



University of British Columbia
CPSC 314 Computer Graphics
Jan-Apr 2013

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Final Review

<http://www.ugrad.cs.ubc.ca/~cs314/Vjan2013>

Final

- exam notes
 - exam will be timed for 2.5 hours, but reserve entire 3-hour block of time just in case
 - closed book, closed notes
 - except for 2-sided 8.5"x11" sheet of handwritten notes
 - ok to staple midterm sheet + new one back to back
 - calculator: a good idea, but not required
 - graphical OK, smartphones etc not ok
 - IDs out and face up

Final Emphasis

- covers entire course
- includes material from before midterm
 - transformations, viewing/picking
- but heavier weighting for material after last midterm
- post-midterm topics:
 - lighting/shading
 - advanced rendering
 - collision
 - rasterization
 - hidden surfaces / blending
 - textures/procedural
 - clipping
 - color
 - curves
 - visualization

Sample Final

- solutions now posted
 - Spring 06-07 (label was off by one)
- note some material not covered this time
 - projection types like cavalier/cabinet
 - Q1b, Q1c,
 - antialiasing
 - Q1d, Q1l, Q12
 - animation
 - image-based rendering
 - Q1g
 - scientific visualization
 - Q14

Studying Advice

- do problems!
 - work through old homeworks, exams

Reading from OpenGL Red Book

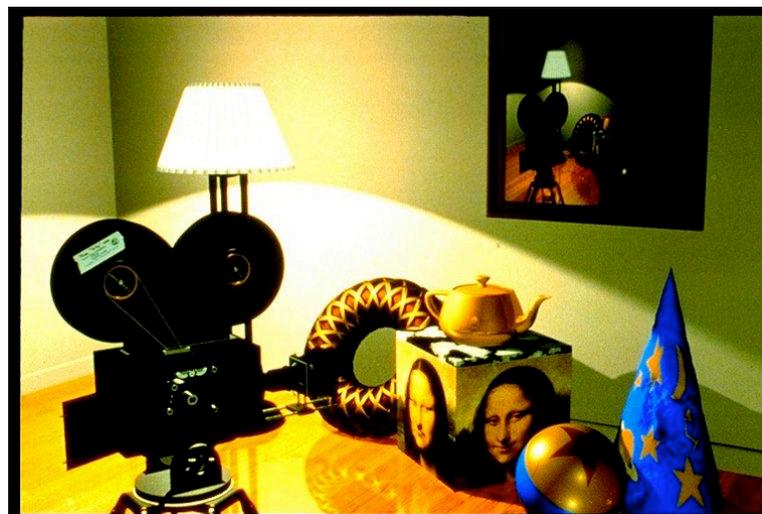
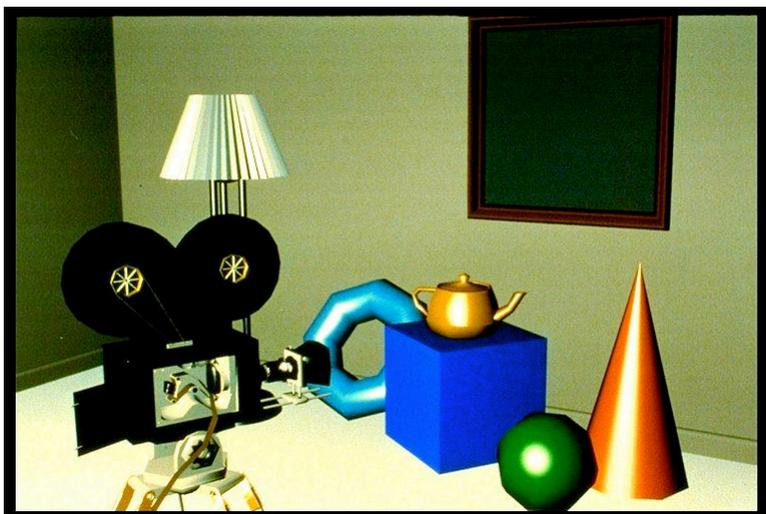
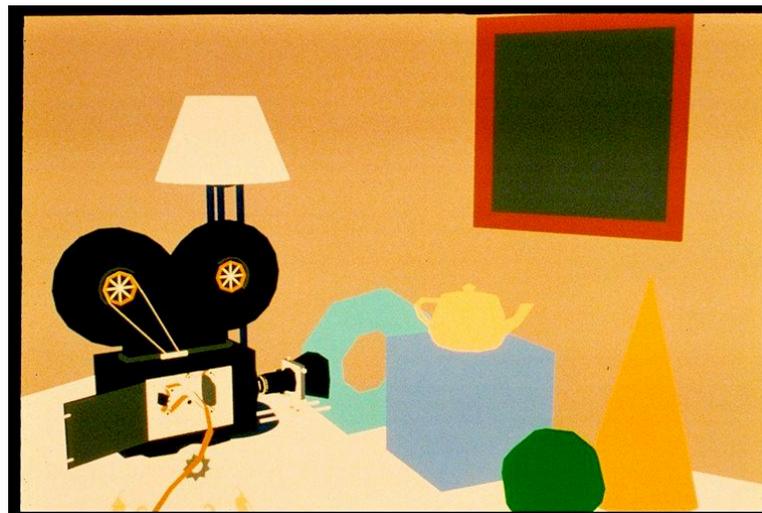
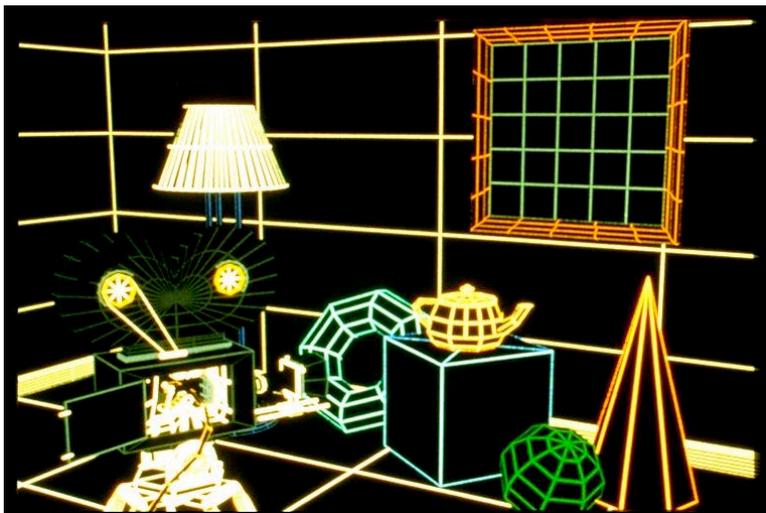
- 1: Introduction to OpenGL
- 2: State Management and Drawing Geometric Objects
- 3: Viewing
- 4: Display Lists
- 5: Color
- 6: Lighting
- 9: Texture Mapping
- 12: Selection and Feedback
- 13: Now That You Know
 - only section Object Selection Using the Back Buffer
- Appendix: Basics of GLUT (Aux in v 1.1)
- Appendix: Homogeneous Coordinates and Transformation Matrices

Reading from Shirley: Foundations of CG

- 1: Intro *
- 2: Misc Math *
- 3: Raster Algs *
 - through 3.3
- 4: Ray Tracing *
- 5: Linear Algebra *
 - except for 5.4
- 6: Transforms *
 - except 6.1.6
- 7: Viewing *
- 8: Graphics Pipeline *
 - 8.1 through 8.1.6, 8.2.3-8.2.5, 8.2.7, 8.4
- 10: Surface Shading *
- 11: Texture Mapping *
- 13: More Ray Tracing *
 - only 13.1
- 12: Data Structures *
 - only 12.2-12.4
- 15: Curves and Surfaces *
- 17: Computer Animation *
 - only 17.6-17.7
- 21: Color *
- 22: Visual Perception *
 - only 22.2.2 and 22.2.4
- 27: Visualization *

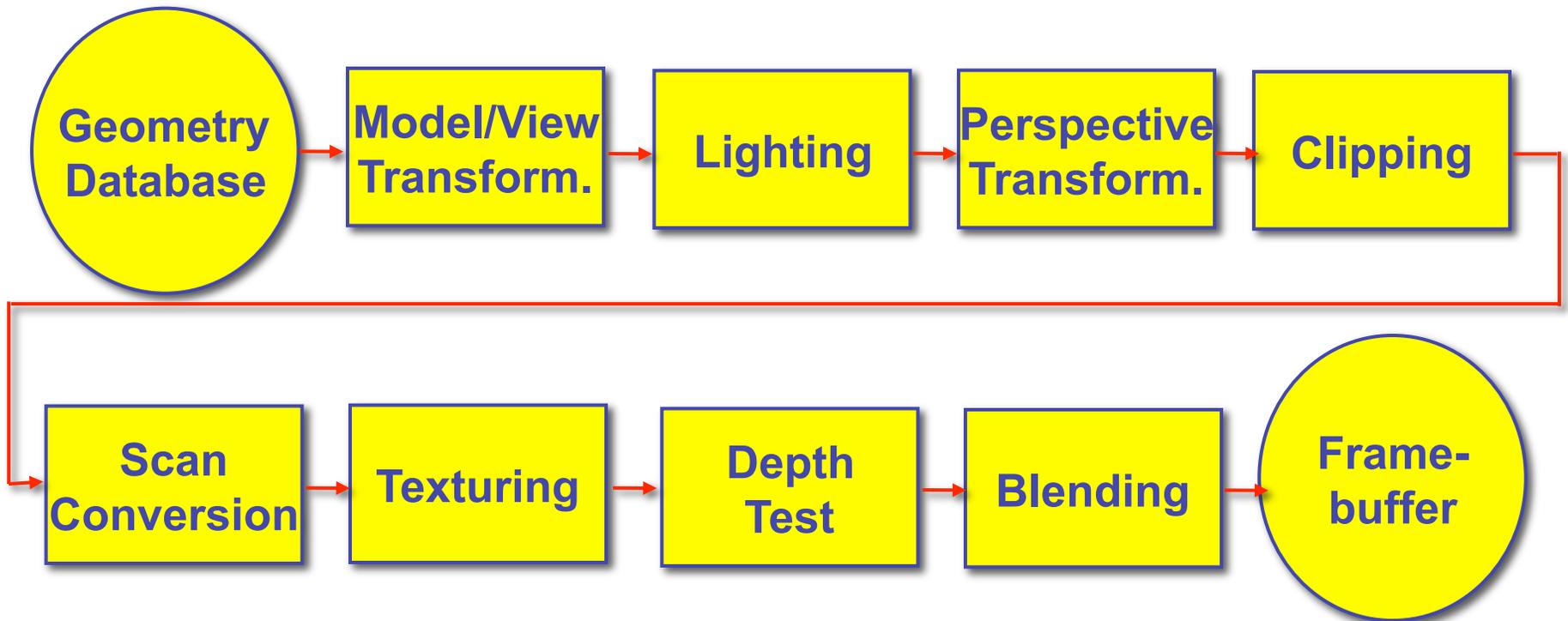
Review – Fast!!

Review: Rendering Capabilities



www.siggraph.org/education/materials/HyperGraph/shutbug.htm

Review: Rendering Pipeline



Review: OpenGL

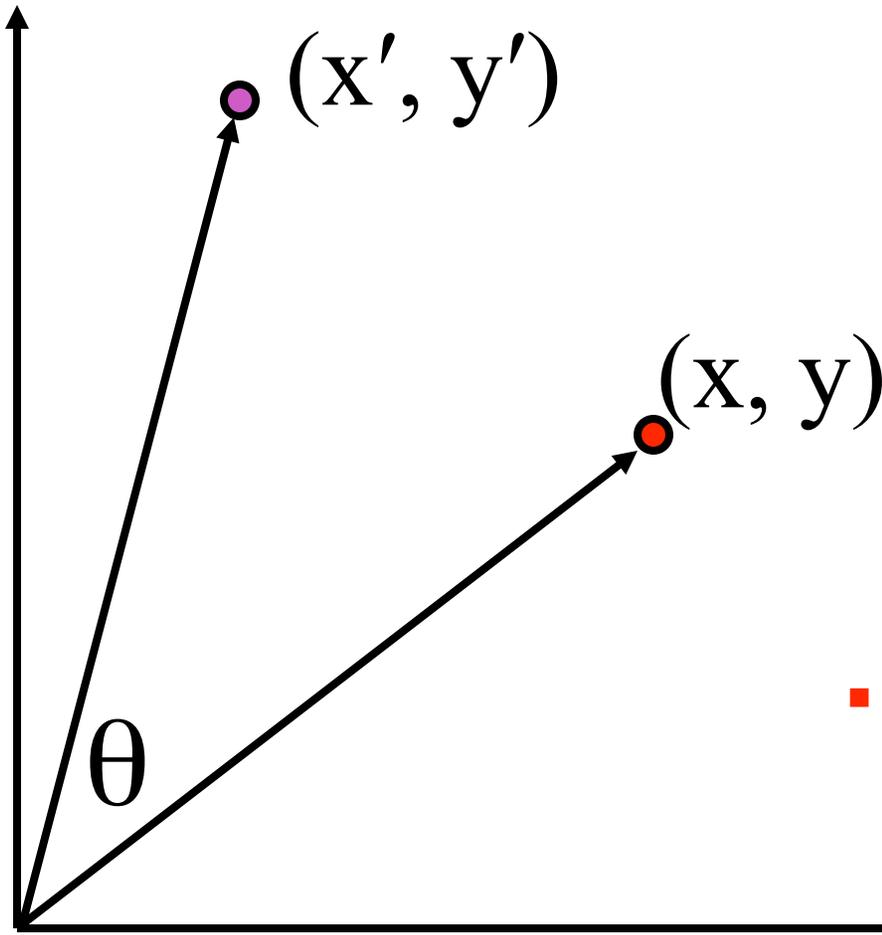
- pipeline processing, set state as needed

```
void display()
{
    glClearColor(0.0, 0.0, 0.0, 0.0);
    glClear(GL_COLOR_BUFFER_BIT);
    glColor3f(0.0, 1.0, 0.0);
    glBegin(GL_POLYGON);
        glVertex3f(0.25, 0.25, -0.5);
        glVertex3f(0.75, 0.25, -0.5);
        glVertex3f(0.75, 0.75, -0.5);
        glVertex3f(0.25, 0.75, -0.5);
    glEnd();
    glFlush();
}
```

Review: Event-Driven Programming

- main loop not under your control
 - vs. procedural
- control flow through event **callbacks**
 - redraw the window now
 - key was pressed
 - mouse moved
- callback functions called from main loop when events occur
 - mouse/keyboard state setting vs. redrawing

Review: 2D Rotation



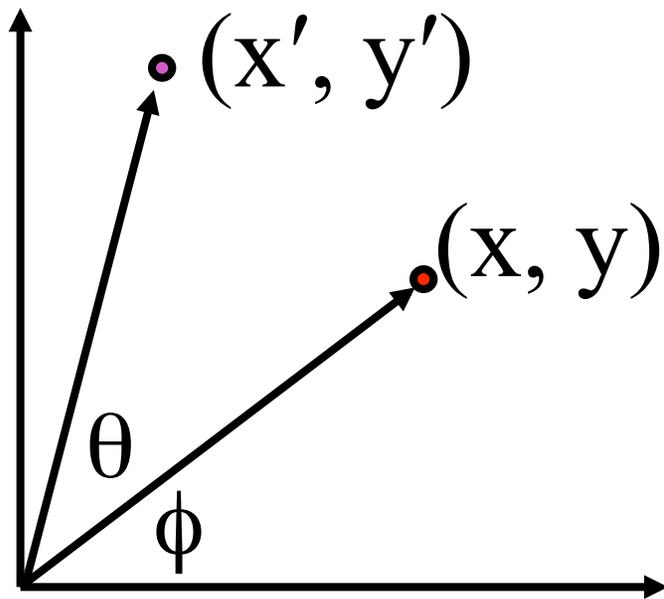
$$x' = x \cos(\theta) - y \sin(\theta)$$

$$y' = x \sin(\theta) + y \cos(\theta)$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

■ counterclockwise, RHS

Review: 2D Rotation From Trig Identities



$$x = r \cos(\phi)$$

$$y = r \sin(\phi)$$

$$x' = r \cos(\phi + \theta)$$

$$y' = r \sin(\phi + \theta)$$

Trig Identity...

$$x' = r \cos(\phi) \cos(\theta) - r \sin(\phi) \sin(\theta)$$

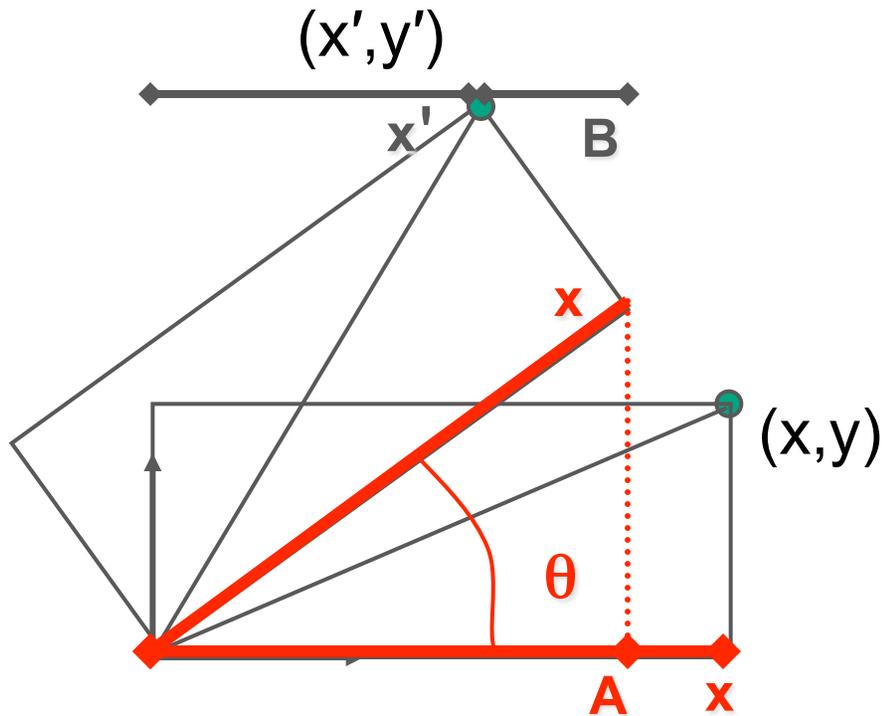
$$y' = r \sin(\phi) \cos(\theta) + r \cos(\phi) \sin(\theta)$$

Substitute...

$$x' = x \cos(\theta) - y \sin(\theta)$$

$$y' = x \sin(\theta) + y \cos(\theta)$$

Review: 2D Rotation: Another Derivation



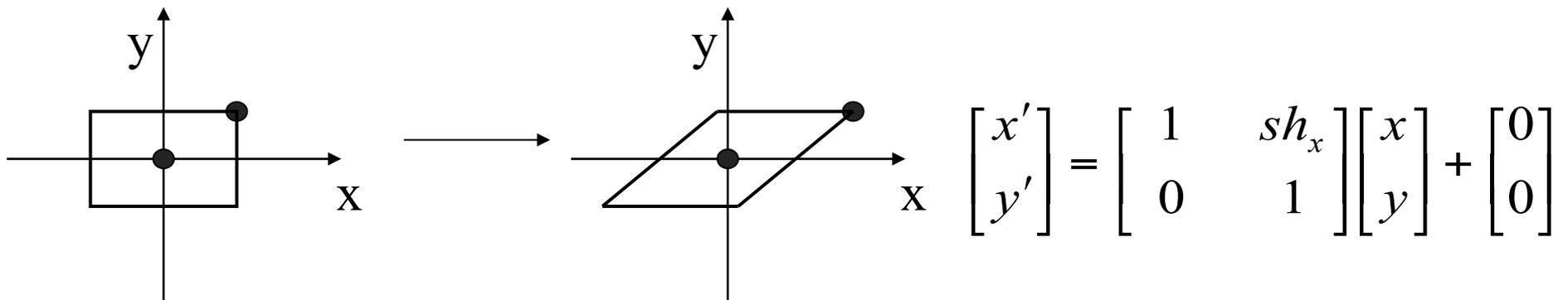
$$x' = x \cos \theta - y \sin \theta$$

$$y' = x \sin \theta + y \cos \theta$$

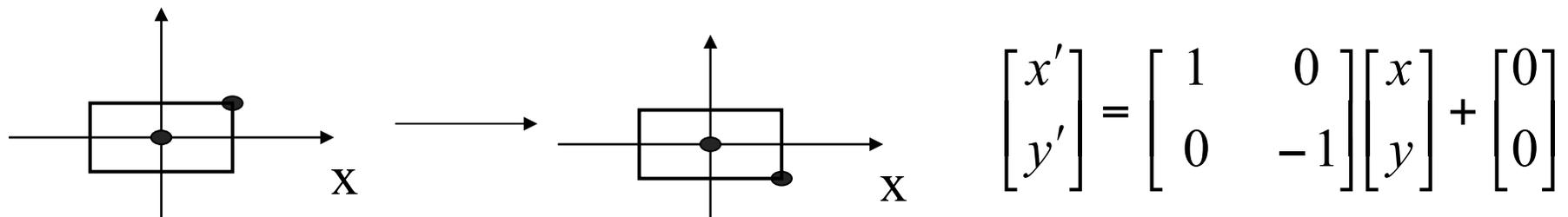
$$x' = A - B$$
$$A = x \cos \theta$$

Review: Shear, Reflection

- shear along x axis
 - push points to right in proportion to height



- reflect across x axis
 - mirror



Review: 2D Transformations

matrix multiplication

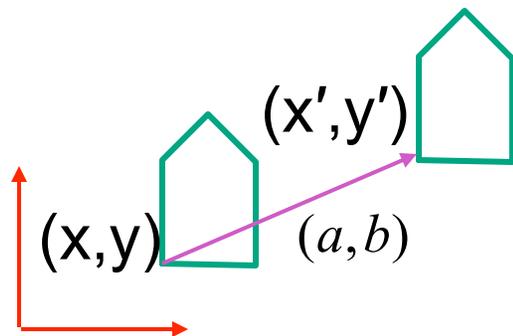
$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \underbrace{\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}} \begin{bmatrix} x \\ y \end{bmatrix}$$

scaling matrix

matrix multiplication

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \underbrace{\begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}} \begin{bmatrix} x \\ y \end{bmatrix}$$

rotation matrix



vector addition

$$\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} x + a \\ y + b \end{bmatrix} = \begin{bmatrix} x' \\ y' \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} a & b \\ c & d \end{bmatrix}} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x' \\ y' \end{bmatrix}$$

translation multiplication matrix??

Review: Linear Transformations

- linear transformations are combinations of

- shear

- scale

- rotate

- reflect

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$x' = ax + by$$

$$y' = cx + dy$$

- properties of linear transformations

- satisfies $T(s\mathbf{x} + t\mathbf{y}) = sT(\mathbf{x}) + tT(\mathbf{y})$

- origin maps to origin

- lines map to lines

- parallel lines remain parallel

- ratios are preserved

- closed under composition

Review: Affine Transformations

- affine transforms are combinations of

- linear transformations
- translations

$$\begin{bmatrix} x' \\ y' \\ w \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

- properties of affine transformations

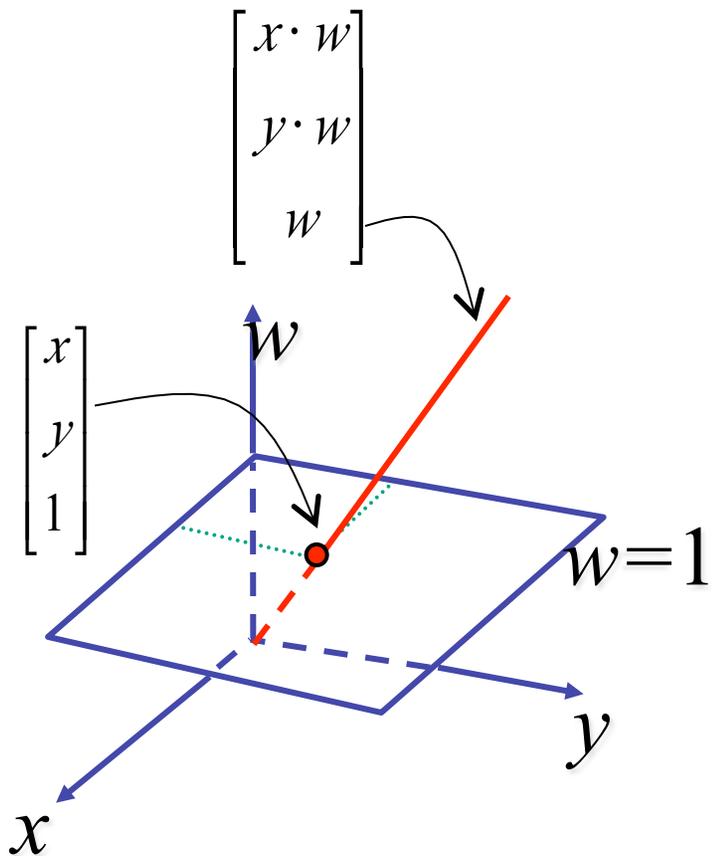
- origin does not necessarily map to origin
- lines map to lines
- parallel lines remain parallel
- ratios are preserved
- closed under composition

Review: Homogeneous Coordinates

homogeneous

cartesian

$$(x, y, w) \xrightarrow{/w} \left(\frac{x}{w}, \frac{y}{w} \right)$$



- **homogenize** to convert homog. 3D point to cartesian 2D point:
 - divide by w to get $(x/w, y/w, 1)$
 - projects line to point onto $w=1$ plane
 - like normalizing, one dimension up
- when $w=0$, consider it as direction
 - points at infinity
 - these points cannot be homogenized
 - lies on x - y plane
- $(0,0,0)$ is undefined

Review: 3D Homog Transformations

- use 4x4 matrices for 3D transformations

translate(a,b,c)

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & & a \\ & 1 & b \\ & & 1 & c \\ & & & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

scale(a,b,c)

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} a & & & \\ & b & & \\ & & c & \\ & & & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Rotate(x,θ)

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & & & \\ & \cos \theta & -\sin \theta & \\ & \sin \theta & \cos \theta & \\ & & & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Rotate(y,θ)

$$\begin{bmatrix} \cos \theta & & \sin \theta & \\ & 1 & & \\ -\sin \theta & & \cos \theta & \\ & & & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Rotate(z,θ)

$$\begin{bmatrix} \cos \theta & -\sin \theta & & \\ \sin \theta & \cos \theta & & \\ & & 1 & \\ & & & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Review: 3D Shear

- general shear $shear(hxy, hxz, hyx, hyz, hzx, hzy) = \begin{bmatrix} 1 & hyx & hzx & 0 \\ hxy & 1 & hzy & 0 \\ hxz & hyz & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
- "x-shear" usually means shear along x in direction of some other axis
 - **correction: not** shear along some axis in direction of x
 - to avoid ambiguity, always say "shear along <axis> in direction of <axis>"

$$shearAlongXinDirectionOfY(h) = \begin{bmatrix} 1 & h & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$shearAlongXinDirectionOfZ(h) = \begin{bmatrix} 1 & 0 & h & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$shearAlongYinDirectionOfX(h) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ h & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

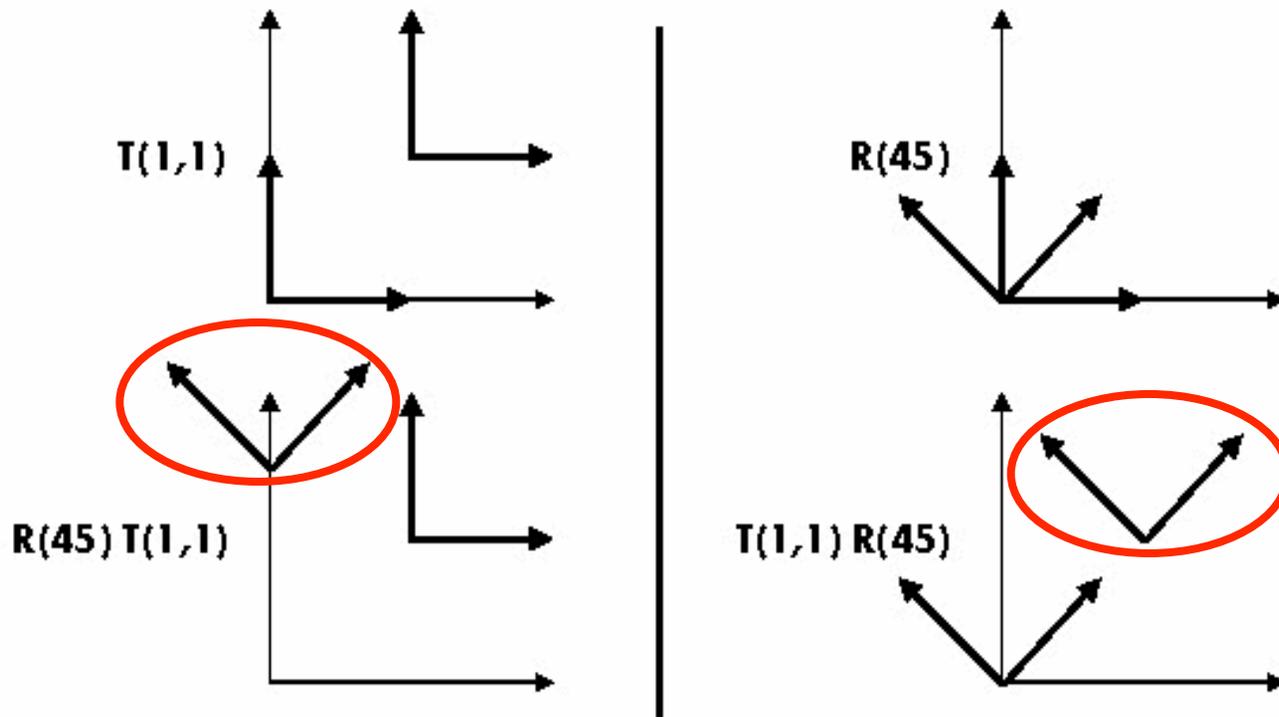
$$shearAlongYinDirectionOfZ(h) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & h & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$shearAlongZinDirectionOfX(h) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ h & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$shearAlongZinDirectionOfY(h) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & h & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Review: Composing Transformations

ORDER MATTERS!



$T_a T_b = T_b T_a$, but $R_a R_b \neq R_b R_a$ and $T_a R_b \neq R_b T_a$

- translations commute
- rotations around same axis commute
- rotations around different axes do not commute
- rotations and translations do not commute

Review: Composing Transformations

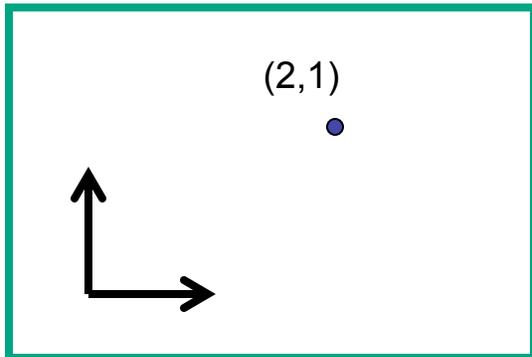
$$\mathbf{p}' = \mathbf{TRp}$$

- which direction to read?
 - right to left
 - interpret operations wrt fixed coordinates
 - **moving object**
 - left to right OpenGL pipeline ordering!
 - interpret operations wrt local coordinates
 - **changing coordinate system**
 - OpenGL updates current matrix with postmultiply
 - `glTranslatef(2,3,0);`
 - `glRotatef(-90,0,0,1);`
 - `glVertexf(1,1,1);`
 - specify vector last, in final coordinate system
 - first matrix to affect it is specified second-to-last

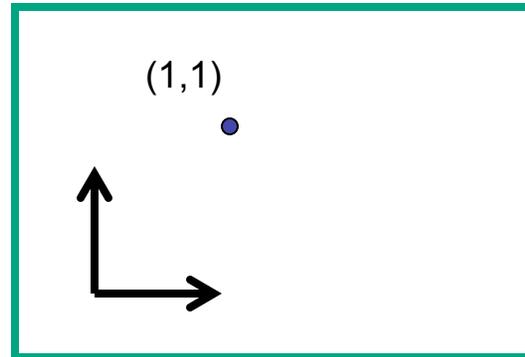
Review: Interpreting Transformations

$$\mathbf{p}' = \mathbf{TRp}$$

translate by $(-1,0)$

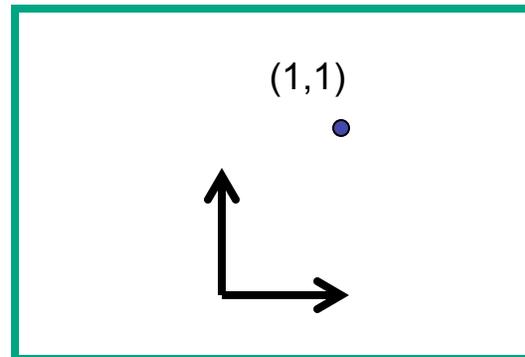


right to left: **moving object**



intuitive?

left to right: **changing coordinate system**



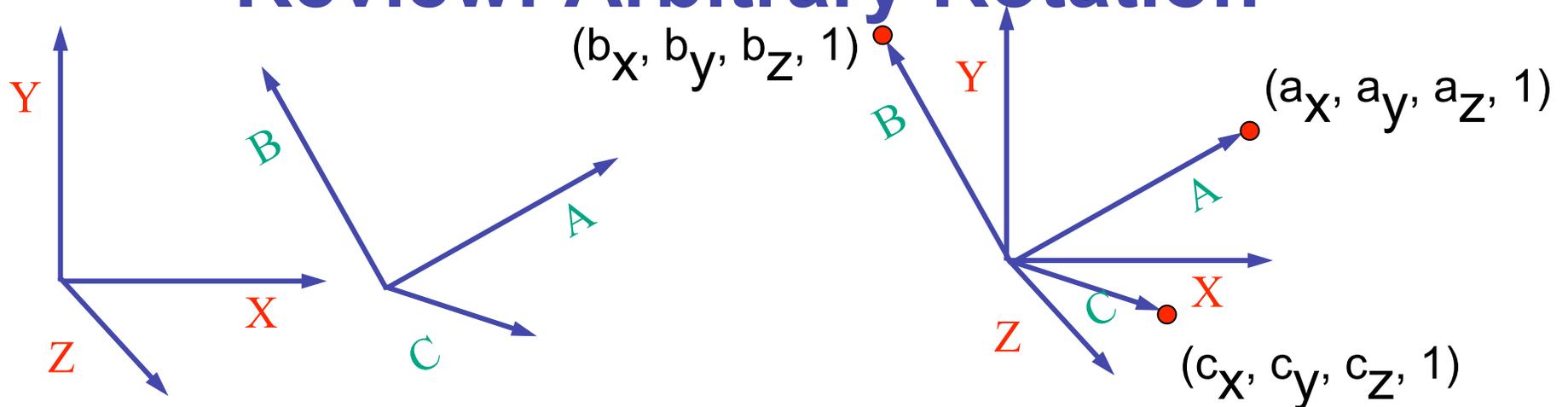
OpenGL

- same relative position between object and basis vectors

Review: General Transform Composition

- transformation of geometry into coordinate system where operation becomes simpler
 - typically translate to origin
- perform operation
- transform geometry back to original coordinate system

Review: Arbitrary Rotation

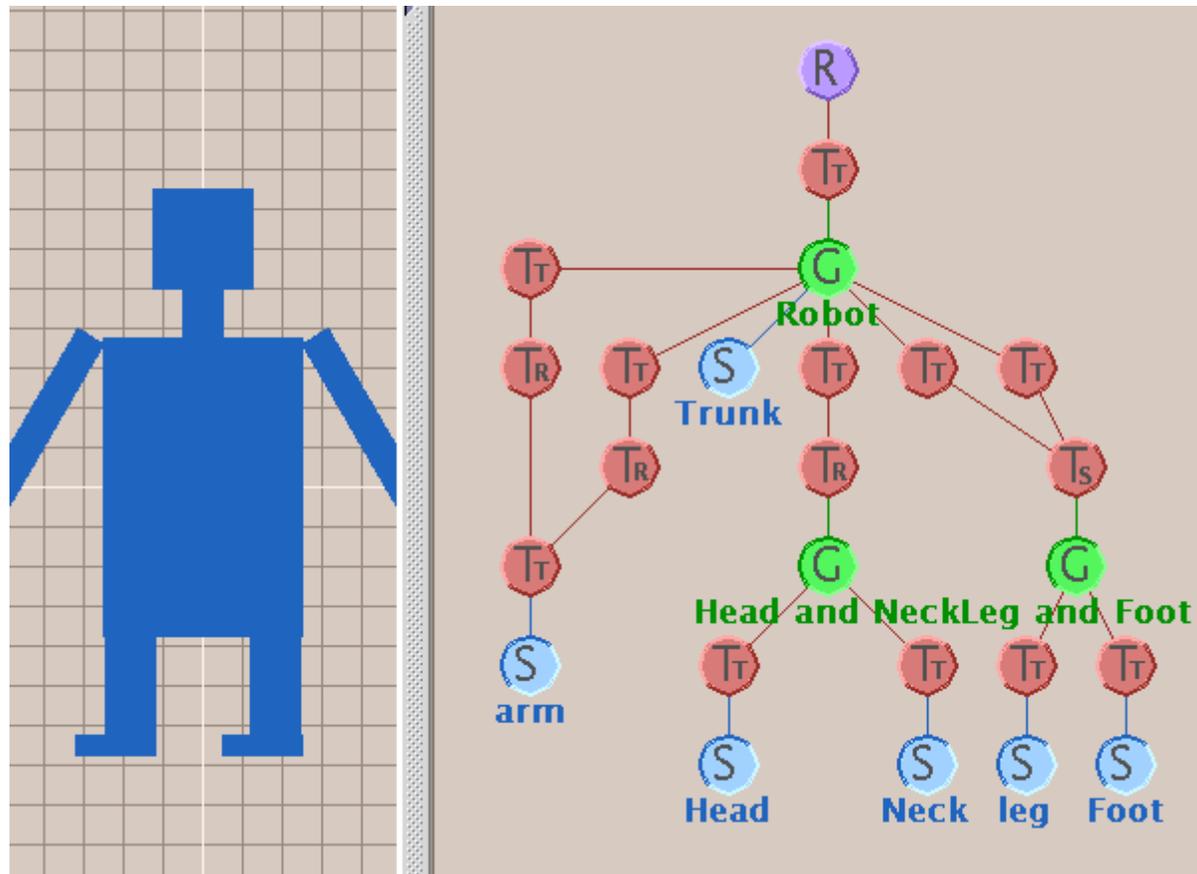


- arbitrary rotation: change of basis
 - given two **orthonormal** coordinate systems XYZ and ABC
 - A 's location in the XYZ coordinate system is $(a_x, a_y, a_z, 1), \dots$
- transformation from one to the other is matrix R whose **columns** are A, B, C :

$$R(X) = \begin{bmatrix} a_x & b_x & c_x & 0 \\ a_y & b_y & c_y & 0 \\ a_z & b_z & c_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} = (a_x, a_y, a_z, 1) = A$$

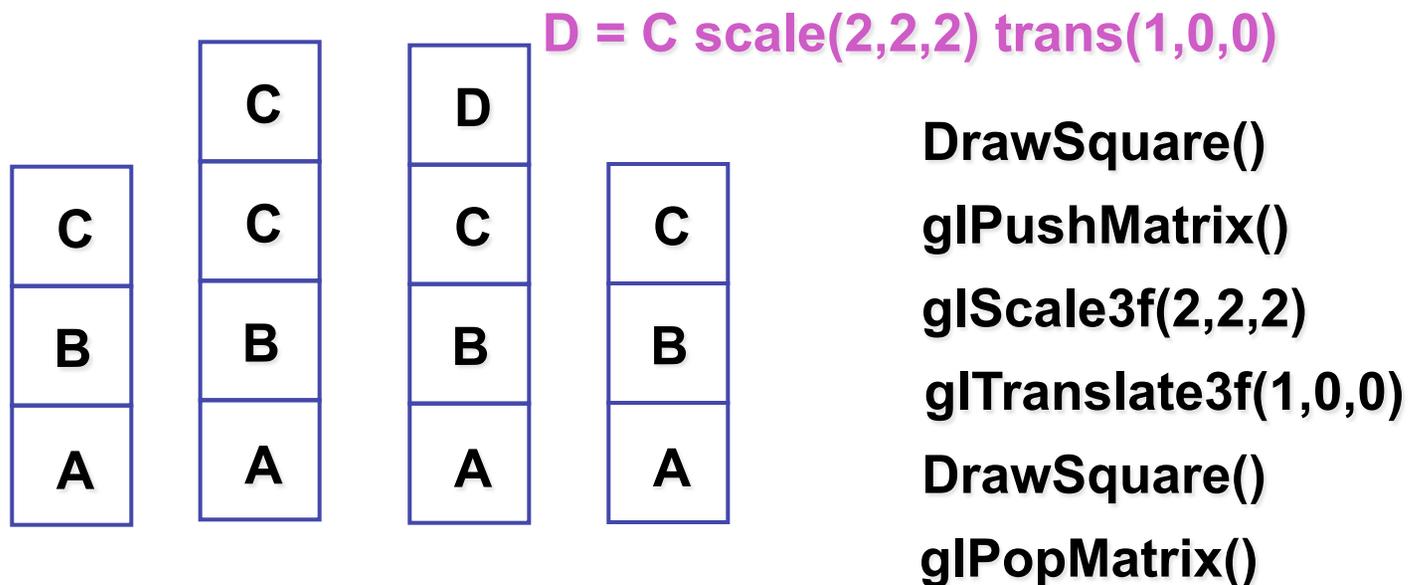
Review: Transformation Hierarchies

- transforms apply to graph nodes beneath them
- design structure so that object doesn't fall apart
- instancing



Review: Matrix Stacks

- OpenGL matrix calls postmultiply matrix M onto current matrix P, overwrite it to be PM
 - or can save intermediate states with stack
 - no need to compute inverse matrices all the time
 - modularize changes to pipeline state
 - avoids accumulation of numerical errors

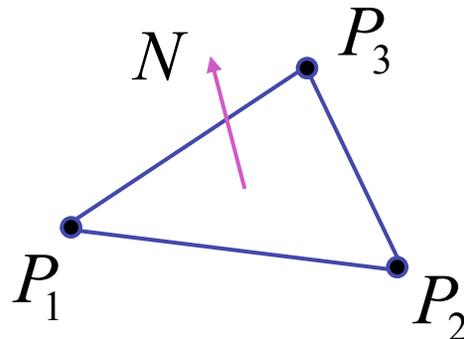


Review: Display Lists

- precompile/cache block of OpenGL code for reuse
 - usually more efficient than **immediate mode**
 - exact optimizations depend on driver
 - good for multiple instances of same object
 - but cannot change contents, not parametrizable
 - good for static objects redrawn often
 - display lists persist across multiple frames
 - interactive graphics: objects redrawn every frame from new viewpoint from moving camera
 - can be nested hierarchically
- snowman example
 - 3x performance improvement, 36K polys
 - <http://www.lighthouse3d.com/opengl/displaylists>

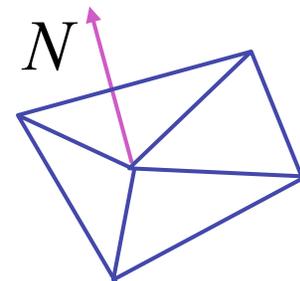
Review: Normals

- polygon:



$$N = (P_2 - P_1) \times (P_3 - P_1)$$

- assume vertices ordered CCW when viewed from visible side of polygon
- normal for a vertex
 - specify polygon orientation
 - used for lighting
 - supplied by model (i.e., sphere), or computed from neighboring polygons



Review: Transforming Normals

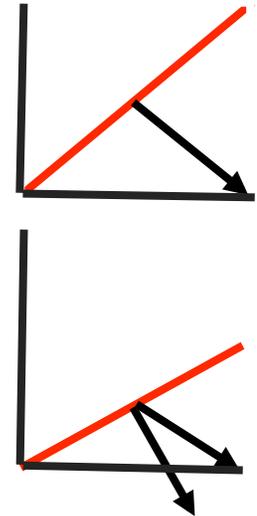
- cannot transform normals using same matrix as points
 - nonuniform scaling would cause to be not perpendicular to desired plane!

$$\begin{array}{l} P \\ N \end{array} \xrightarrow{\quad} \begin{array}{l} P' = MP \\ N' = QN \end{array}$$

given M ,
what should Q be?

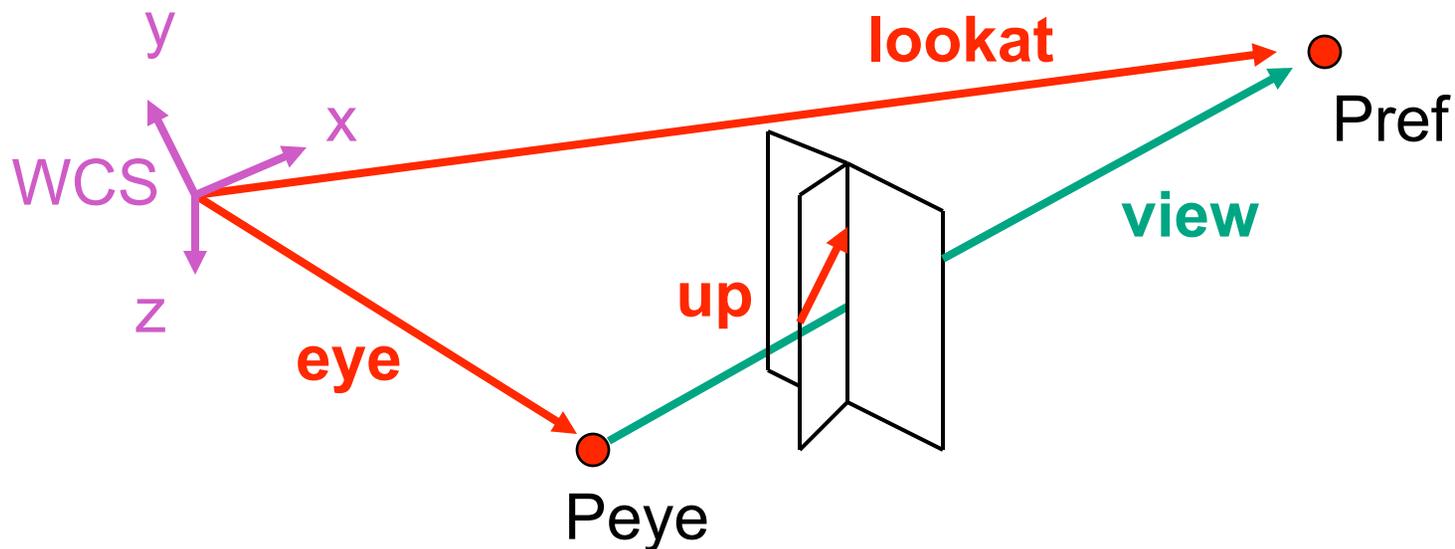
$$Q = (M^{-1})^T$$

inverse transpose of the modelling transformation



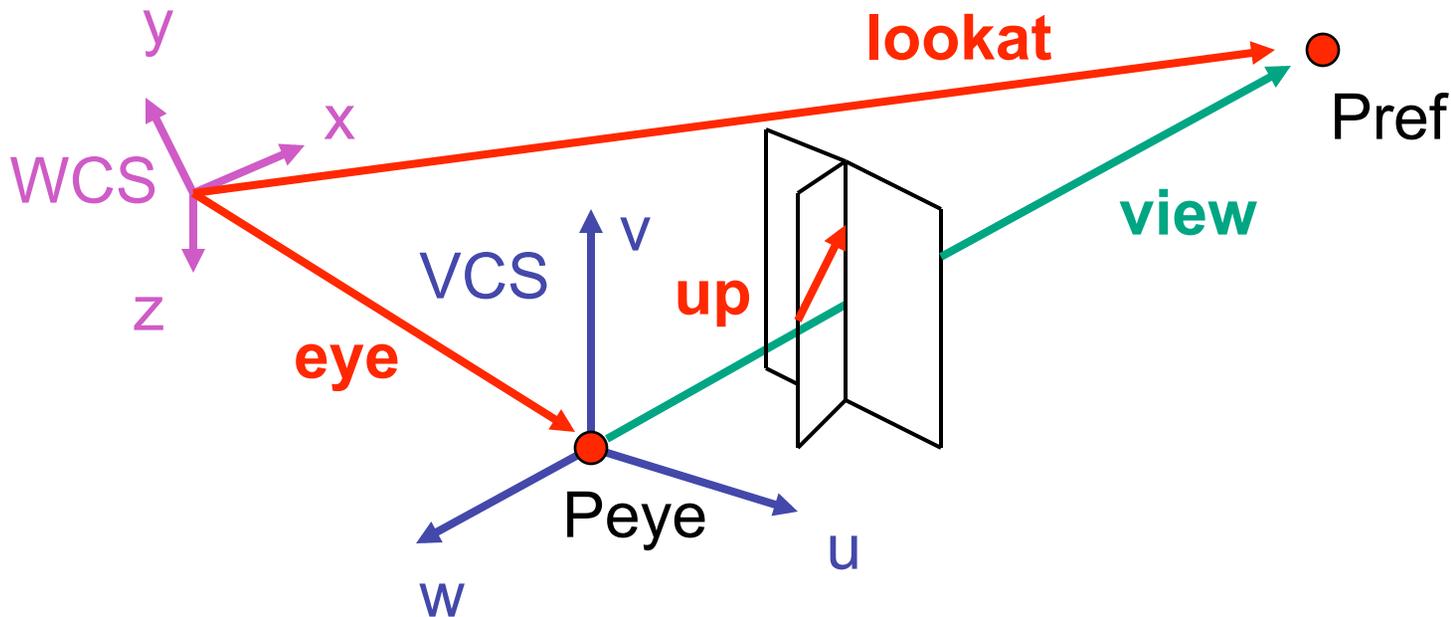
Review: Camera Motion

- rotate/translate/scale difficult to control
- arbitrary viewing position
 - eye point, gaze/lookat direction, up vector



Review: Constructing Lookat

- translate from origin to **eye**
- rotate **view** vector (**lookat** – **eye**) to **w** axis
- rotate around **w** to bring **up** into **vw**-plane



Review: V2W vs. W2V

- $M_{V2W} = \mathbf{T}\mathbf{R}$

$$\mathbf{T} = \begin{bmatrix} 1 & 0 & 0 & e_x \\ 0 & 1 & 0 & e_y \\ 0 & 0 & 1 & e_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \mathbf{R} = \begin{bmatrix} u_x & v_x & w_x & 0 \\ u_y & v_y & w_y & 0 \\ u_z & v_z & w_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- we derived position of camera as object in world
 - invert for gluLookAt: go from world to camera!

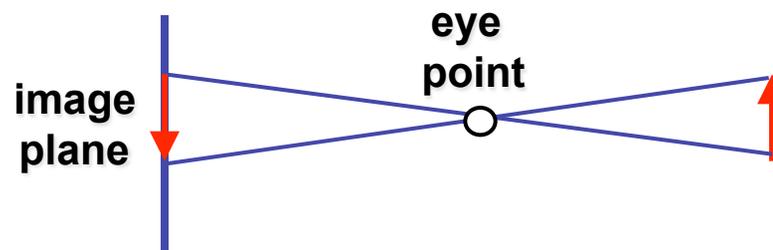
- $M_{W2V} = (M_{V2W})^{-1} = \mathbf{R}^{-1}\mathbf{T}^{-1}$

$$\mathbf{R}^{-1} = \begin{bmatrix} u_x & u_y & u_z & 0 \\ v_x & v_y & v_z & 0 \\ w_x & w_y & w_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \mathbf{T}^{-1} = \begin{bmatrix} 1 & 0 & 0 & -e_x \\ 0 & 1 & 0 & -e_y \\ 0 & 0 & 1 & -e_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

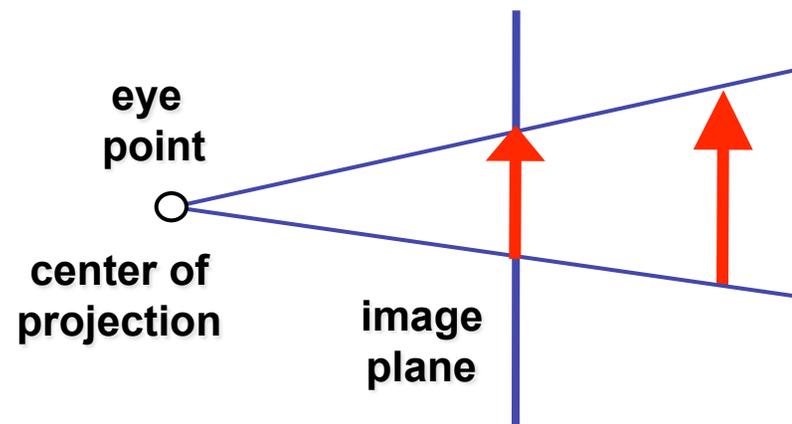
$$\mathbf{M}_{W2V} = \begin{bmatrix} u_x & u_y & u_z & -\mathbf{e} \cdot \mathbf{u} \\ v_x & v_y & v_z & -\mathbf{e} \cdot \mathbf{v} \\ w_x & w_y & w_z & -\mathbf{e} \cdot \mathbf{w} \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} u_x & u_y & u_z & -e_x * u_x + -e_y * u_y + -e_z * u_z \\ v_x & v_y & v_z & -e_x * v_x + -e_y * v_y + -e_z * v_z \\ w_x & w_y & w_z & -e_x * w_x + -e_y * w_y + -e_z * w_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad 35$$

Review: Graphics Cameras

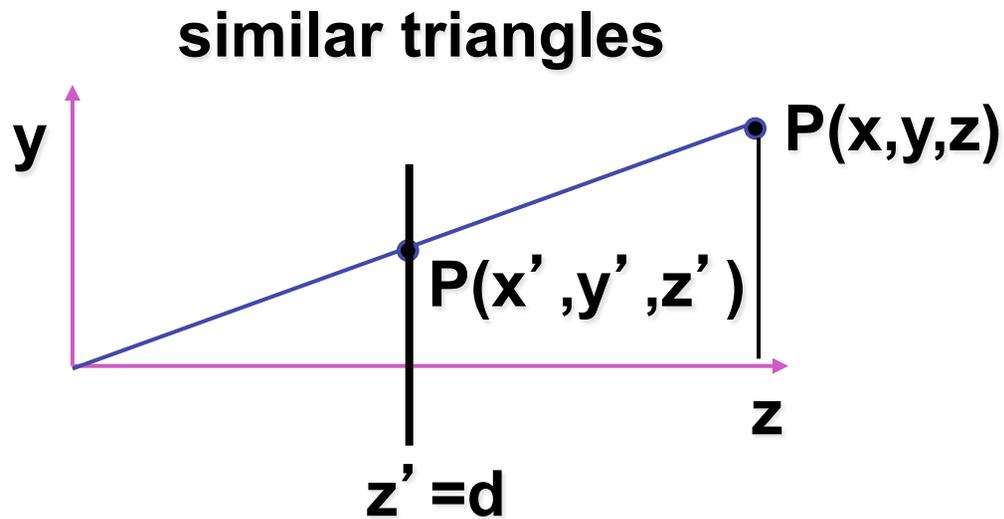
- real pinhole camera: image inverted



- computer graphics camera: convenient equivalent



Review: Basic Perspective Projection



$$\frac{y'}{d} = \frac{y}{z} \rightarrow y' = \frac{y \cdot d}{z}$$

$$x' = \frac{x \cdot d}{z} \quad z' = d$$

$$\begin{bmatrix} x \\ z/d \\ y \\ z/d \\ d \end{bmatrix}$$

homogeneous
coords

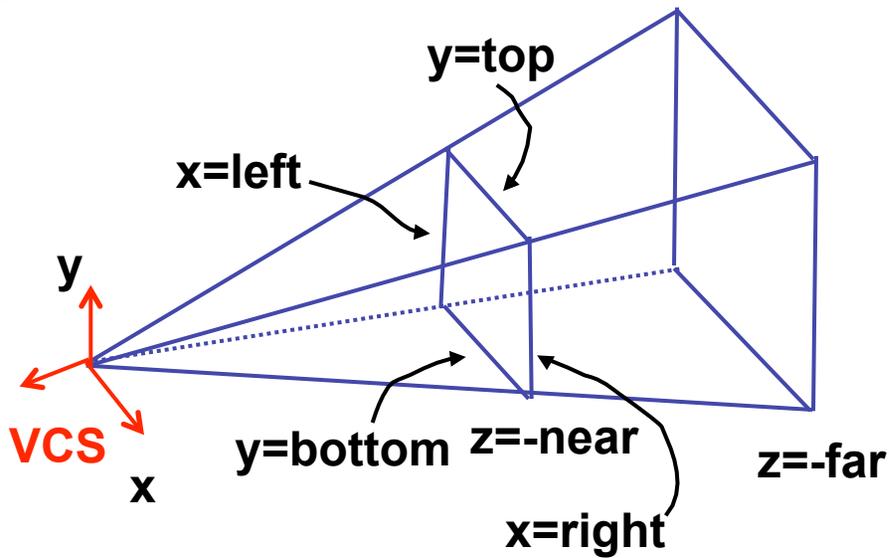


$$\begin{bmatrix} x \\ y \\ z \\ z/d \end{bmatrix}$$

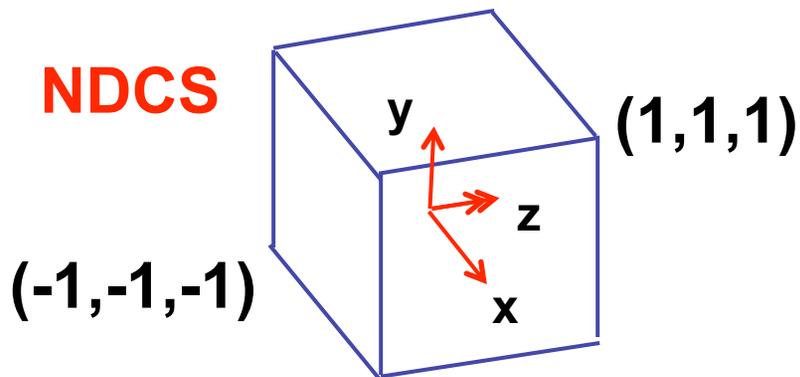
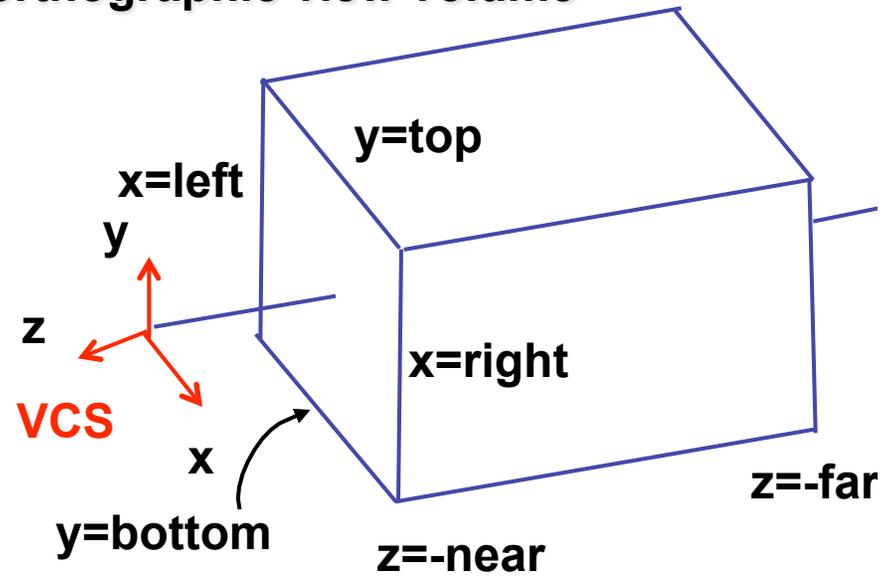
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{bmatrix}$$

Review: From VCS to NDCS

perspective view volume



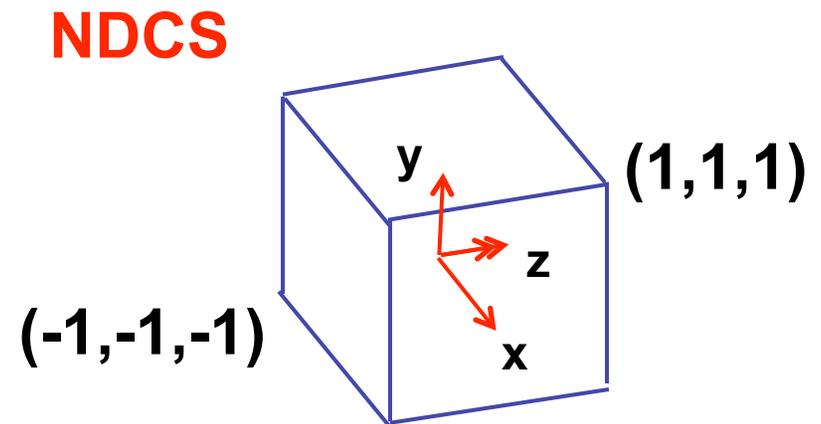
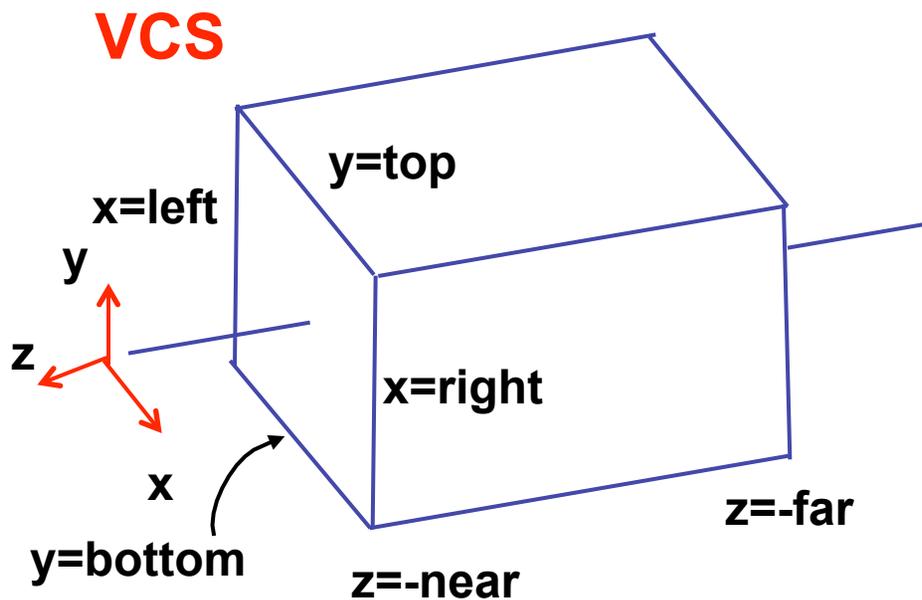
orthographic view volume



- orthographic camera
 - center of projection at infinity
 - no perspective convergence

Review: Orthographic Derivation

- scale, translate, reflect for new coord sys



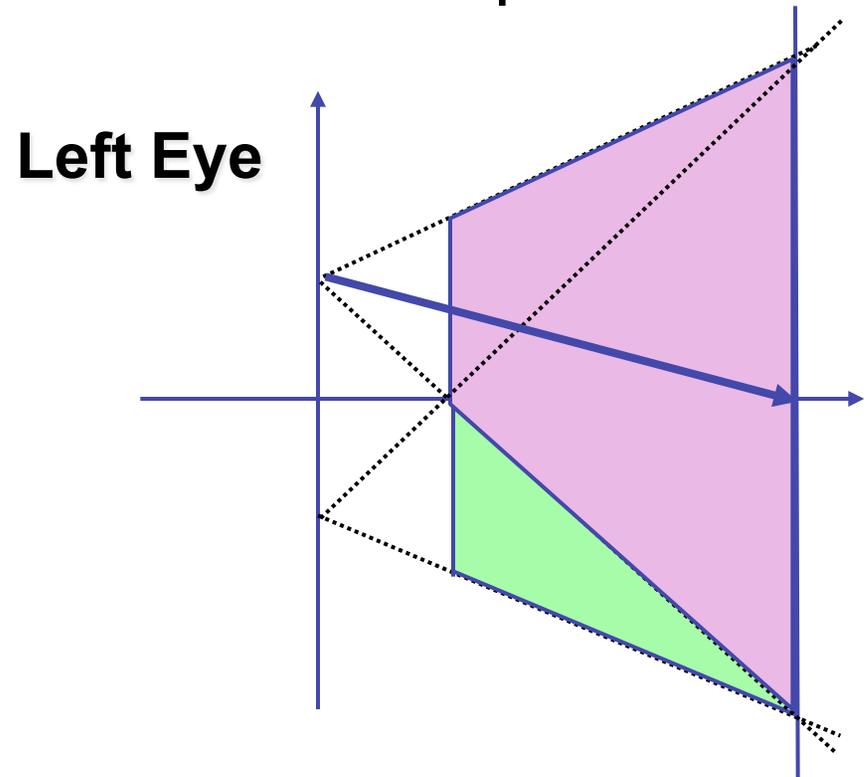
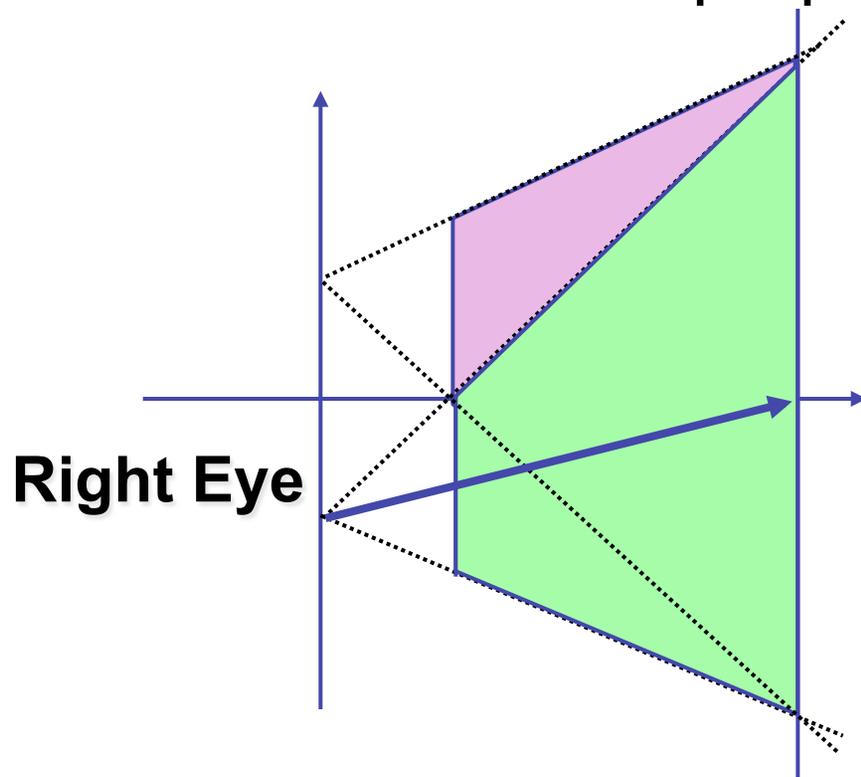
Review: Orthographic Derivation

- scale, translate, reflect for new coord sys

$$P' = \begin{bmatrix} \frac{2}{\text{right} - \text{left}} & 0 & 0 & -\frac{\text{right} + \text{left}}{\text{right} - \text{left}} \\ 0 & \frac{2}{\text{top} - \text{bot}} & 0 & -\frac{\text{top} + \text{bot}}{\text{top} - \text{bot}} \\ 0 & 0 & \frac{-2}{\text{far} - \text{near}} & -\frac{\text{far} + \text{near}}{\text{far} - \text{near}} \\ 0 & 0 & 0 & 1 \end{bmatrix} P$$

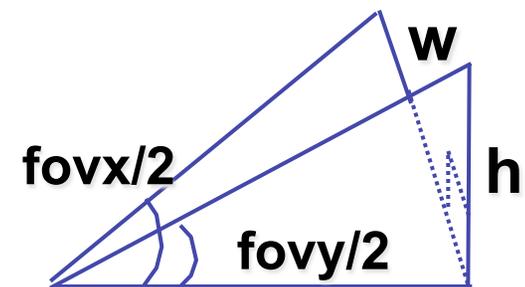
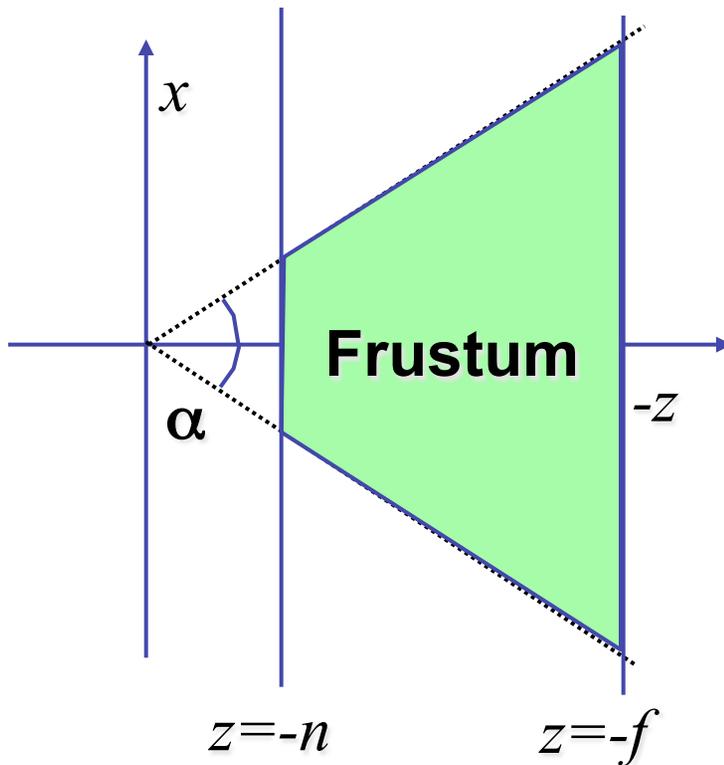
Review: Asymmetric Frusta

- our formulation allows asymmetry
 - why bother? binocular stereo
 - view vector not perpendicular to view plane



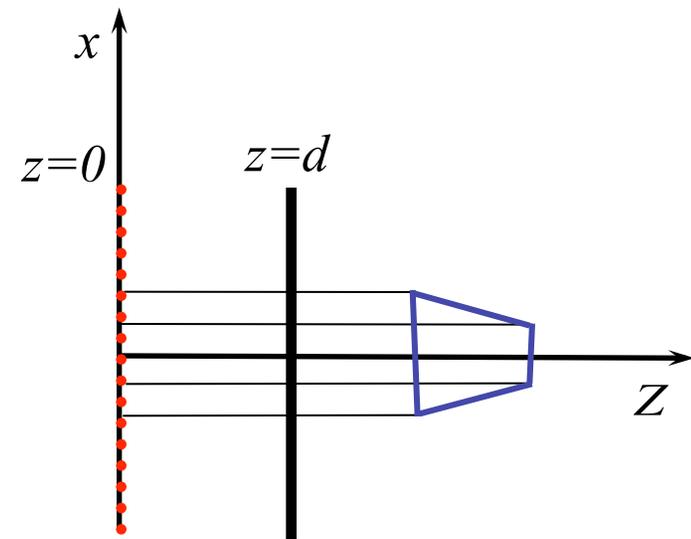
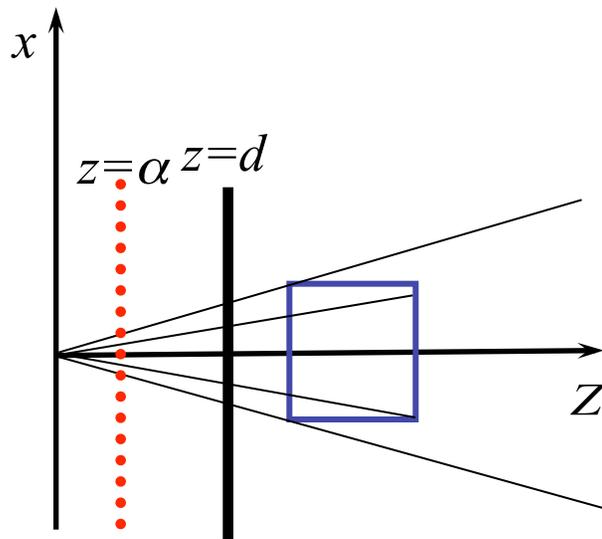
Review: Field-of-View Formulation

- FOV in one direction + aspect ratio (w/h)
 - determines FOV in other direction
 - also set near, far (reasonably intuitive)

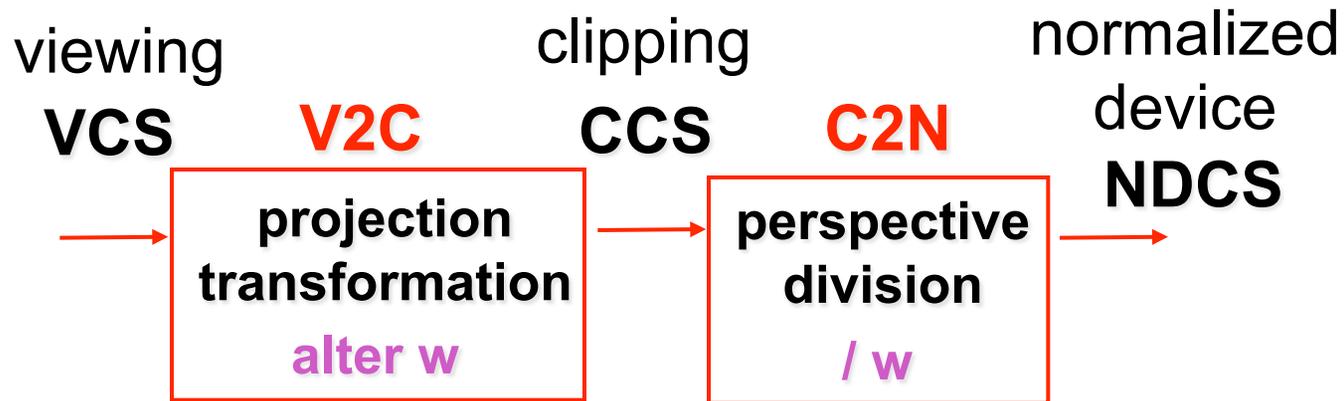


Review: Projection Normalization

- warp perspective view volume to orthogonal view volume
 - render all scenes with orthographic projection!
 - aka perspective warp



Review: Separate Warp From Homogenization

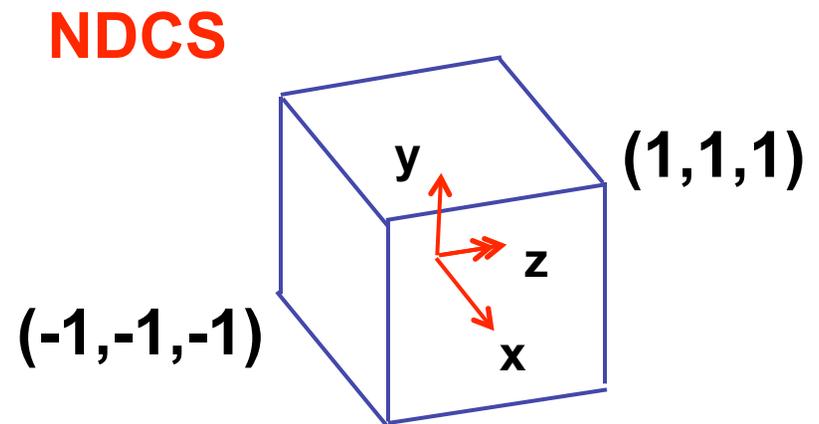
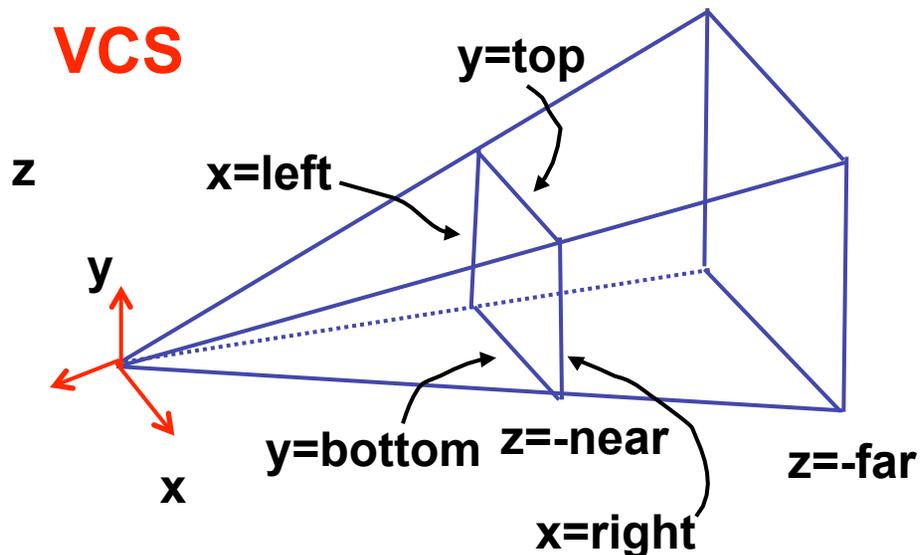


- warp requires only standard matrix multiply
 - distort such that orthographic projection of distorted objects is desired persp projection
 - w is changed
 - clip after warp, before divide
 - division by w : homogenization

Review: Perspective Derivation

- shear
- scale
- projection-normalization

$$\begin{bmatrix} \frac{2n}{r-l} & 0 & \frac{r+l}{r-l} & 0 \\ 0 & \frac{2n}{t-b} & \frac{t+b}{t-b} & 0 \\ 0 & 0 & \frac{-(f+n)}{f-n} & \frac{-2fn}{f-n} \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

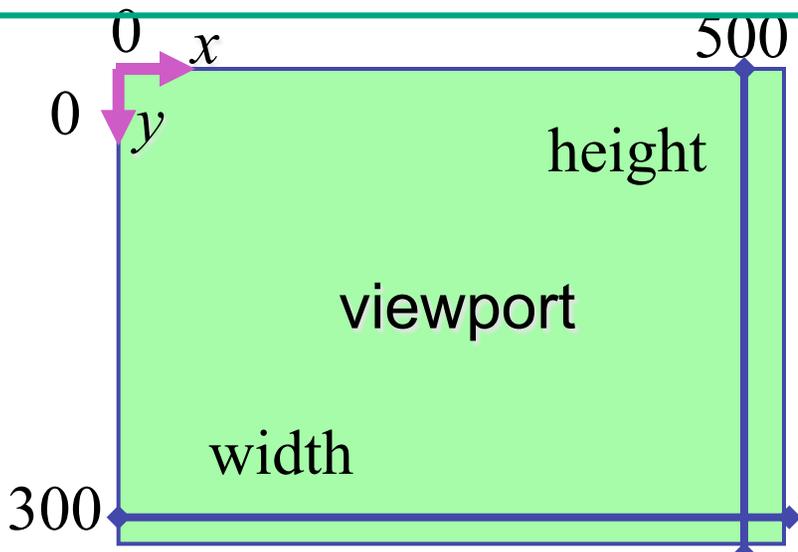
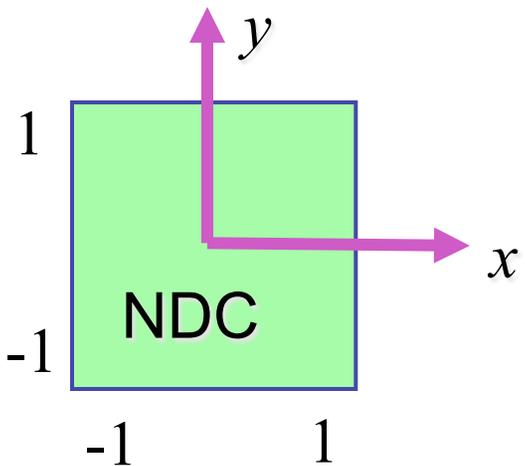


Review: N2D Transformation

$$\begin{bmatrix} x_D \\ y_D \\ z_D \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & \frac{width}{2} - \frac{1}{2} \\ 0 & 1 & 0 & \frac{height}{2} - \frac{1}{2} \\ 0 & 0 & 1 & \frac{depth}{2} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} width \\ 2 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ \frac{height}{2} \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \frac{depth}{2} \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_N \\ y_N \\ z_N \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{width(x_N + 1) - 1}{2} \\ \frac{height(-y_N + 1) - 1}{2} \\ \frac{depth(z_N + 1)}{2} \\ 1 \end{bmatrix}$$

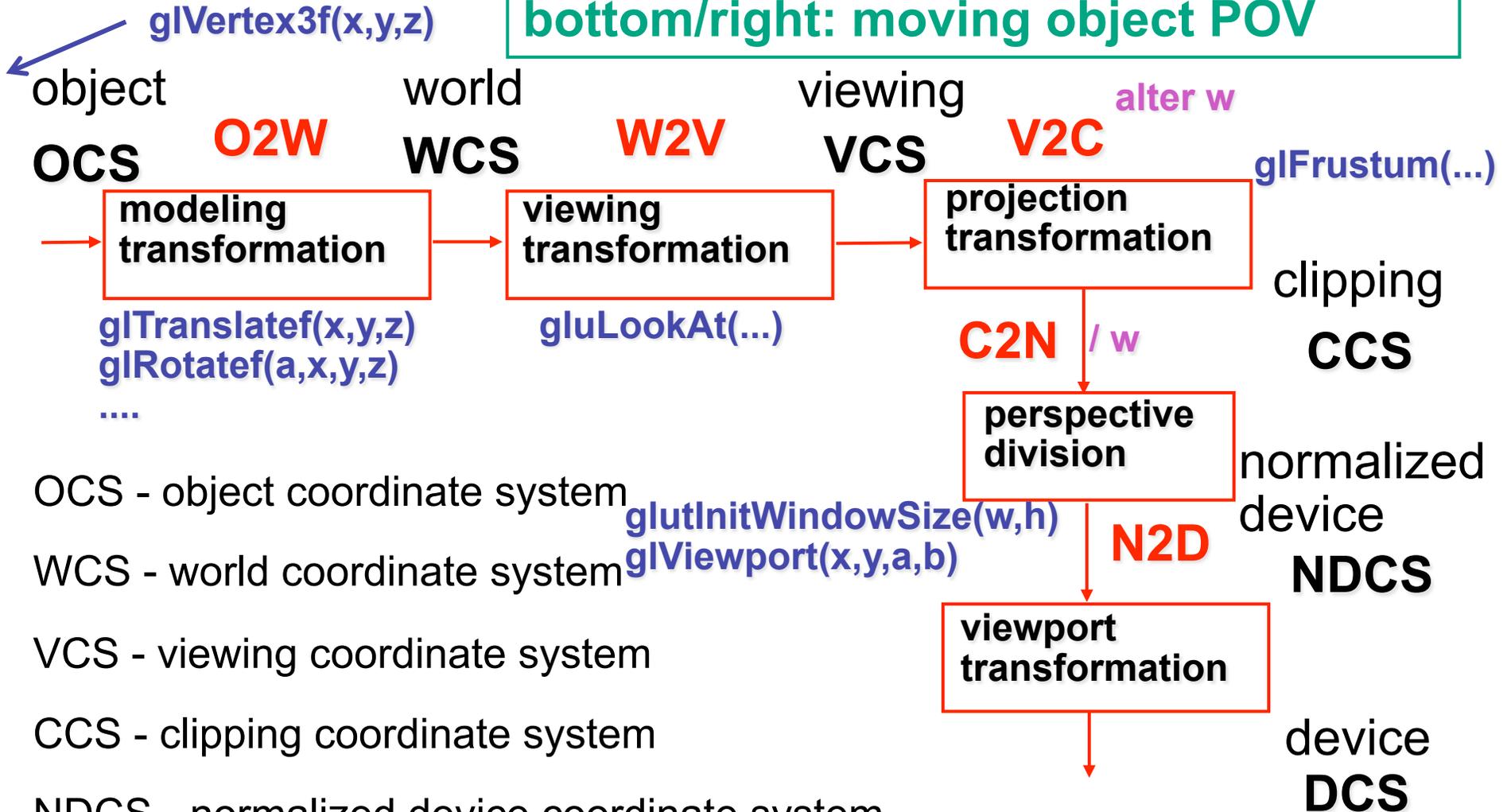
reminder:
NDC z range is -1 to 1

Display z range is 0 to 1.
glDepthRange(n,f) can constrain further, but *depth* = 1 is both max and default



Review: Projective Rendering Pipeline

following pipeline from top/left to bottom/right: moving object POV



- OCS - object coordinate system
- WCS - world coordinate system
- VCS - viewing coordinate system
- CCS - clipping coordinate system
- NDCS - normalized device coordinate system
- DCS - device coordinate system

Review: OpenGL Example

go back from end of pipeline to beginning: coord frame POV!



CCS

```
glMatrixMode( GL_PROJECTION );  
glLoadIdentity();  
gluPerspective( 45, 1.0, 0.1, 200.0 );
```

VCS

```
glMatrixMode( GL_MODELVIEW );  
glLoadIdentity();  
glTranslatef( 0.0, 0.0, -5.0 );
```

V2W

WCS

```
glPushMatrix();  
glTranslate( 4, 4, 0 );
```

W2O

OCS1

```
glutSolidTeapot(1);  
glPopMatrix();  
glTranslate( 2, 2, 0 );
```

W2O

OCS2

```
glutSolidTeapot(1);
```

- transformations that are applied to object first are specified last

Review: Coord Sys: Frame vs Point

read down: transforming between coordinate frames, from frame A to frame B

read up: transforming points, up from frame B coords to frame A coords

OpenGL command order

D2N

DCS display
`glViewport(x,y,a,b)`

N2D

N2V

NDCS normalized device
`glFrustum(...)`

V2N

V2W

VCS viewing
`gluLookAt(...)`

W2V

W2O

WCS world
`glRotatef(a,x,y,z)`

O2W

OCS object
`glVertex3f(x,y,z)`

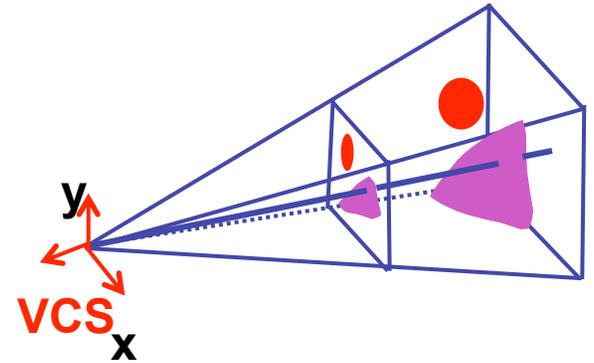
pipeline interpretation

Review: Coord Sys: Frame vs Point

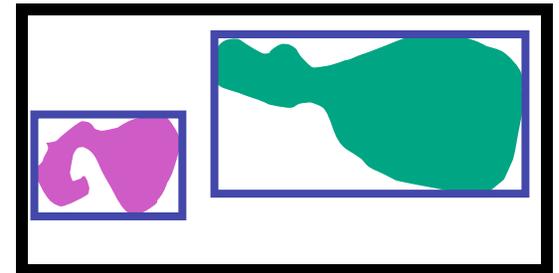
- is `gluLookat` viewing transformation V2W or W2V?
depends on which way you read!
 - coordinate frames: V2W
 - takes you from view to world coordinate frame
 - points/objects: W2V
 - point is transformed from world to view coords when multiply by `gluLookAt` matrix
- H2 uses the object/pipeline POV
 - Q1/4 is W2V (`gluLookAt`)
 - Q2/5-6 is V2N (`glFrustum`)
 - Q3/7 is N2D (`glViewport`)

Review: Picking Methods

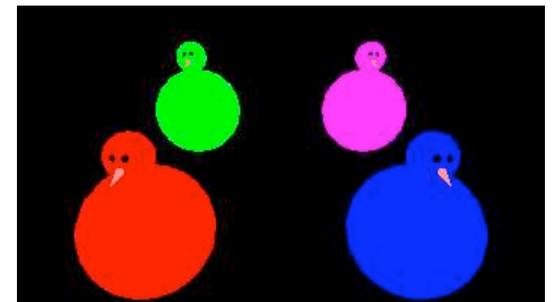
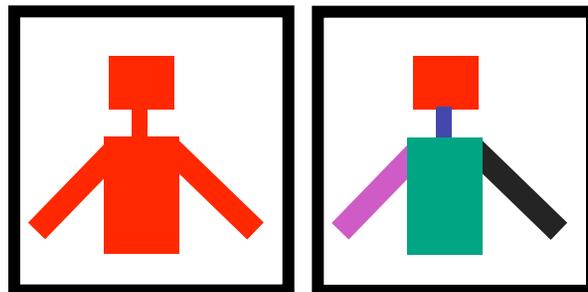
- manual ray intersection



- bounding extents

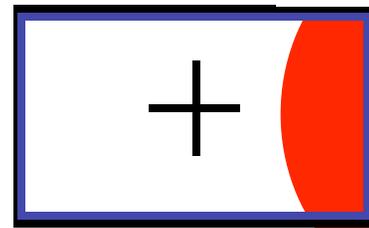
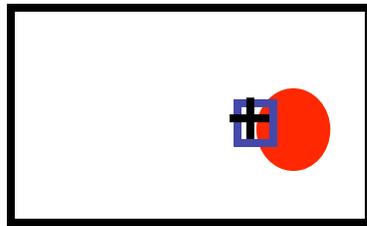


- backbuffer coding



Review: Select/Hit Picking

- assign (hierarchical) integer key/name(s)
- small region around cursor as new viewport



- redraw in selection mode
 - equivalent to casting pick “tube”
 - store keys, depth for drawn objects in hit list
- examine hit list
 - usually use frontmost, but up to application

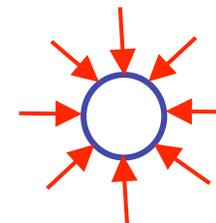
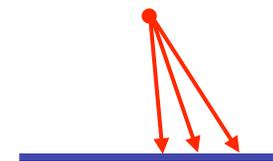
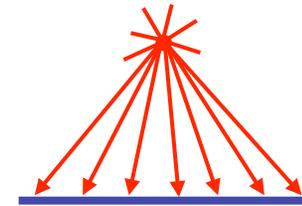
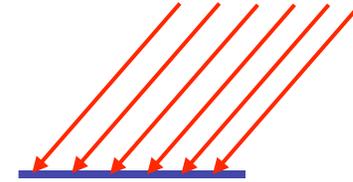
Review: Hit List

- `glSelectBuffer(bufferSize, *buffer)`
 - where to store hit list data
- on hit, copy entire contents of name stack to output buffer.
- hit record
 - number of names on stack
 - minimum and maximum depth of object vertices
 - depth lies in the z-buffer range $[0,1]$
 - multiplied by $2^{32} - 1$ then rounded to nearest int

Post-Midterm Material

Review: Light Sources

- directional/parallel lights
 - point at infinity: $(x,y,z,0)^T$
- point lights
 - finite position: $(x,y,z,1)^T$
- spotlights
 - position, direction, angle
- ambient lights

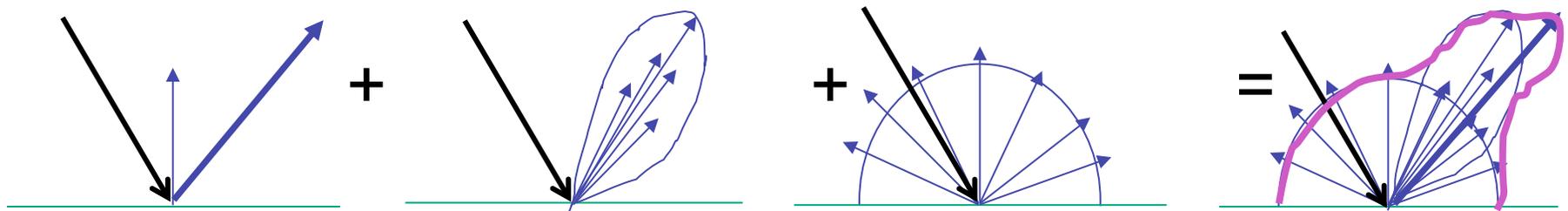


Review: Light Source Placement

- geometry: positions and directions
 - standard: world coordinate system
 - effect: lights fixed wrt world geometry
 - alternative: camera coordinate system
 - effect: lights attached to camera (car headlights)

Review: Reflectance

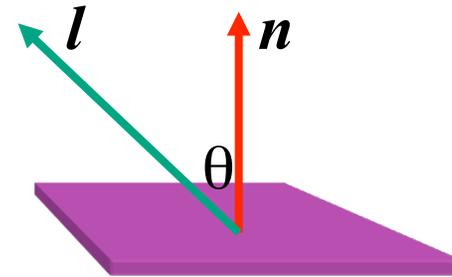
- *specular*: perfect mirror with no scattering
- *gloss*: mixed, partial specularity
- *diffuse*: all directions with equal energy



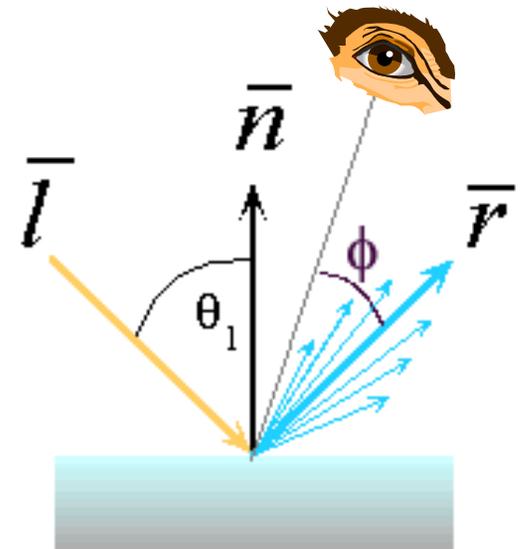
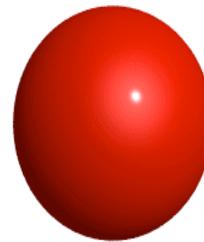
specular + glossy + diffuse =
reflectance distribution

Review: Reflection Equations

$$I_{\text{diffuse}} = k_d I_{\text{light}} (\mathbf{n} \cdot \mathbf{l})$$



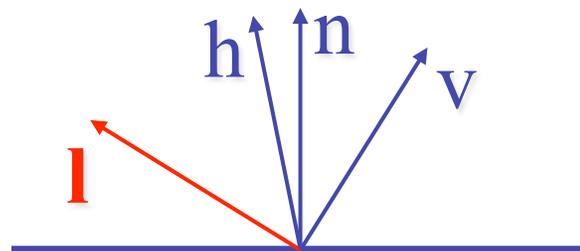
$$I_{\text{specular}} = k_s I_{\text{light}} (\mathbf{v} \cdot \mathbf{r})^{n_{\text{shiny}}}$$



$$\mathbf{R} = 2 (\mathbf{N} (\mathbf{N} \cdot \mathbf{L})) - \mathbf{L}$$

$$I_{\text{specular}} = k_s I_{\text{light}} (\mathbf{h} \cdot \mathbf{n})^{n_{\text{shiny}}}$$

$$\mathbf{h} = (\mathbf{l} + \mathbf{v}) / 2$$



Review: Reflection Equations

full Phong lighting model

- combine ambient, diffuse, specular components

$$\mathbf{I}_{\text{total}} = \mathbf{k}_a \mathbf{I}_{\text{ambient}} + \sum_{i=1}^{\# \text{lights}} \mathbf{I}_i (\mathbf{k}_d (\mathbf{n} \cdot \mathbf{l}_i) + \mathbf{k}_s (\mathbf{v} \cdot \mathbf{r}_i)^{n_{\text{shiny}}})$$

- Blinn-Phong lighting

$$\mathbf{I}_{\text{total}} = \mathbf{k}_a \mathbf{I}_{\text{ambient}} + \sum_{i=1}^{\# \text{lights}} \mathbf{I}_i (\mathbf{k}_d (\mathbf{n} \cdot \mathbf{l}_i) + \mathbf{k}_s (\mathbf{h} \cdot \mathbf{n}_i)^{n_{\text{shiny}}})$$

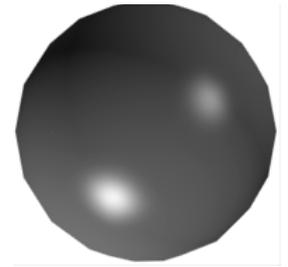
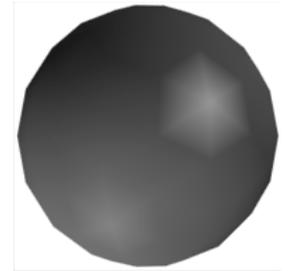
- don't forget to normalize all lighting vectors!! $\mathbf{n}, \mathbf{l}, \mathbf{r}, \mathbf{v}, \mathbf{h}$

Review: Lighting

- lighting models
 - ambient
 - normals don't matter
 - Lambert/diffuse
 - angle between surface normal and light
 - Phong/specular
 - surface normal, light, and viewpoint

Review: Shading Models

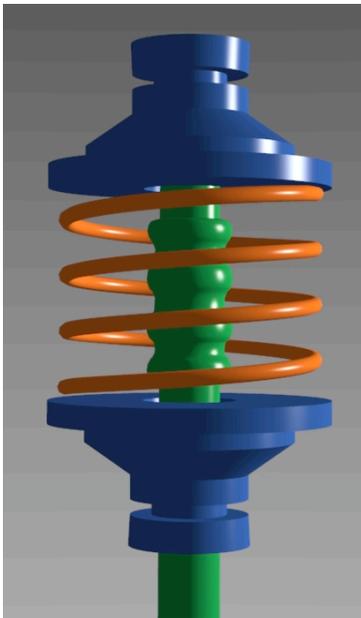
- flat shading
 - for each polygon
 - compute Phong lighting just once
- Gouraud shading
 - compute Phong lighting at the vertices
 - for each pixel in polygon, interpolate colors
- Phong shading
 - for each pixel in polygon
 - interpolate normal
 - perform Phong lighting



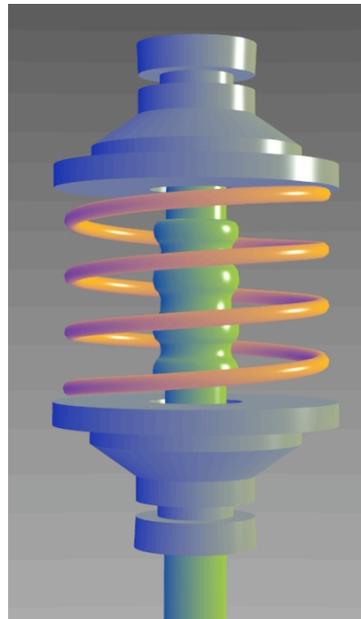
Review: Non-Photorealistic Shading

- cool-to-warm shading: $k_w = \frac{1 + \mathbf{n} \cdot \mathbf{l}}{2}, c = k_w c_w + (1 - k_w) c_c$
- draw silhouettes: if $(\mathbf{e} \cdot \mathbf{n}_0)(\mathbf{e} \cdot \mathbf{n}_1) \leq 0$, \mathbf{e} =edge-eye vector
- draw creases: if $(\mathbf{n}_0 \cdot \mathbf{n}_1) \leq \text{threshold}$

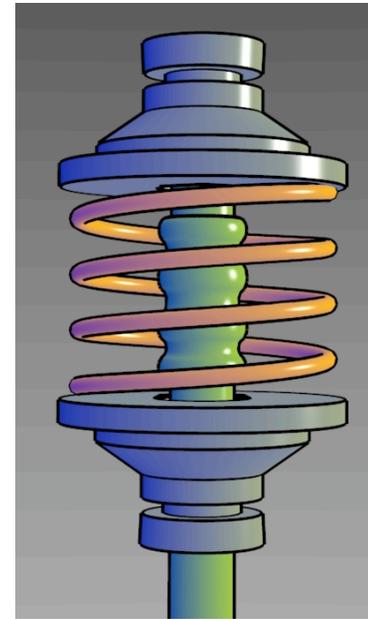
standard



cool-to-warm



with edges/creases



Review: Specifying Normals

- OpenGL state machine
 - uses last normal specified
 - if no normals specified, assumes all identical

- per-vertex normals

```
glNormal3f(1,1,1);  
glVertex3f(3,4,5);  
glNormal3f(1,1,0);  
glVertex3f(10,5,2);
```

- per-face normals

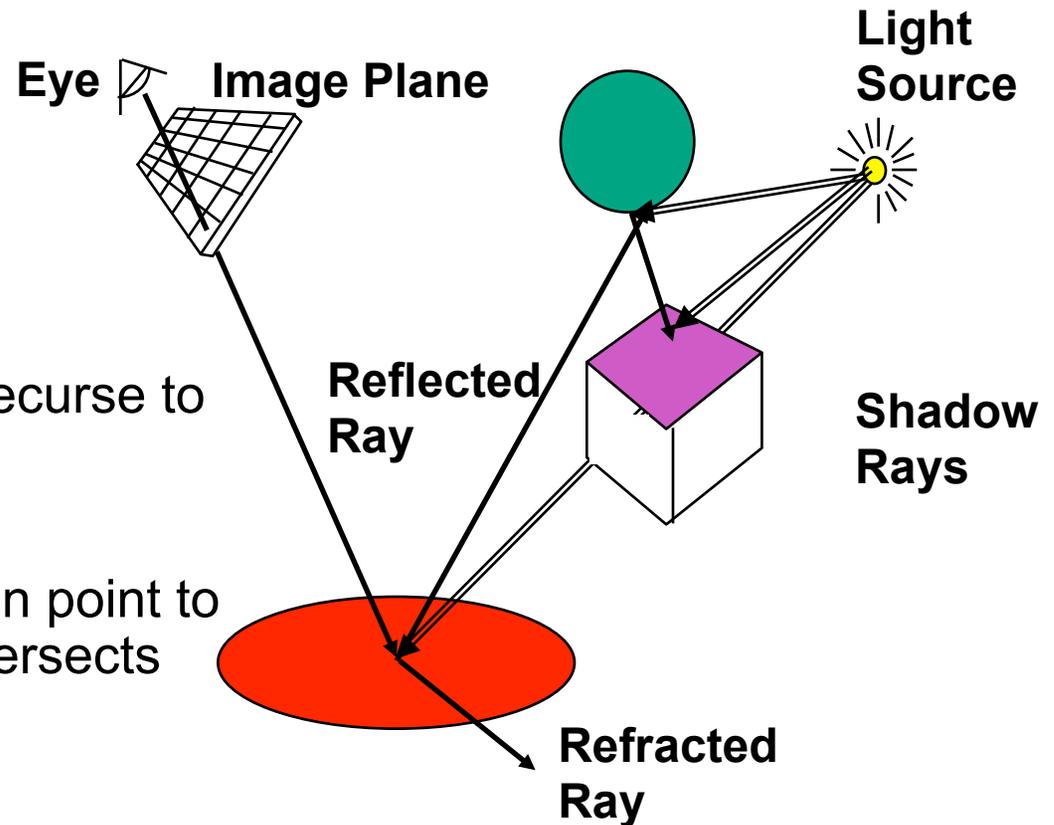
```
glNormal3f(1,1,1);  
glVertex3f(3,4,5);  
glVertex3f(10,5,2);
```

- normal interpreted as direction from vertex location
- can automatically normalize (computational cost)

```
glEnable(GL_NORMALIZE);
```

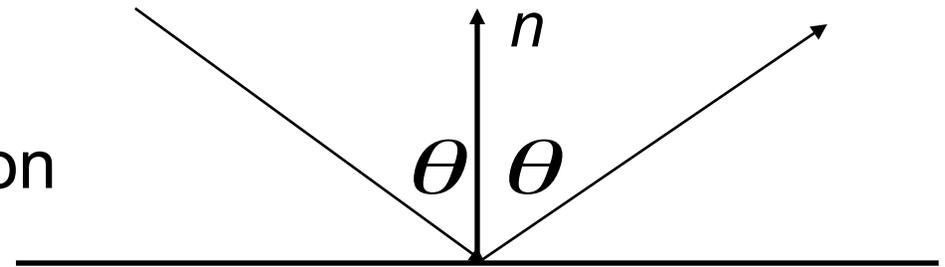
Review: Recursive Ray Tracing

- ray tracing can handle
 - reflection (chrome/mirror)
 - refraction (glass)
 - shadows
- one primary ray per pixel
- spawn secondary rays
 - reflection, refraction
 - if another object is hit, recurse to find its color
 - shadow
 - cast ray from intersection point to light source, check if intersects another object
 - termination criteria
 - no intersection (ray exits scene)
 - max bounces (recursion depth)
 - attenuated below threshold



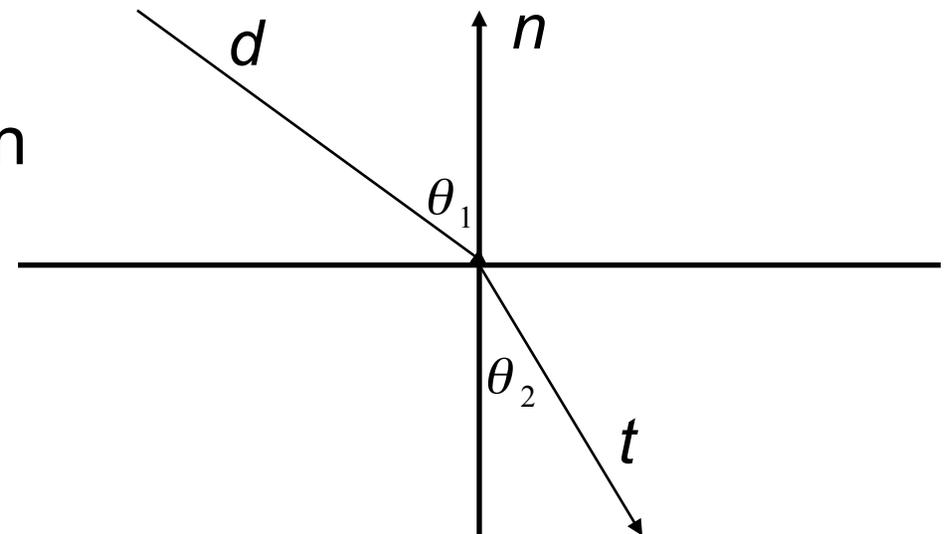
Review: Reflection and Refraction

- reflection: mirror effects
 - perfect specular reflection



- refraction: at boundary
- Snell's Law
 - light ray bends based on refractive indices c_1, c_2

$$c_1 \sin \theta_1 = c_2 \sin \theta_2$$



Review: Ray Tracing

- issues:
 - generation of rays
 - intersection of rays with geometric primitives
 - geometric transformations
 - lighting and shading
 - efficient data structures so we don't have to test intersection with *every* object

Review: Radiosity

- capture indirect diffuse-diffuse light exchange
- model light transport as flow with conservation of energy until convergence
 - view-independent, calculate for whole scene then browse from any viewpoint
- divide surfaces into small patches
- loop: check for light exchange between all pairs
 - form factor: orientation of one patch wrt other patch ($n \times n$ matrix)



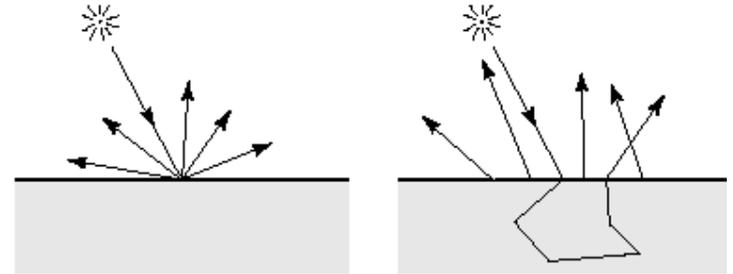
escience.anu.edu.au/lecture/cg/GlobalIllumination/Image/discrete.jpg



escience.anu.edu.au/lecture/cg/GlobalIllumination/Image/continuous.jpg

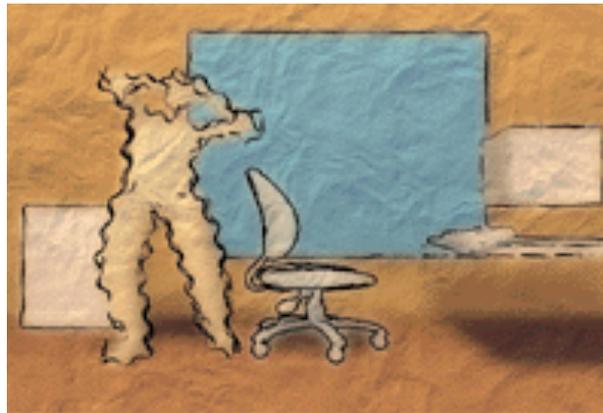
Review: Subsurface Scattering

- light enters and leaves at *different* locations on the surface
 - bounces around inside
- technical Academy Award, 2003
 - Jensen, Marschner, Hanrahan



Review: Non-Photorealistic Rendering

- simulate look of hand-drawn sketches or paintings, using digital models



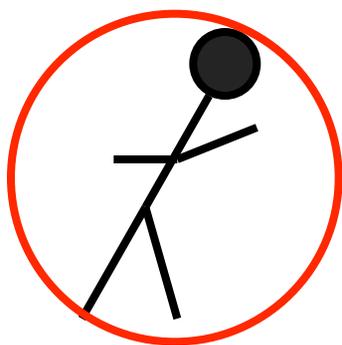
www.red3d.com/cwr/npr/

Review: Collision Detection

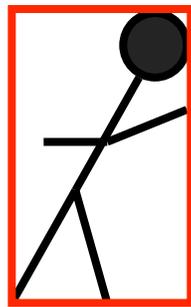
- boundary check
 - perimeter of world vs. viewpoint or objects
 - 2D/3D absolute coordinates for bounds
 - simple point in space for viewpoint/objects
- set of fixed barriers
 - walls in maze game
 - 2D/3D absolute coordinate system
- set of moveable objects
 - one object against set of items
 - missile vs. several tanks
 - multiple objects against each other
 - punching game: arms and legs of players
 - room of bouncing balls

Review: Collision Proxy Tradeoffs

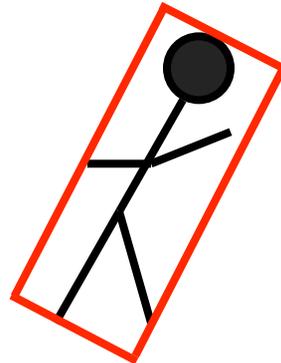
- **collision proxy (bounding volume)** is piece of geometry used to represent complex object for purposes of finding collision
- proxies exploit facts about human perception
 - we are bad at determining collision correctness
 - especially many things happening quickly



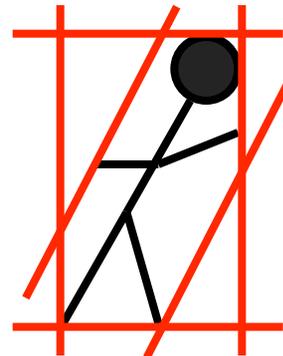
Sphere



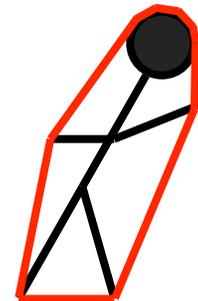
AABB



OBB



6-dof



Convex Hull



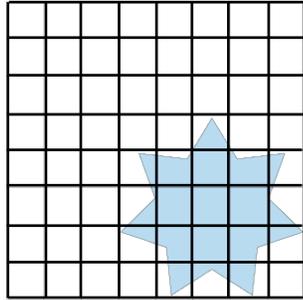
increasing complexity & tightness of fit



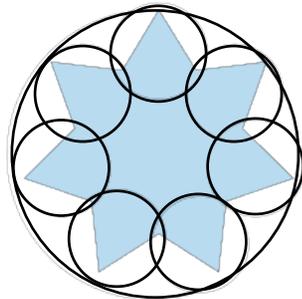
decreasing cost of (overlap tests + proxy update)

Review: Spatial Data Structures

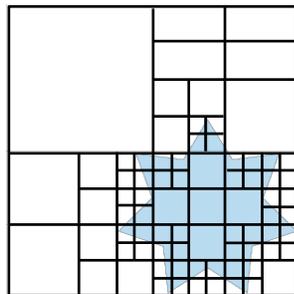
uniform grids



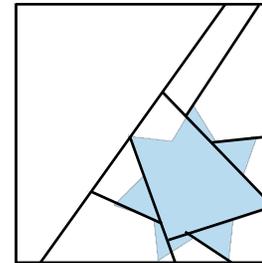
bounding volume hierarchies



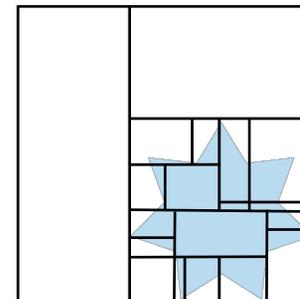
octrees



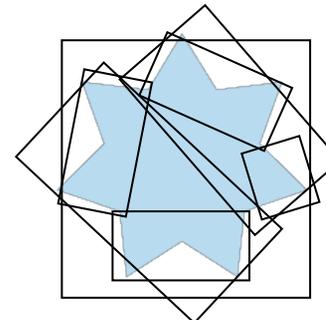
BSP trees



kd-trees

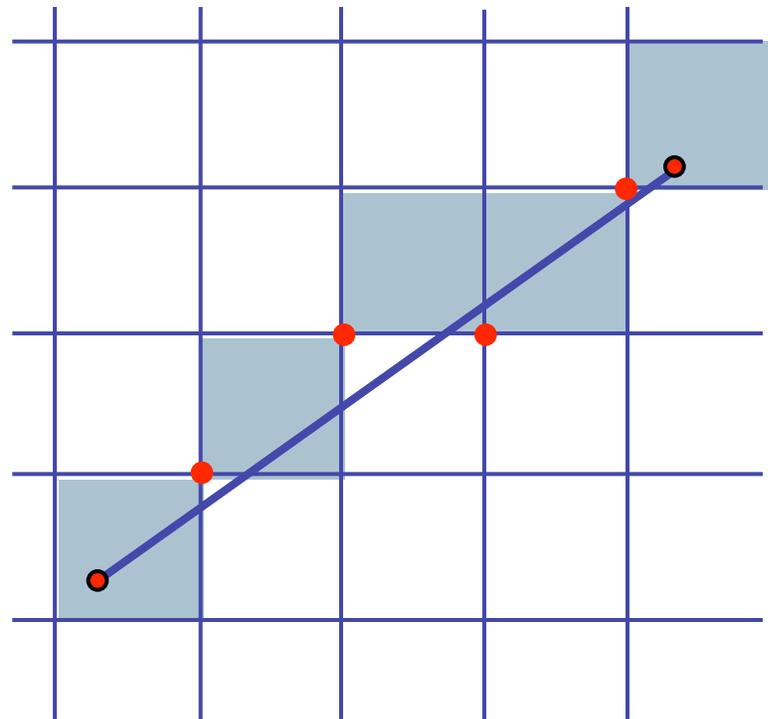
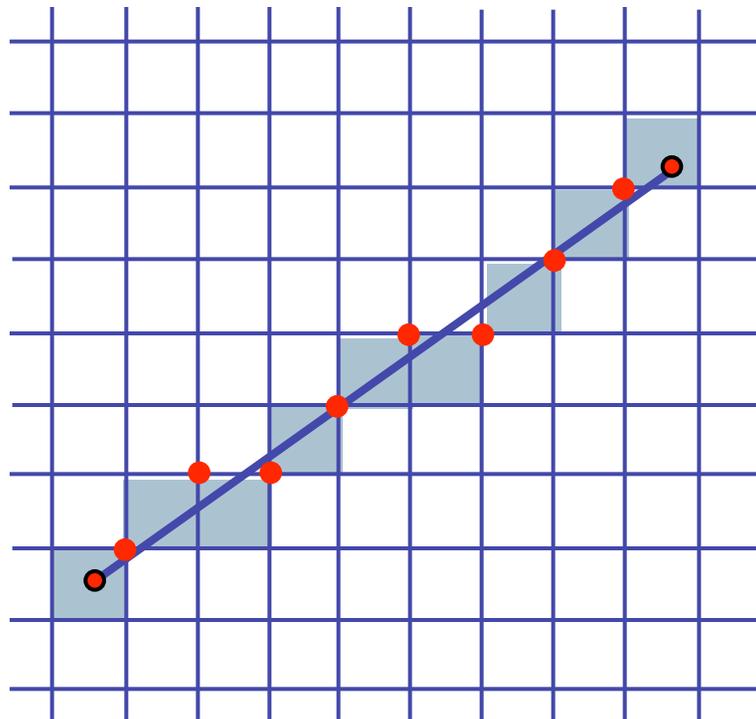


OBB trees



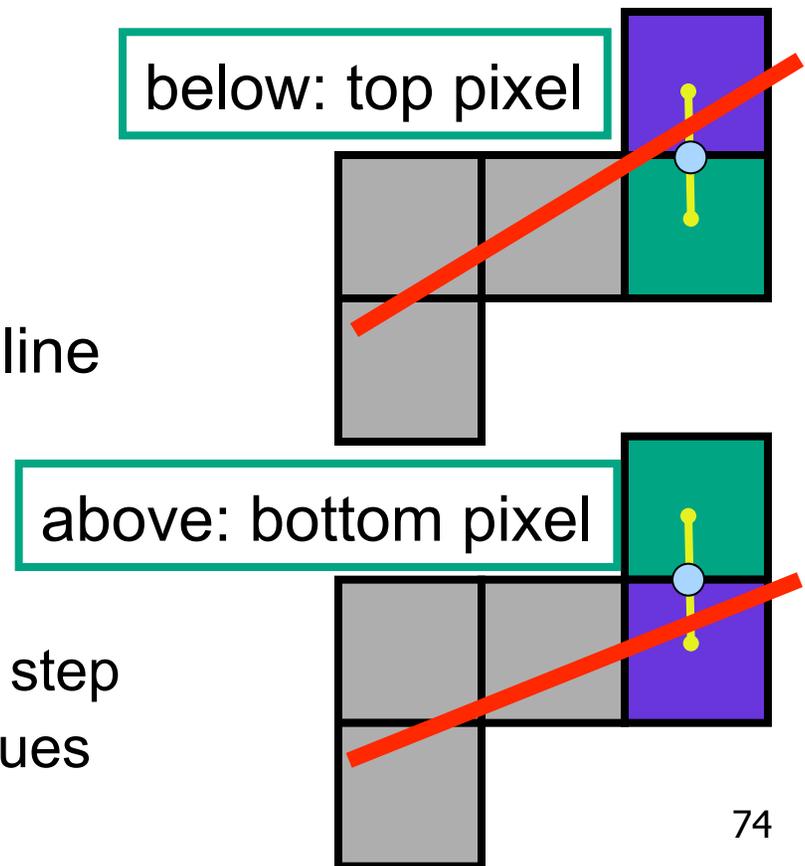
Review: Scan Conversion

- convert continuous rendering primitives into discrete fragments/pixels
 - given vertices in DCS, fill in the pixels
- display coordinates required to provide scale for discretization



Review: Midpoint Algorithm

- we're moving horizontally along x direction (first octant)
 - only two choices: draw at current y value, or move up vertically to $y+1$?
 - check if midpoint between two possible pixel centers above or below line
 - candidates
 - top pixel: $(x+1, y+1)$
 - bottom pixel: $(x+1, y)$
 - midpoint: $(x+1, y+.5)$
- check if midpoint above or below line
 - below: pick top pixel
 - above: pick bottom pixel
- key idea behind Bresenham
 - reuse computation from previous step
 - integer arithmetic by doubling values

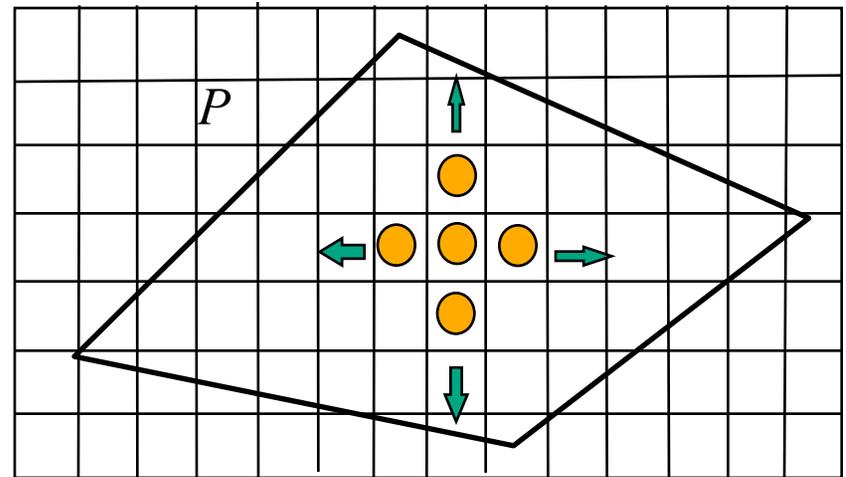
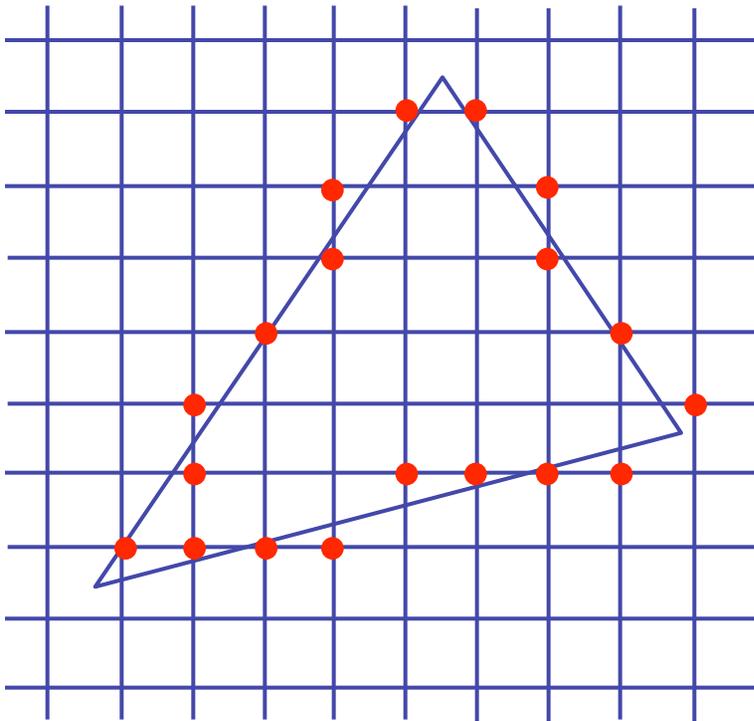


Review: Bresenham - Reuse Computation, Integer Only

```
y=y0;
dx = x1-x0;
dy = y1-y0;
d = 2*dy-dx;
incKeepY = 2*dy;
incIncreaseY = 2*dy-2*dx;
for (x=x0; x <= x1; x++) {
    draw(x,y);
    if (d>0) then {
        y = y + 1;
        d += incIncreaseY;
    } else {
        d += incKeepY;
    }
}
```

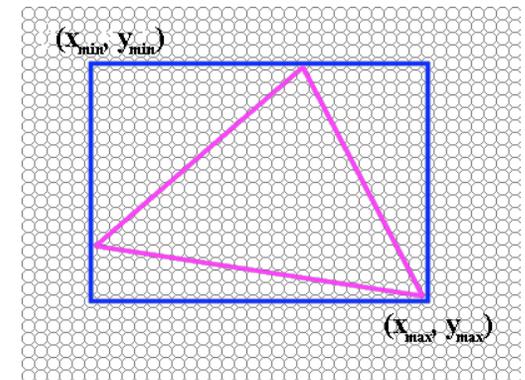
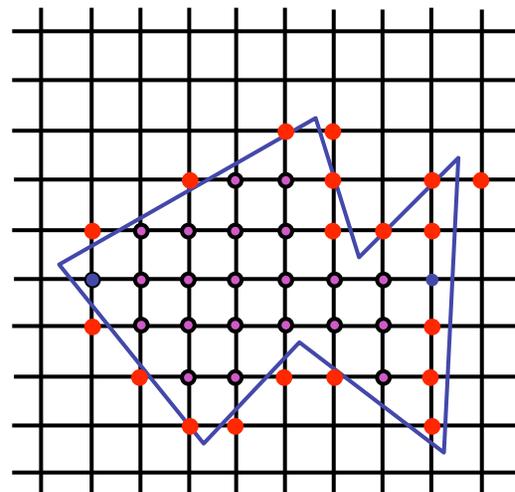
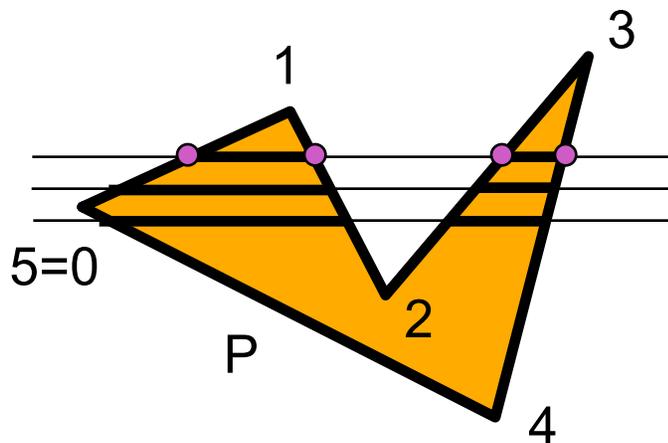
Review: Flood Fill

- simple algorithm
 - draw edges of polygon
 - use flood-fill to draw interior



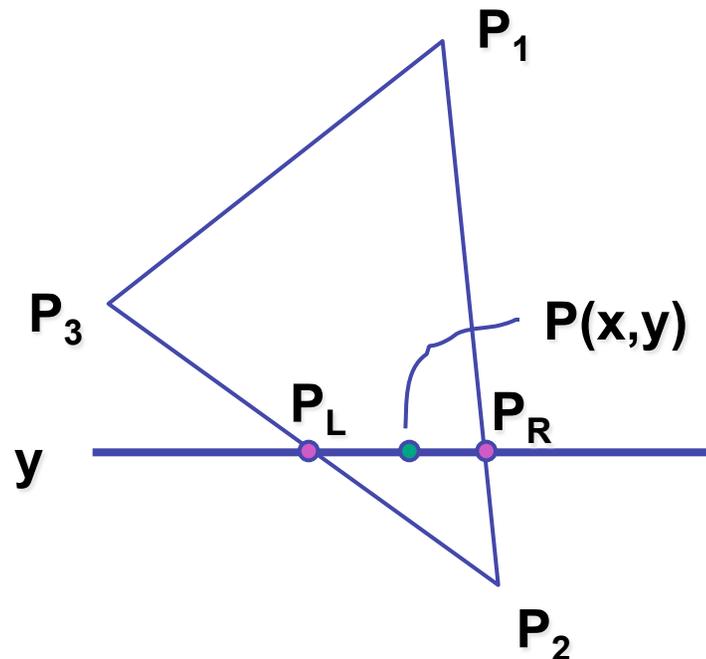
Review: Scanline Algorithms

- **scanline**: a line of pixels in an image
 - set pixels inside polygon boundary along horizontal lines one pixel apart vertically
 - parity test: draw pixel if edgecount is odd
 - optimization: only loop over axis-aligned bounding box of x_{min}/x_{max} , y_{min}/y_{max}



Review: Bilinear Interpolation

- interpolate quantity along L and R edges, as a function of y
 - then interpolate quantity as a function of x



Review: Barycentric Coordinates

- non-orthogonal coordinate system based on triangle itself
 - origin: P_1 , basis vectors: $(P_2 - P_1)$ and $(P_3 - P_1)$

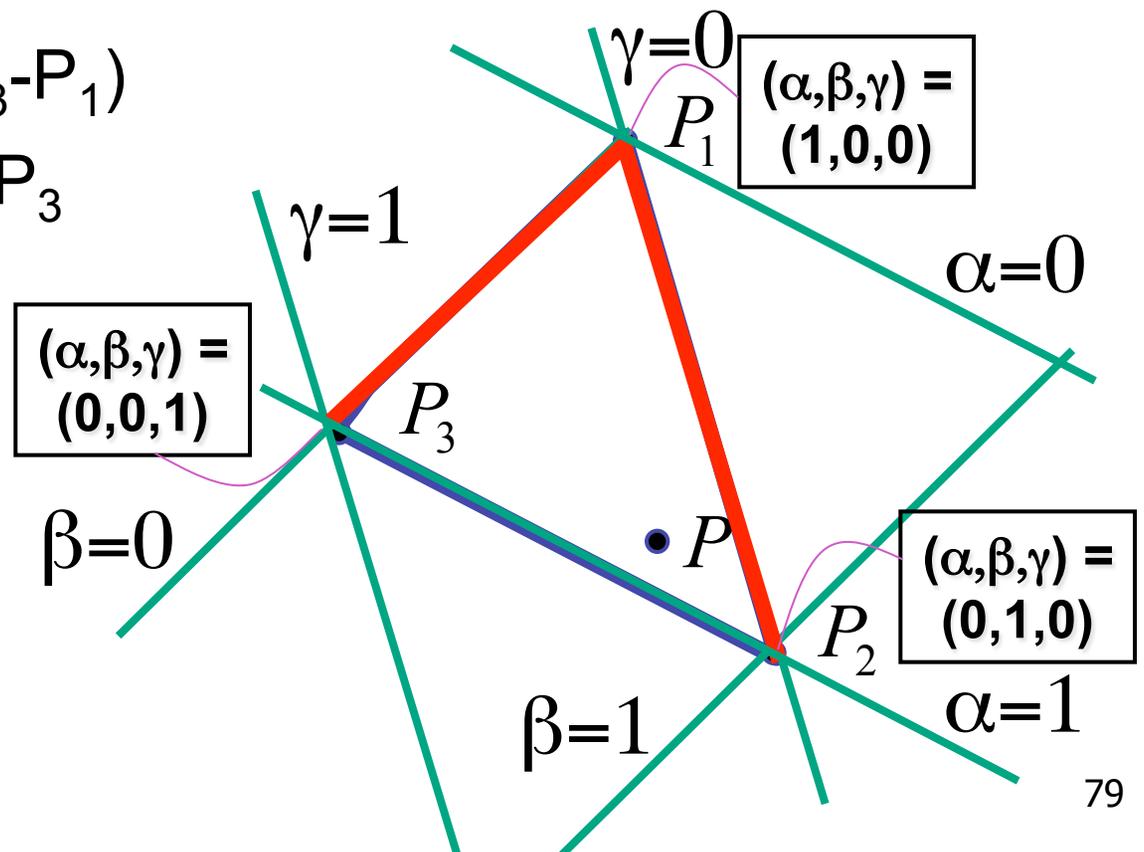
$$P = P_1 + \beta(P_2 - P_1) + \gamma(P_3 - P_1)$$

$$P = (1 - \beta - \gamma)P_1 + \beta P_2 + \gamma P_3$$

$$P = \alpha P_1 + \beta P_2 + \gamma P_3$$

$$\alpha + \beta + \gamma = 1$$

$$0 \leq \alpha, \beta, \gamma \leq 1$$



Review: Computing Barycentric Coordinates

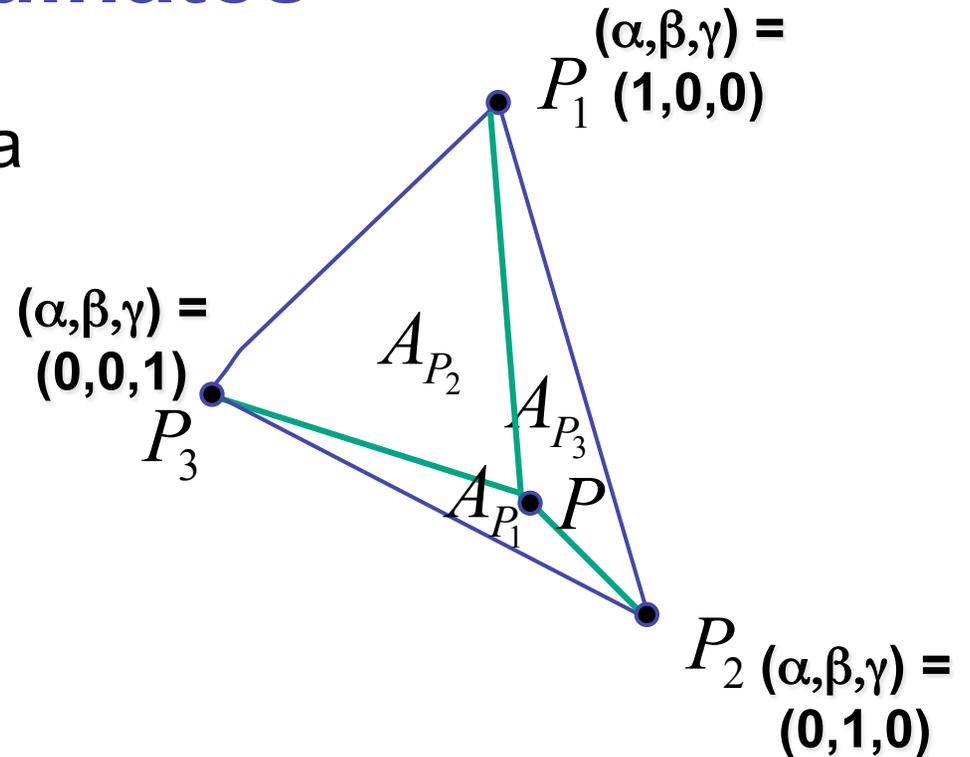
- 2D triangle area
 - half of parallelogram area
 - from cross product

$$A = A_{P_1} + A_{P_2} + A_{P_3}$$

$$\alpha = A_{P_1} / A$$

$$\beta = A_{P_2} / A$$

$$\gamma = A_{P_3} / A$$



weighted combination of three points

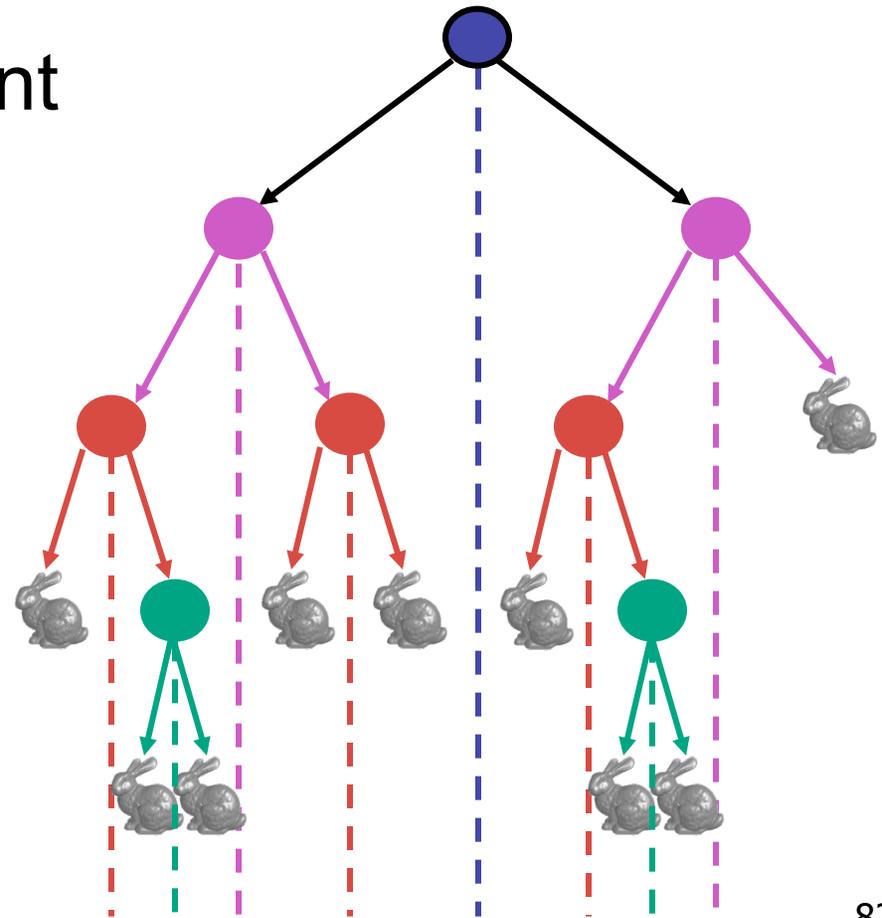
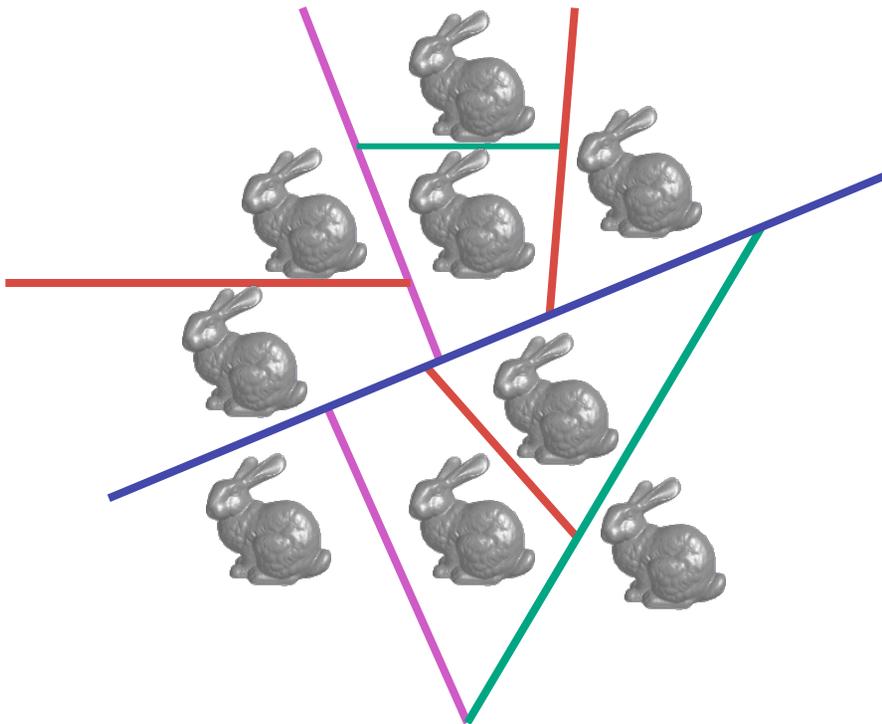
Review: Painter's Algorithm

- draw objects from back to front
- problems: no valid visibility order for
 - intersecting polygons
 - cycles of non-intersecting polygons possible



Review: BSP Trees

- preprocess: create binary tree
 - recursive spatial partition
 - viewpoint independent



Review: Z-Buffer Algorithm

- augment color framebuffer with **Z-buffer** or **depth buffer** which stores Z value at each pixel
 - at frame beginning, initialize all pixel depths to ∞
 - when rasterizing, interpolate depth (Z) across polygon
 - check Z-buffer before storing pixel color in framebuffer and storing depth in Z-buffer
 - don't write pixel if its Z value is more distant than the Z value already stored there

Review: Depth Test Precision

- reminder: perspective transformation maps eye-space (view) z to NDC z

$$\begin{bmatrix} E & 0 & A & 0 \\ 0 & F & B & 0 \\ 0 & 0 & C & D \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} Ex + Az \\ Fy + Bz \\ Cz + D \\ -z \end{bmatrix} = \begin{bmatrix} -\left(\frac{Ex}{z} + Az\right) \\ -\left(\frac{Fy}{z} + Bz\right) \\ -\left(C + \frac{D}{z}\right) \\ 1 \end{bmatrix}$$

- thus $z_{NDC} = -\left(C + \frac{D}{z_{eye}}\right)$

- depth buffer essentially stores $1/z$
 - high precision for near, low precision for distant

Review: Integer Depth Buffer

- reminder from picking: depth stored as integer
 - depth lies in the DCS z range $[0,1]$
 - format: multiply by $2^n - 1$ then round to nearest int
 - where n = number of bits in depth buffer
- 24 bit depth buffer = $2^{24} = 16,777,216$ possible values
 - small numbers near, large numbers far
- consider depth from VCS: $(1 \ll N) * (a + b / z)$
 - N = number of bits of Z precision
 - $a = z_{Far} / (z_{Far} - z_{Near})$
 - $b = z_{Far} * z_{Near} / (z_{Near} - z_{Far})$
 - z = distance from the eye to the object

Review: Object Space Algorithms

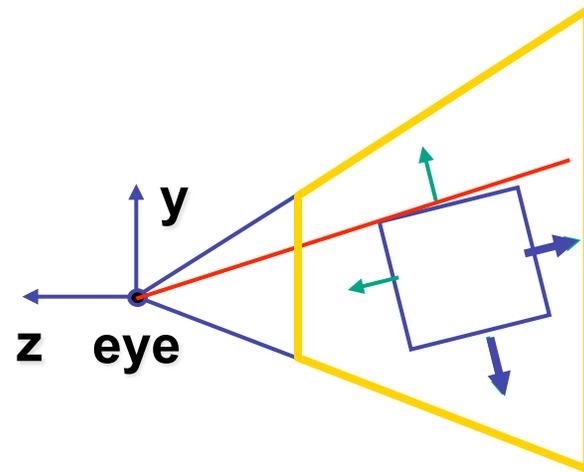
- determine visibility on object or polygon level
 - using camera coordinates
- resolution independent
 - explicitly compute visible portions of polygons
- early in pipeline
 - after clipping
- requires depth-sorting
 - painter's algorithm
 - BSP trees

Review: Image Space Algorithms

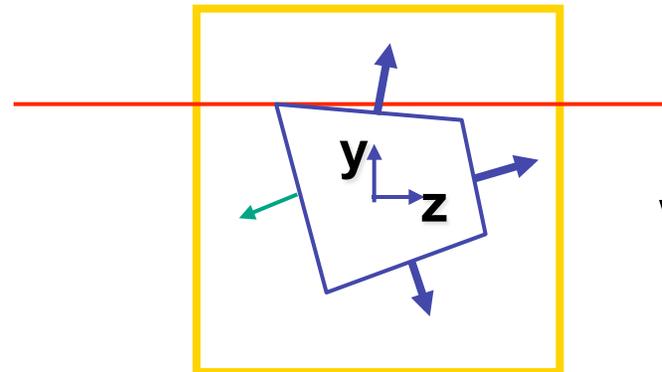
- perform visibility test for in screen coordinates
 - limited to resolution of display
 - Z-buffer: check every pixel independently
- performed late in rendering pipeline

Review: Back-face Culling

VCS



NDCS



eye

works to cull if $N_z > 0$

Review: Invisible Primitives

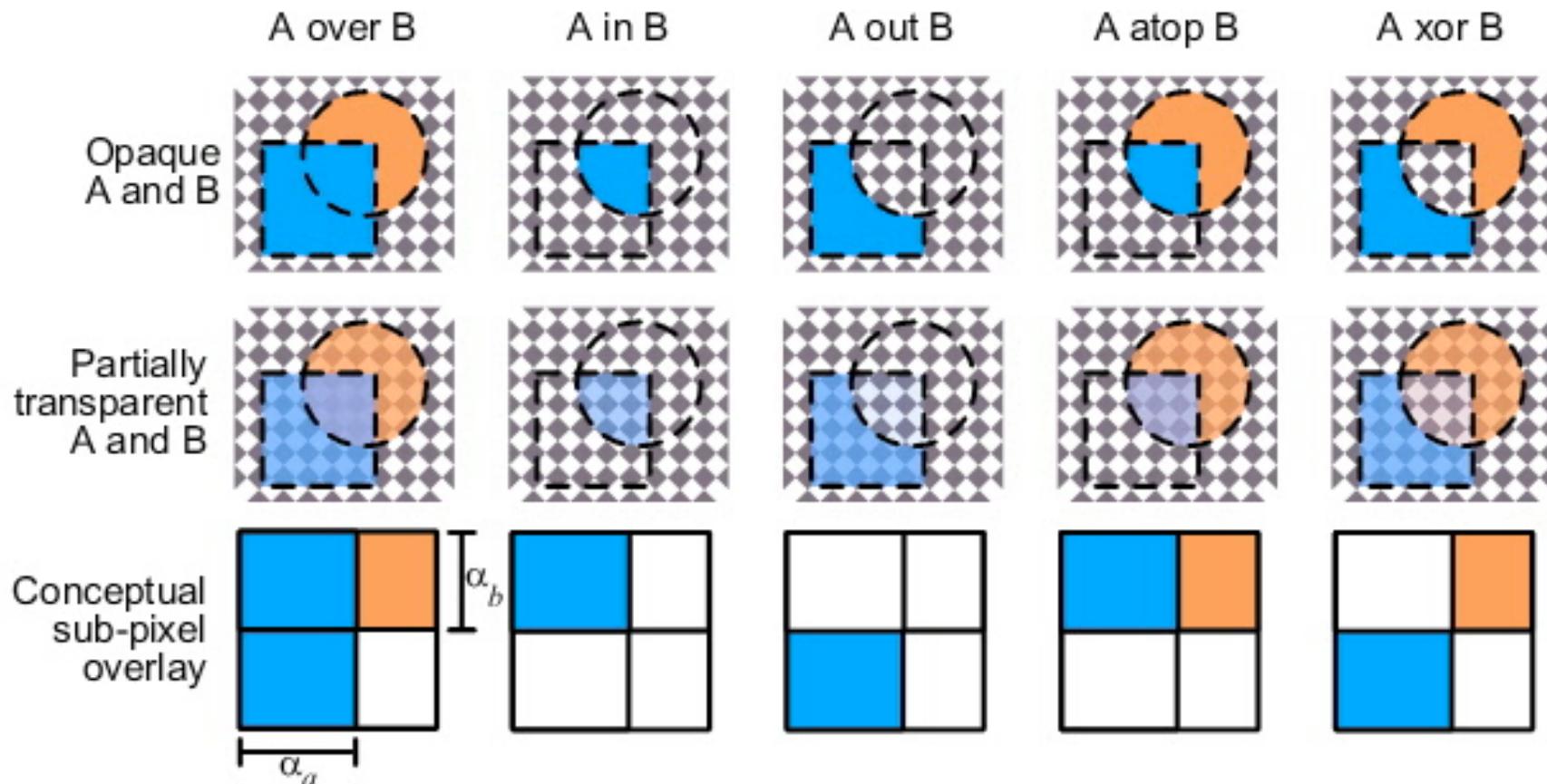
- *why might a polygon be invisible?*
 - polygon outside the *field of view / frustum*
 - solved by **clipping**
 - polygon is *backfacing*
 - solved by **backface culling**
 - polygon is *occluded* by object(s) nearer the viewpoint
 - solved by **hidden surface removal**

Review: Alpha and Premultiplication

- specify opacity with alpha channel α
 - $\alpha=1$: opaque, $\alpha=.5$: translucent, $\alpha=0$: transparent
- how to express a pixel is half covered by a red object?
 - obvious way: store color independent from transparency (r,g,b,α)
 - intuition: alpha as transparent colored glass
 - 100% transparency can be represented with many different RGB values
 - pixel value is $(1,0,0,.5)$
 - upside: easy to change opacity of image, very intuitive
 - downside: compositing calculations are more difficult - not associative
 - elegant way: premultiply by α so store $(\alpha r, \alpha g, \alpha b, \alpha)$
 - intuition: alpha as screen/mesh
 - RGB specifies how much color object contributes to scene
 - alpha specifies how much object obscures whatever is behind it (coverage)
 - alpha of .5 means half the pixel is covered by the color, half completely transparent
 - only one 4-tuple represents 100% transparency: $(0,0,0,0)$
 - pixel value is $(.5, 0, 0, .5)$
 - upside: compositing calculations easy (& additive blending for glowing!)
 - downside: less intuitive

Review: Complex Compositing

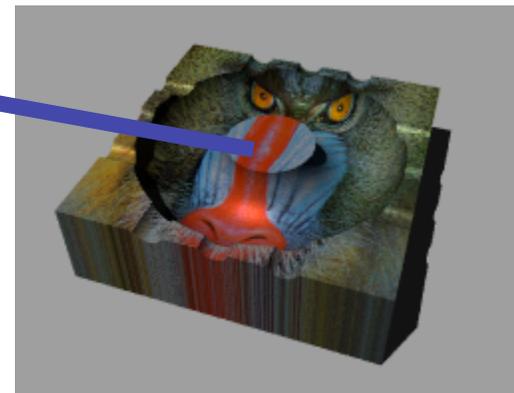
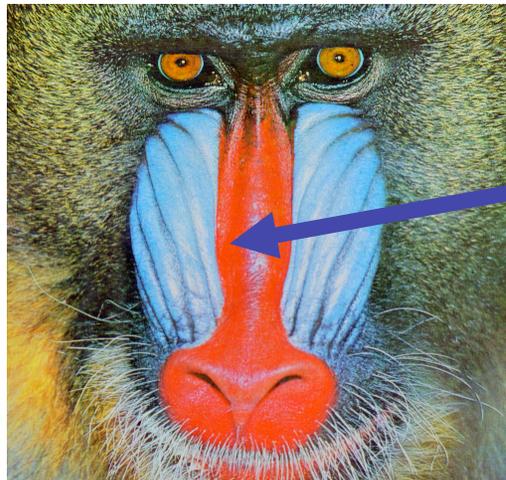
- foreground color **A**, background color **B**
- how might you combine multiple elements?
 - Compositing Digital Images, Porter and Duff, Siggraph '84
 - pre-multiplied alpha allows all cases to be handled simply



Review: Texture Coordinates

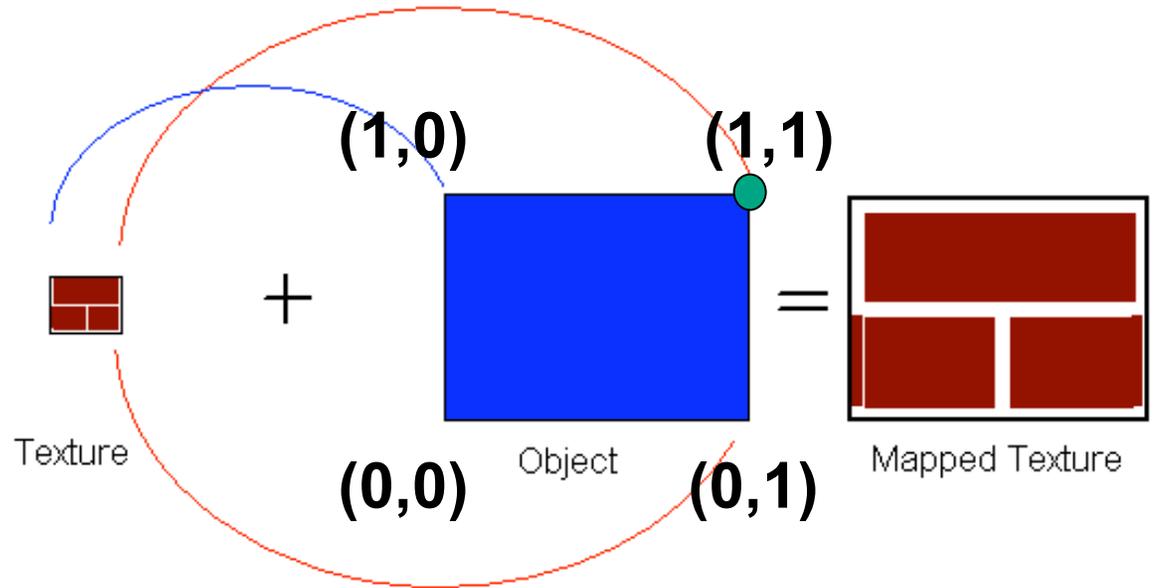
- texture image: 2D array of color values (**texels**)
- assigning **texture coordinates** (s,t) at vertex with object coordinates (x,y,z,w)
 - use interpolated (s,t) for texel lookup at each pixel
 - use value to modify a polygon's color
 - or other surface property
 - specified by programmer or artist

`glTexCoord2f (s , t)`
`glVertexf (x , y , z , w)`

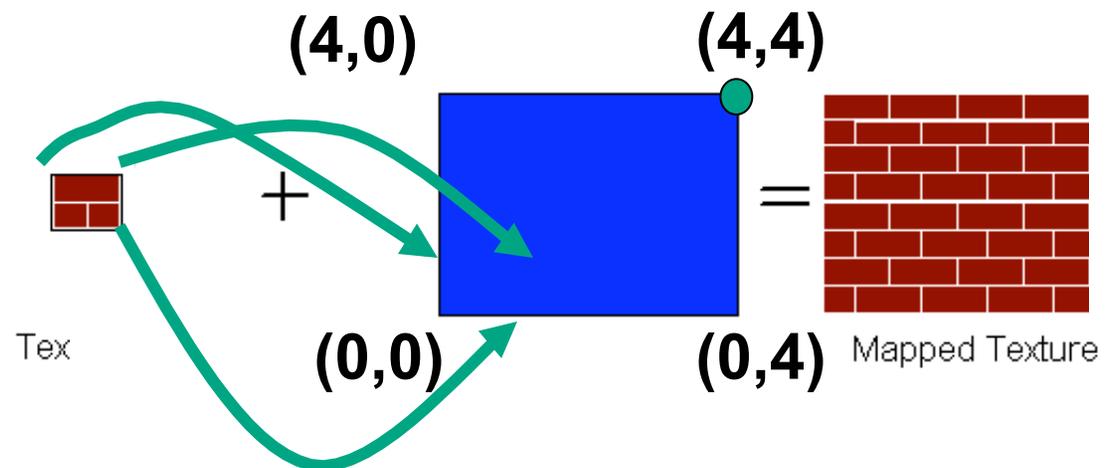


Review: Tiled Texture Map

```
glTexCoord2d(1, 1);  
glVertex3d (x, y, z);
```

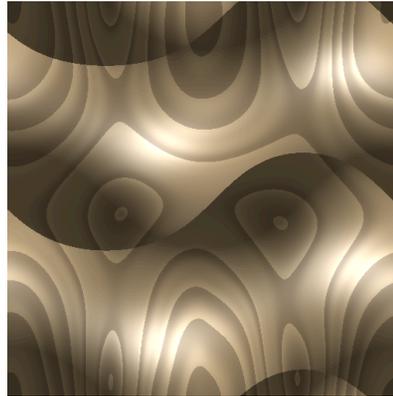


```
glTexCoord2d(4, 4);  
glVertex3d (x, y, z);
```



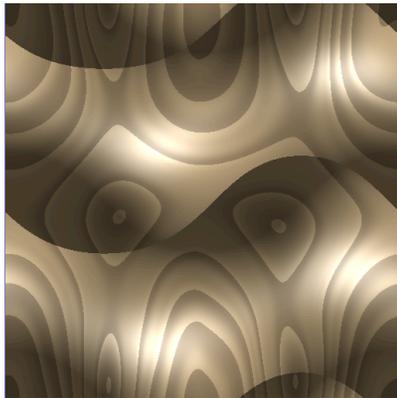
Review: Fractional Texture Coordinates

texture
image



$(0,1)$

$(1,1)$

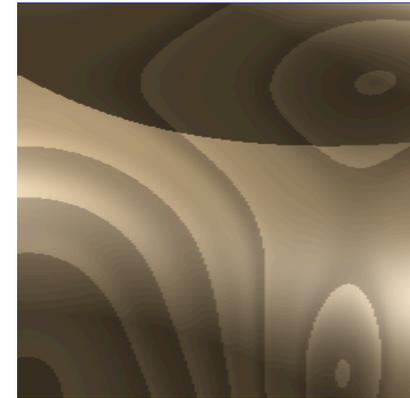


$(0,0)$

$(1,0)$

$(0,.5)$

$(.25,.5)$



$(0,0)$

$(.25,0)$

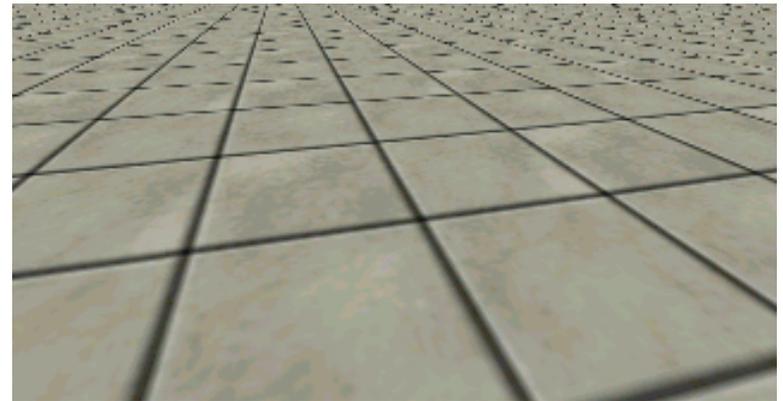
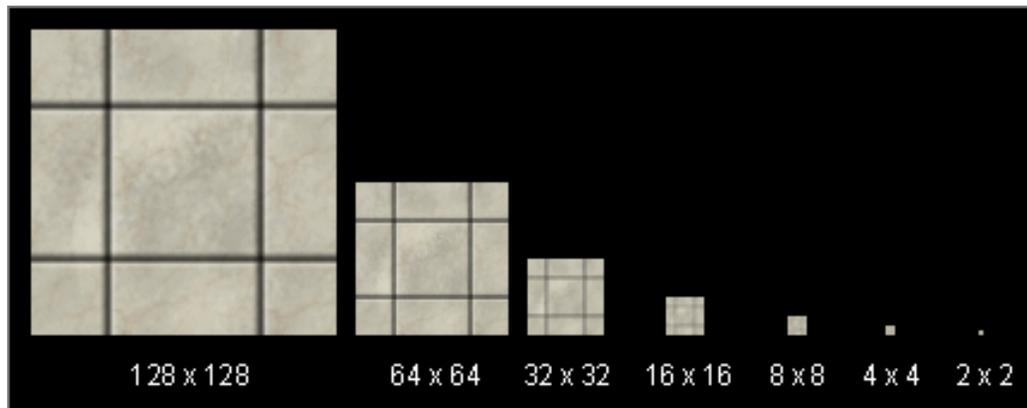
Review: Texture

- action when s or t is outside [0...1] interval
 - tiling
 - clamping
- functions
 - replace/decal
 - modulate
 - blend
- texture matrix stack

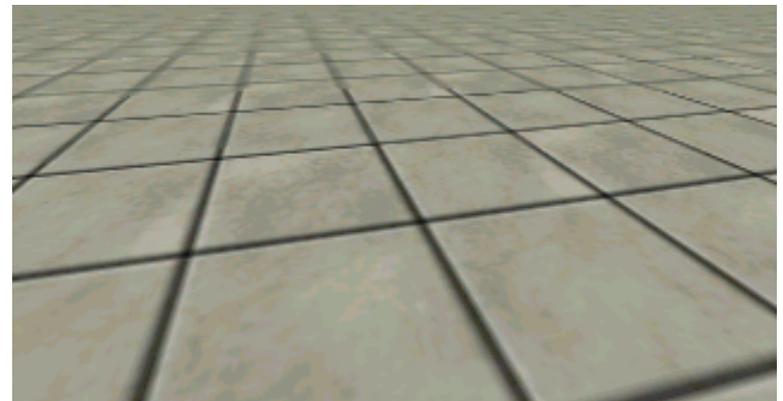
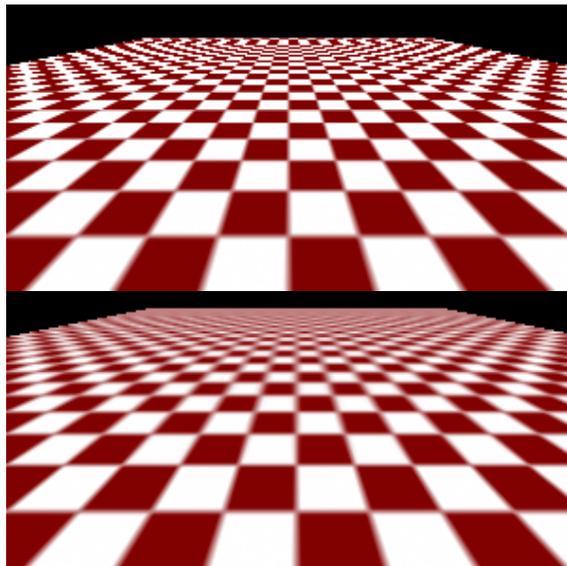
```
glMatrixMode ( GL_TEXTURE ) ;
```

Review: MIPmapping

- image pyramid, precompute averaged versions



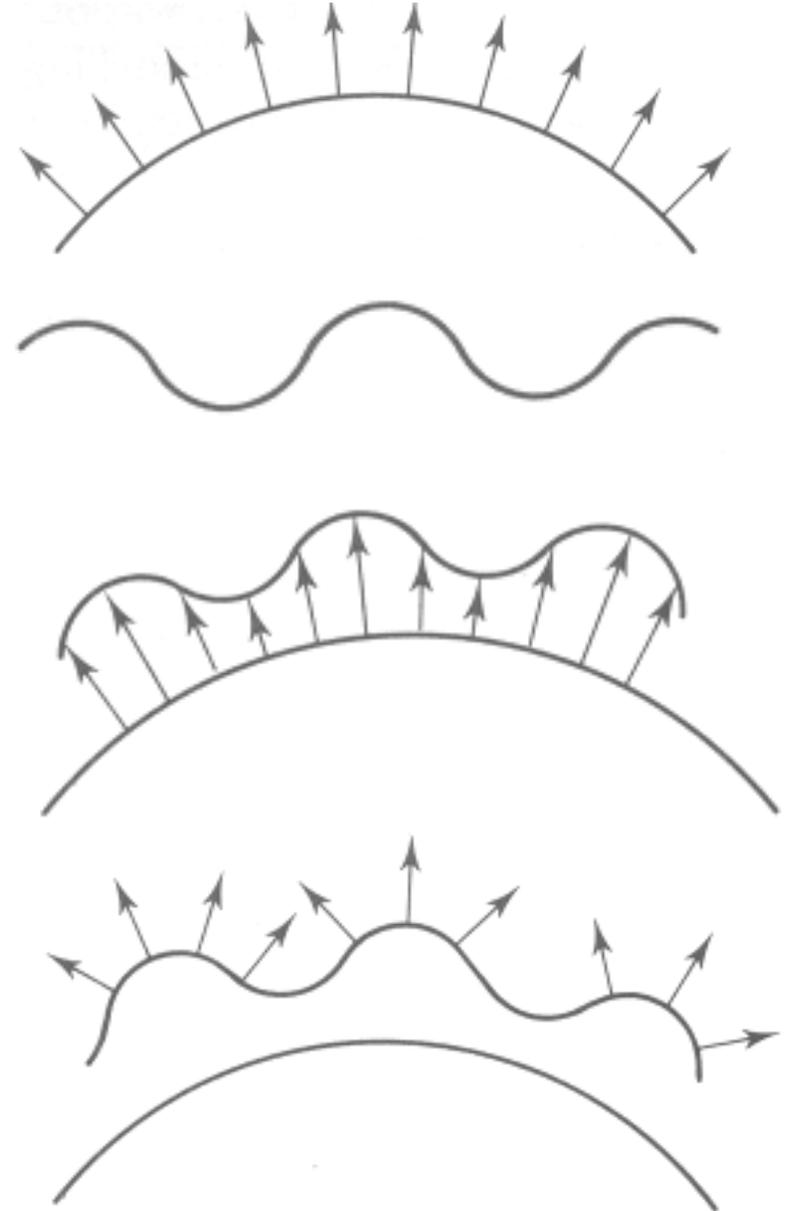
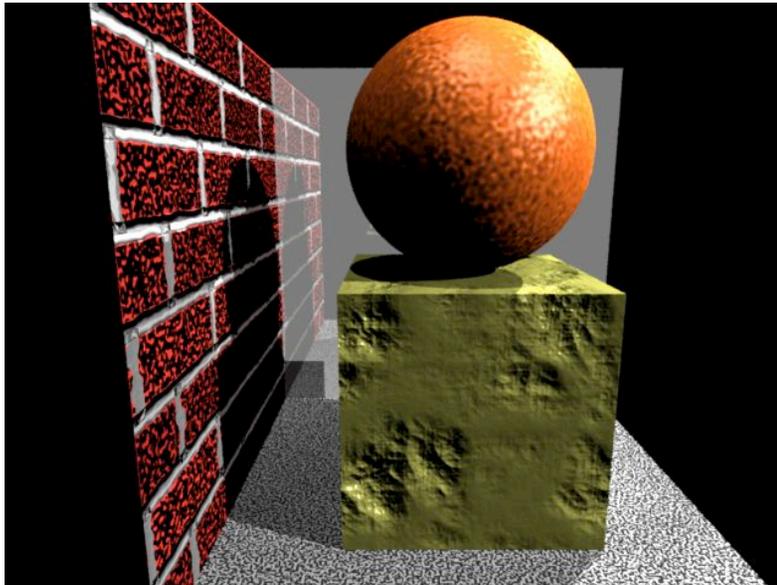
Without MIP-mapping



With MIP-mapping⁹⁷

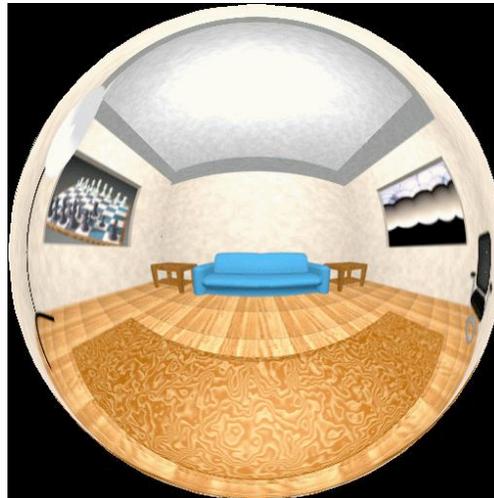
Review: Bump Mapping: Normals As Texture

- create illusion of complex geometry model
- control shape effect by locally perturbing surface normal



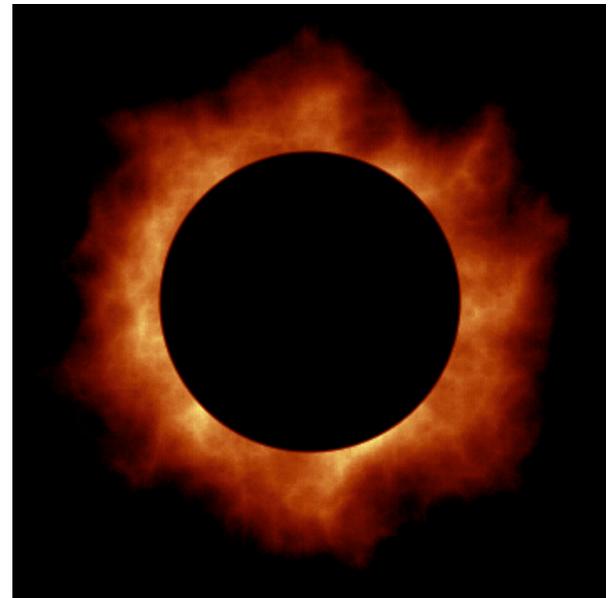
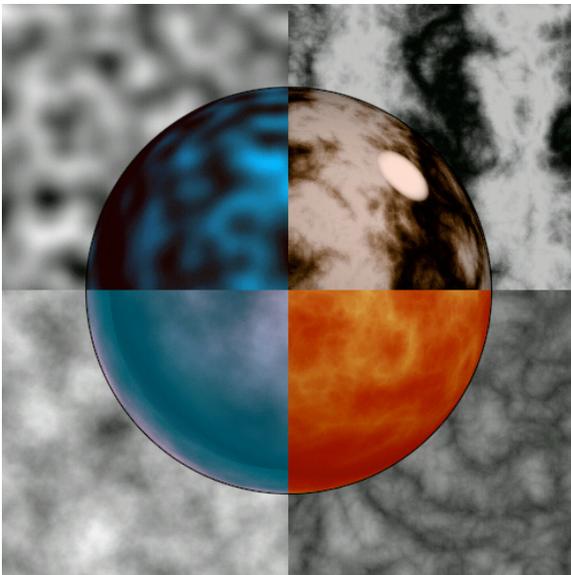
Review: Environment Mapping

- cheap way to achieve reflective effect
 - generate image of surrounding
 - map to object as texture
- sphere mapping: texture is distorted fisheye view
 - point camera at mirrored sphere
 - use spherical texture coordinates



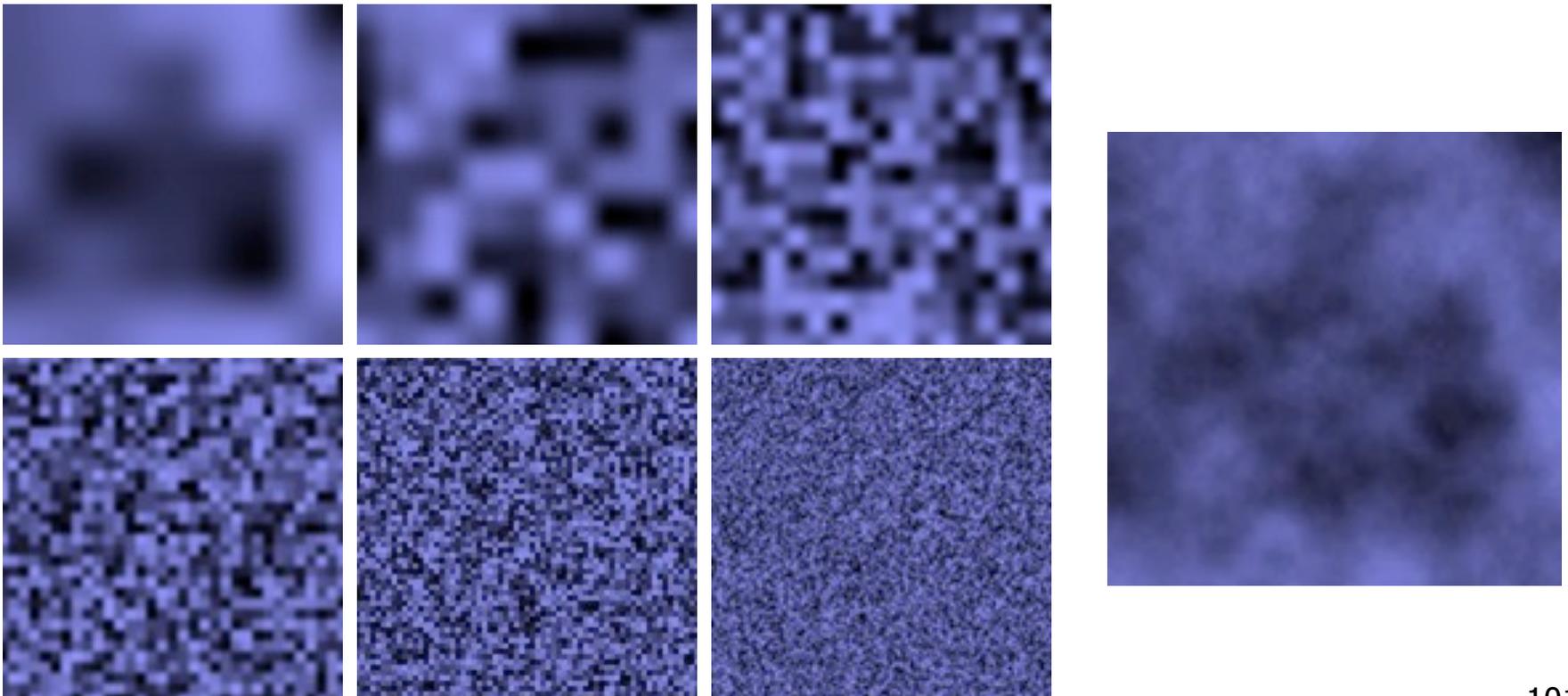
Review: Perlin Noise: Procedural Textures

```
function marble(point)
  x = point.x + turbulence(point);
  return marble_color(sin(x))
```



Review: Perlin Noise

- coherency: smooth not abrupt changes
- turbulence: multiple feature sizes

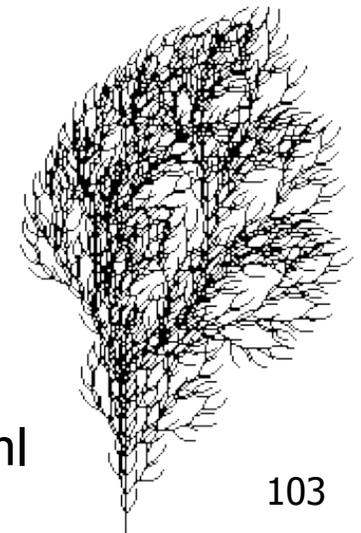
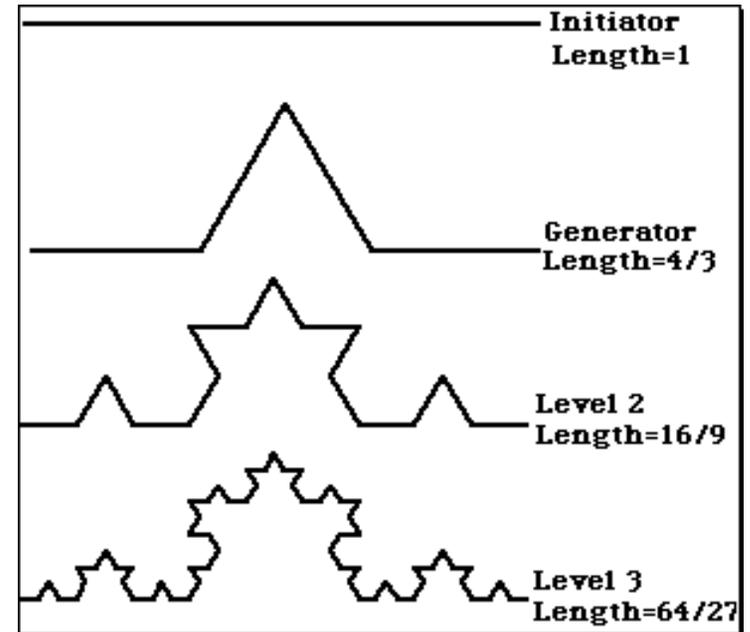


Review: Procedural Modeling

- textures, geometry
 - nonprocedural: explicitly stored in memory
- procedural approach
 - compute something on the fly
 - not load from disk
 - often less memory cost
 - visual richness
 - adaptable precision
- noise, fractals, particle systems

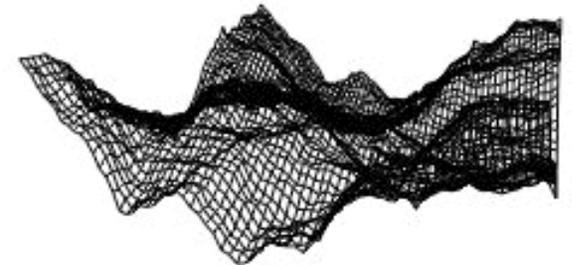
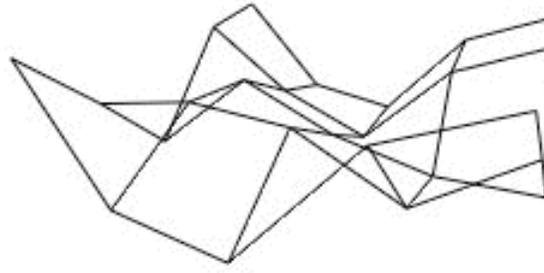
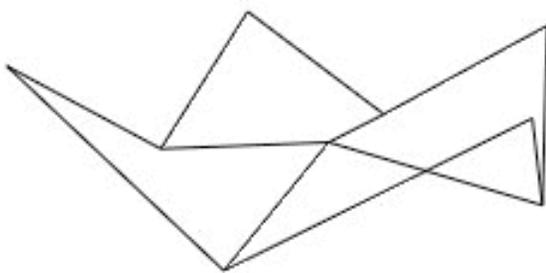
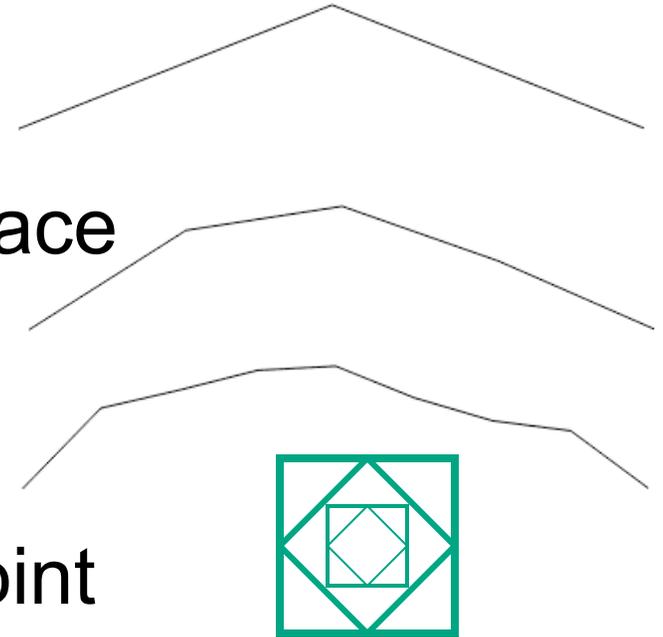
Review: Language-Based Generation

- L-Systems
 - F: forward, R: right, L: left
 - Koch snowflake:
 $F = FLFRRFLF$
 - Mariano's Bush:
 $F = FF - [-F + F + F] + [+F - F - F]$
 - angle 16



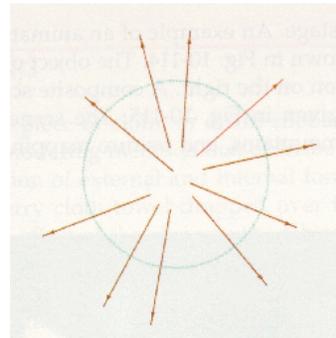
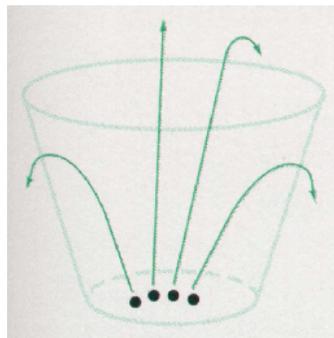
Review: Fractal Terrain

- 1D: midpoint displacement
 - divide in half, randomly displace
 - scale variance by half
- 2D: diamond-square
 - generate new value at midpoint
 - average corner values + random displacement
 - scale variance by half each time



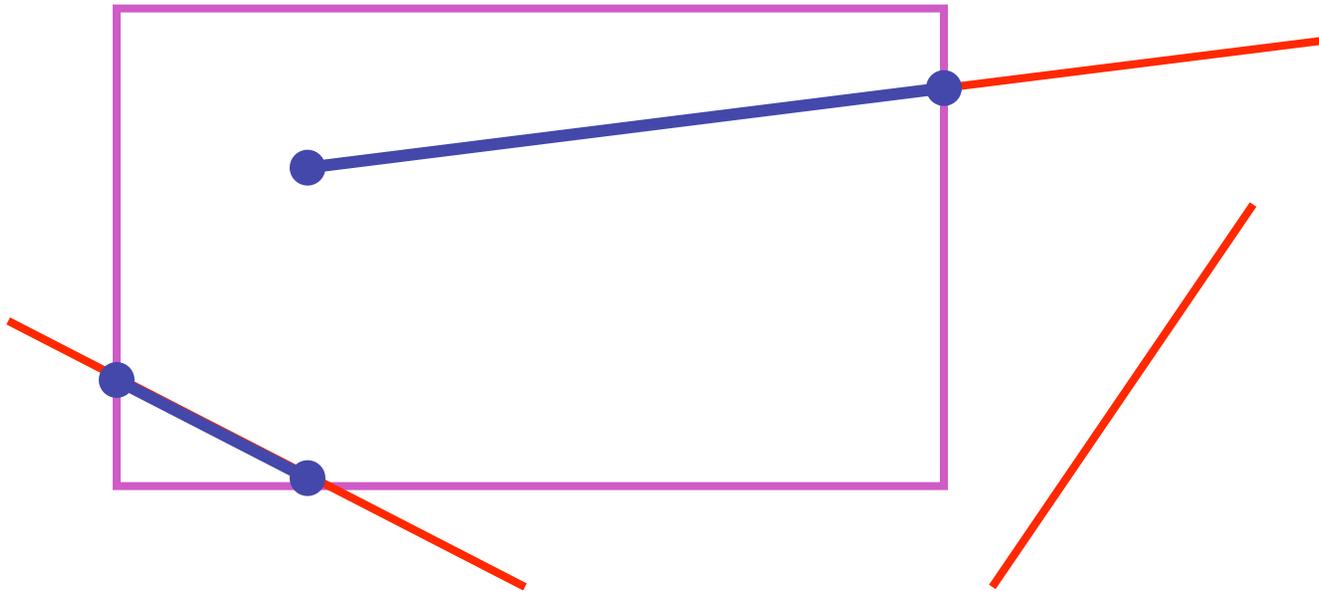
Review: Particle Systems

- changeable/fluid stuff
 - fire, steam, smoke, water, grass, hair, dust, waterfalls, fireworks, explosions, flocks
- life cycle
 - generation, dynamics, death
- rendering tricks
 - avoid hidden surface computations



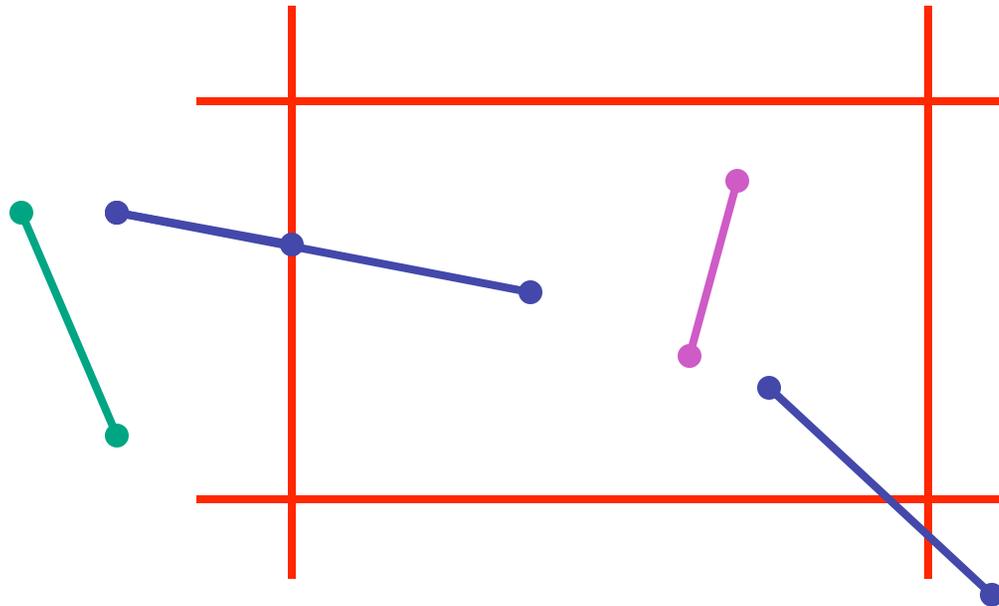
Review: Clipping

- analytically calculating the portions of primitives within the viewport



Review: Clipping Lines To Viewport

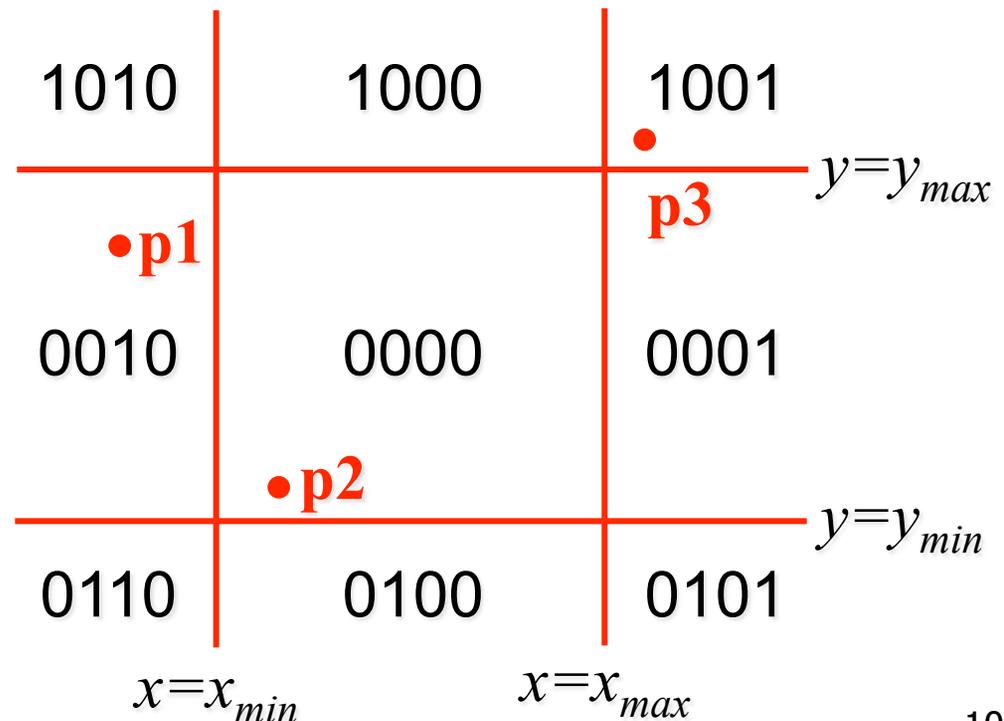
- combining trivial accepts/rejects
 - trivially **accept** lines with both endpoints **inside all edges of the viewport**
 - trivially **reject** lines with both endpoints **outside the same edge of the viewport**
 - otherwise, reduce to trivial cases by **splitting into two segments**



Review: Cohen-Sutherland Line Clipping

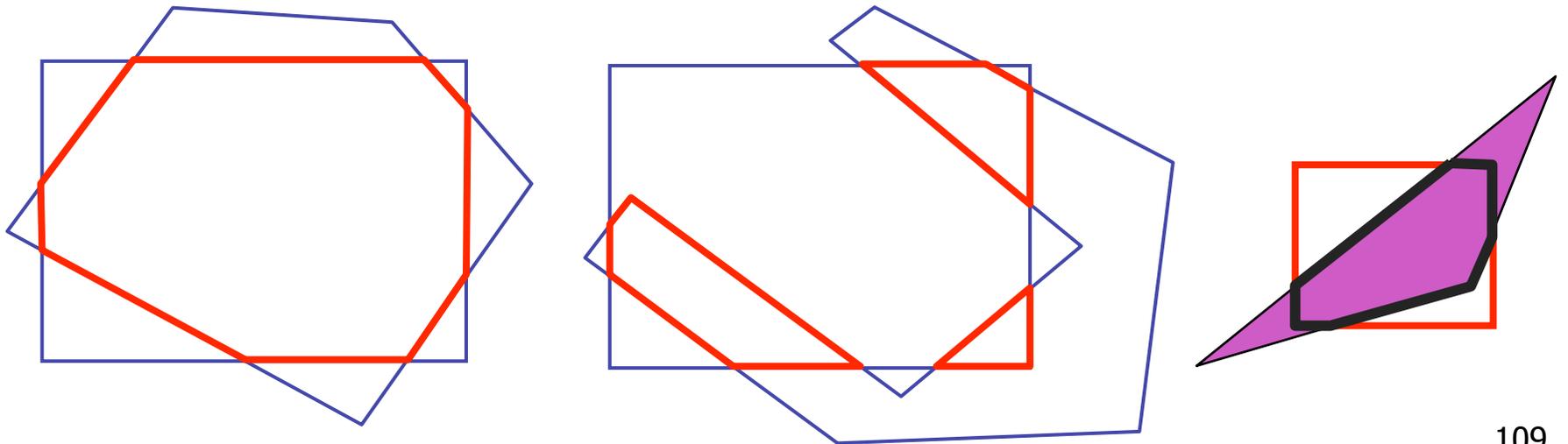
- outcodes
 - 4 flags encoding position of a point relative to top, bottom, left, and right boundary

- $OC(p1) == 0 \ \&\&$
 $OC(p2) == 0$
 - trivial accept
- $(OC(p1) \ \&$
 $OC(p2)) \neq 0$
 - trivial reject



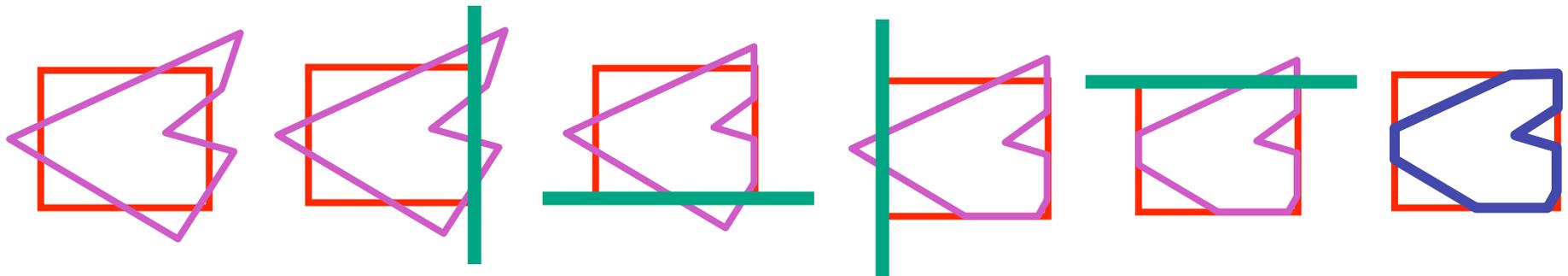
Review: Polygon Clipping

- not just clipping all boundary lines
 - may have to introduce new line segments



Review: Sutherland-Hodgeman Clipping

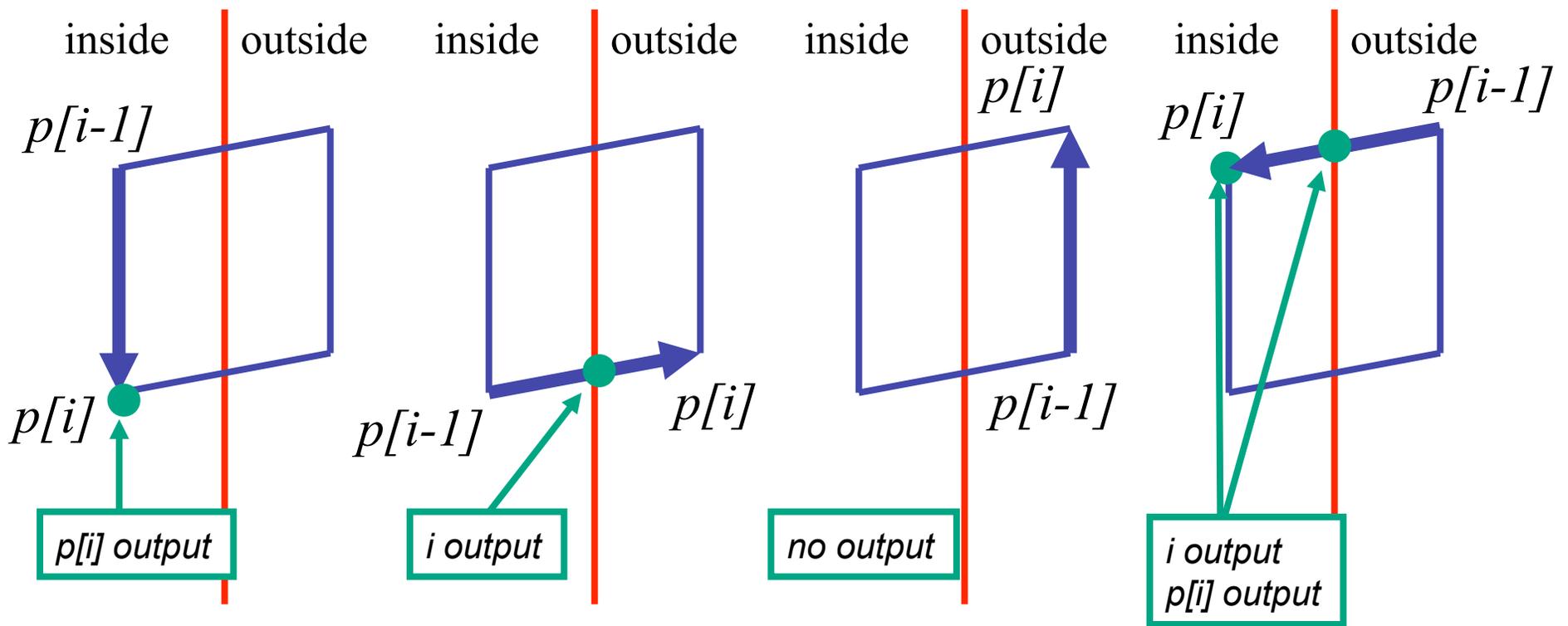
- for each viewport edge
 - clip the polygon against the edge equation for new vertex list
 - after doing all edges, the polygon is fully clipped



- for each polygon vertex
 - decide what to do based on 4 possibilities
 - is vertex inside or outside?
 - is previous vertex inside or outside?

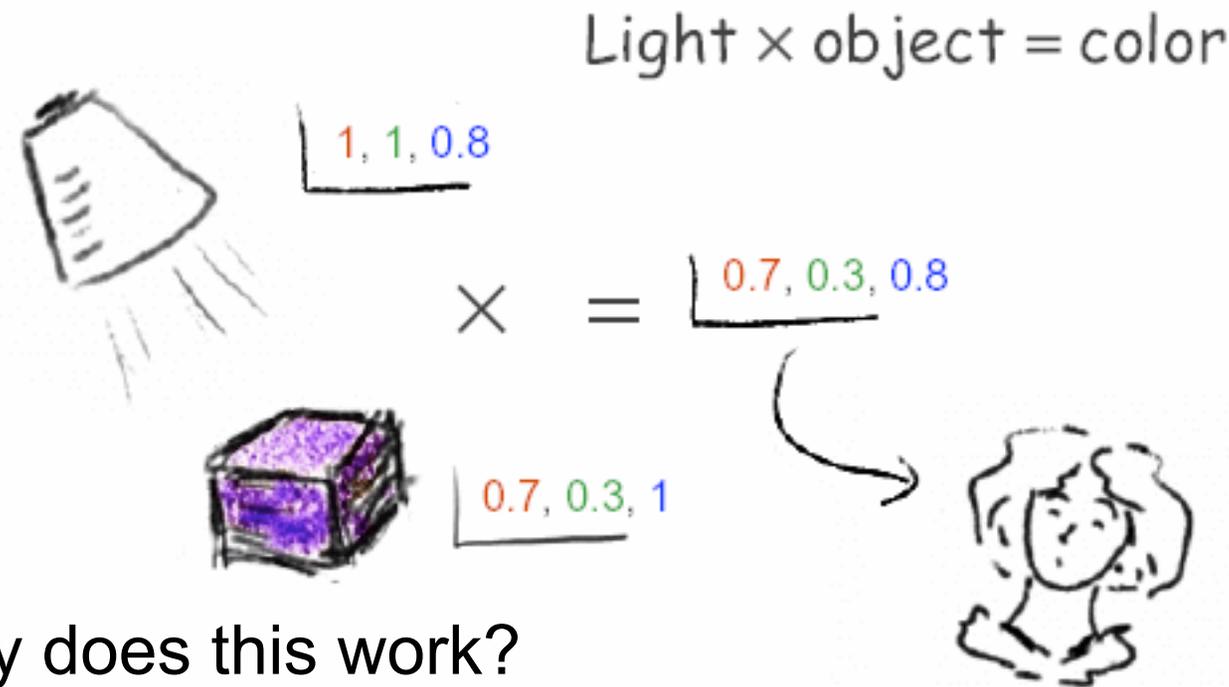
Review: Sutherland-Hodgeman Clipping

- edge from $p[i-1]$ to $p[i]$ has four cases
 - decide what to add to output vertex list



Review: RGB Component Color

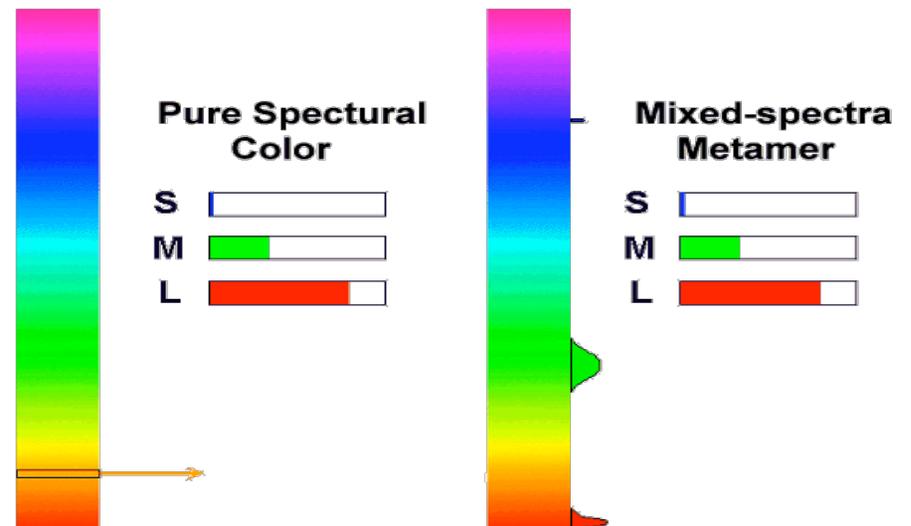
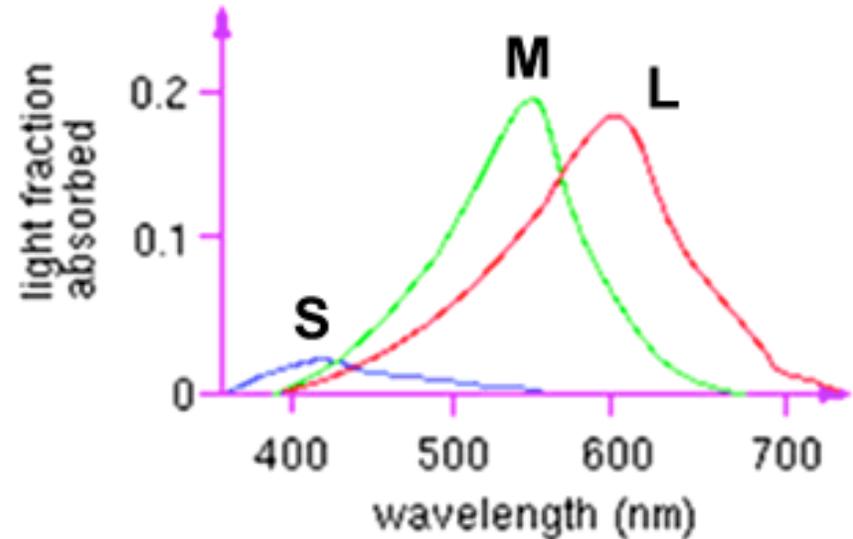
- simple model of color using RGB triples
- component-wise multiplication
 - $(a_0, a_1, a_2) * (b_0, b_1, b_2) = (a_0 * b_0, a_1 * b_1, a_2 * b_2)$



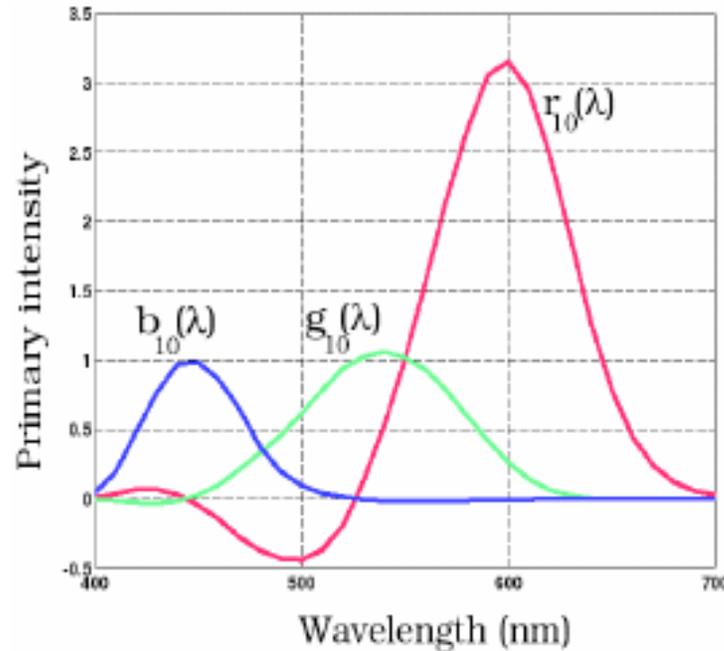
- why does this work?
 - must dive into light, human vision, color spaces

Review: Trichromacy and Metamers

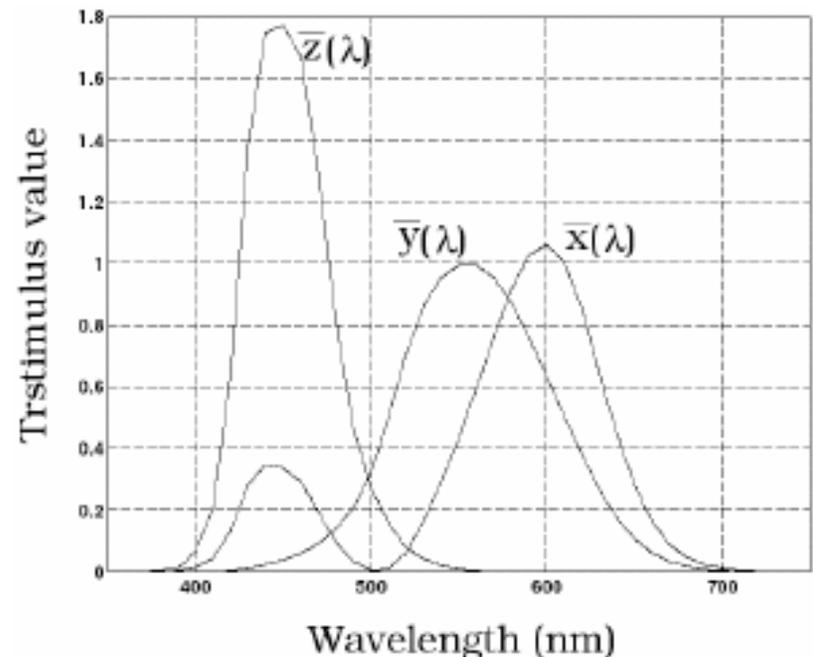
- three types of cones
- color is combination of cone stimuli
 - metamer: identically perceived color caused by very different spectra



Review: Measured vs. CIE Color Spaces



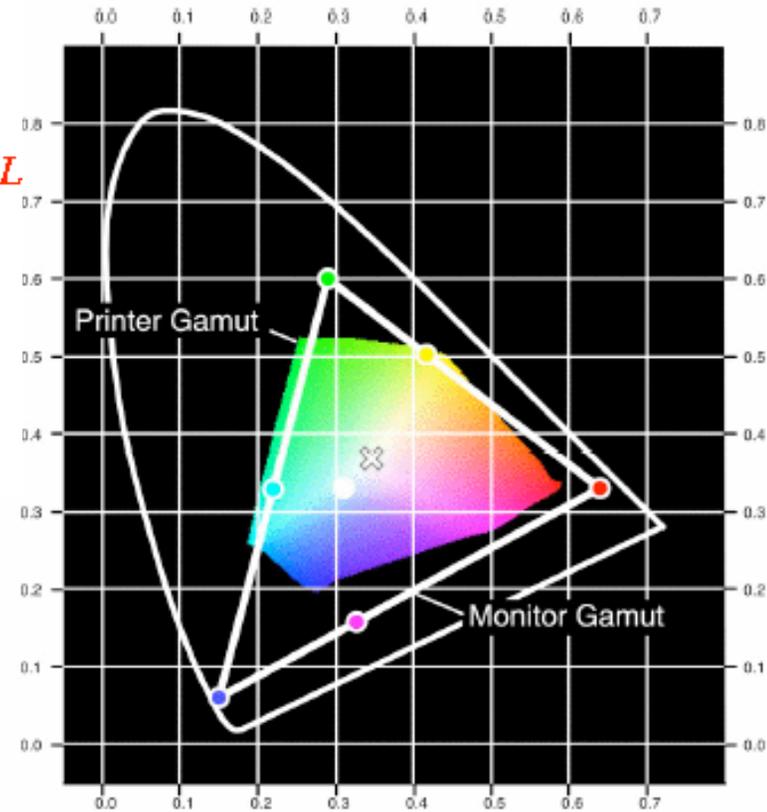
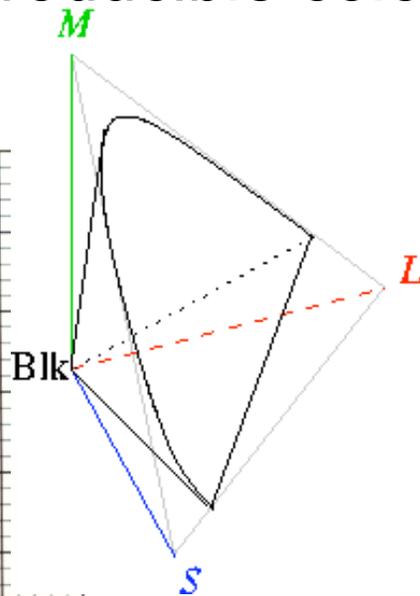
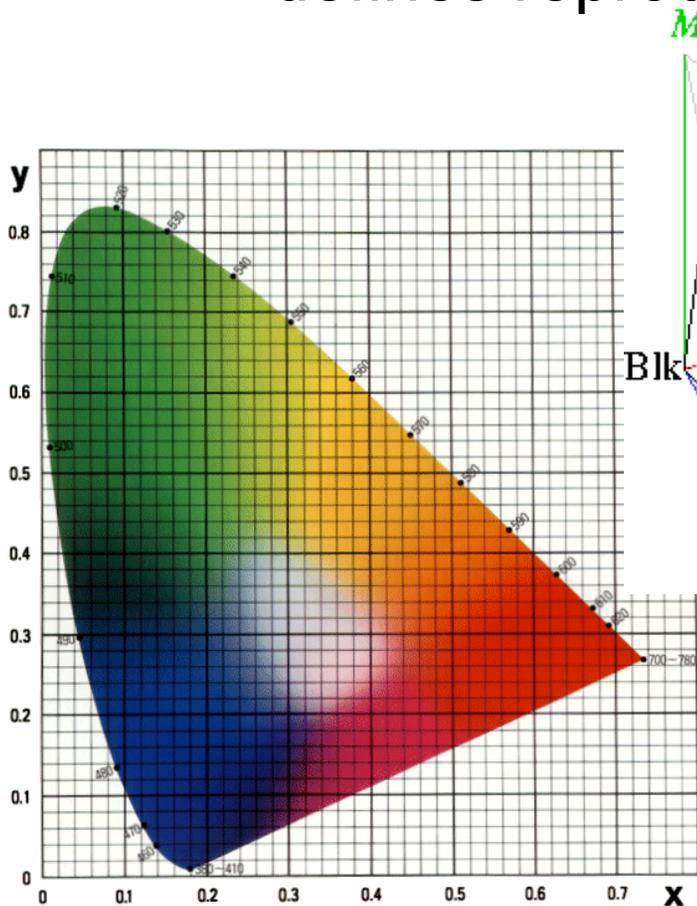
- measured basis
 - monochromatic lights
 - physical observations
 - negative lobes



- transformed basis
 - “imaginary” lights
 - all positive, unit area
 - Y is luminance

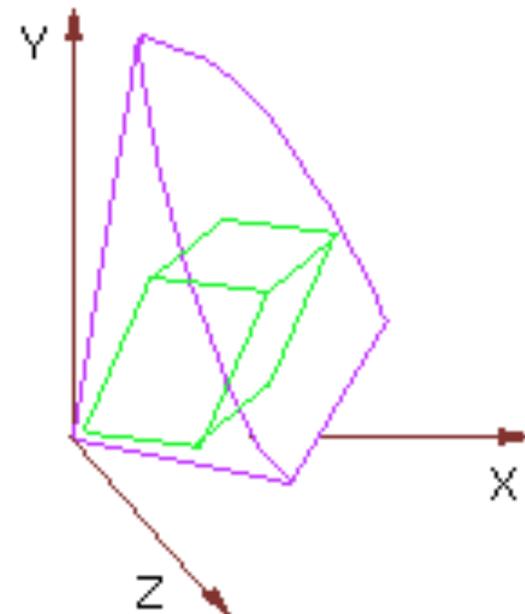
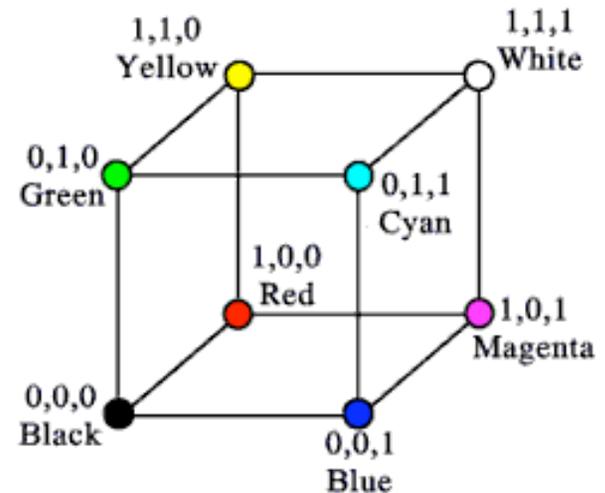
Review: Chromaticity Diagram and Gamuts

- plane of equal brightness showing chromaticity
- gamut is polygon, device primaries at corners
 - defines reproducible color range



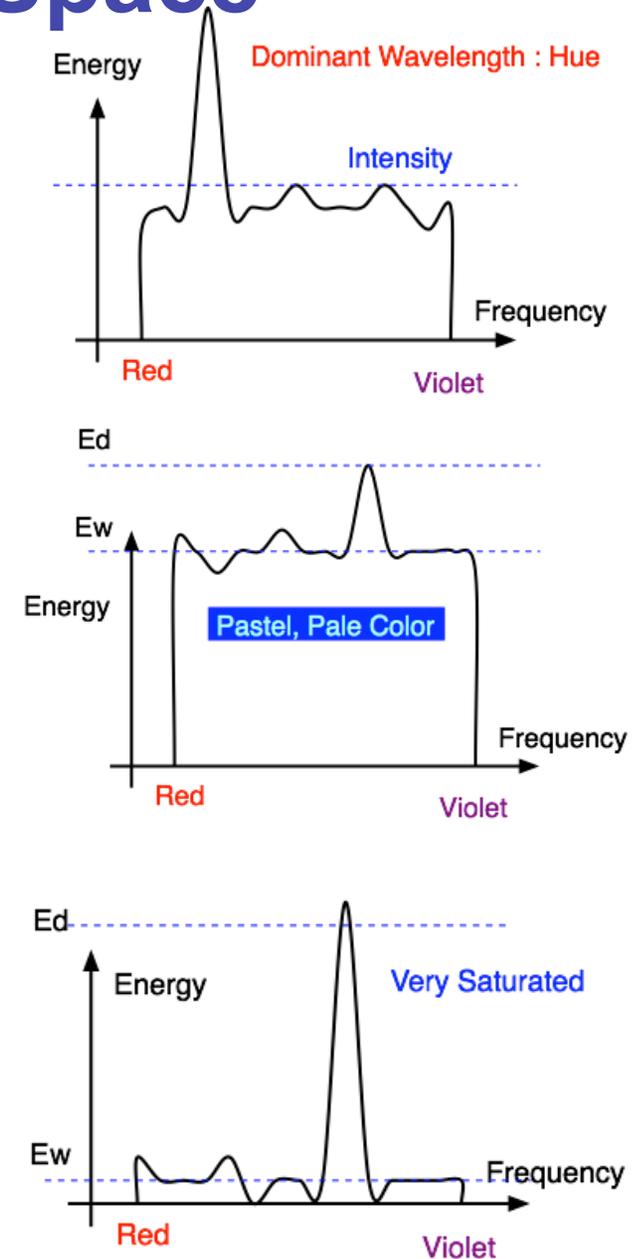
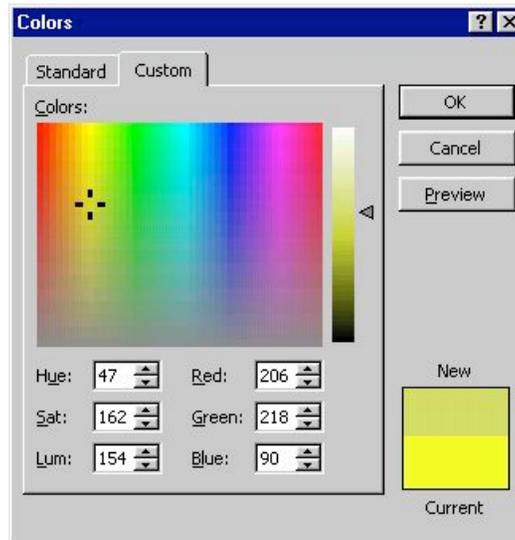
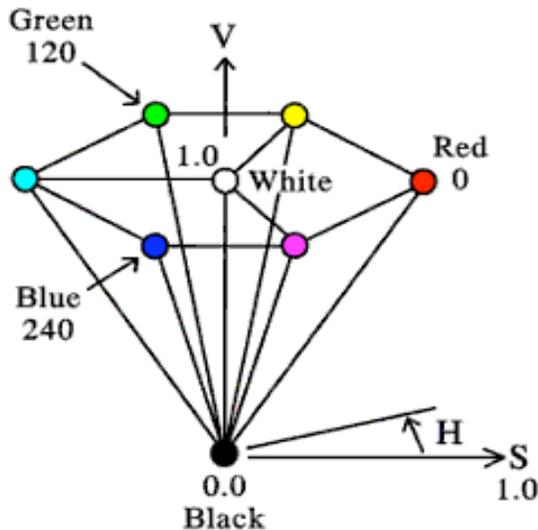
Review: RGB Color Space (Color Cube)

- define colors with (r, g, b) amounts of red, green, and blue
 - used by OpenGL
 - hardware-centric
- RGB color cube sits within CIE color space
 - subset of perceivable colors
 - scale, rotate, shear cube



Review: HSV Color Space

- hue: dominant wavelength, “color”
- saturation: how far from grey
- value/brightness: how far from black/white
- cannot convert to RGB with matrix alone



Review: HSI/HSV and RGB

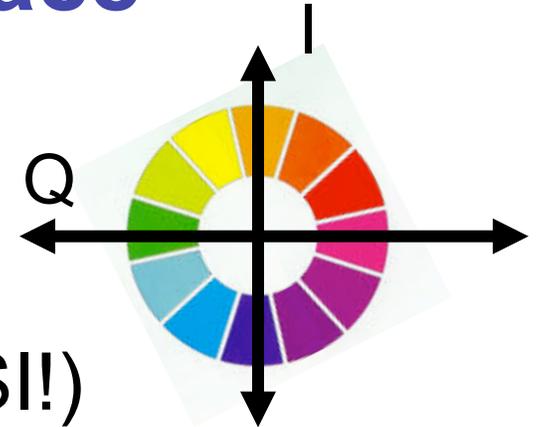
- HSV/HSI conversion from RGB
 - hue same in both
 - value is max, intensity is average

$$H = \cos^{-1} \left[\frac{\frac{1}{2} [(R - G) + (R - B)]}{\sqrt{(R - G)^2 + (R - B)(G - B)}} \right] \begin{array}{l} \text{if } (B > G), \\ H = 360 - H \end{array}$$

$$\bullet \text{HSI: } S = 1 - \frac{\min(R, G, B)}{I} \quad I = \frac{R + G + B}{3}$$

$$\bullet \text{HSV: } S = 1 - \frac{\min(R, G, B)}{V} \quad V = \max(R, G, B)$$

Review: YIQ Color Space



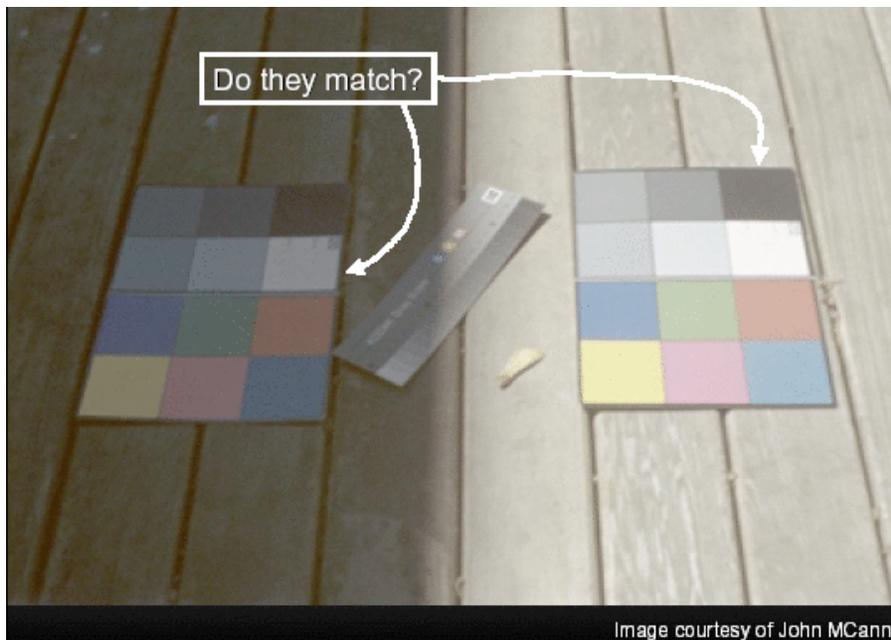
- color model used for color TV
 - Y is luminance (same as CIE)
 - I & Q are color (not same I as HSI!)
 - using Y backwards compatible for B/W TVs
 - conversion from RGB is linear

$$\begin{bmatrix} Y \\ I \\ Q \end{bmatrix} = \begin{bmatrix} 0.30 & 0.59 & 0.11 \\ 0.60 & -0.28 & -0.32 \\ 0.21 & -0.52 & 0.31 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

- green is much lighter than red, and red lighter than blue

Review: Color Constancy

- automatic “white balance” from change in illumination
- vast amount of processing behind the scenes!
- colorimetry vs. perception

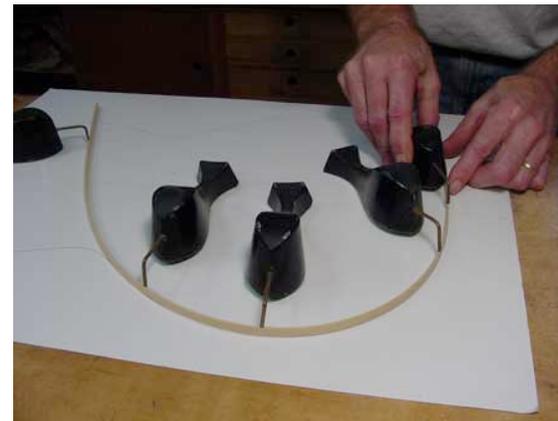


Review: Splines

- *spline* is parametric curve defined by *control points*
 - *knots*: control points that lie on curve
 - engineering drawing: spline was flexible wood, control points were physical weights



A Duck (weight)



Ducks trace out curve

Review: Hermite Spline

- user provides
 - endpoints
 - derivatives at endpoints

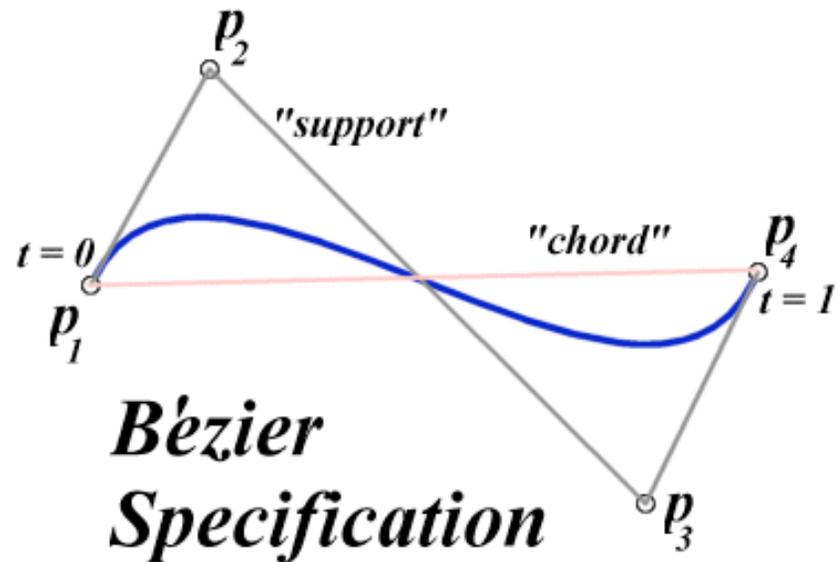


Review: Bézier Curves

- four control points, two of which are knots
 - more intuitive definition than derivatives
- curve will always remain within convex hull (bounding region) defined by control points



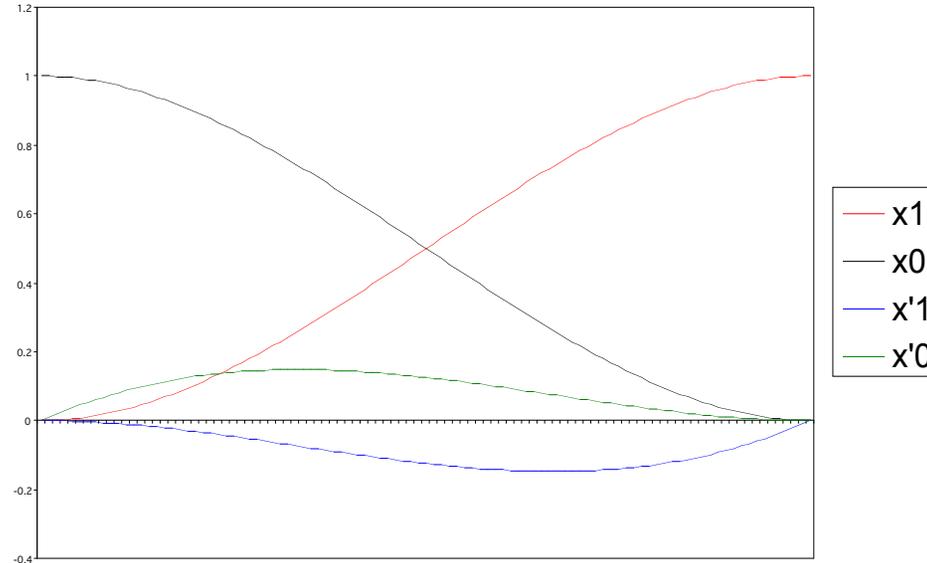
Hermite Specification



Bézier Specification

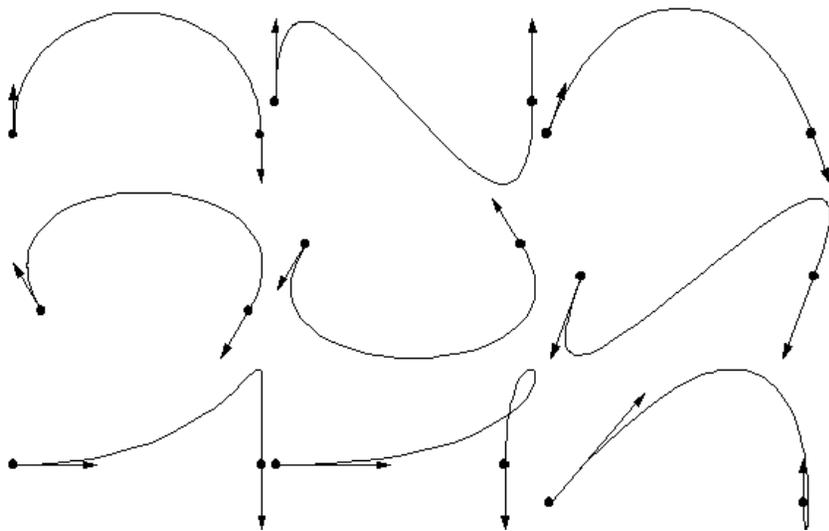
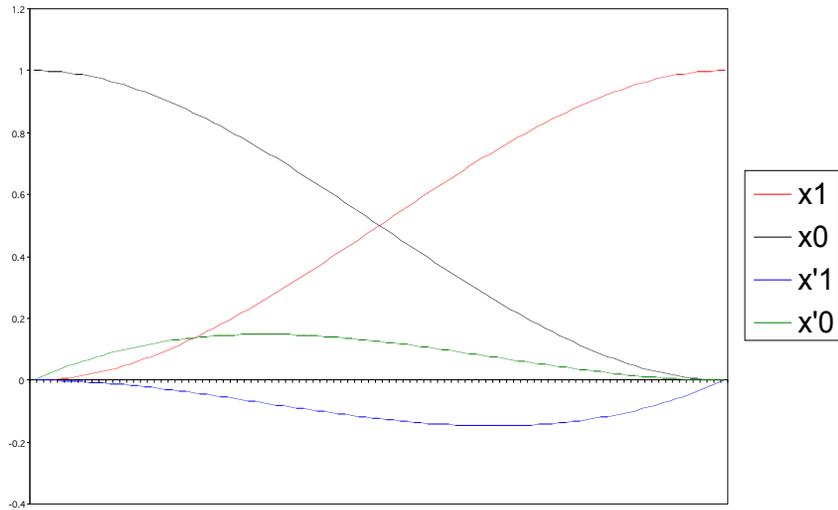
Review: Basis Functions

- point on curve obtained by multiplying each control point by some **basis function** and summing

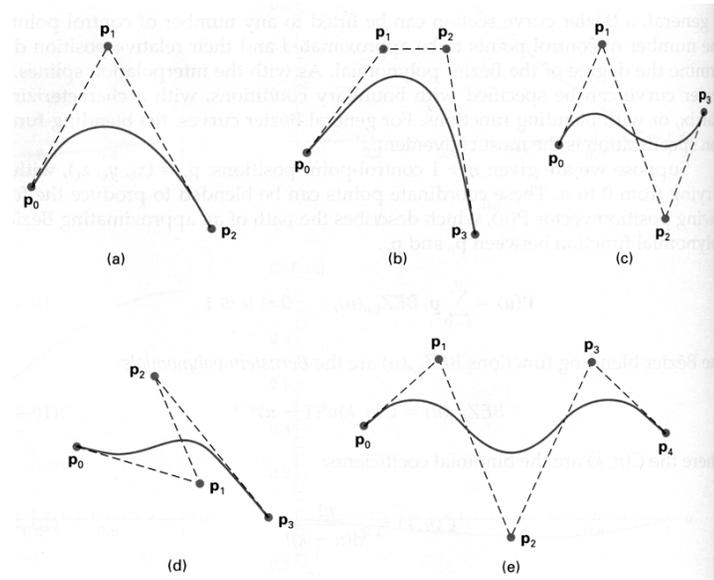
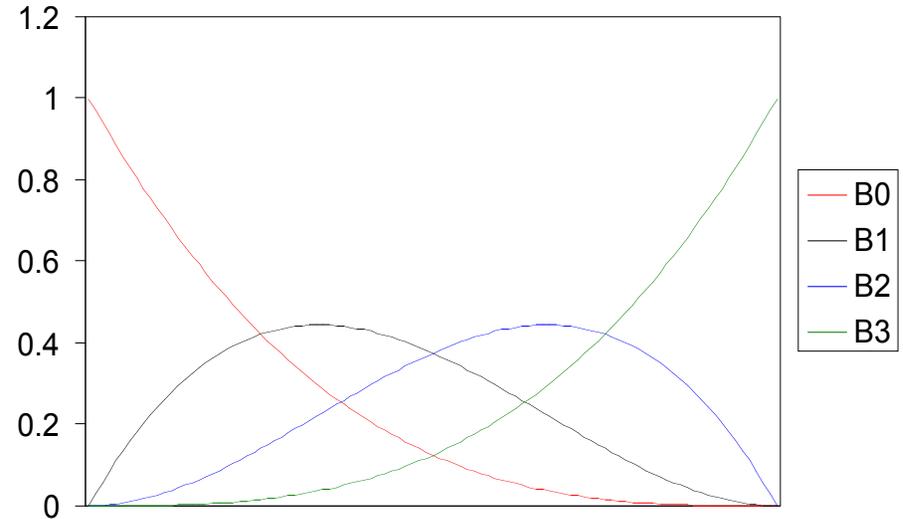


Review: Comparing Hermite and Bézier

Hermite

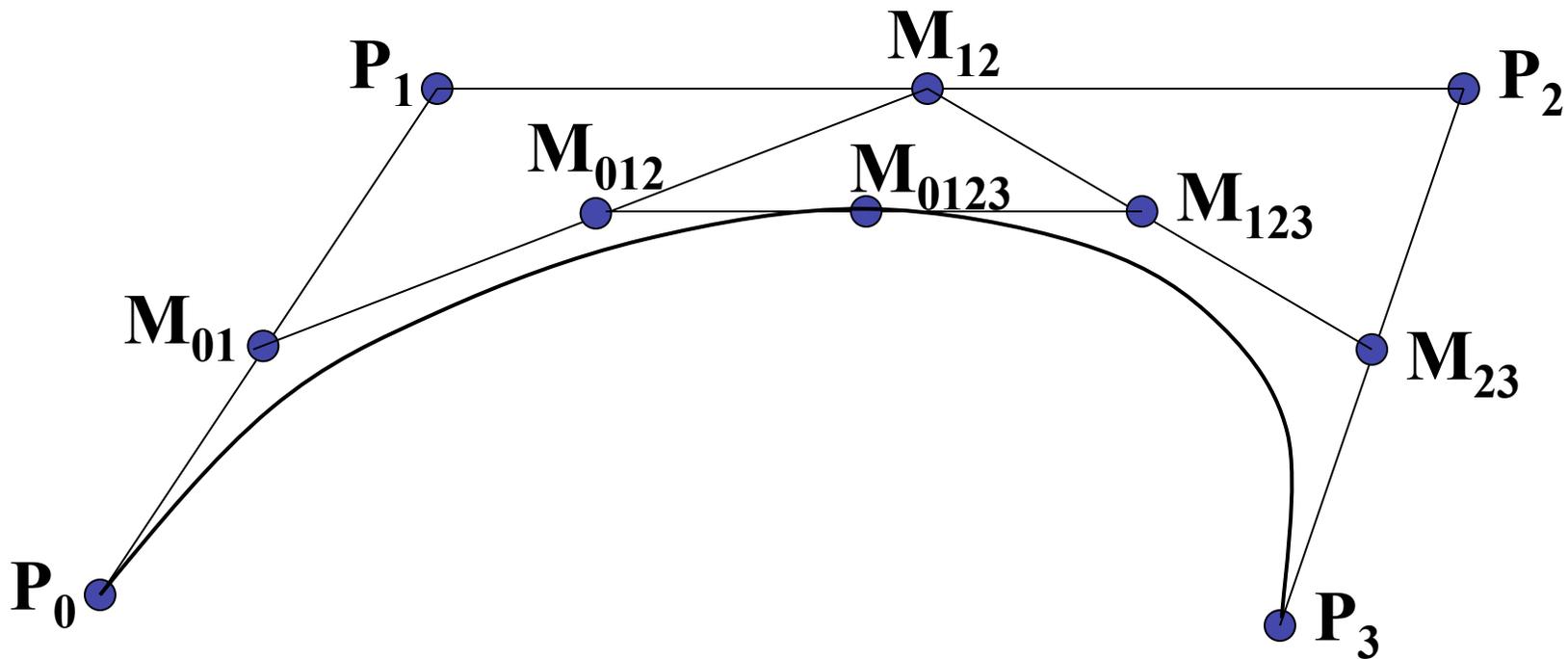


Bézier



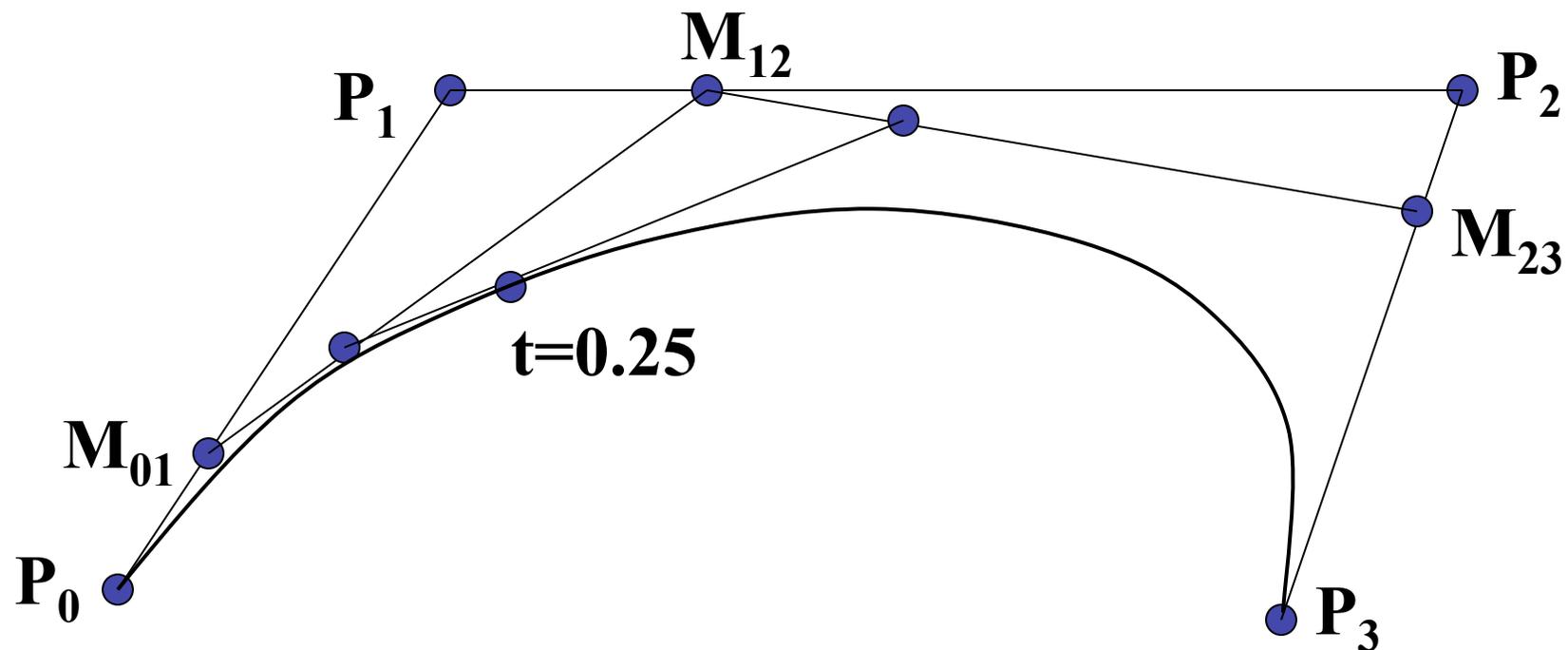
Review: Sub-Dividing Bézier Curves

- find the midpoint of the line joining M_{012} , M_{123} . call it M_{0123}



Review: de Casteljau's Algorithm

- can find the point on Bézier curve for any parameter value t with similar algorithm
 - for $t=0.25$, instead of taking midpoints take points 0.25 of the way



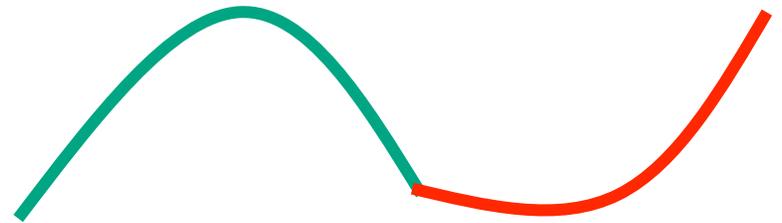
demo: www.saltire.com/applets/advanced_geometry/spline/spline.htm

Review: Continuity

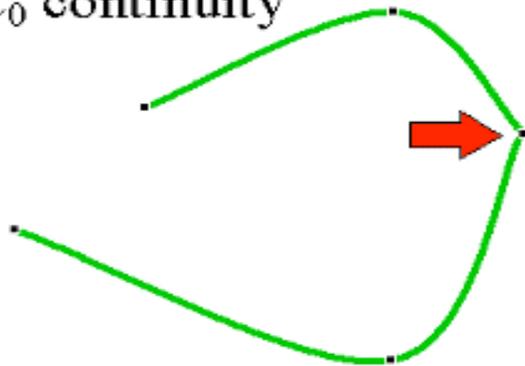
- piecewise Bézier: no continuity guarantees

- continuity definitions

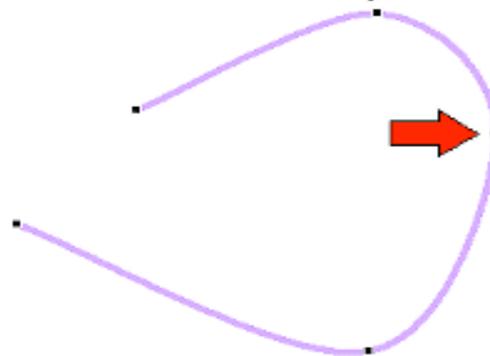
- C^0 : share join point
- C^1 : share continuous derivatives
- C^2 : share continuous second derivatives



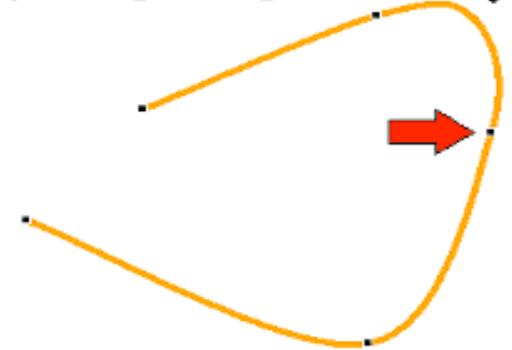
C_0 continuity



C_0 & C_1 continuity

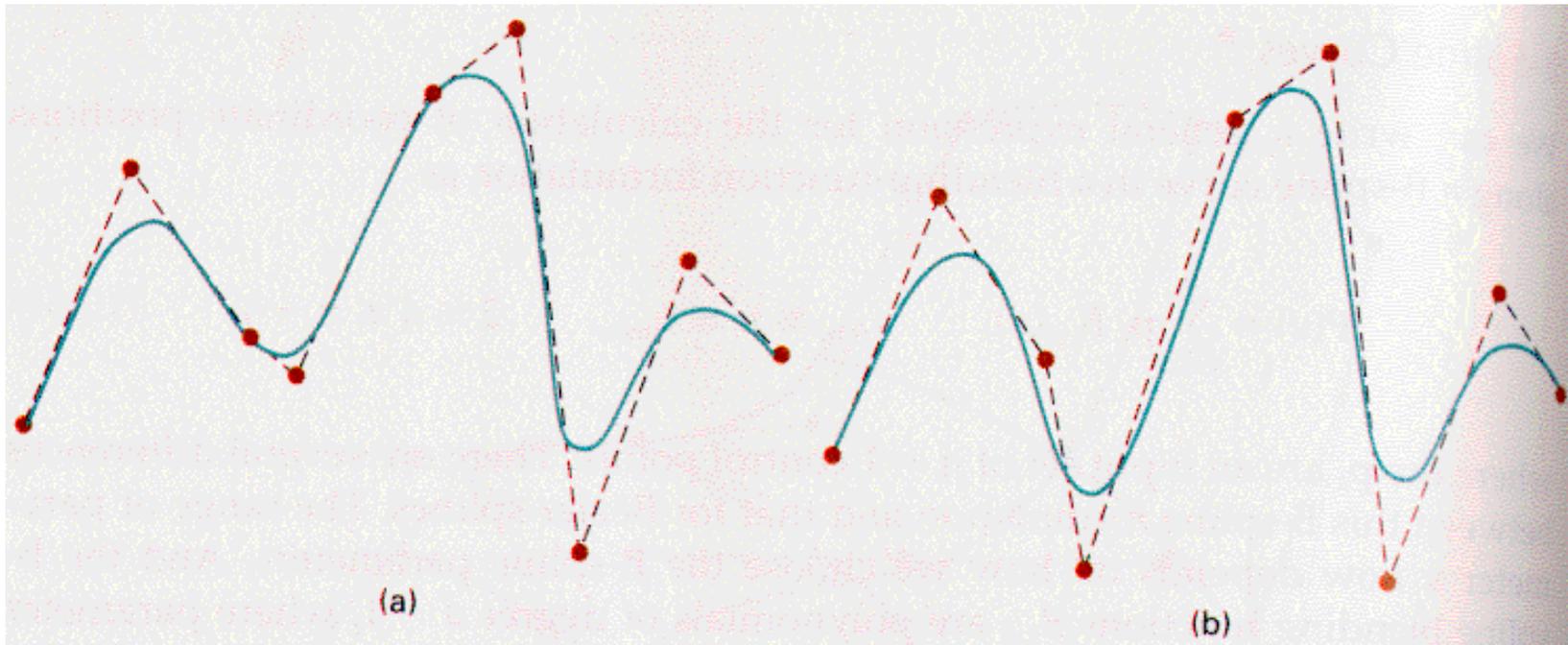


C_0 & C_1 & C_2 continuity



Review: B-Spline

- C_0 , C_1 , and C_2 continuous
- piecewise: locality of control point influence



Review: Visual Encoding

marks: geometric primitives

points lines areas

attributes

position



size



grey level



texture



color



orientation



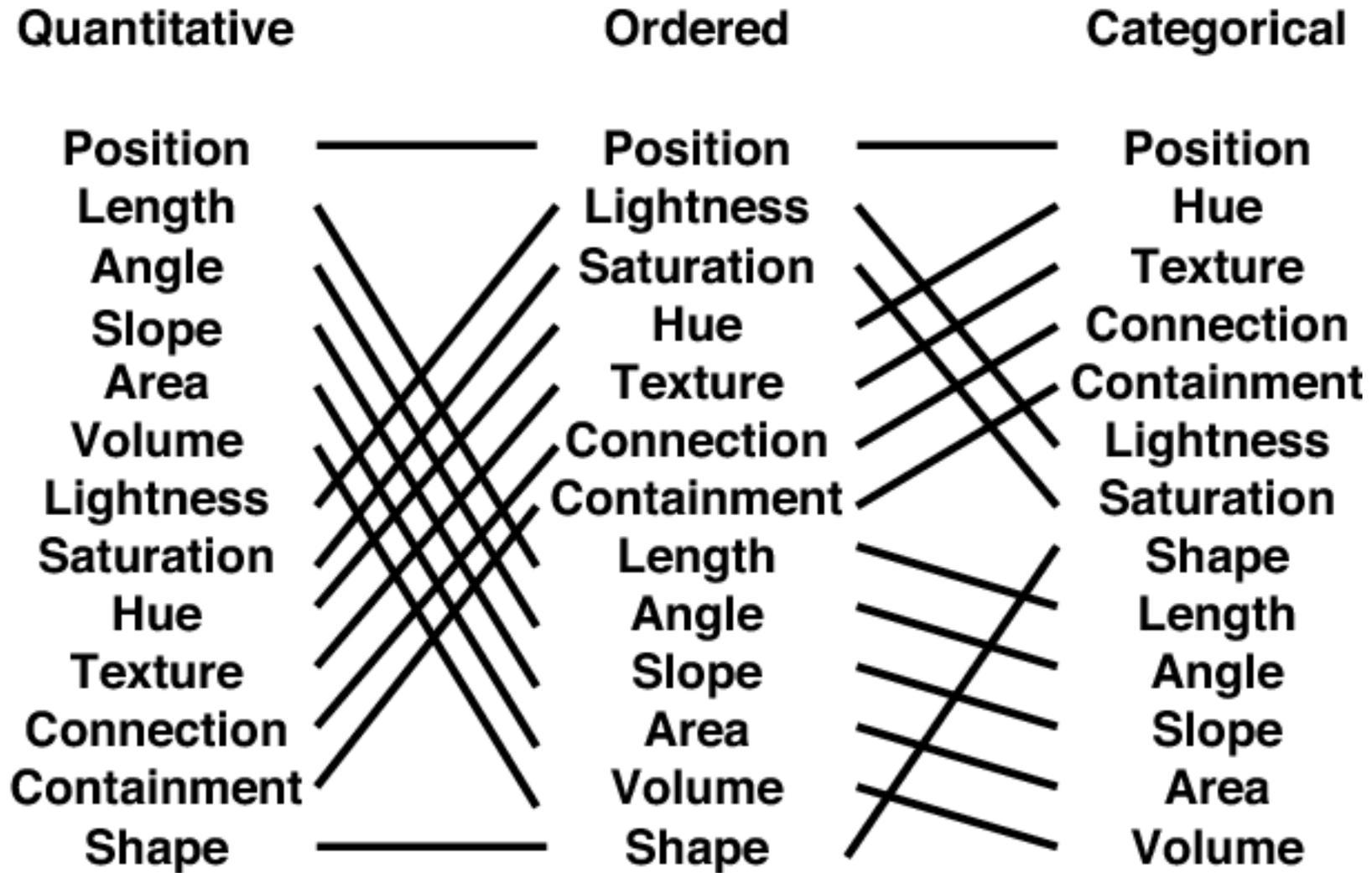
shape



- attributes

- parameters control mark appearance
- separable channels flowing from retina to brain

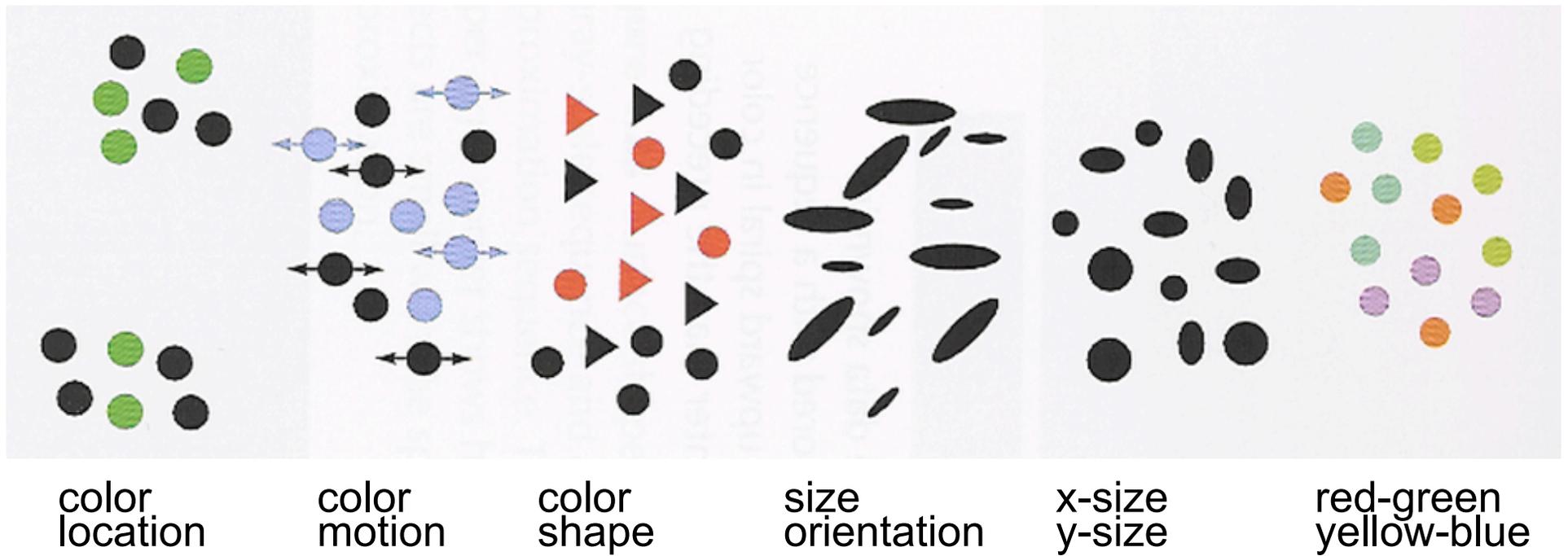
Review: Channel Ranking By Data Type



[Mackinlay, Automating the Design of Graphical Presentations of Relational Information. ACM, 1981.]

Review: Integral vs. Separable Channels

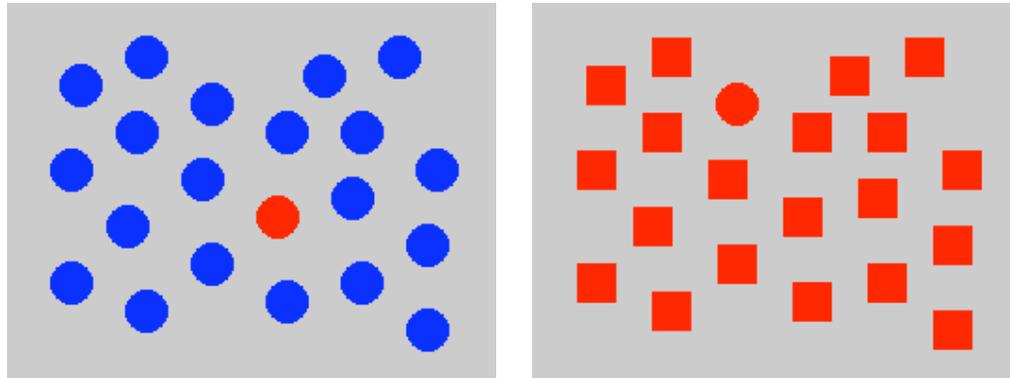
- not all channels separable



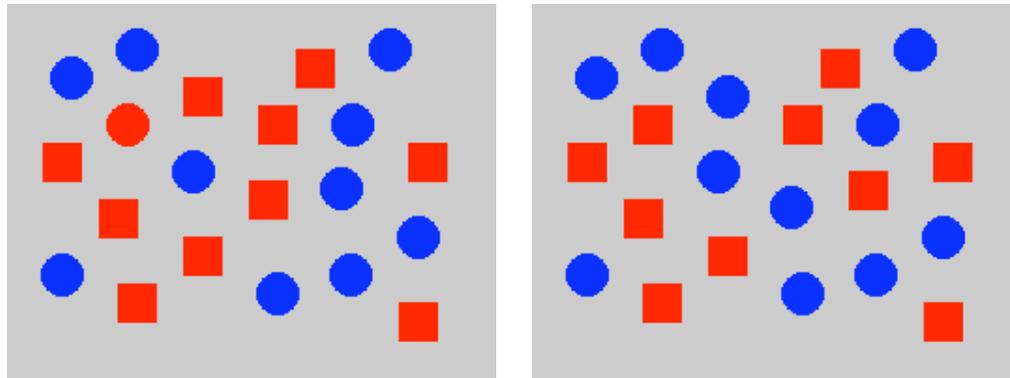
[Colin Ware, Information Visualization: Perception for Design. Morgan Kaufmann 1999.]

Review: Preattentive Visual Channels

- color alone, shape alone: preattentive

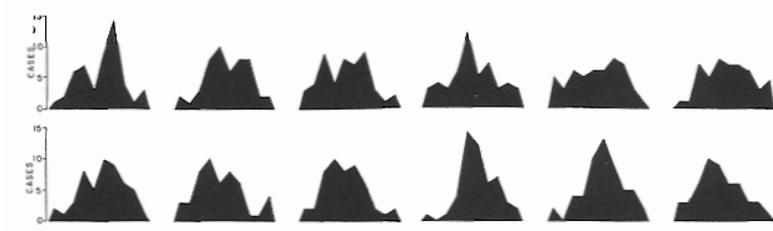


- combined color and shape: requires attention
 - search speed linear with distractor count



Review: InfoVis Techniques

- 3D often worse than 2D for abstract data
 - perspective distortion, occlusion
 - transform, use linked views
- animation often worse than small multiples



- aggregation and filtering
 - focus+context
- dimensionality reduction
- parallel coordinates

Beyond 314: Other Graphics Courses

- 424: Geometric Modelling
 - was offered this year
- 426: Computer Animation
 - will be offered next year
- 514: Image-Based Rendering - Heidrich
- 526: Algorithmic Animation - van de Panne
- 530P: Sensorimotor Computation - Pai
- 533A: Digital Geometry – Sheffer
- 547: Information Visualization - Munzner