CPSC 314, Written Homework 2

solution and grading criteria by Yufeng Zhu
Value: 4% of final grade
Total Points: 100

Marking Issue

1. *Problem 1 to 7*:

- For solutions who have intermediate steps, 2 points deduction for each incorrect derivation and 2 points for correctness of the final result. An incorrect derivation will not effect later ones, I will always assume your previous steps are all correct.

- For solutions who have no deriving steps, I will just give marks by judging how the results match the expected ones. Basically 2 points deduction for each mismatching.

2. *Problem 8*:

- 10 points for each key step as listed in the solution. Partial credits might be given based on individual cases.

1. (10 pts) Give the camera/viewing transformation matrix for an eye position (2,3,1), a lookat point (4, 5, -5) and an up vector (0,-1,0).

   Solution

$$
\begin{aligned}
\vec{g} &= \vec{l} - \vec{e} = (4,5,-5) - (2,3,1) = (2,2,-6) \\
\vec{w} &= -\frac{\vec{g}}{\|\vec{g}\|} = -\frac{(2,2,-6)}{\sqrt{44}} = \left(-\frac{1}{\sqrt{11}}, -\frac{1}{\sqrt{11}}, \frac{3}{\sqrt{11}}\right) \\
\vec{u} &= \frac{\vec{t}\times\vec{w}}{\|\vec{t}\times\vec{w}\|} = \frac{\left(-\frac{3}{\sqrt{11}}, 0, -\frac{1}{\sqrt{11}}\right)}{\sqrt{\frac{10}{11}}} = \left(-\frac{3}{\sqrt{10}}, 0, -\frac{1}{\sqrt{10}}\right) \\
\vec{v} &= \vec{w}\times\vec{u} = \left(\frac{1}{\sqrt{110}}, -\frac{10}{\sqrt{110}}, -\frac{3}{\sqrt{110}}\right) \\
\mathbf{R}^{-1} &= \begin{pmatrix} -\frac{3}{\sqrt{10}} & 0 & -\frac{1}{\sqrt{10}} & 0 \\ \frac{1}{\sqrt{110}} & -\frac{10}{\sqrt{110}} & -\frac{3}{\sqrt{110}} & 0 \\ -\frac{1}{\sqrt{11}} & -\frac{1}{\sqrt{11}} & \frac{3}{\sqrt{11}} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \\
\mathbf{T}^{-1} &= \begin{pmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \\
\mathbf{M}_{W2V} = \mathbf{R}^{-1}\mathbf{T}^{-1} &= \begin{pmatrix} -\frac{3}{\sqrt{10}} & 0 & -\frac{1}{\sqrt{10}} & \frac{7}{\sqrt{10}} \\ \frac{1}{\sqrt{110}} & -\frac{10}{\sqrt{110}} & -\frac{3}{\sqrt{110}} & \frac{31}{\sqrt{110}} \\ -\frac{1}{\sqrt{11}} & -\frac{1}{\sqrt{11}} & \frac{3}{\sqrt{11}} & \frac{2}{\sqrt{11}} \\ 0 & 0 & 0 & 1 \end{pmatrix} \\
&= \begin{pmatrix} -0.95 & 0 & -0.32 & 2.21 \\ 0.10 & -0.95 & -0.29 & 2.96 \\ -0.30 & -0.30 & 0.90 & 0.60 \\ 0 & 0 & 0 & 1 \end{pmatrix}
\end{aligned} \tag{1}
$$

2. (10 pts) Give the perspective projection matrix for a view volume with a near plane of 3, far plane of 15, a left plane of 2, a right plane of -2, a top plane of 3, and a bottom plane of -3.

   Solution

$$
\mathbf{M}_P = \begin{pmatrix} \frac{2n}{r-l} & 0 & \frac{r+l}{r-l} & 0 \\ 0 & \frac{2n}{t-b} & \frac{t+b}{t-b} & 0 \\ 0 & 0 & -\frac{f+n}{f-n} & -\frac{2fn}{f-n} \\ 0 & 0 & -1 & 0 \end{pmatrix} = \begin{pmatrix} -\frac{3}{2} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -\frac{3}{2} & -\frac{15}{2} \\ 0 & 0 & -1 & 0 \end{pmatrix} \tag{2}
$$

3. (10 pts) Give the NDC-to-display transformation matrix for a viewport 900 pixels wide and 800 pixels high, with the origin in the upper left of the display.

Solution

$$\mathbf{M}_{N2D} = \begin{pmatrix} 1 & 0 & 0 & \frac{899}{2} \\ 0 & 1 & 0 & \frac{799}{2} \\ 0 & 0 & 1 & \frac{1}{2} \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 450 & 0 & 0 & 0 \\ 0 & 400 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 450 & 0 & 0 & \frac{899}{2} \\ 0 & -400 & 0 & \frac{799}{2} \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{3}$$

4. (10 pts) In world coordinates, a point is (4, 4, -6). Give its coordinates in the camera coordinate system, after the viewing transformation from problem 1 above has been applied to it.

Solution

$$\vec{p}_1 = \mathbf{M}_{W2V}\vec{p}_0 = \begin{pmatrix} -\frac{3}{\sqrt{10}} & 0 & -\frac{1}{\sqrt{10}} & \frac{7}{\sqrt{10}} \\ \frac{1}{\sqrt{110}} & -\frac{10}{\sqrt{110}} & -\frac{3}{\sqrt{110}} & \frac{31}{\sqrt{110}} \\ -\frac{1}{\sqrt{11}} & -\frac{1}{\sqrt{11}} & \frac{3}{\sqrt{11}} & \frac{2}{\sqrt{11}} \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 4 \\ 4 \\ -6 \\ 1 \end{pmatrix} = \begin{pmatrix} 0.32 \\ 1.24 \\ -7.24 \\ 1 \end{pmatrix} \tag{4}$$

5. (10 pts) Then give its coordinates in the clipping coordinate system, after the perspective warp for the frustum specified in problem 2 has been applied to the tetrahedron points in camera coordinates (that is, the answer from problem 4).

Solution

$$\vec{p}_2 = \mathbf{M}_P\vec{p}_1 = \begin{pmatrix} -0.47 \\ 1.24 \\ 3.35 \\ 7.24 \end{pmatrix} \tag{5}$$

6. (10 pts) Then give its coordinates in the normalized device coordinate system, after the perspective divide has been applied to the answer from problem 5.

Solution

$$\vec{p}_3 = \vec{p}_2/\vec{p}_2[4] = \vec{p}_2/7.24 = \begin{pmatrix} -0.07 \\ 0.17 \\ 0.46 \\ 1 \end{pmatrix} \tag{6}$$

7. (10 pts) Finally, give its coordinates in the display coordinate system, after the viewport transformation of problem 3 has been applied to the answer from problem 6.

Solution

$$\vec{p}_4 = \mathbf{M}_{N2D}\vec{p}_3 = \begin{pmatrix} 420.00 \\ 330.98 \\ 0.73 \\ 1 \end{pmatrix} \tag{7}$$

8. (30 pts) Derive the values C = -(f+n) / (f-n) and D = -2fn/(f-n) in the perspective to NDCS matrix, where z'= Cz+D.

Solution

$$\begin{aligned} z' &= Cz + D \\ w' &= -z \\ f_{NDC} &:= [-n, -f] \mapsto [-1, 1] \end{aligned} \tag{8}$$

$$\begin{aligned} (8) \Rightarrow \frac{z'}{w'} &= \begin{cases} -1, & z = -n \\ 1, & z = -f \end{cases} \\ &\Rightarrow \begin{cases} \frac{-Cn+D}{n} = -1 \\ \frac{-Cf+D}{f} = 1 \end{cases} \\ &\Rightarrow \begin{cases} D = Cn - n, (a) \\ D = Cf + f, (b) \end{cases} \\ (a) - (b) &\Rightarrow Cn - n = Cf + f \Rightarrow C = -\frac{f+n}{f-n}, (c) \\ \textit{Plug (c) into (a)} &\Rightarrow D = -\frac{2fn}{f-n} \end{aligned} \tag{9}$$

$$(9) \Rightarrow \begin{cases} C = -\frac{f+n}{f-n} \\ D = -\frac{2fn}{f-n} \end{cases} \tag{10}$$