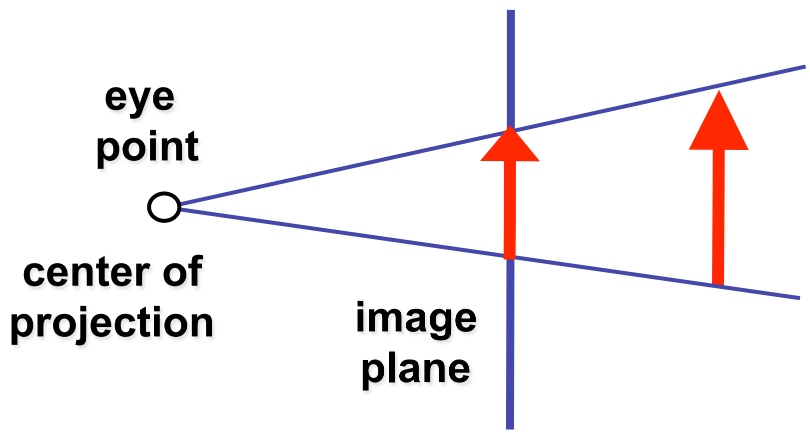
http://en.wikipedia.org/wiki/Image:DOF-ShallowDepthofField.jpg

Graphics Cameras

- real pinhole camera: image inverted

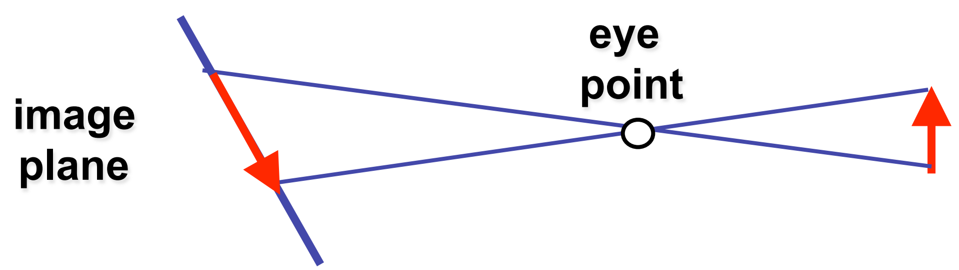


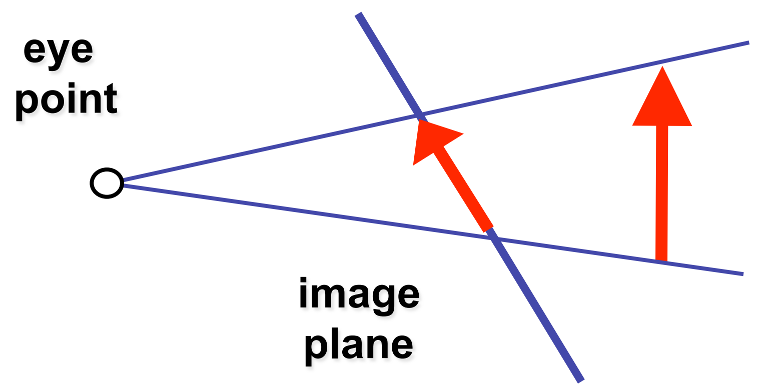
- computer graphics camera: convenient equivalent



General Projection

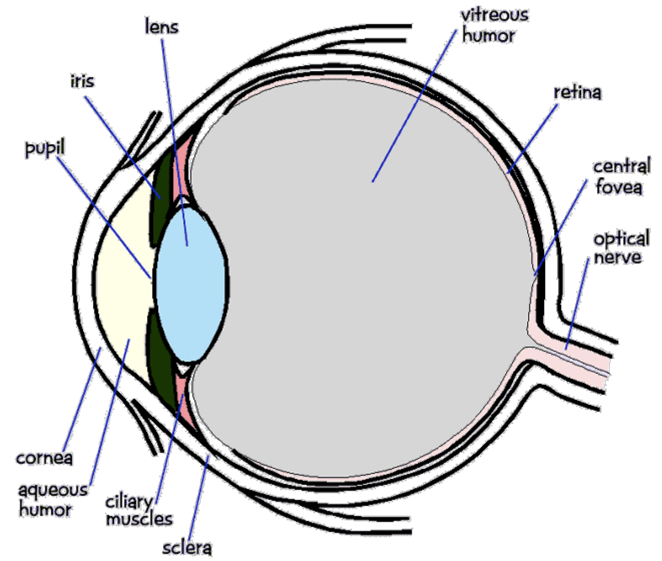
- image plane need not be perpendicular to view plane





Perspective Projection

- our camera must model perspective



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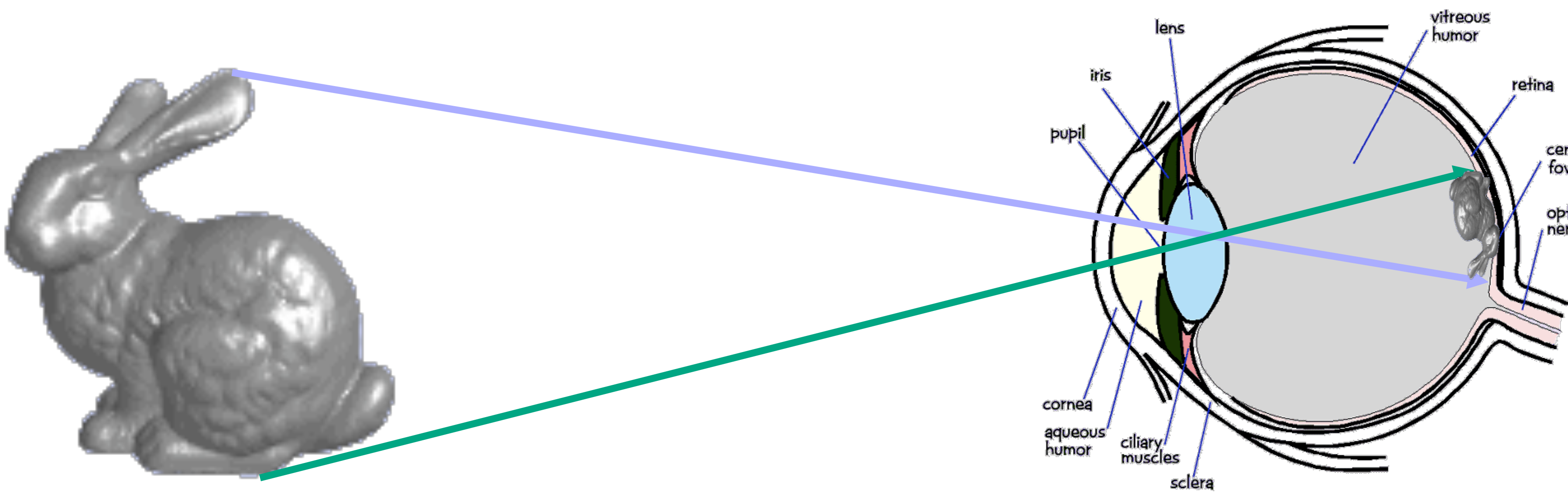


Projective Transformations

- planar geometric projections
  - planar: onto a plane
  - geometric: using straight lines
  - projections: 3D -> 2D
- aka projective mappings

- counterexamples?

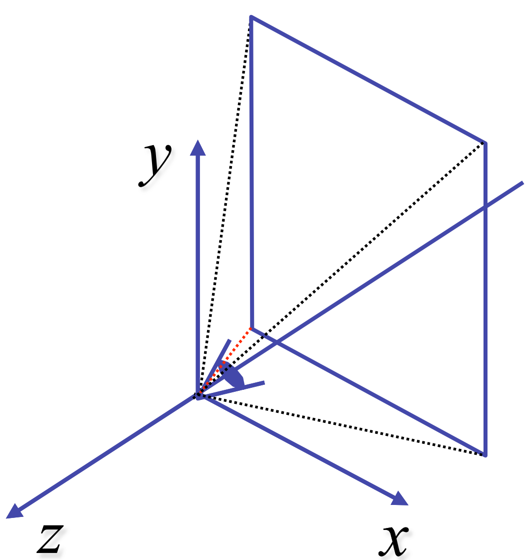
Projective Transformations

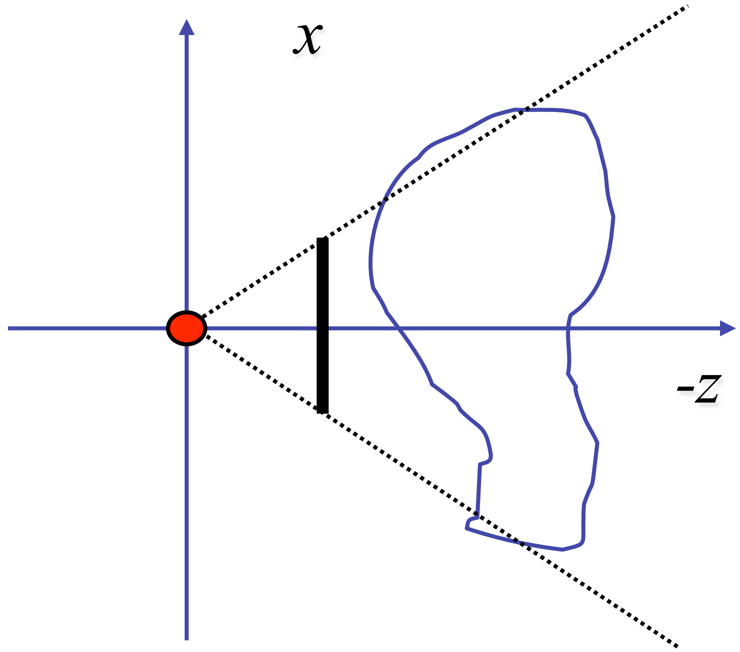
- properties
  - lines mapped to lines and triangles to triangles
  - parallel lines do NOT remain parallel
    - e.g. rails vanishing at infinity

  - affine combinations are NOT preserved
    - e.g. center of a line does not map to center of projected line (perspective foreshortening)

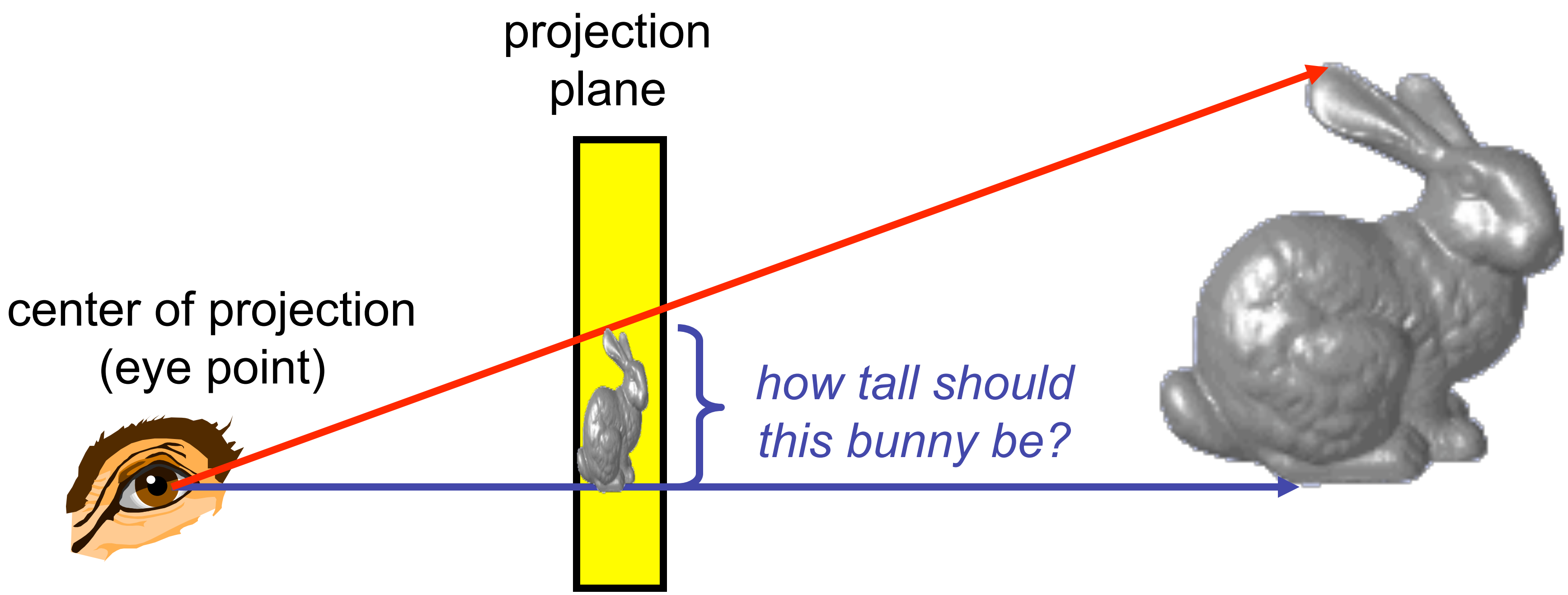
Perspective Projection

- project all geometry
  - through common center of projection (eye point)
  - onto an image plane



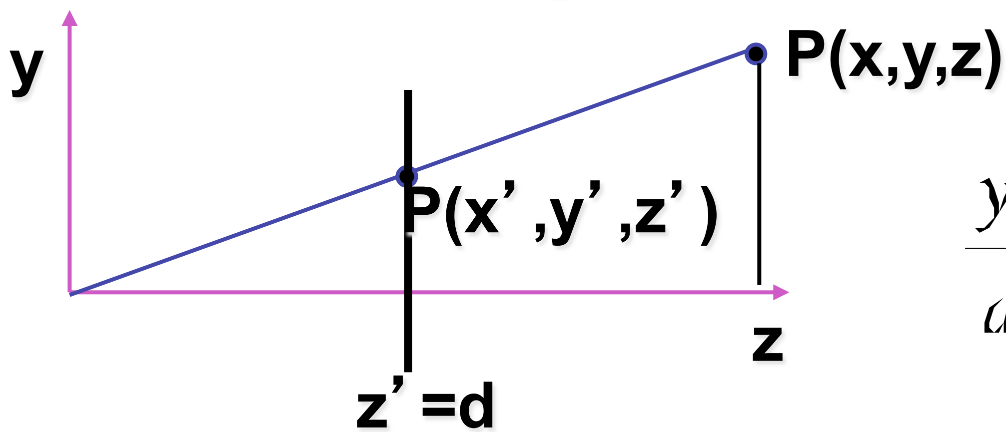


Perspective Projection



Basic Perspective Projection

similar triangles



$$\frac{y'}{d} = \frac{y}{z} \rightarrow y' = \frac{y \cdot d}{z}$$

$$\frac{x'}{d} = \frac{x}{z} \rightarrow x' = \frac{x \cdot d}{z}$$

but $z' = d$

- nonuniform foreshortening
- not affine

Perspective Projection

- desired result for a point $[x, y, z, 1]^T$ projected onto the view plane:

$$\frac{x'}{d} = \frac{x}{z}, \quad \frac{y'}{d} = \frac{y}{z}$$

$$x' = \frac{x \cdot d}{z} = \frac{x}{z/d}, \quad y' = \frac{y \cdot d}{z} = \frac{y}{z/d}, \quad z' = d$$

- what could a matrix look like to do this?

Simple Perspective Projection Matrix

$$\begin{bmatrix} \dfrac{x}{z/d} \\ \dfrac{y}{z/d} \\ d \end{bmatrix}$$

Simple Perspective Projection Matrix

$$\begin{bmatrix} \dfrac{x}{z/d} \\ \dfrac{y}{z/d} \\ d \end{bmatrix}$$

is homogenized version of

$$\begin{bmatrix} x \\ y \\ z \\ z/d \end{bmatrix}$$

where w = z/d

Simple Perspective Projection Matrix

$$\begin{bmatrix} \frac{x}{z/d} \\ \frac{y}{z/d} \\ d \end{bmatrix}$$

is homogenized version of

$$\begin{bmatrix} x \\ y \\ z \\ z/d \end{bmatrix}$$

where w = z/d

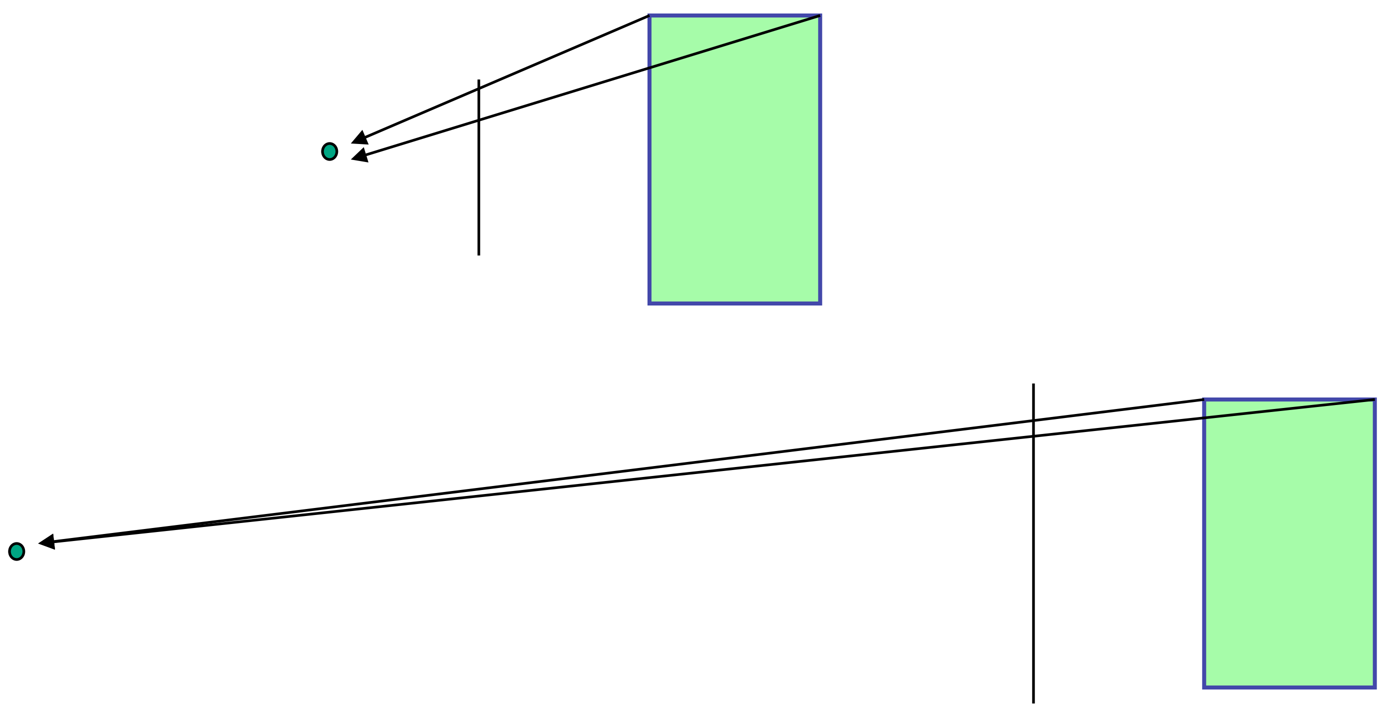
$$\begin{bmatrix} x \\ y \\ z \\ z/d \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Perspective Projection

- expressible with 4x4 homogeneous matrix
  - use previously untouched bottom row
- perspective projection is irreversible
  - many 3D points can be mapped to same (x, y, d) on the projection plane
  - no way to retrieve the unique z values

Moving COP to Infinity

- as COP moves away, lines approach parallel
- when COP at infinity, orthographic view



Orthographic Camera Projection

- camera's back plane parallel to lens
- infinite focal length
- no perspective convergence

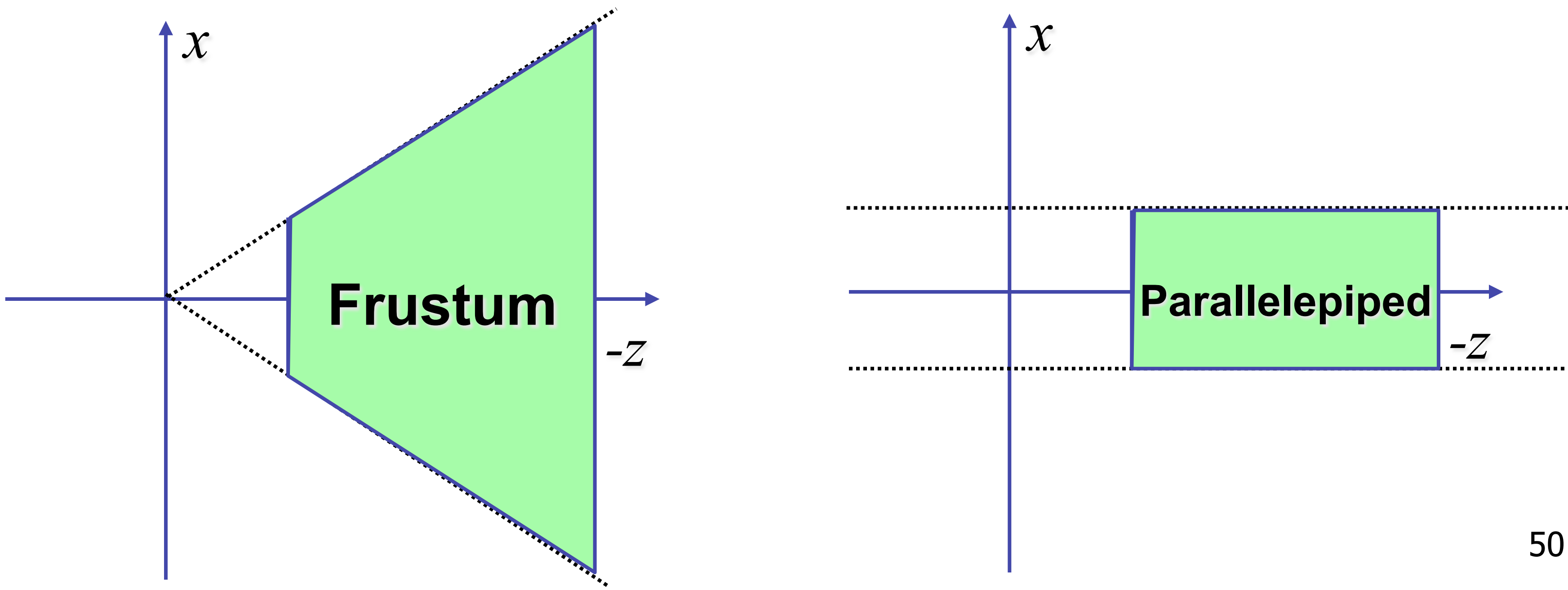
- just throw away z values

$$\begin{bmatrix} x_p \\ y_p \\ z_p \end{bmatrix} = \begin{bmatrix} x \\ y \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x_p \\ y_p \\ z_p \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Perspective to Orthographic

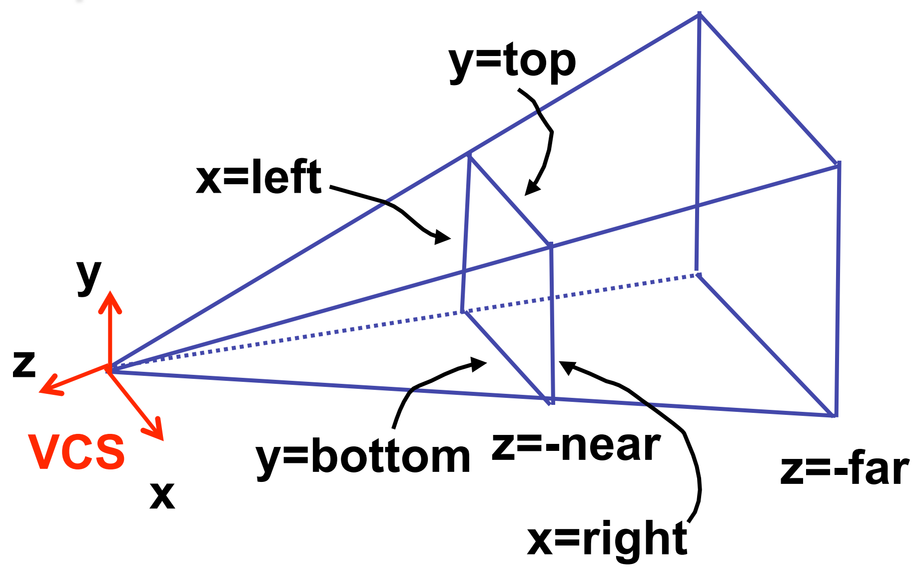
- transformation of space
  - center of projection moves to infinity
  - view volume transformed
    - from frustum (truncated pyramid) to parallelepiped (box)



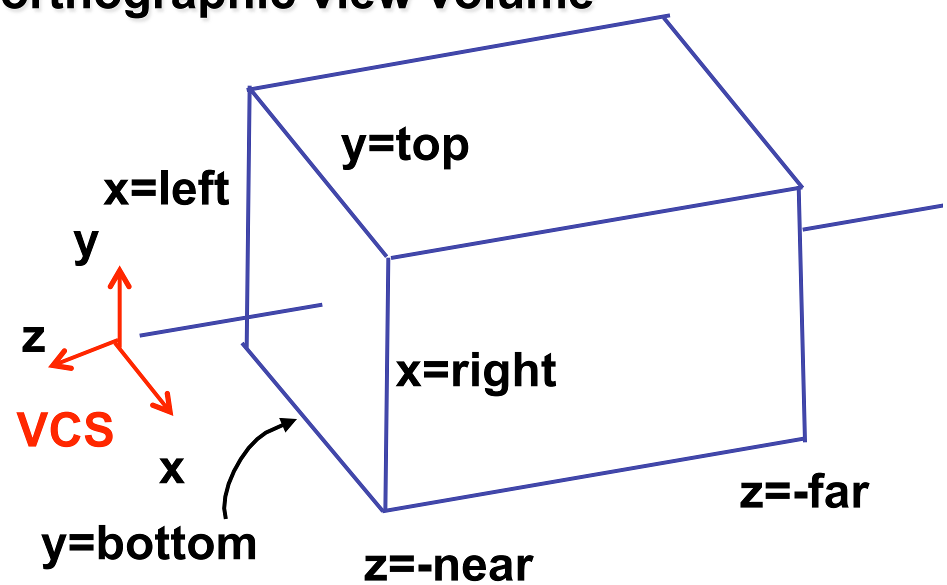
View Volumes

- specifies field-of-view, used for clipping
- restricts domain of ***z*** stored for visibility test

perspective view volume



orthographic view volume

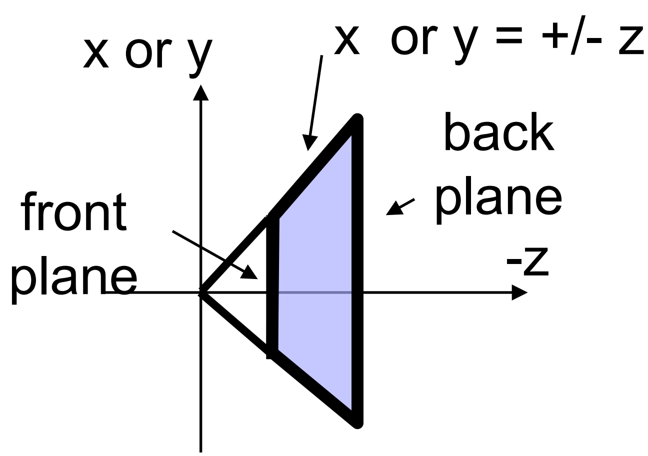


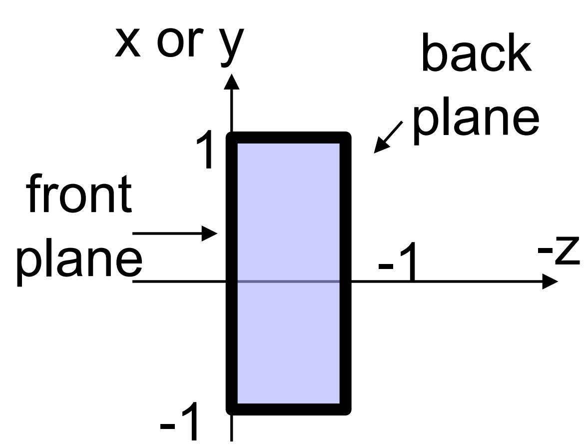
Canonical View Volumes

- standardized viewing volume representation

perspective

orthographic
orthogonal
parallel





Why Canonical View Volumes?

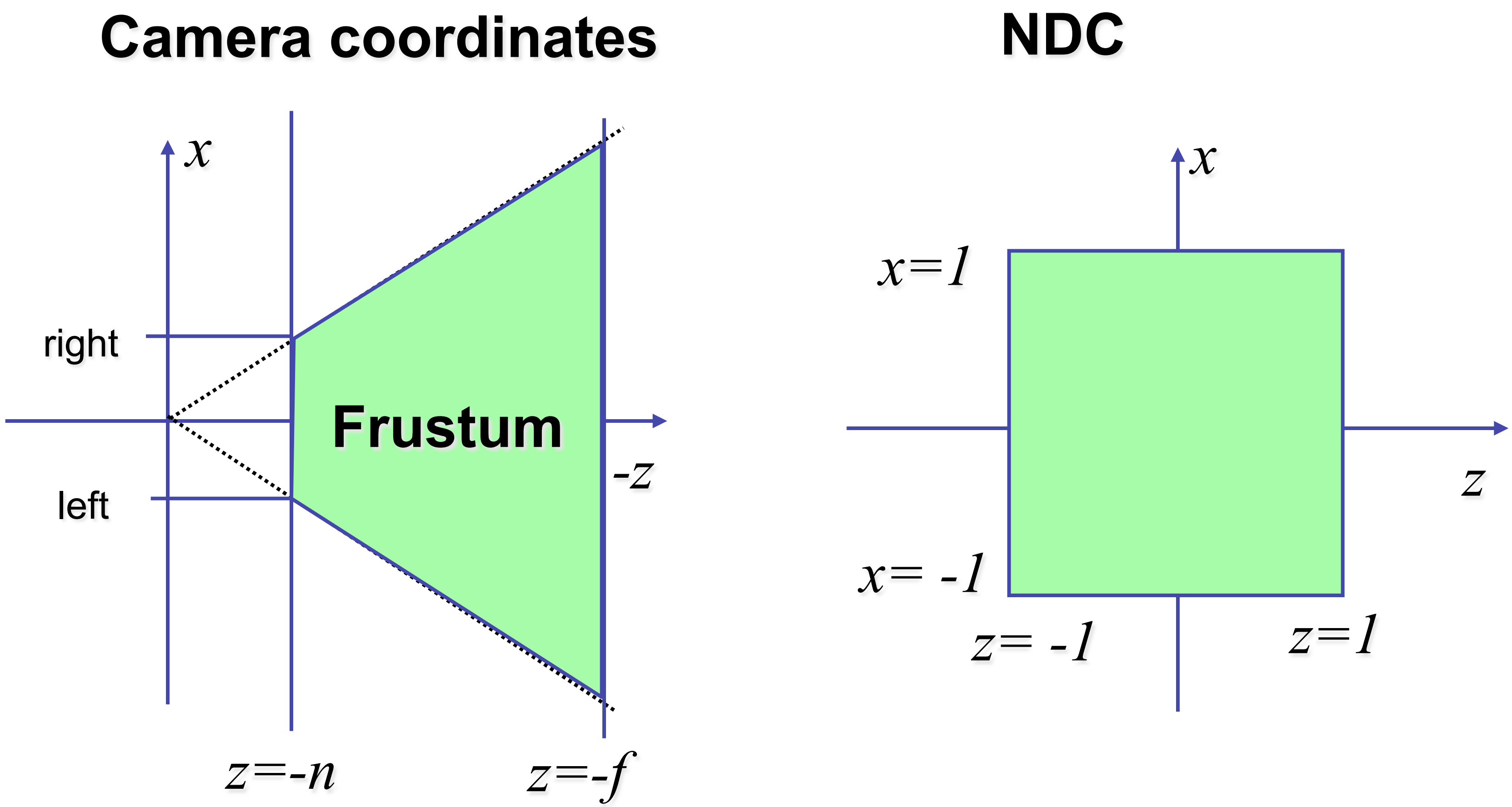
- permits standardization
  - clipping
    - easier to determine if an arbitrary point is enclosed in volume with canonical view volume vs. clipping to six arbitrary planes
  - rendering
    - projection and rasterization algorithms can be reused

Normalized Device Coordinates

- convention
  - viewing frustum mapped to specific parallelepiped
    - Normalized Device Coordinates (NDC)
    - same as clipping coords
  - only objects inside the parallelepiped get rendered
  - which parallelepiped?
    - depends on rendering system

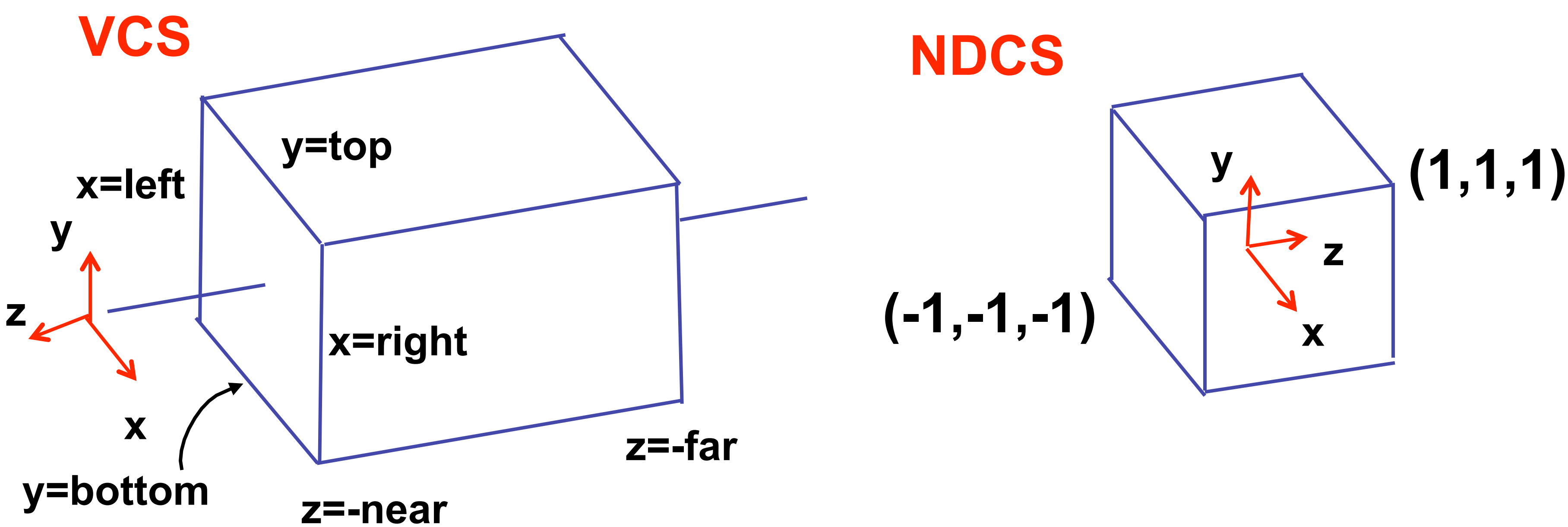
Normalized Device Coordinates

left/right x =+/- 1, top/bottom y =+/- 1, near/far z =+/- 1



Understanding Z

- z axis flip changes coord system handedness
  - RHS before projection (eye/view coords)
  - LHS after projection (clip, norm device coords)

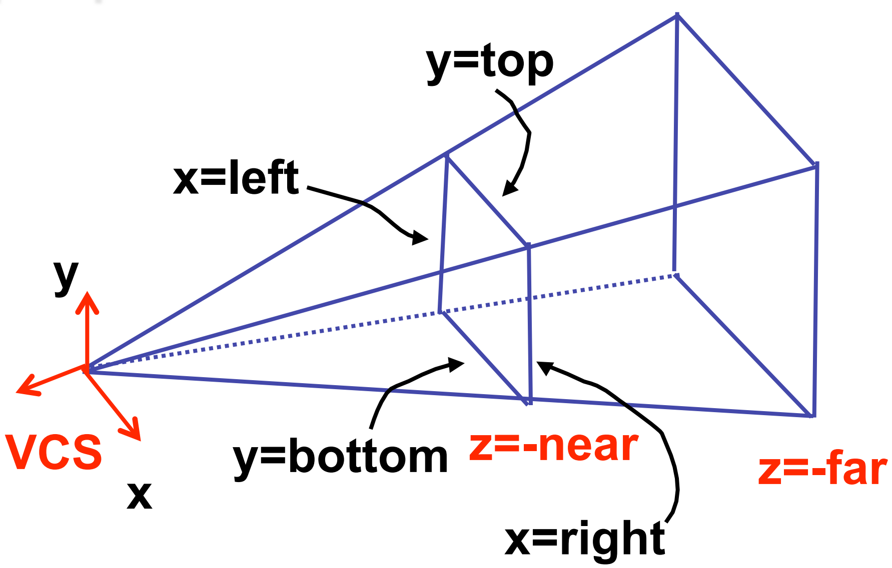


Understanding Z

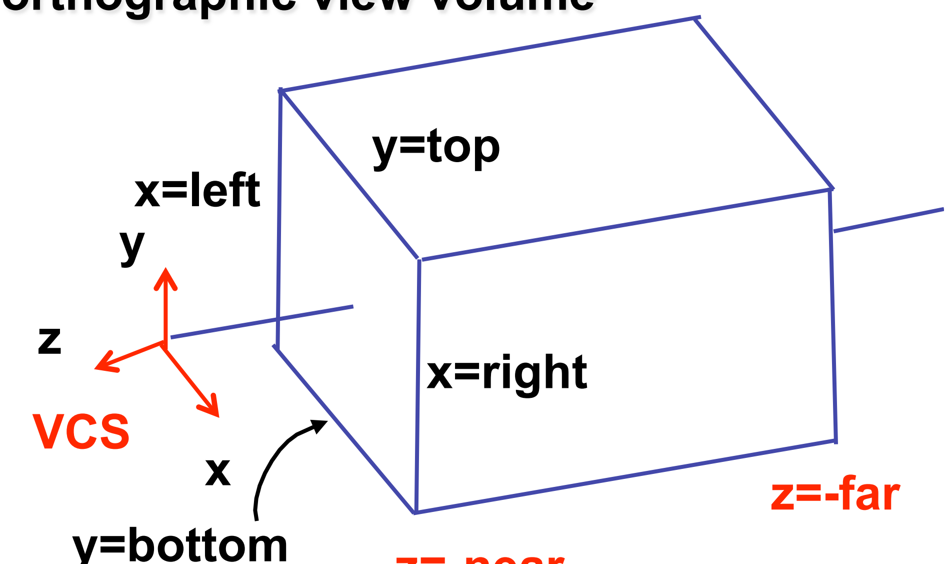
near, far always positive in OpenGL calls

```
glOrtho(left,right,bot,top,near,far);
glFrustum(left,right,bot,top,near,far);
glPerspective(fovy,aspect,near,far);
```

perspective view volume



orthographic view volume

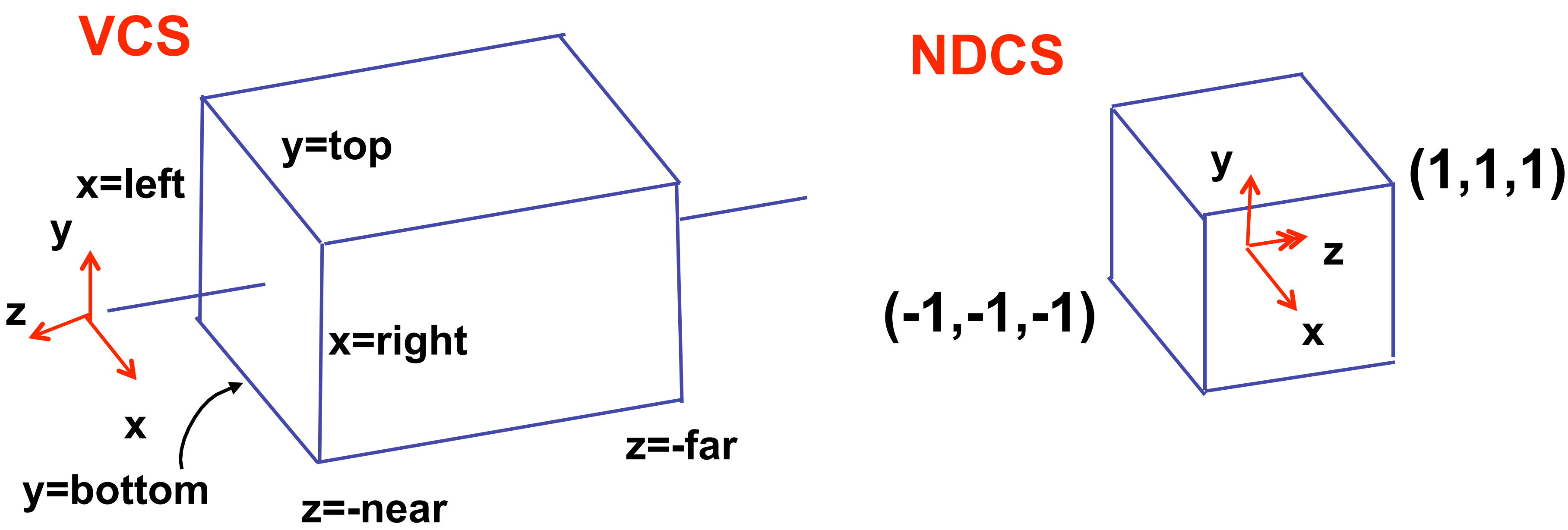


Understanding Z

- why near and far plane?
  - near plane:
    - avoid singularity (division by zero, or very small numbers)
  - far plane:
    - store depth in fixed-point representation (integer), thus have to have fixed range of values (0…1)
    - avoid/reduce numerical precision artifacts for distant objects

Orthographic Derivation

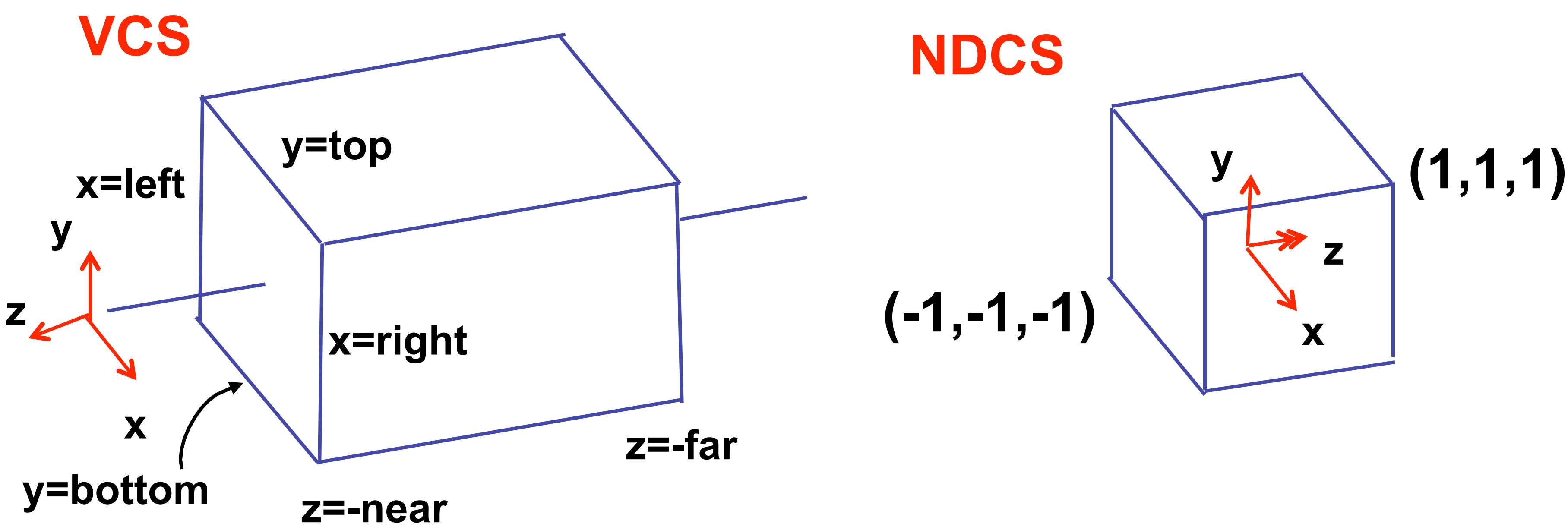
- scale, translate, reflect for new coord sys



Orthographic Derivation

- scale, translate, reflect for new coord sys

$$y' = a \cdot y + b \qquad \begin{array}{l} y = top \rightarrow y' = 1 \\ y = bot \rightarrow y' = -1 \end{array}$$



Orthographic Derivation

- scale, translate, reflect for new coord sys

$$y' = a \cdot y + b$$

$$y = top \rightarrow y' = 1 \qquad 1 = a \cdot top + b$$

$$y = bot \rightarrow y' = -1 \qquad -1 = a \cdot bot + b$$

$$b = 1 - a \cdot top, b = -1 - a \cdot bot$$

$$1 - a \cdot top = -1 - a \cdot bot$$

$$1 - (-1) = -a \cdot bot - (-a \cdot top)$$

$$2 = a(-bot + top)$$

$$a = \frac{2}{top - bot}$$

$$1 = \frac{2}{top - bot} top + b$$

$$b = 1 - \frac{2 \cdot top}{top - bot}$$

$$b = \frac{(top - bot) - 2 \cdot top}{top - bot}$$

$$b = \frac{-top - bot}{top - bot}$$

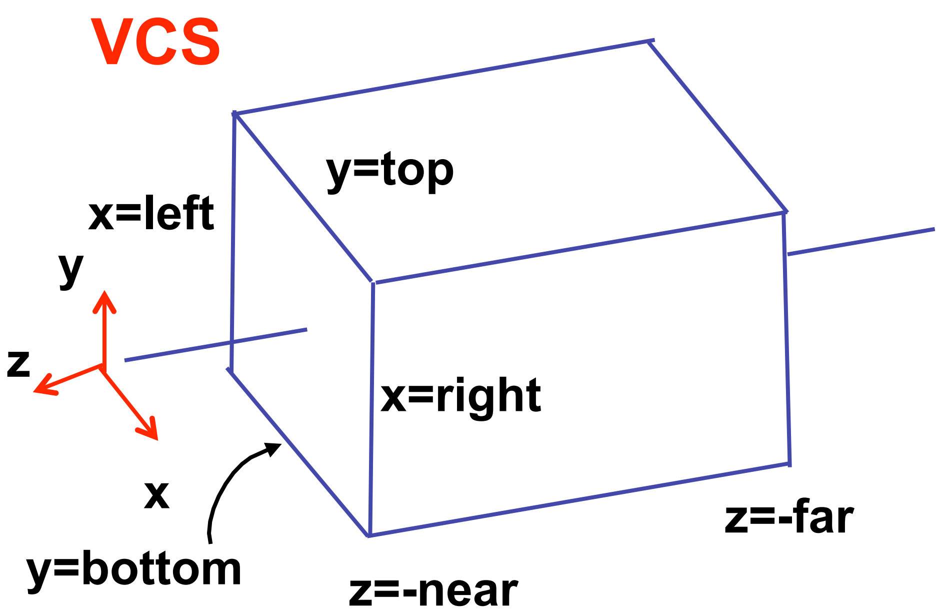
Orthographic Derivation

- scale, translate, reflect for new coord sys

$$y' = a \cdot y + b$$

$$y = top \rightarrow y' = 1$$

$$y = bot \rightarrow y' = -1$$



$$a = \frac{2}{top - bot}$$

$$b = -\frac{top + bot}{top - bot}$$

same idea for right/left, far/near

Orthographic Derivation

- scale, translate, reflect for new coord sys

$$P' = \begin{bmatrix} \frac{2}{right-left} & 0 & 0 & -\frac{right+left}{right-left} \\ 0 & \frac{2}{top-bot} & 0 & -\frac{top+bot}{top-bot} \\ 0 & 0 & \frac{-2}{far-near} & -\frac{far+near}{far-near} \\ 0 & 0 & 0 & 1 \end{bmatrix} P$$

Orthographic Derivation

- scale, translate, reflect for new coord sys

$$P' = \begin{bmatrix} \frac{2}{right-left} & 0 & 0 & -\frac{right+left}{right-left} \\ 0 & \frac{2}{top-bot} & 0 & -\frac{top+bot}{top-bot} \\ 0 & 0 & \frac{-2}{far-near} & -\frac{far+near}{far-near} \\ 0 & 0 & 0 & 1 \end{bmatrix} P$$

Orthographic Derivation

- scale, translate, reflect for new coord sys

$$P' = \begin{bmatrix} \frac{2}{right-left} & 0 & 0 & -\frac{right+left}{right-left} \\ 0 & \frac{2}{top-bot} & 0 & -\frac{top+bot}{top-bot} \\ 0 & 0 & \frac{-2}{far-near} & -\frac{far+near}{far-near} \\ 0 & 0 & 0 & 1 \end{bmatrix} P$$

Orthographic Derivation

- scale, translate, reflect for new coord sys

$$P' = \begin{bmatrix} \frac{2}{right-left} & 0 & 0 & -\frac{right+left}{right-left} \\ 0 & \frac{2}{top-bot} & 0 & -\frac{top+bot}{top-bot} \\ 0 & 0 & \frac{-2}{far-near} & -\frac{far+near}{far-near} \\ 0 & 0 & 0 & 1 \end{bmatrix} P$$

Orthographic OpenGL

```
glMatrixMode(GL_PROJECTION);
glLoadIdentity();
glOrtho(left,right,bot,top,near,far);
```

Demo

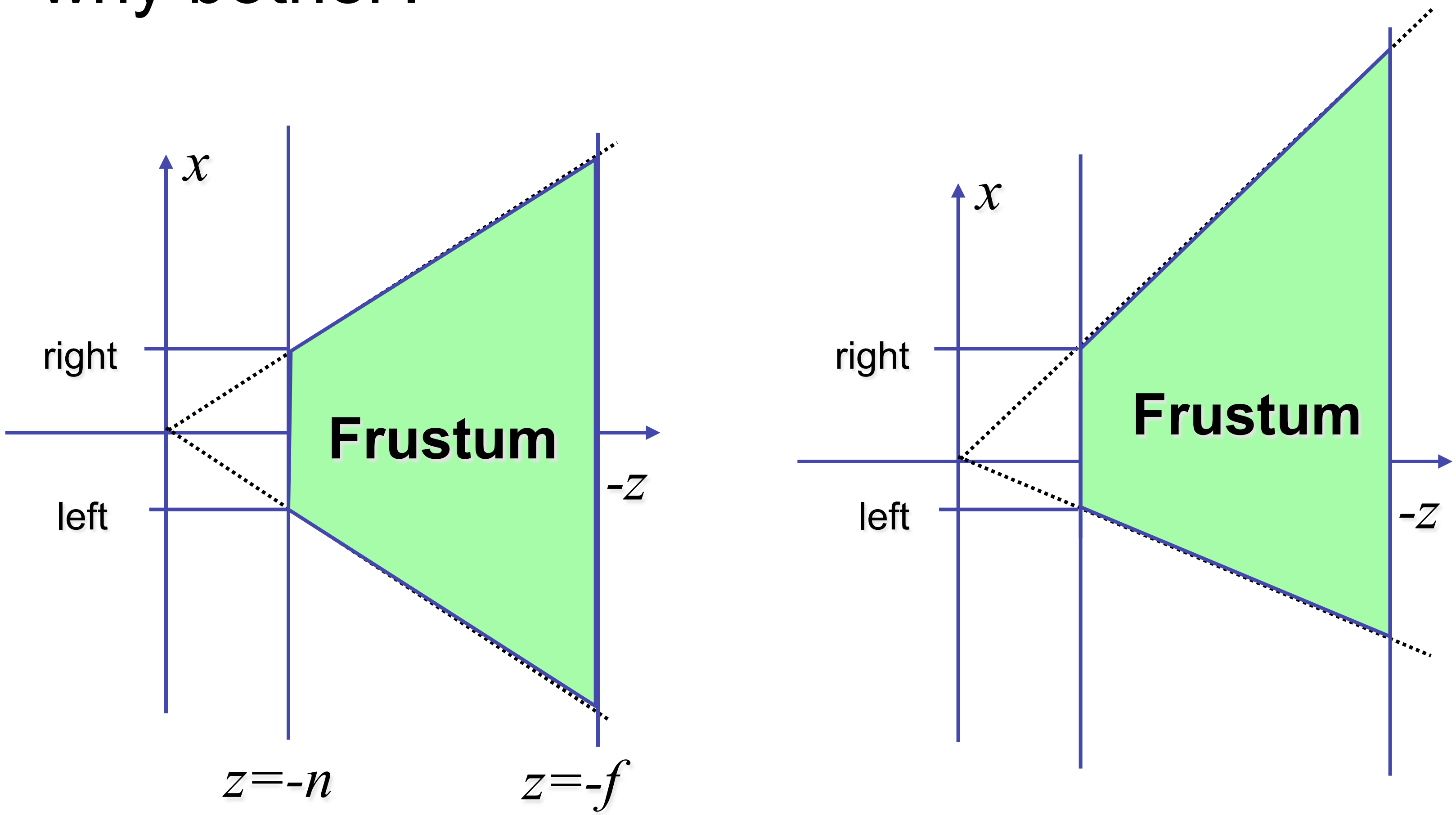
- Brown applets: viewing techniques
  - parallel/orthographic cameras
  - projection cameras

- http://www.cs.brown.edu/exploratories/freeSoftware/catalogs/viewing_techniques.html

Projections II

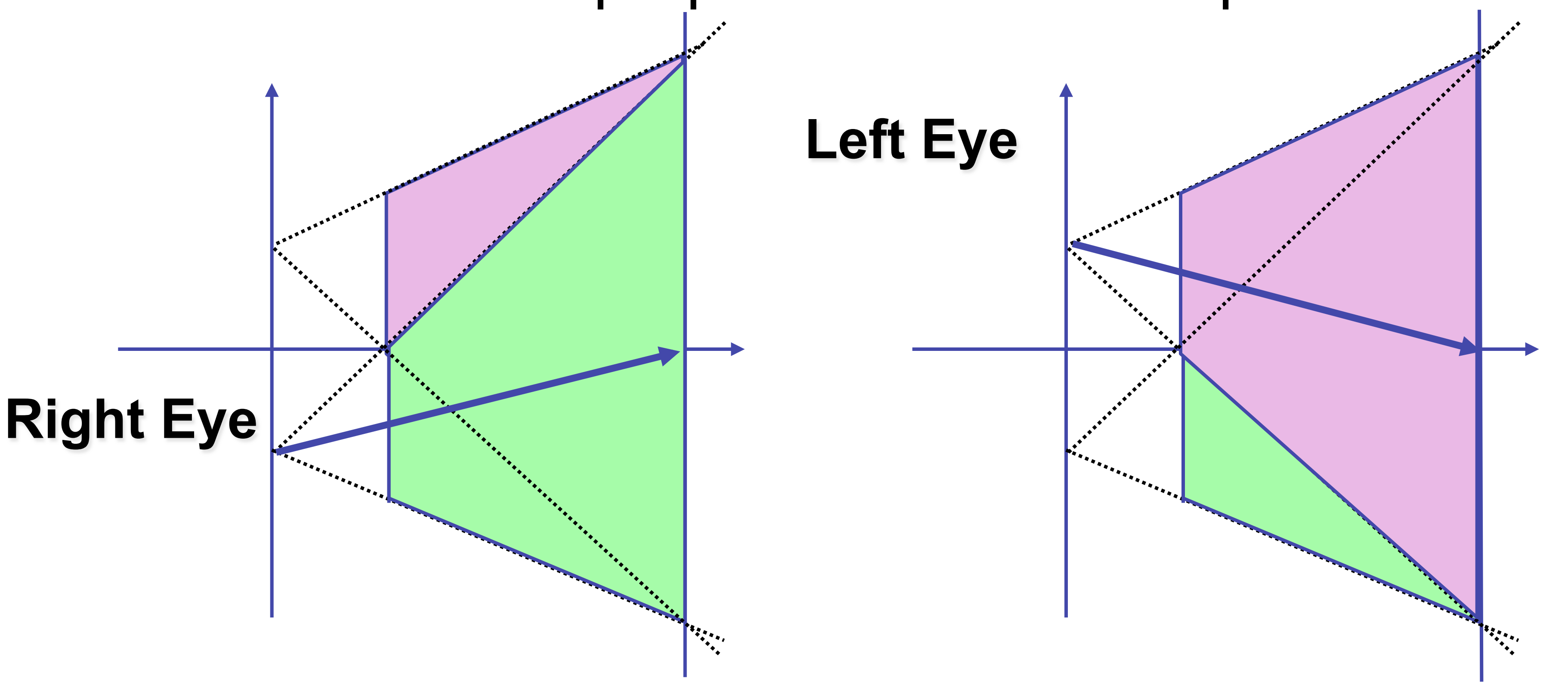
Asymmetric Frusta

- our formulation allows asymmetry
  - why bother?



Asymmetric Frusta

- our formulation allows asymmetry
  - why bother? binocular stereo
    - view vector not perpendicular to view plane

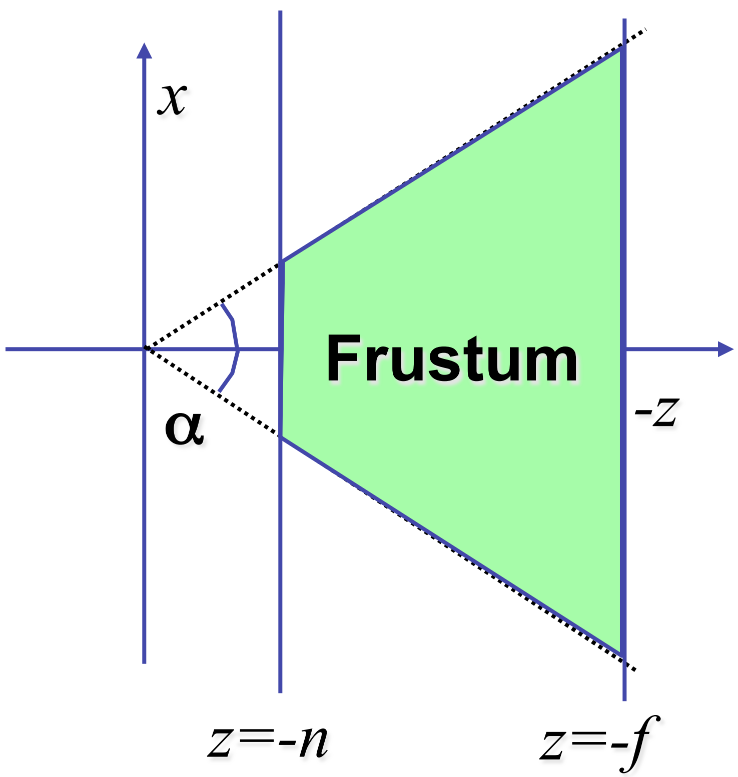


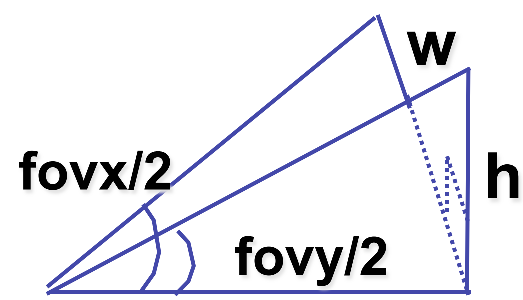
Simpler Formulation

- left, right, bottom, top, near, far
  - nonintuitive
  - often overkill
- look through window center
  - symmetric frustum
- constraints
  - left = -right, bottom = -top

Field-of-View Formulation

- FOV in one direction + aspect ratio (w/h)
  - determines FOV in other direction
  - also set near, far (reasonably intuitive)





Perspective OpenGL

```
glMatrixMode(GL_PROJECTION);
glLoadIdentity();

glFrustum(left,right,bot,top,near,far);
```

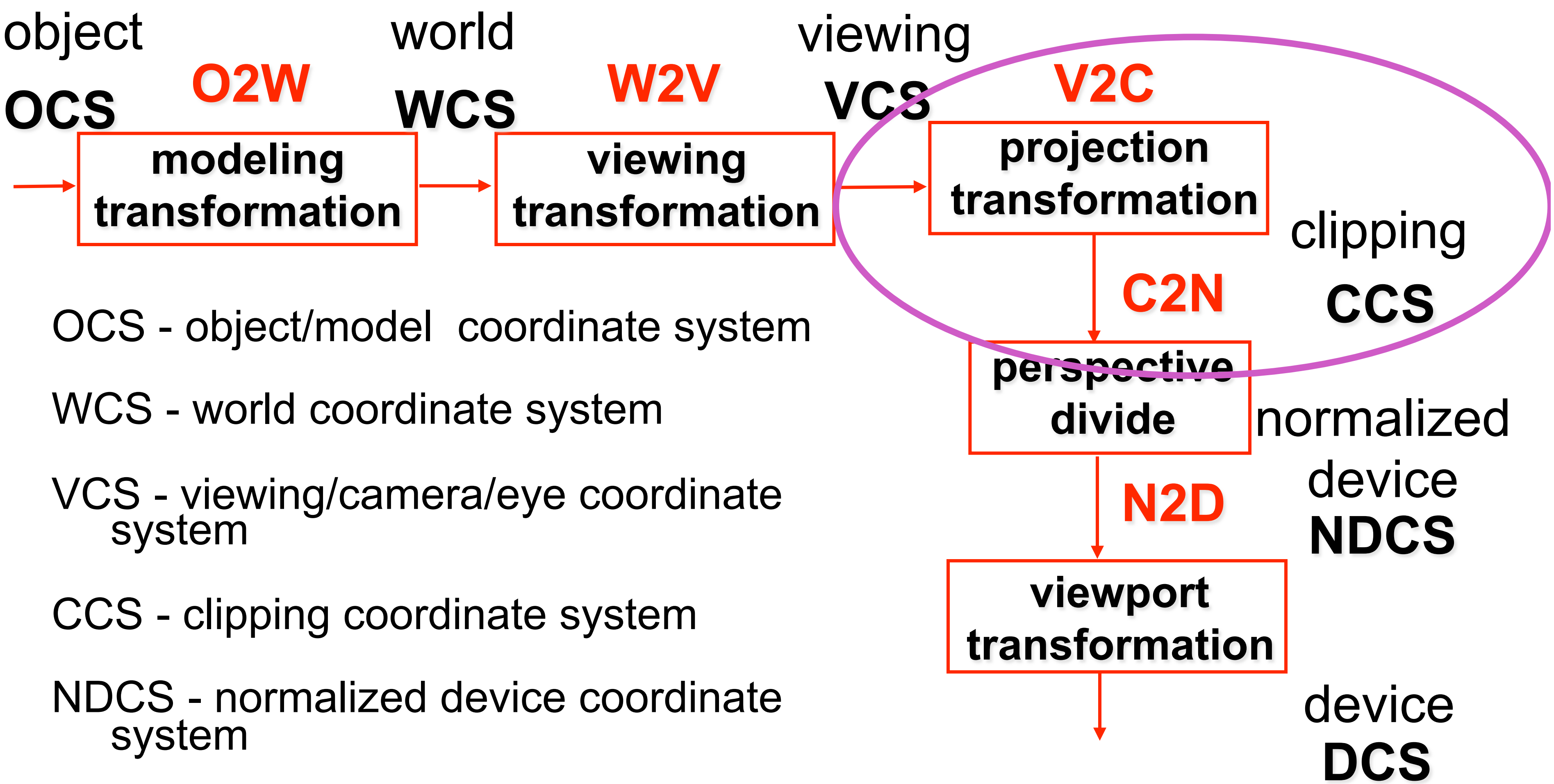
or

```
glPerspective(fovy,aspect,near,far);
```

Demo: Frustum vs. FOV

- Nate Robins tutorial (take 2):
    - http://www.xmission.com/~nate/tutors.html

Projective Rendering Pipeline



OCS - object/model coordinate system

WCS - world coordinate system

VCS - viewing/camera/eye coordinate system

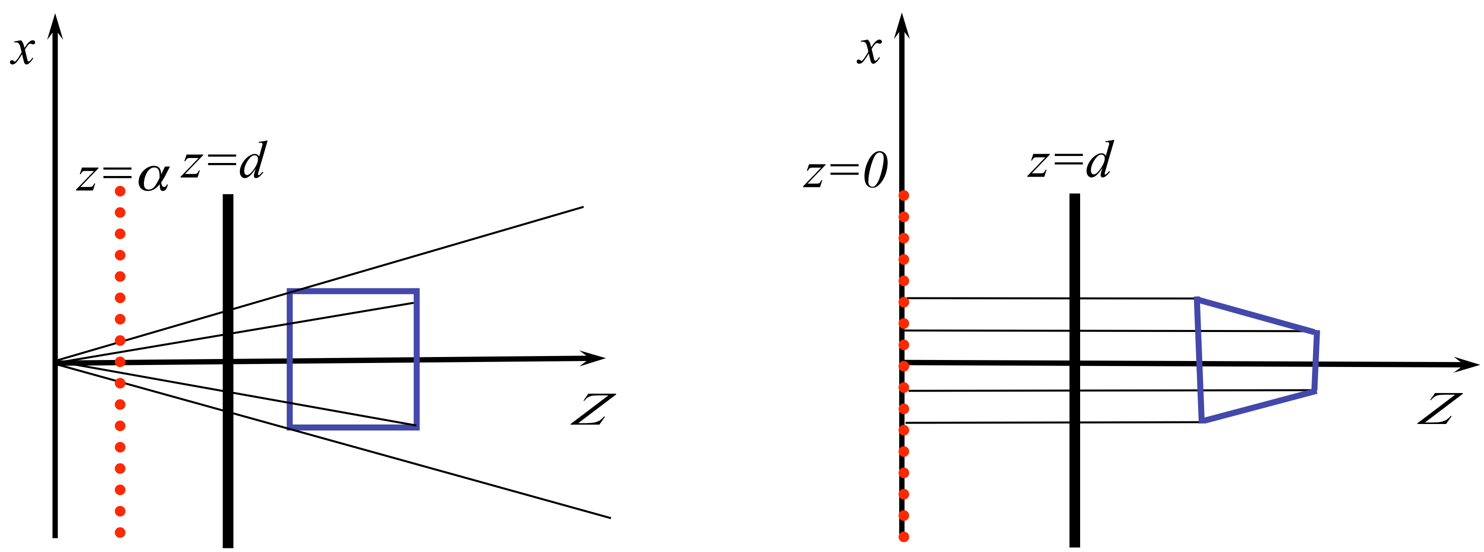
CCS - clipping coordinate system

NDCS - normalized device coordinate system

DCS - device/display/screen coordinate system

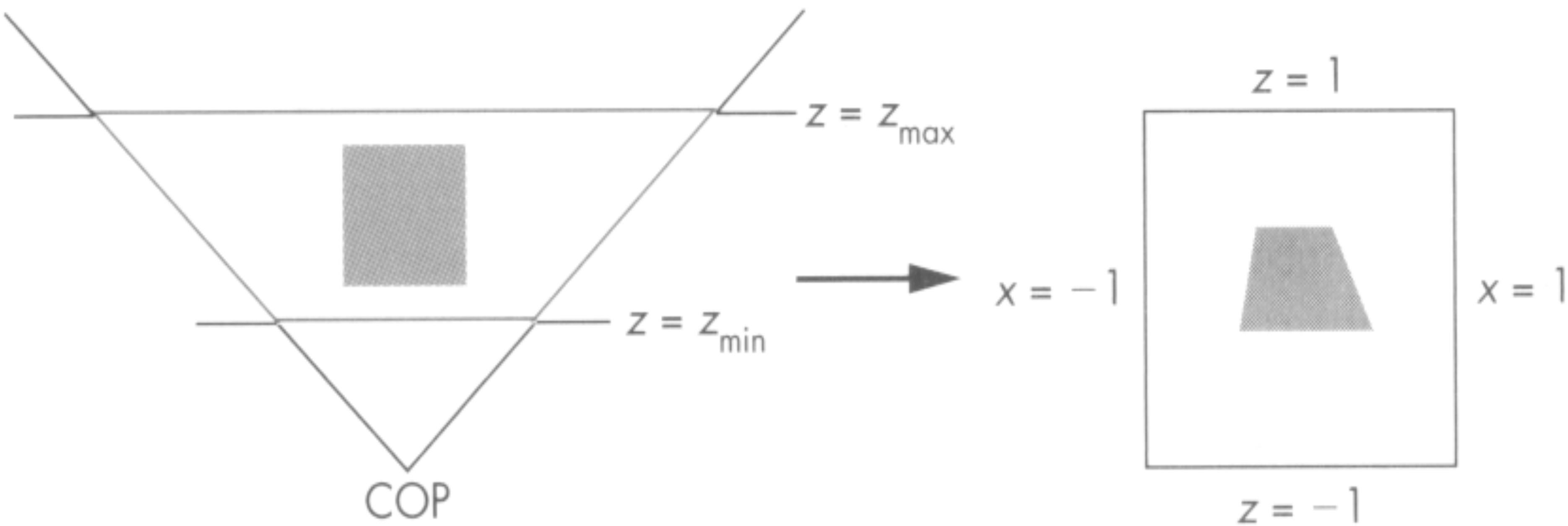
Projection Warp

- warp perspective view volume to orthogonal view volume
  - render all scenes with orthographic projection!
  - aka perspective warp

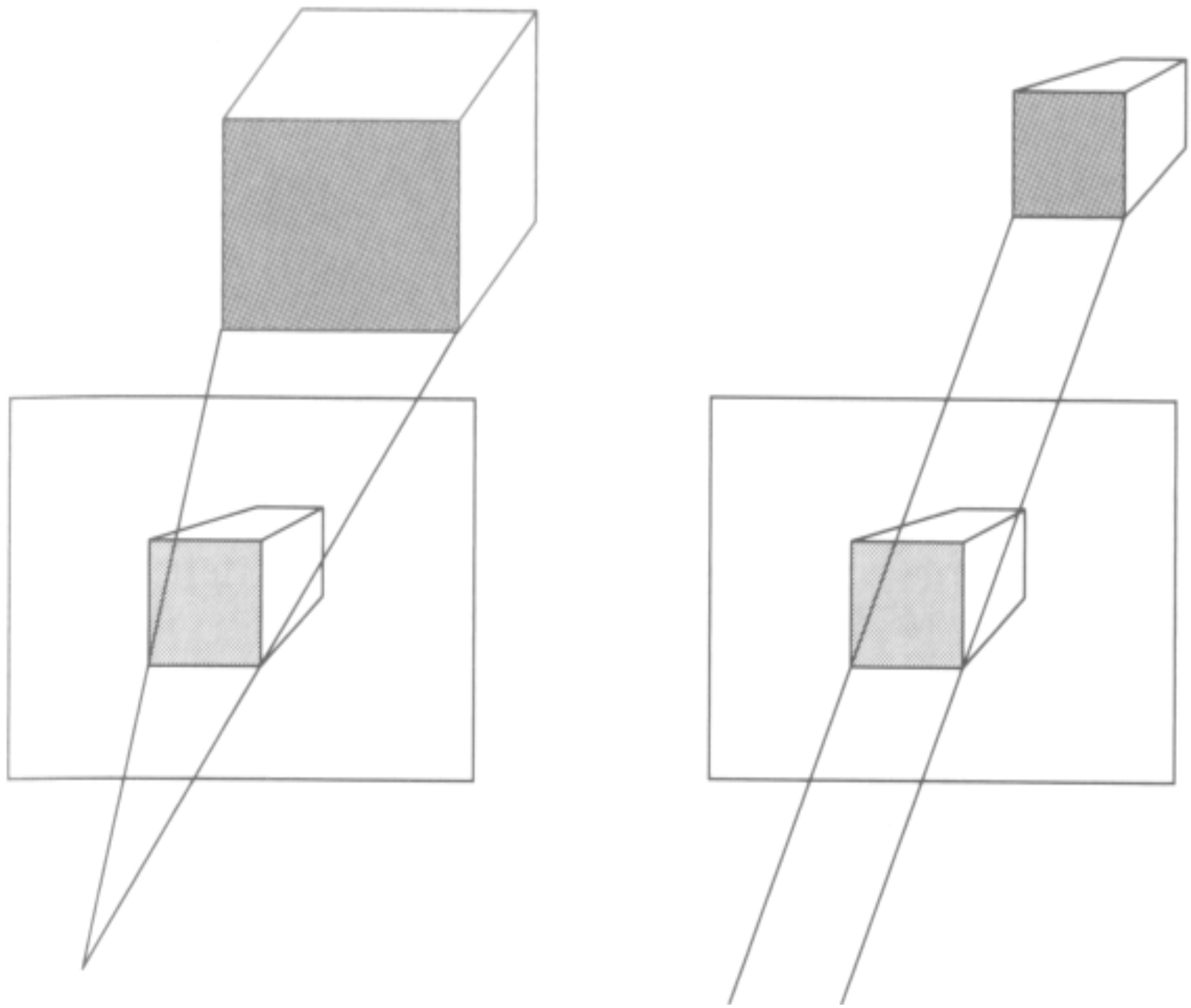


Perspective Warp

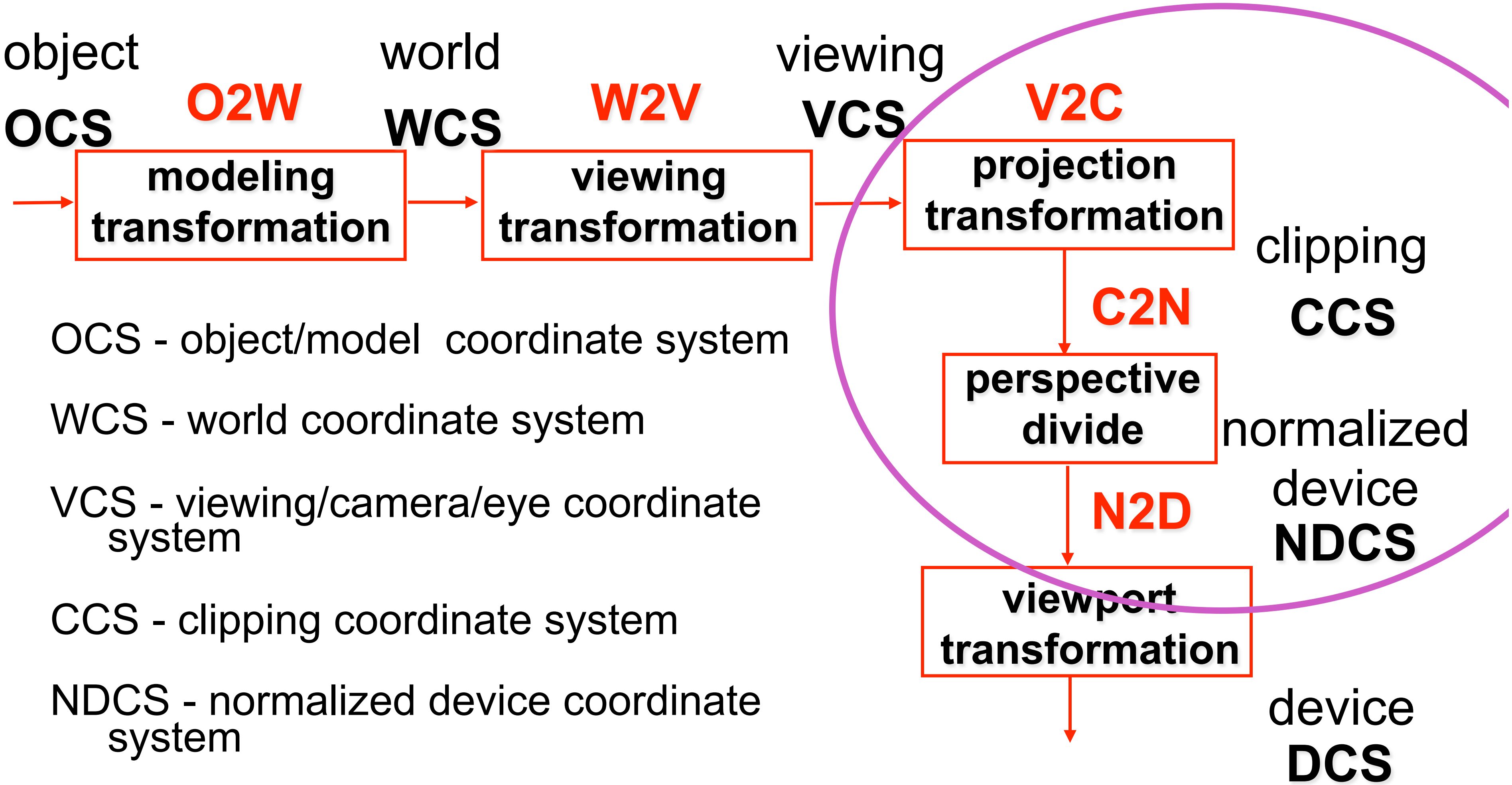
- perspective viewing frustum transformed to cube
- orthographic rendering of cube produces same image as perspective rendering of original



Predistortion



Projective Rendering Pipeline



OCS - object/model coordinate system

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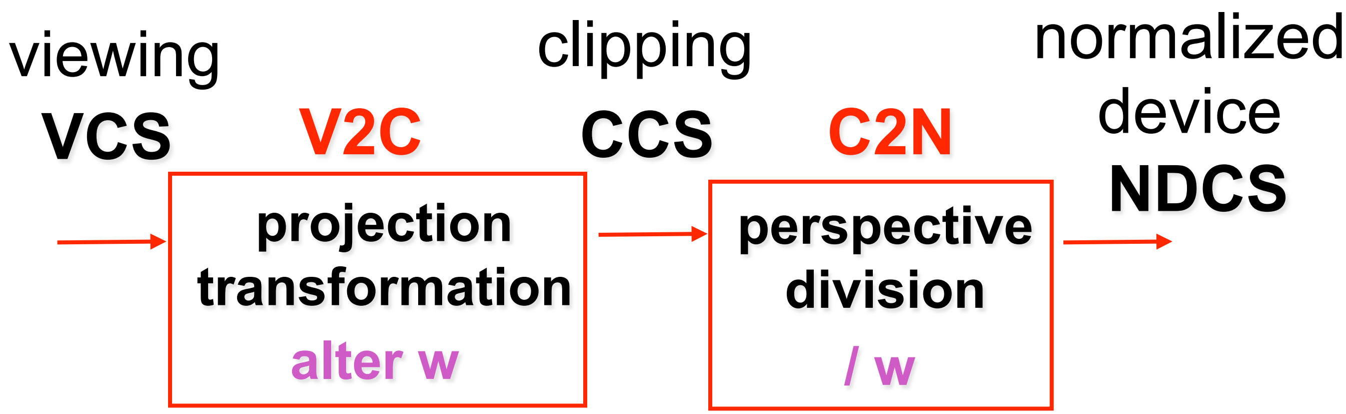
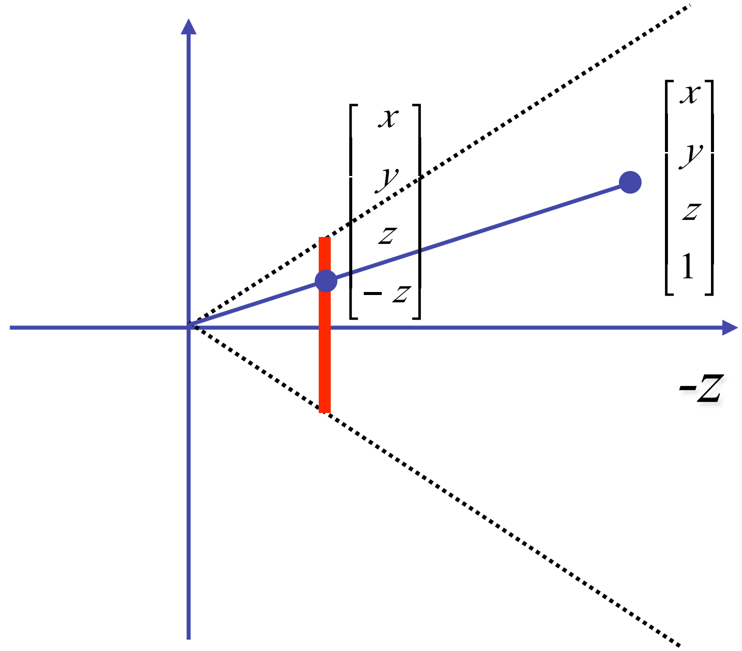
Separate Warp From Homogenization



- warp requires only standard matrix multiply
  - distort such that orthographic projection of distorted objects is desired persp projection
    - w is changed
  - clip after warp, before divide
  - division by w: homogenization

Perspective Divide Example

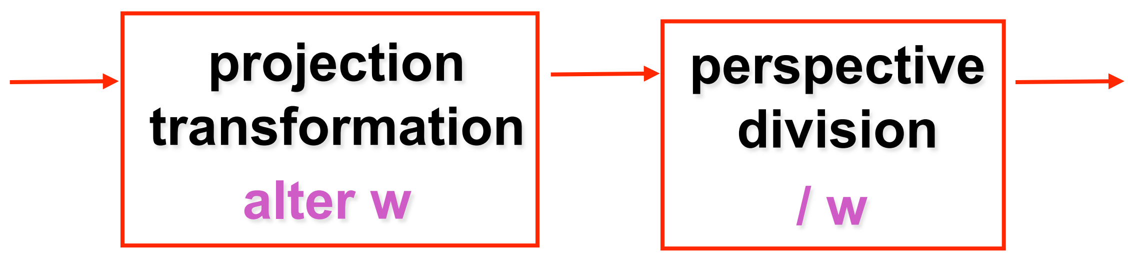
- specific example
  - assume image plane at $z = -1$
  - a point $[x,y,z,1]^T$ projects to $[-x/z,-y/z,-z/z,1]^T \equiv [x,y,z,-z]^T$



Perspective Divide Example

$$T\left(\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \\ -z \end{bmatrix} \equiv \begin{bmatrix} -x/z \\ -y/z \\ -1 \\ 1 \end{bmatrix}$$

- after homogenizing, once again w=1



Perspective Normalization

- matrix formulation

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{d}{d-\alpha} & \frac{-\alpha \cdot d}{d-\alpha} \\ 0 & 0 & \frac{1}{d} & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ \left(\frac{(z-\alpha) \cdot d}{d-\alpha} \right) \\ \frac{z}{d} \end{bmatrix} \qquad \begin{bmatrix} x_p \\ y_p \\ z_p \end{bmatrix} = \begin{bmatrix} \frac{x}{z/d} \\ \frac{y}{z/d} \\ \frac{d^2}{d-\alpha} \left(1 - \frac{\alpha}{z} \right) \end{bmatrix}$$

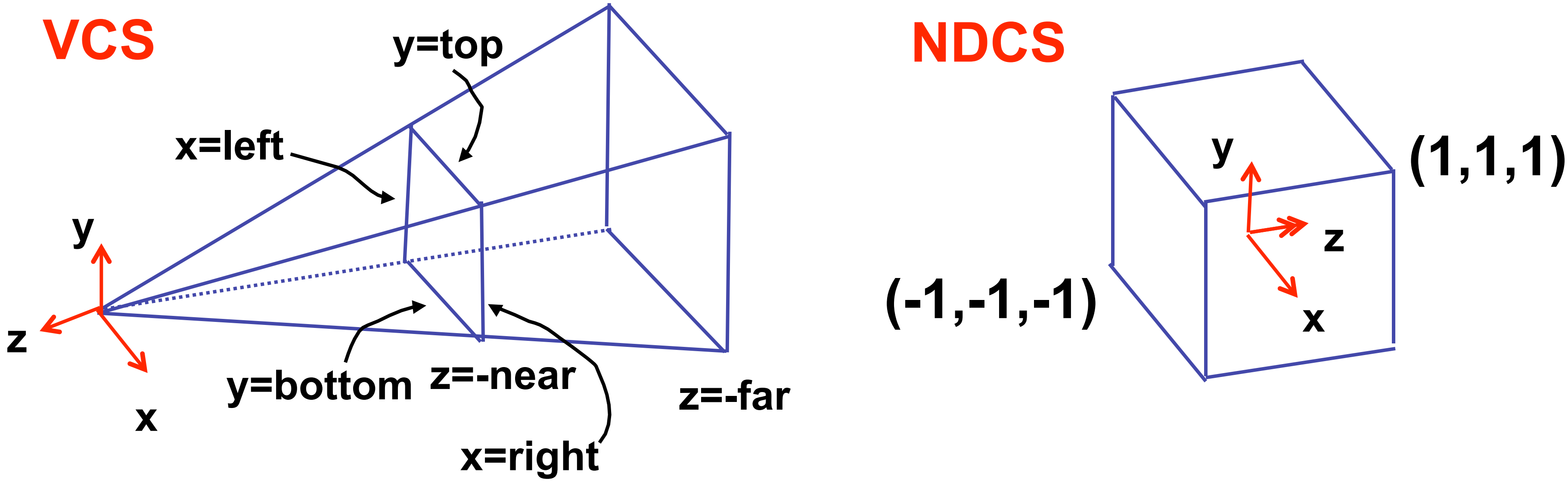
- warp and homogenization both preserve relative depth (z coordinate)

Demo

- Brown applets: viewing techniques
  - parallel/orthographic cameras
  - projection cameras

- http://www.cs.brown.edu/exploratories/freeSoftware/catalogs/viewing_techniques.html

Perspective To NDCS Derivation



Perspective Derivation

simple example earlier:

$$\begin{bmatrix} x' \\ y' \\ z' \\ w' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

complete: shear, scale, projection-normalization

$$\begin{bmatrix} x' \\ y' \\ z' \\ w' \end{bmatrix} = \begin{bmatrix} E & 0 & A & 0 \\ 0 & F & B & 0 \\ 0 & 0 & C & D \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Perspective Derivation

earlier:

$$\begin{bmatrix} x' \\ y' \\ z' \\ w' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

complete: shear, scale, projection-normalization

$$\begin{bmatrix} x' \\ y' \\ z' \\ w' \end{bmatrix} = \begin{bmatrix} E & 0 & A & 0 \\ 0 & F & B & 0 \\ 0 & 0 & C & D \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Perspective Derivation

earlier:

$$\begin{bmatrix} x' \\ y' \\ z' \\ w' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

complete: shear, scale, projection-normalization

$$\begin{bmatrix} x' \\ y' \\ z' \\ w' \end{bmatrix} = \begin{bmatrix} E & 0 & A & 0 \\ 0 & F & B & 0 \\ 0 & 0 & C & D \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Recorrection: Perspective Derivation

L/R sign error

$$\begin{bmatrix} x' \\ y' \\ z' \\ w' \end{bmatrix} = \begin{bmatrix} E & 0 & A & 0 \\ 0 & F & B & 0 \\ 0 & 0 & C & D \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$x' = Ex + Az$$
$$y' = Fy + Bz$$
$$z' = Cz + D$$
$$w' = -z$$

$$x = left \rightarrow x'/w' = -1$$
$$x = right \rightarrow x'/w' = 1$$
$$y = top \rightarrow y'/w' = 1$$
$$y = bottom \rightarrow y'/w' = -1$$
$$z = -near \rightarrow z'/w' = -1$$
$$z = -far \rightarrow z'/w' = 1$$

z axis flip!

$$y' = Fy + Bz, \quad \frac{y'}{w'} = \frac{Fy + Bz}{w'}, \quad 1 = \frac{Fy + Bz}{w'}, \quad 1 = \frac{Fy + Bz}{-z},$$

$$1 = F\frac{y}{-z} + B\frac{z}{-z}, \quad 1 = F\frac{y}{-z} - B, \quad 1 = F\frac{top}{-(-near)} - B,$$

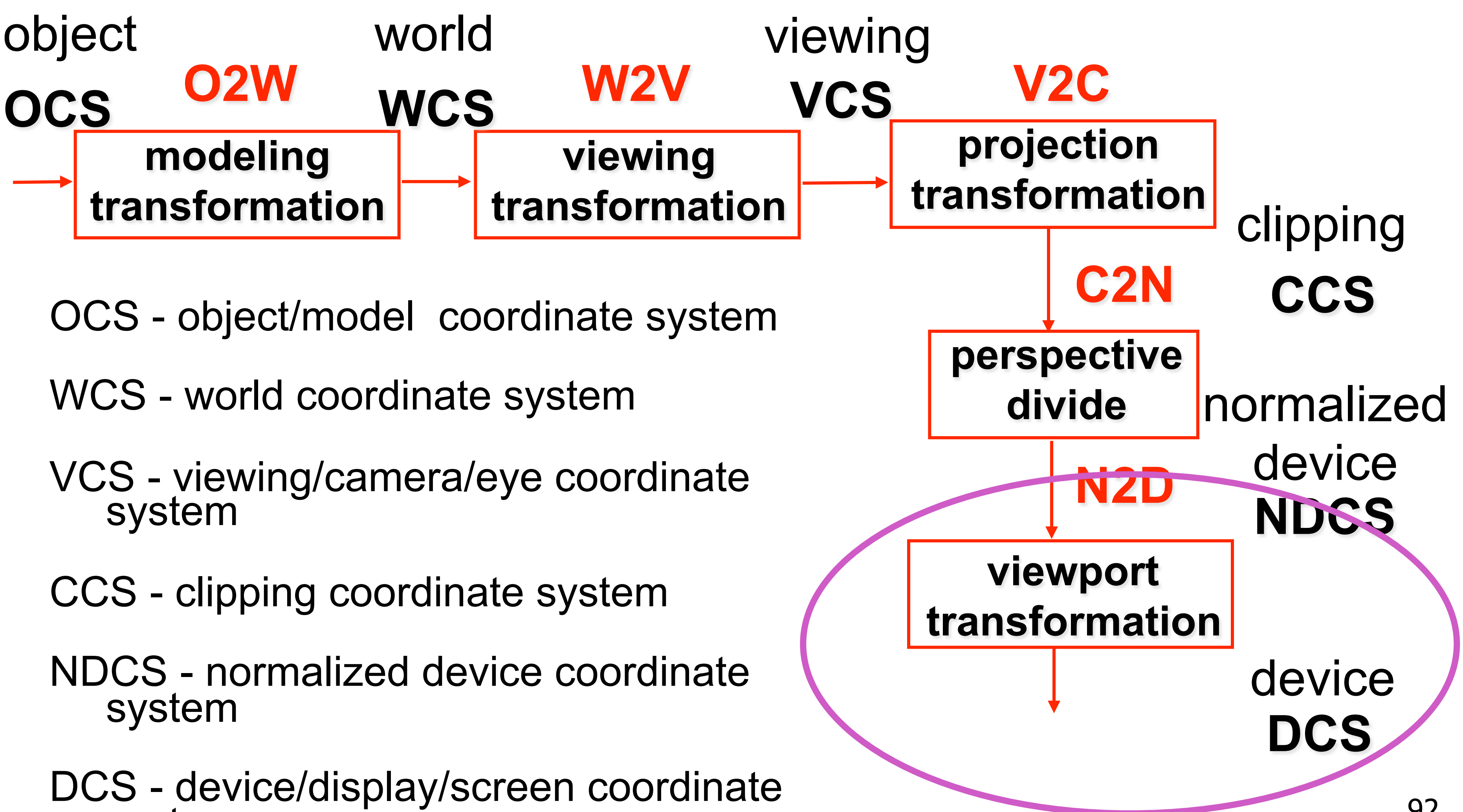
$$1 = F\frac{top}{near} - B$$

Perspective Derivation

- similarly for other 5 planes
- 6 planes, 6 unknowns

$$\begin{bmatrix} \frac{2n}{r-l} & 0 & \frac{r+l}{r-l} & 0 \\ 0 & \frac{2n}{t-b} & \frac{t+b}{t-b} & 0 \\ 0 & 0 & \frac{-(f+n)}{f-n} & \frac{-2fn}{f-n} \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

Projective Rendering Pipeline



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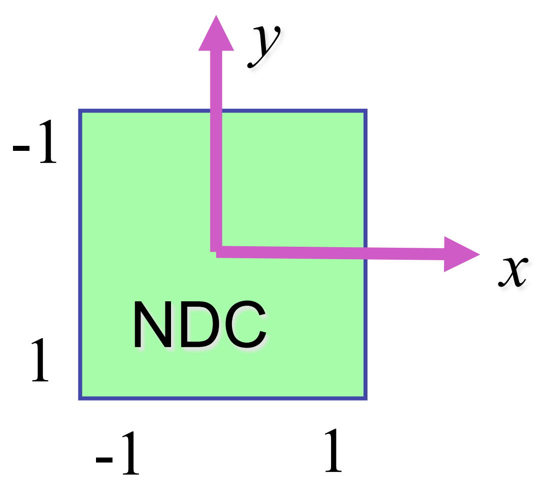
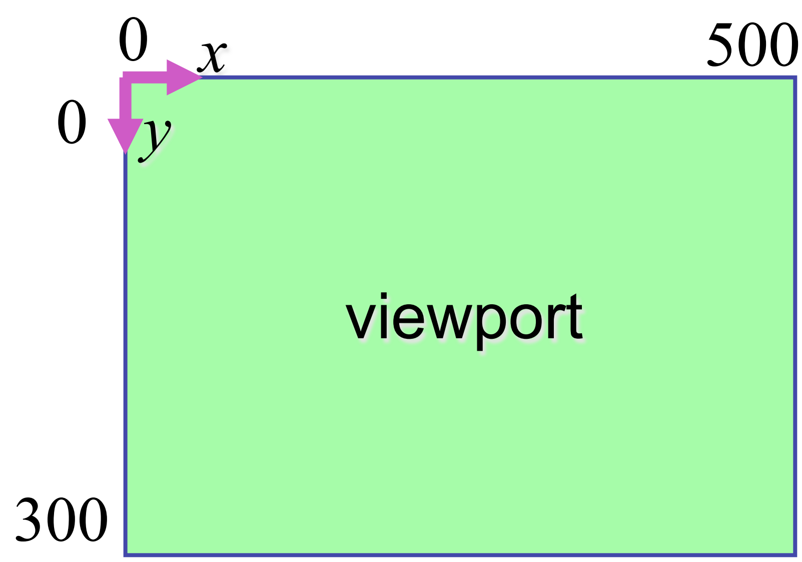
NDC to Device Transformation

- map from NDC to pixel coordinates on display
  - NDC range is x = -1...1, y = -1...1, z = -1...1
  - typical display range: x = 0...500, y = 0...300
    - maximum is size of actual screen
    - z range max and default is (0, 1), use later for visibility

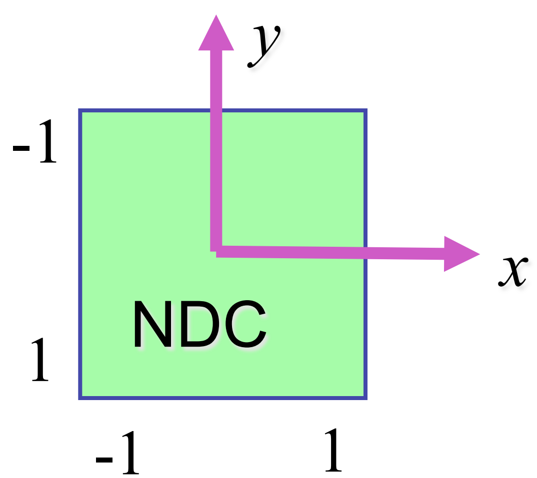
```
glViewport(0,0,w,h);
glDepthRange(0,1); // depth = 1 by default
```

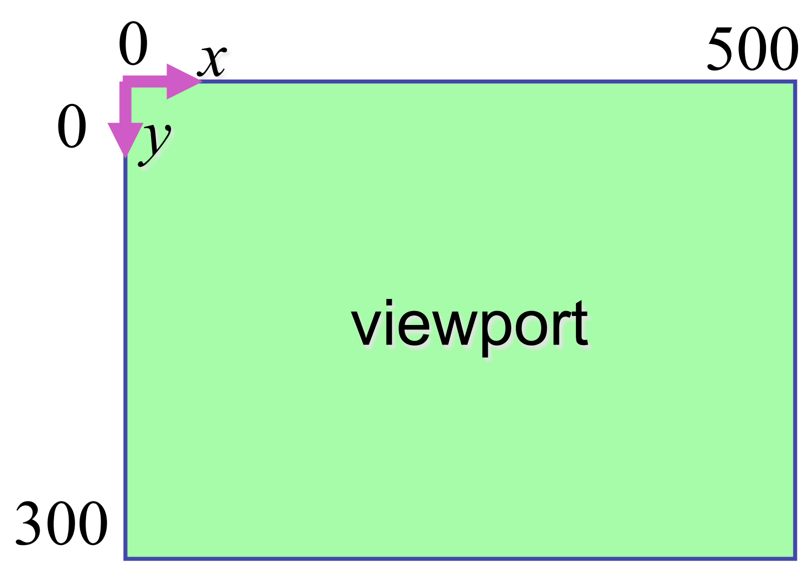




Origin Location

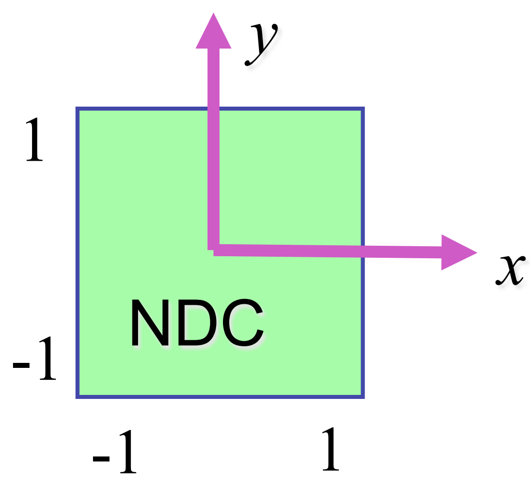
- yet more (possibly confusing) conventions
  - OpenGL origin: lower left
  - most window systems origin: upper left
- then must reflect in y
- when interpreting mouse position, have to flip your y coordinates

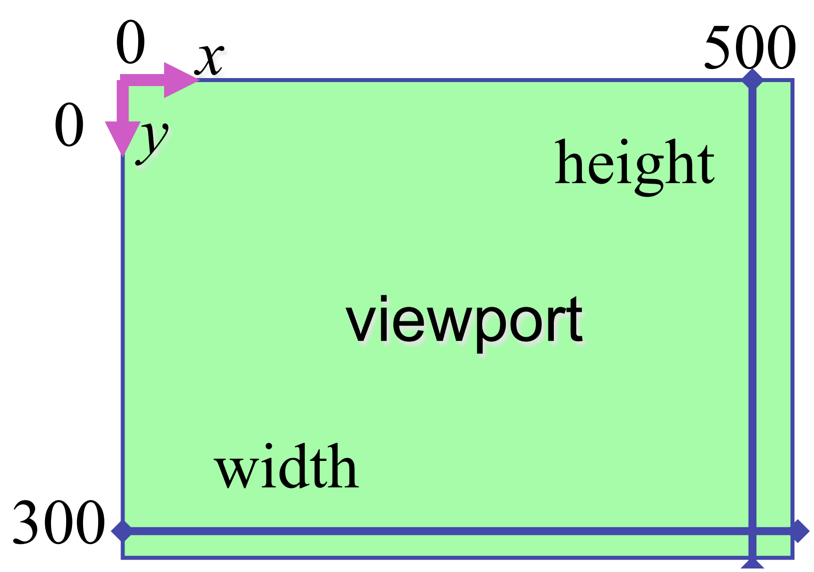




N2D Transformation

- general formulation
  - reflect in y for upper vs. lower left origin
  - scale by width, height, depth
  - translate by width/2, height/2, depth/2
    - FCG includes additional translation for pixel centers at (.5, .5) instead of (0,0)



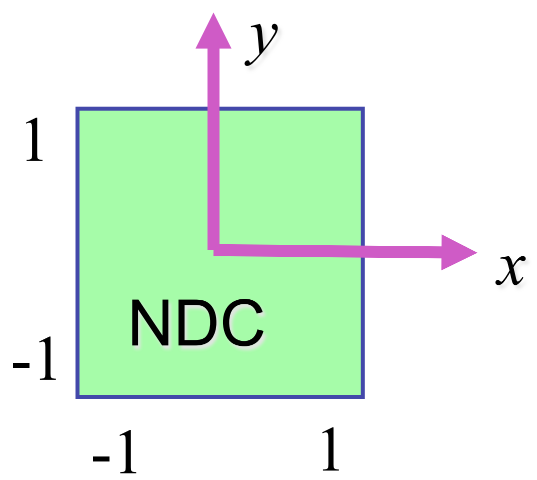


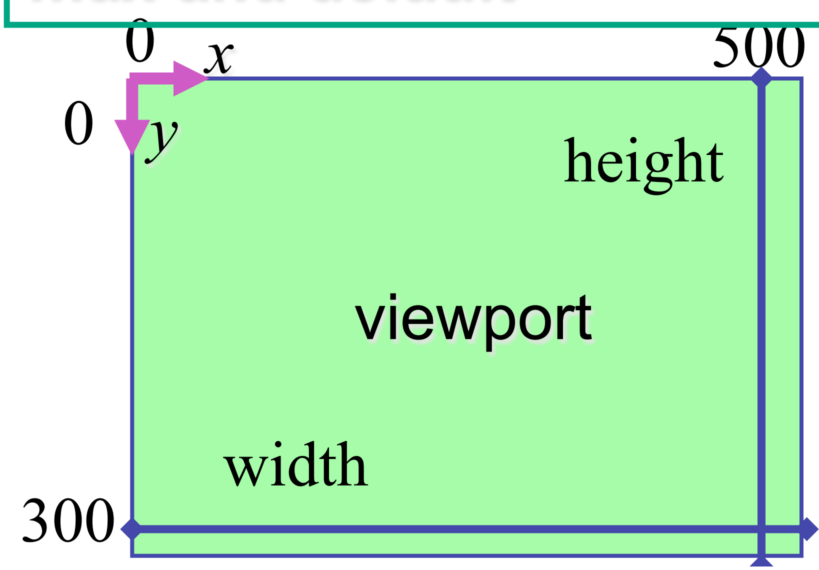
N2D Transformation

$$\begin{bmatrix} x_D \\ y_D \\ z_D \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & \frac{width}{2} - \frac{1}{2} \\ 0 & 1 & 0 & \frac{height}{2} - \frac{1}{2} \\ 0 & 0 & 1 & \frac{depth}{2} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{width}{2} & 0 & 0 & 0 \\ 0 & \frac{height}{2} & 0 & 0 \\ 0 & 0 & \frac{depth}{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_N \\ y_N \\ z_N \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{width(x_N + 1) - 1}{2} \\ \frac{height(-y_N + 1) - 1}{2} \\ \frac{depth(z_N + 1)}{2} \\ 1 \end{bmatrix}$$

Display z range is 0 to 1. glDepthRange(n,f) can constrain further, but *depth* = 1 is both max and default

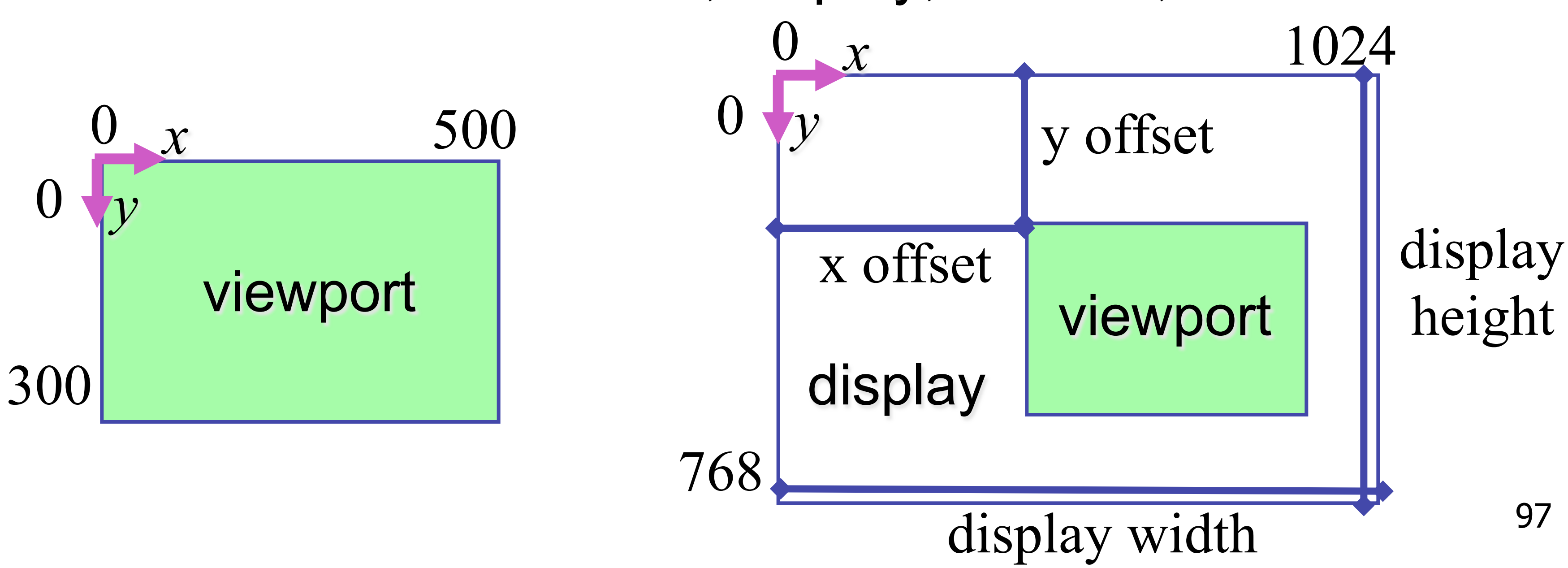
reminder:
NDC z range is -1 to 1



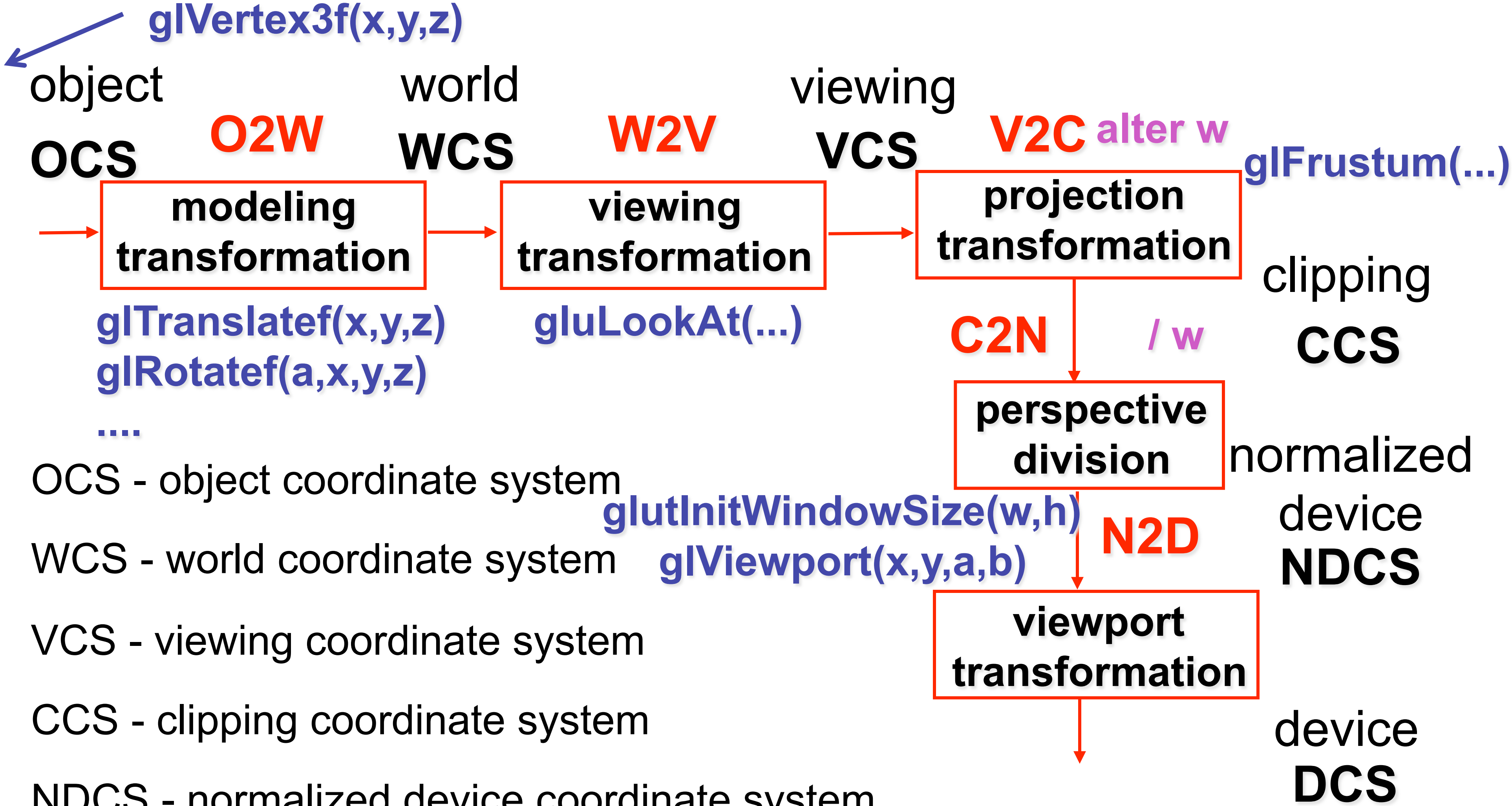


Device vs. Screen Coordinates

- viewport/window location wrt actual display not available within OpenGL
  - usually don't care
    - use relative information when handling mouse events, not absolute coordinates
  - could get actual display height/width, window offsets from OS
- loose use of terms: device, display, window, screen...



Projective Rendering Pipeline



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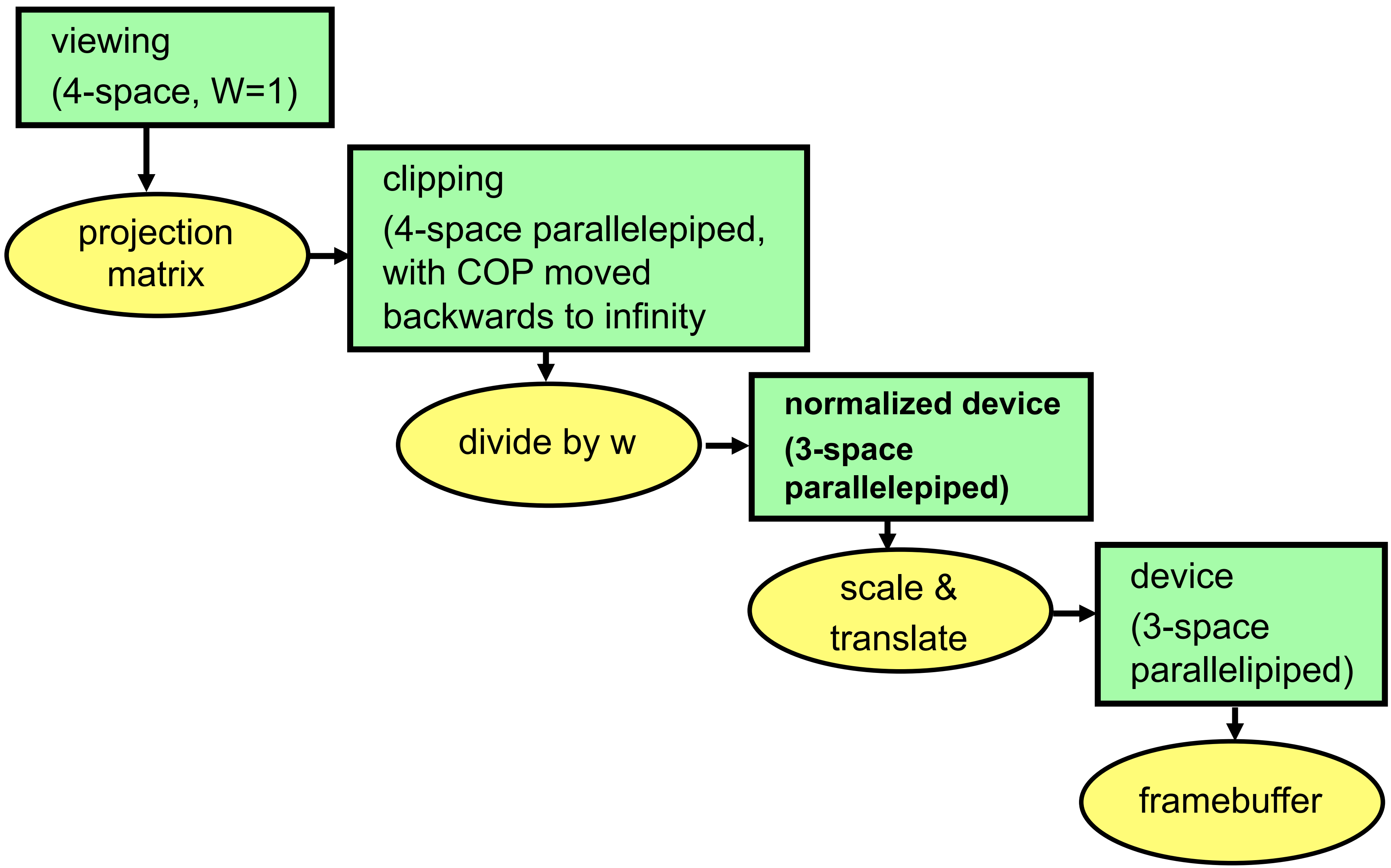
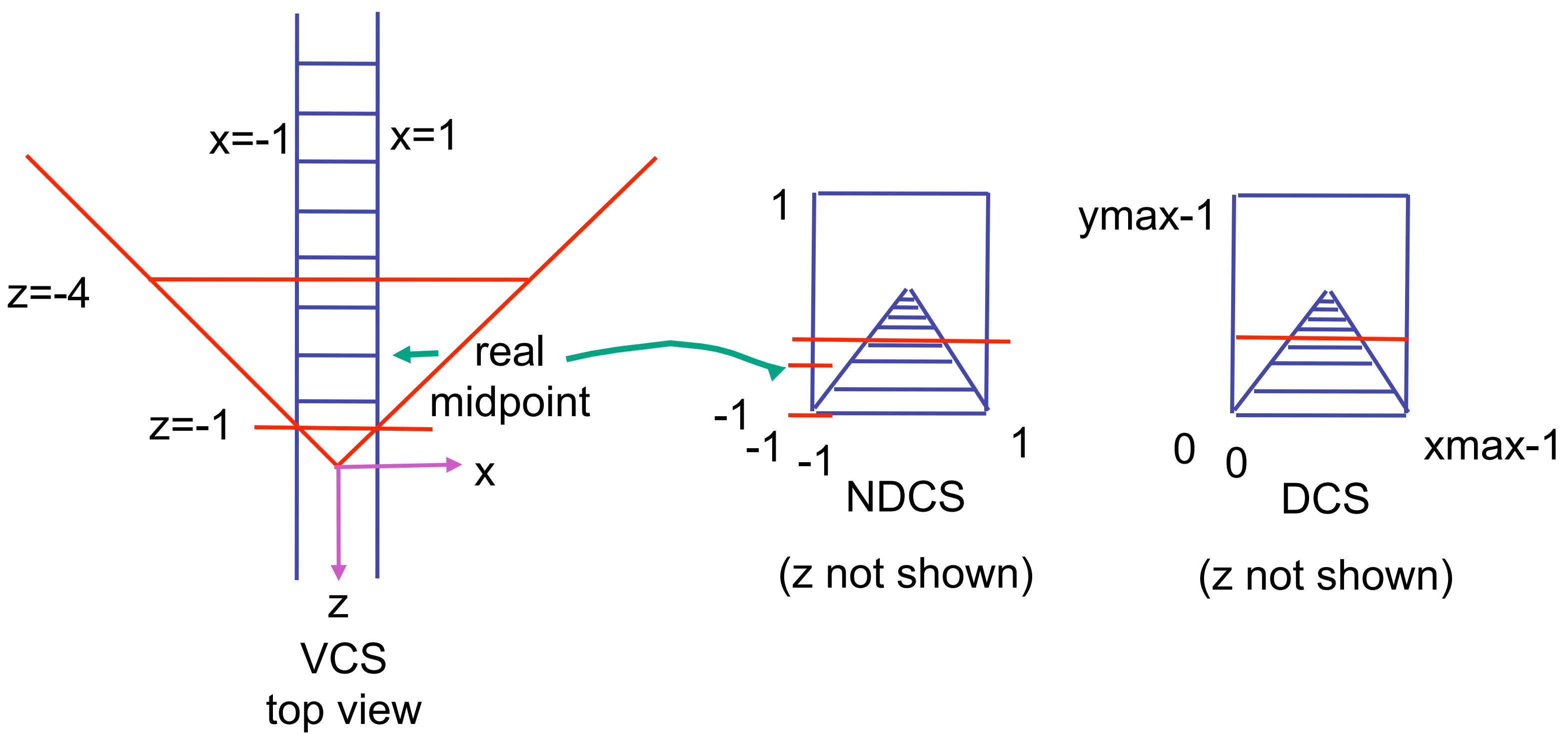
Coordinate Systems



Perspective Example

tracks in VCS:
- left x=-1, y=-1
- right x=1, y=-1

view volume
- left = -1, right = 1
- bot = -1, top = 1
- near = 1, far = 4



Perspective Example

view volume

- left = -1, right = 1
- bot = -1, top = 1
- near = 1, far = 4

$$\begin{bmatrix} \frac{2n}{r-l} & 0 & \frac{r+l}{r-l} & 0 \\ 0 & \frac{2n}{t-b} & \frac{t+b}{t-b} & 0 \\ 0 & 0 & \frac{-(f+n)}{f-n} & \frac{-2fn}{f-n} \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -5/3 & -8/3 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

Perspective Example

$$\begin{bmatrix} 1 \\ -1 \\ -5z_{VCS}/3 - 8/3 \\ -z_{VCS} \end{bmatrix} = \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & -5/3 & -8/3 \\ & & -1 & \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ z_{VCS} \\ 1 \end{bmatrix}$$

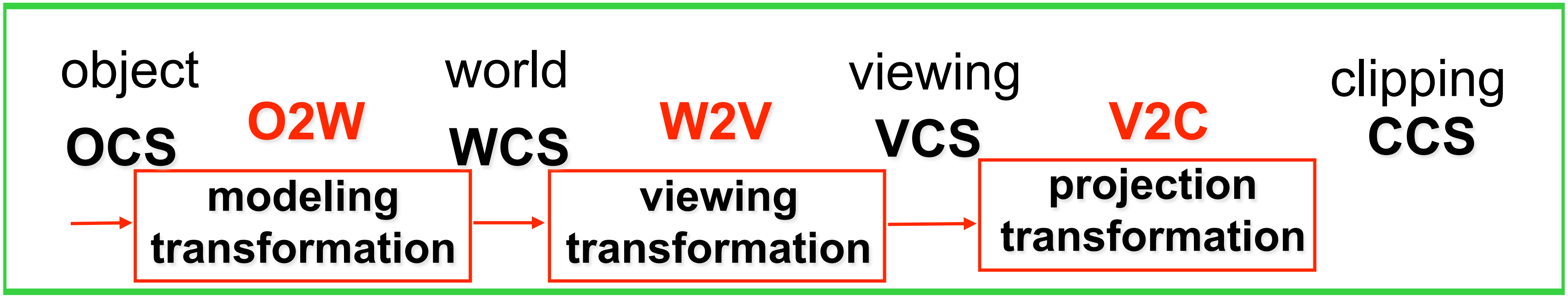
/ w

$$x_{NDCS} = -1/z_{VCS}$$

$$y_{NDCS} = 1/z_{VCS}$$

$$z_{NDCS} = \frac{5}{3} + \frac{8}{3z_{VCS}}$$

OpenGL Example




```
CCS    glMatrixMode( GL_PROJECTION );
       glLoadIdentity();
       gluPerspective( 45, 1.0, 0.1, 200.0 );
VCS    glMatrixMode( GL_MODELVIEW );
       glLoadIdentity();
       glTranslatef( 0.0, 0.0, -5.0 );
WCS    glPushMatrix()
       glTranslate( 4, 4, 0 );   W2O
OCS1   glutSolidTeapot(1);
       glPopMatrix();
       glTranslate( 2, 2, 0);    W2O
OCS2   glutSolidTeapot(1);
```

- transformations that are applied to object first are specified last

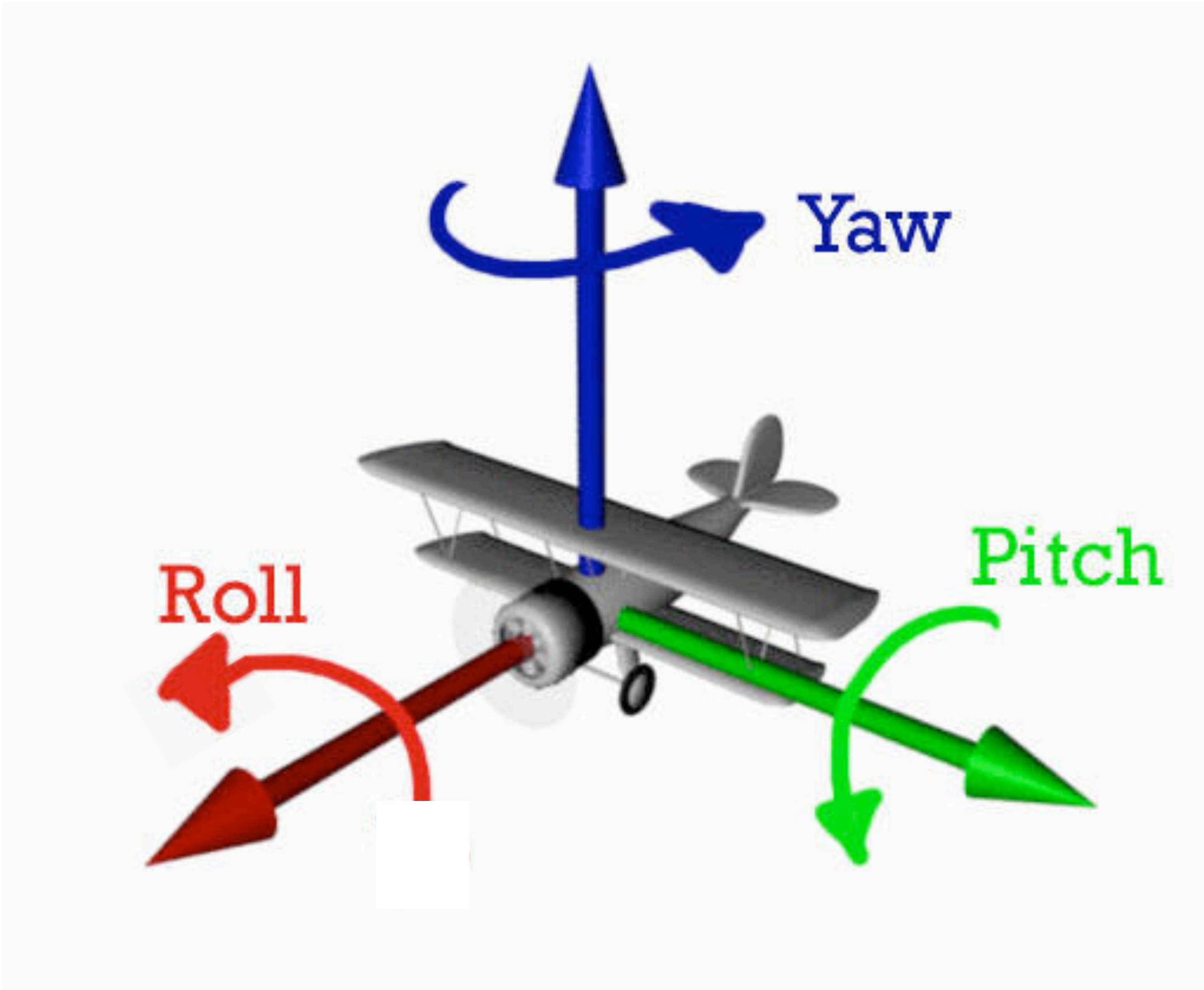
Reading for Next Time

- RB Chap Color

- FCG Sections 3.2-3.3
- FCG Chap 20 Color
- FCG Chap 21.2.2 Visual Perception (Color)

Viewing: More Camera Motion

Fly "Through The Lens": Roll/Pitch/Yaw



Viewing: Incremental Relative Motion

- how to move relative to current camera coordinate system?
  - what you see in the window
- computation in coordinate system used to draw previous frame is simple:
  - incremental change I to current C
  - at time k, want p' = $I_k I_{k-1} I_{k-2} I_{k-3} \dots I_5 I_4 I_3 I_2 I_1 C p$
- each time we just want to premultiply by new matrix
    - p'=ICp
  - but we know that OpenGL only supports postmultiply by new matrix
    - p'=CIp

Viewing: Incremental Relative Motion

- sneaky trick: OpenGL modelview matrix has the info we want!
  - dump out modelview matrix from previous frame with glGetDoublev()
    - C = current camera coordinate matrix
  - wipe the matrix stack with glIdentity()
  - apply incremental update matrix I
  - apply current camera coord matrix C
- must leave the modelview matrix unchanged by object transformations after your display call
  - use push/pop
- using OpenGL for storage and calculation
  - querying pipeline is expensive
    - but safe to do just once per frame

Caution: OpenGL Matrix Storage

- OpenGL internal matrix storage is columnwise, not rowwise

```
a   e   i   m
b   f   j   n
c   g   k   o
d   h   l   p
```

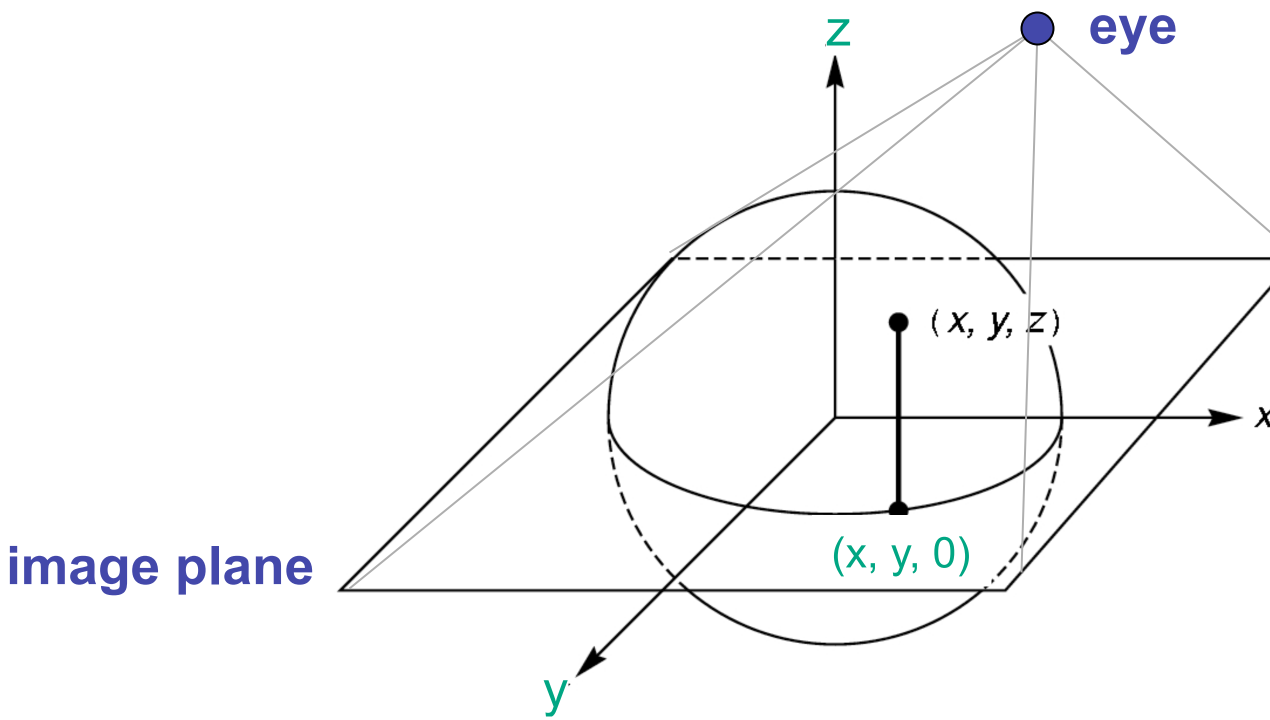
  - opposite of standard C/C++/Java convention
  - possibly confusing if you look at the matrix from glGetDoublev()!

Viewing: Virtual Trackball

- interface for spinning objects around
  - drag mouse to control rotation of view volume
    - orbit/spin metaphor
    - vs. flying/driving
- rolling glass trackball
  - center at screen origin, surrounds world
  - hemisphere “sticks up” in z, out of screen
  - rotate ball = spin world

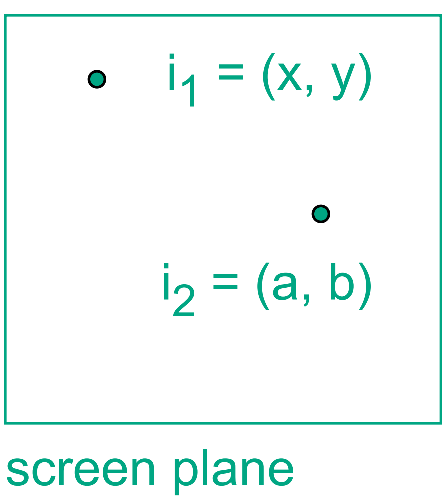
Clarify: Virtual Trackball

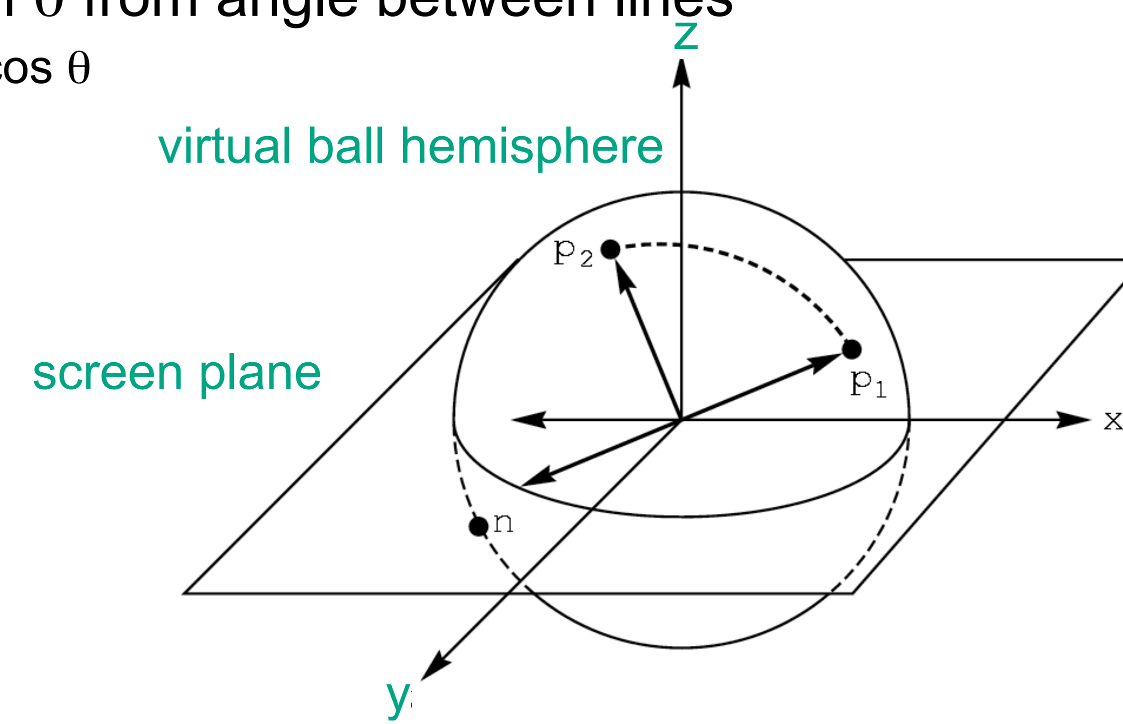
- know screen click: (x, y, 0)
- want to infer point on trackball: (x,y,z)
  - ball is unit sphere, so ||x, y, z|| = 1.0
  - solve for z



Clarify: Trackball Rotation

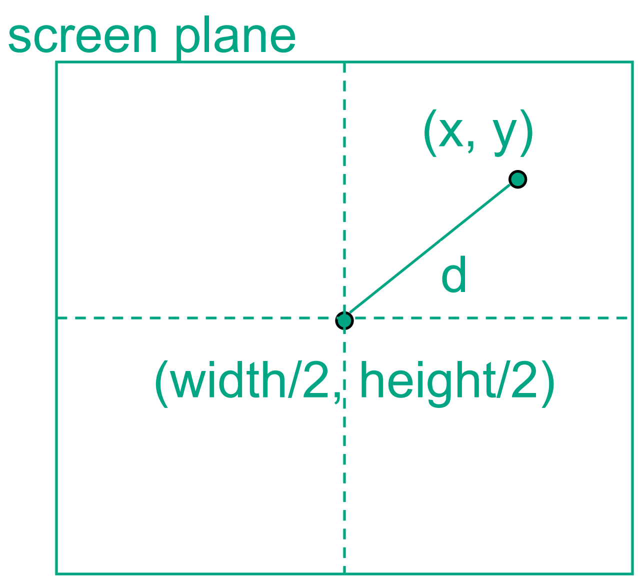
- user drags between two points on image plane
  - mouse down at i_1 = (x, y), mouse up at i_2 = (a, b)
- find corresponding points on virtual ball
  - $\mathbf{p_1}$ = (x, y, z), $\mathbf{p_2}$ = (a, b, c)
- compute rotation angle and axis for ball
  - axis of rotation is plane normal: cross product $\mathbf{p_1} \times \mathbf{p_2}$
  - amount of rotation θ from angle between lines
    - $\mathbf{p_1} \bullet \mathbf{p_2} = |\mathbf{p_1}|\,|\mathbf{p_2}| \cos\theta$

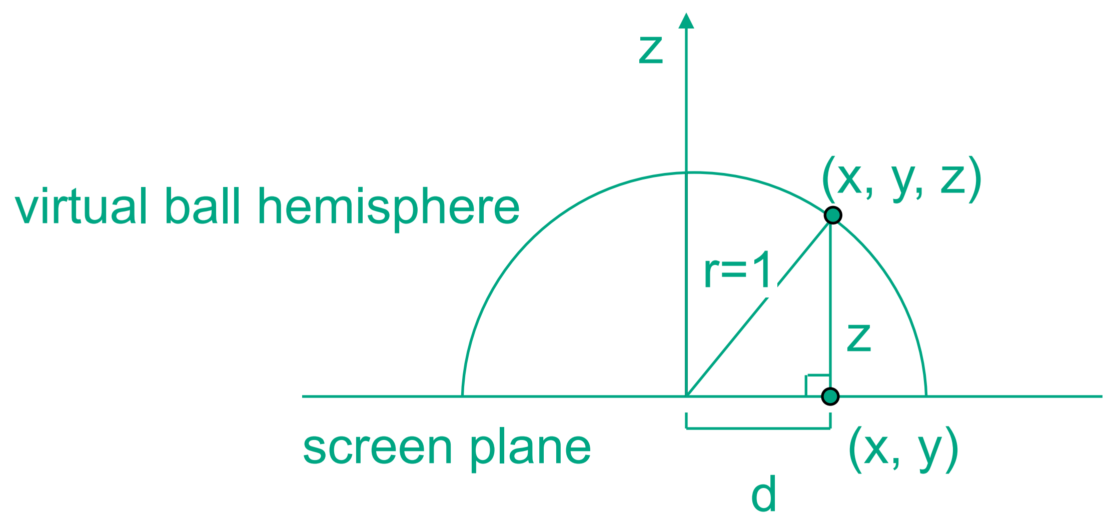




Clarify: Trackball Rotation

- finding location on ball corresponding to click on image plane
  - ball radius r is 1





Trackball Computation

- user defines two points
  - place where first clicked $\mathbf{p_1}$ = (x, y, z)
  - place where released $\mathbf{p_2}$ = (a, b, c)
- create plane from vectors between points, origin
  - axis of rotation is plane normal: cross product
    - $(\mathbf{p_1} - \mathbf{o}) \times (\mathbf{p_2} - \mathbf{o})$: $\mathbf{p_1} \times \mathbf{p_2}$ if origin = (0,0,0)
  - amount of rotation depends on angle between lines
    - $\mathbf{p_1} \bullet \mathbf{p_2} = |\mathbf{p_1}|\,|\mathbf{p_2}| \cos \theta$
    - $|\mathbf{p_1} \times \mathbf{p_2}| = |\mathbf{p_1}|\,|\mathbf{p_2}| \sin \theta$
- compute rotation matrix, use to rotate world

Picking

Reading

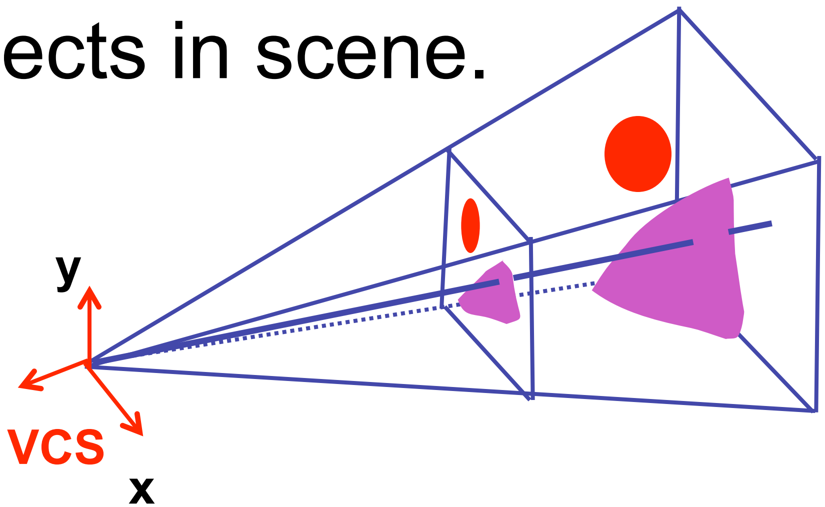
- Red Book
  - Selection and Feedback Chapter
    - all
  - Now That You Know Chapter
    - only Object Selection Using the Back Buffer

Interactive Object Selection

- move cursor over object, click
  - how to decide what is below?
  - inverse of rendering pipeline flow
    - from pixel back up to object
- ambiguity
  - many 3D world objects map to same 2D point
- four common approaches
  - manual ray intersection
  - bounding extents
  - backbuffer color coding
  - selection region with hit list

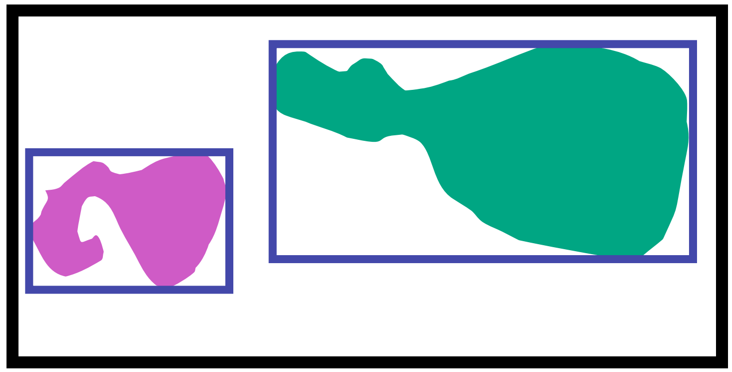
Manual Ray Intersection

- do all computation at application level
  - map selection point to a ray
  - intersect ray with all objects in scene.
- advantages
  - no library dependence
- disadvantages
  - difficult to program
  - slow: work to do depends on total number and complexity of objects in scene



Bounding Extents

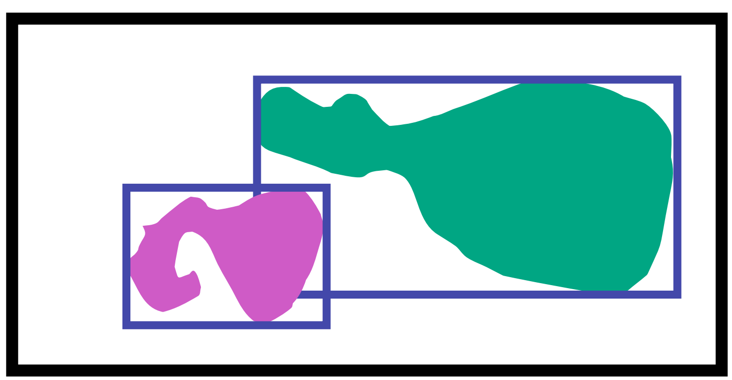
- keep track of axis-aligned bounding rectangles



- advantages
  - conceptually simple
  - easy to keep track of boxes in world space

Bounding Extents

- disadvantages
  - low precision
  - must keep track of object-rectangle relationship
- extensions
  - do more sophisticated bound bookkeeping
    - first level: box check.
    - second level: object check

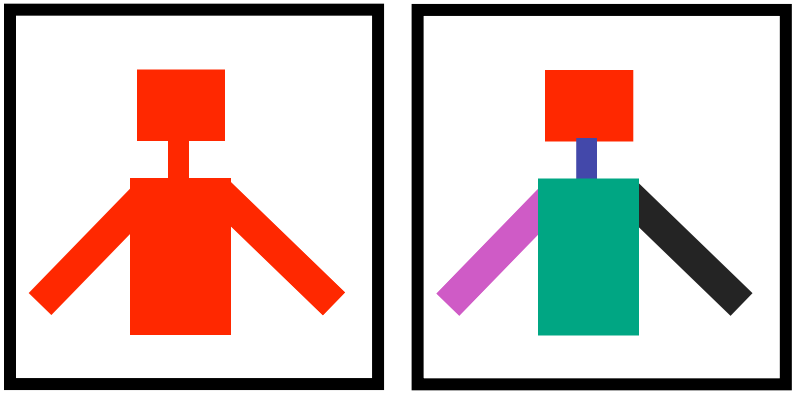


Backbuffer Color Coding

- use backbuffer for picking
  - create image as computational entity
  - never displayed to user
- redraw all objects in backbuffer
  - turn off shading calculations
  - set unique color for each pickable object
    - store in table
  - read back pixel at cursor location
    - check against table

Backbuffer Color Coding

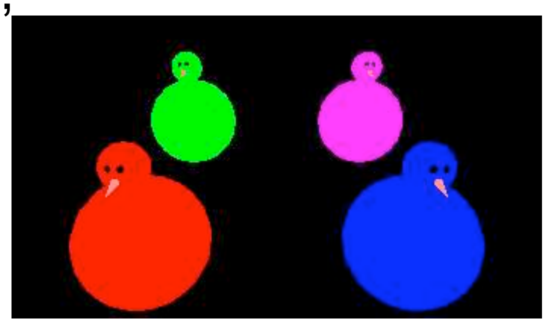
- advantages
  - conceptually simple
  - variable precision
- disadvantages
  - introduce 2x redraw delay
  - backbuffer readback very slow



Backbuffer Example

```
glColor3f(1.0, 1.0, 1.0);
for(int i = 0; i < 2; i++)
    for(int j = 0; j < 2; j++) {
      glPushMatrix();
      glTranslatef(i*3.0,0,-j * 3.0);
      glColor3f(1.0, 1.0, 1.0);
      glCallList(snowman_display_list);
      glPopMatrix();
  }
```

```
for(int i = 0; i < 2; i++)
  for(int j = 0; j < 2; j++) {
    glPushMatrix();
    switch (i*2+j) {
      case 0: glColor3ub(255,0,0);break;
      case 1: glColor3ub(0,255,0);break;
      case 2: glColor3ub(0,0,255);break;
      case 3: glColor3ub(250,0,250);break;
    }
    glTranslatef(i*3.0,0,-j * 3.0)
    glCallList(snowman_display_list);
    glPopMatrix();
}
```



http://www.lighthouse3d.com/opengl/picking/

Select/Hit

- use small region around cursor for viewport
- assign per-object integer keys (names)
- redraw in special mode
- store hit list of objects in region
- examine hit list

- OpenGL support

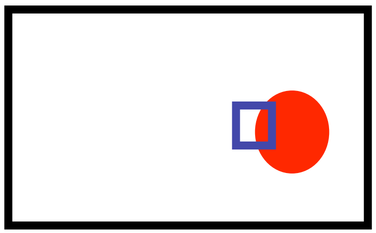
Viewport

- small rectangle around cursor
  - change coord sys so fills viewport

- why rectangle instead of point?
  - people aren't great at positioning mouse
    - Fitts' Law: time to acquire a target is function of the distance to and size of the target
  - allow several pixels of slop

Viewport

- nontrivial to compute
  - invert viewport matrix, set up new orthogonal projection
- simple utility command
  - gluPickMatrix(x,y,w,h,viewport)
    - x,y: cursor point
    - w,h: sensitivity/slop (in pixels)
  - push old setup first, so can pop it later



Render Modes

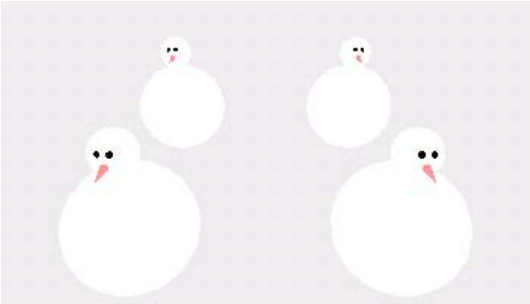
- glRenderMode(mode)
  - GL_RENDER: normal color buffer
    - default
  - GL_SELECT: selection mode for picking
  - (GL_FEEDBACK: report objects drawn)

Name Stack

- again, "names" are just integers
  glInitNames()
- flat list
  glLoadName(name)
- or hierarchy supported by stack
  glPushName(name), glPopName
  - can have multiple names per object

Hierarchical Names Example

```
for(int i = 0; i < 2; i++) {
  glPushName(i);
  for(int j = 0; j < 2; j++) {
    glPushMatrix();
    glPushName(j);
    glTranslatef(i*10.0,0,j * 10.0);
      glPushName(HEAD);
      glCallList(snowManHeadDL);
      glLoadName(BODY);
      glCallList(snowManBodyDL);
      glPopName();
    glPopName();
    glPopMatrix();
  }
  glPopName();
}
```



http://www.lighthouse3d.com/opengl/picking/

Hit List

- glSelectBuffer(buffersize, *buffer)
  - where to store hit list data
- on hit, copy entire contents of name stack to output buffer.
- hit record
  - number of names on stack
  - minimum and maximum depth of object vertices
    - depth lies in the NDC z range [0,1]
    - format: multiplied by 2^32 -1 then rounded to nearest int

Integrated vs. Separate Pick Function

- integrate: use same function to draw and pick
  - simpler to code
  - name stack commands ignored in render mode
- separate: customize functions for each
  - potentially more efficient
  - can avoid drawing unpickable objects

Select/Hit

- advantages
  - faster
    - OpenGL support means hardware acceleration
    - avoid shading overhead
  - flexible precision
    - size of region controllable
  - flexible architecture
    - custom code possible, e.g. guaranteed frame rate
- disadvantages
  - more complex

Hybrid Picking

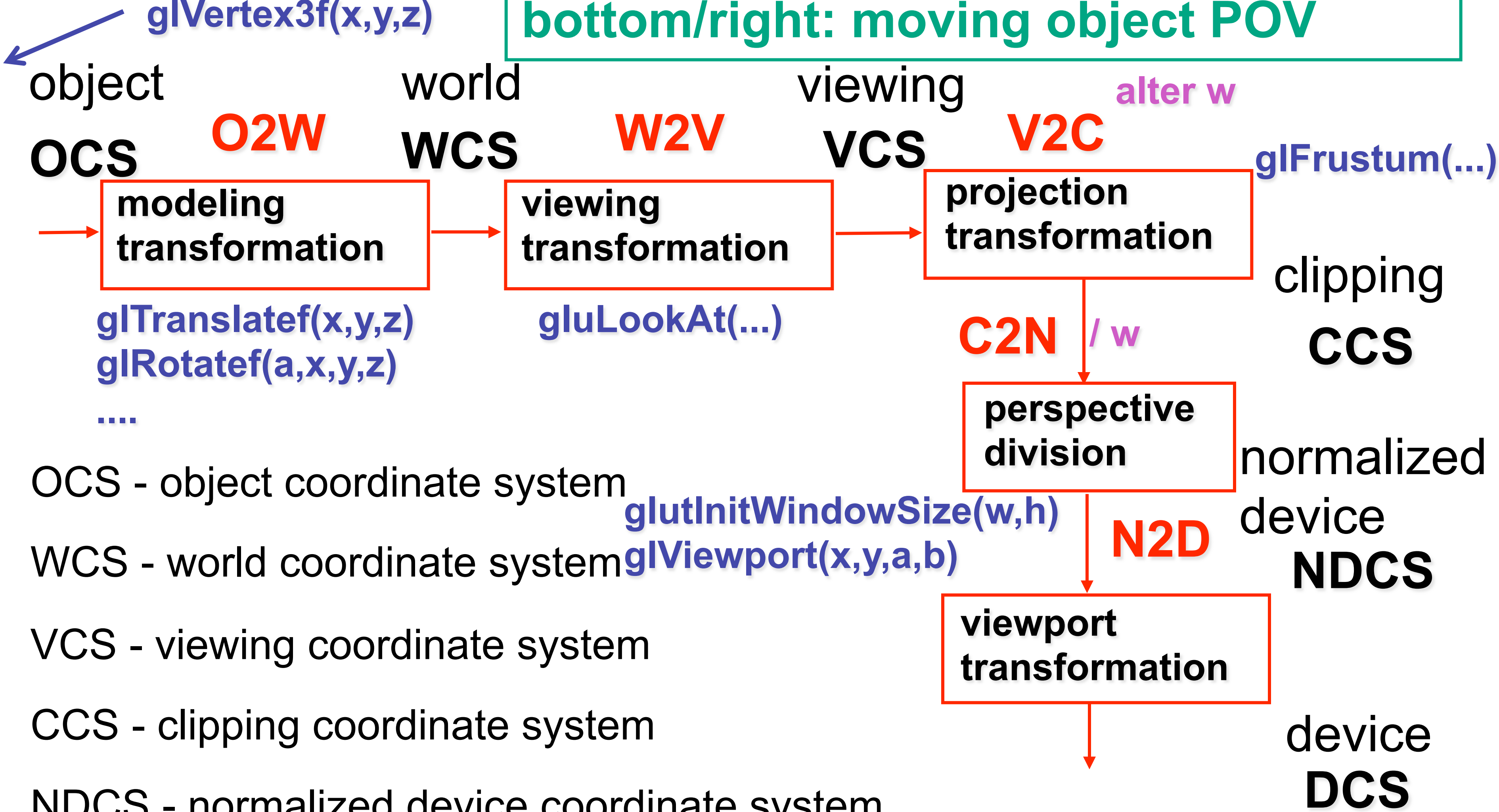
- select/hit approach: fast, coarse
  - object-level granularity
- manual ray intersection: slow, precise
  - exact intersection point
- hybrid: both speed and precision
  - use select/hit to find object
  - then intersect ray with that object

OpenGL Precision Picking Hints

- gluUnproject
  - transform window coordinates to object coordinates given current projection and modelview matrices
  - use to create ray into scene from cursor location
  - call gluUnProject twice with same (x,y) mouse location
    - z = near: (x,y,0)
    - z = far: (x,y,1)
    - subtract near result from far result to get direction vector for ray
- use this ray for line/polygon intersection

Projective Rendering Pipeline

following pipeline from top/left to bottom/right: moving object POV



OCS - object coordinate system

WCS - world coordinate system

VCS - viewing coordinate system

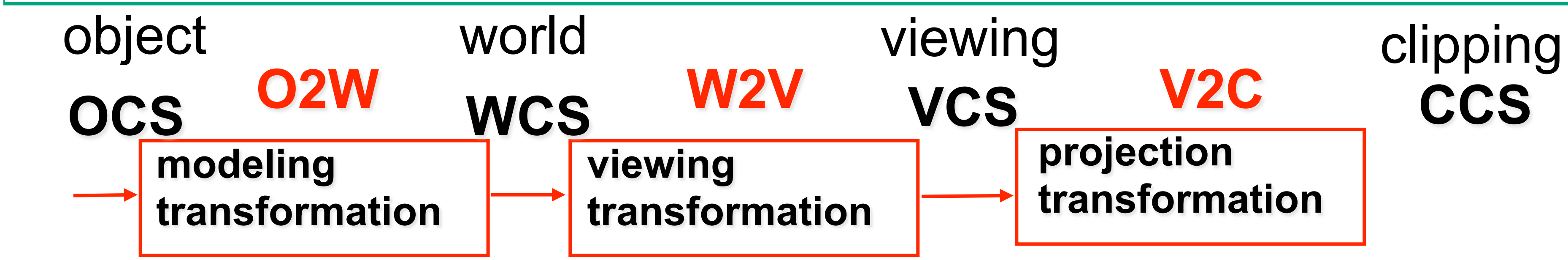
CCS - clipping coordinate system

NDCS - normalized device coordinate system

DCS - device coordinate system

OpenGL Example

go back from end of pipeline to beginning: coord frame POV!




```
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      glLoadIdentity();
      gluPerspective( 45, 1.0, 0.1, 200.0 );
VCS   glMatrixMode( GL_MODELVIEW );
      glLoadIdentity();
      glTranslatef( 0.0, 0.0, -5.0 );   V2W
WCS   glPushMatrix()
      glTranslate( 4, 4, 0 );  W2O
OCS1  glutSolidTeapot(1);
      glPopMatrix();
      glTranslate( 2, 2, 0);   W2O
OCS2  glutSolidTeapot(1);
```

- transformations that are applied to object first are specified last

Coord Sys: Frame vs Point

read down: transforming between coordinate frames, from frame A to frame B

read up: transforming points, up from frame B coords to frame A coords

OpenGL command order

	DCS display		
D2N	glViewport(x,y,a,b)	N2D	
	NDCS normalized device		
N2V	glFrustum(...)	V2N	
	VCS viewing		
V2W	gluLookAt(...)	W2V	
	WCS world		
W2O	glRotatef(a,x,y,z)	O2W	
	OCS object		
	glVertex3f(x,y,z)		

pipeline interpretation

Coord Sys: Frame vs Point

- is gluLookat viewing transformation V2W or W2V? depends on which way you read!
  - coordinate frames: V2W
    - takes you from view to world coordinate frame
  - points/objects: W2V
    - point is transformed from world to view coords when multiply by gluLookAt matrix
- H2 uses the object/pipeline POV
  - Q1/4 is W2V (gluLookAt)
  - Q2/5-6 is V2N (glFrustum)
  - Q3/7 is N2D (glViewport)