Lifted Probabilistic Inference

Guest lecture: StarAI class @ UBC

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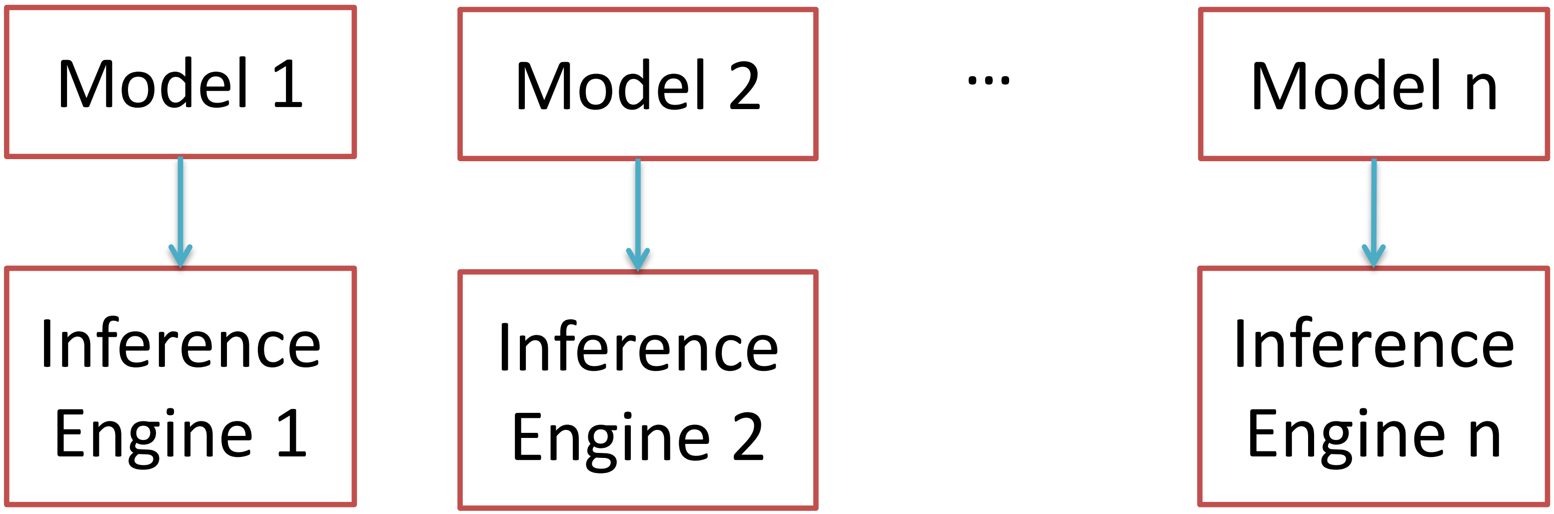
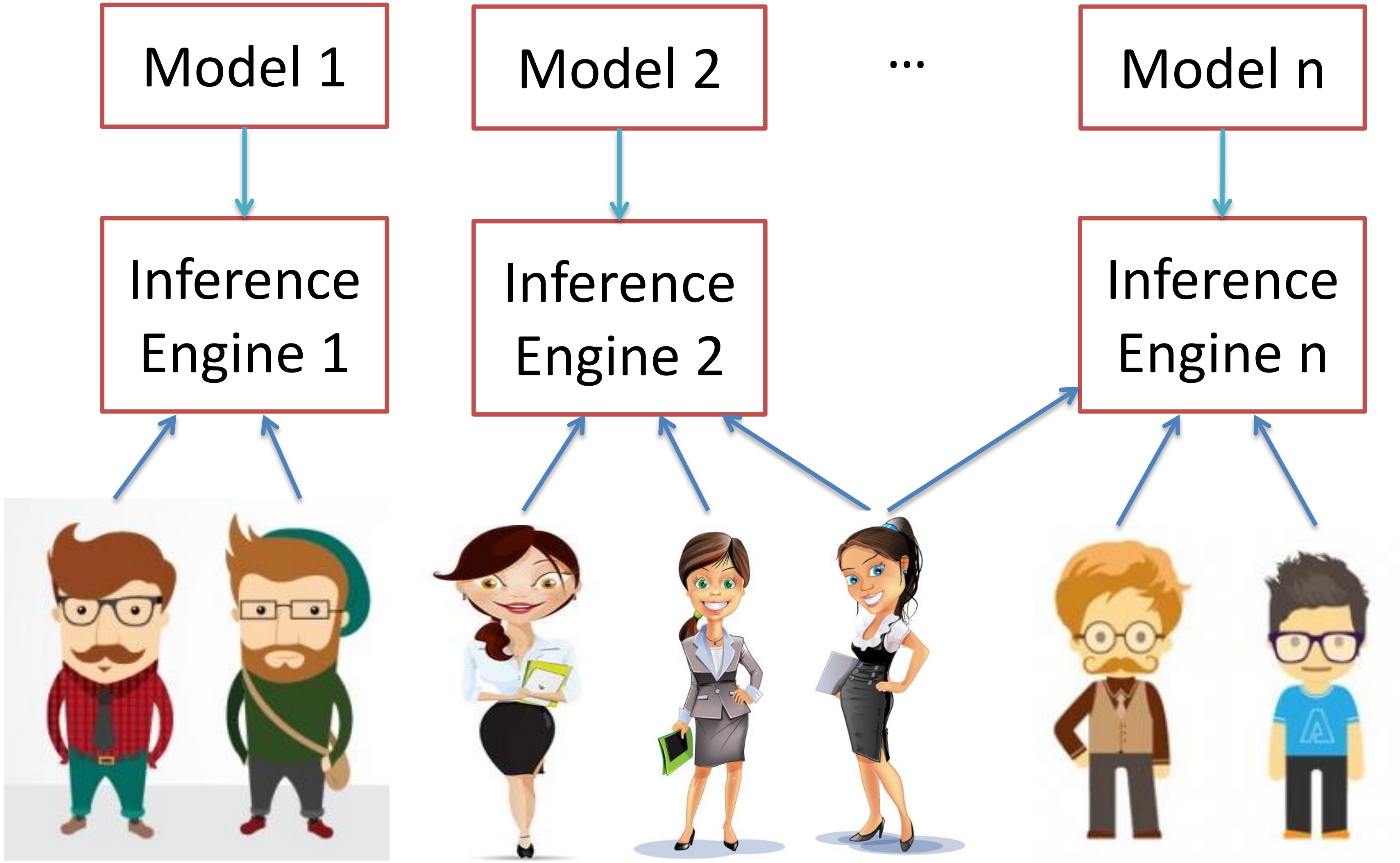
Traditional Works on Inference

Model 1 | Model 2 | … | Model n

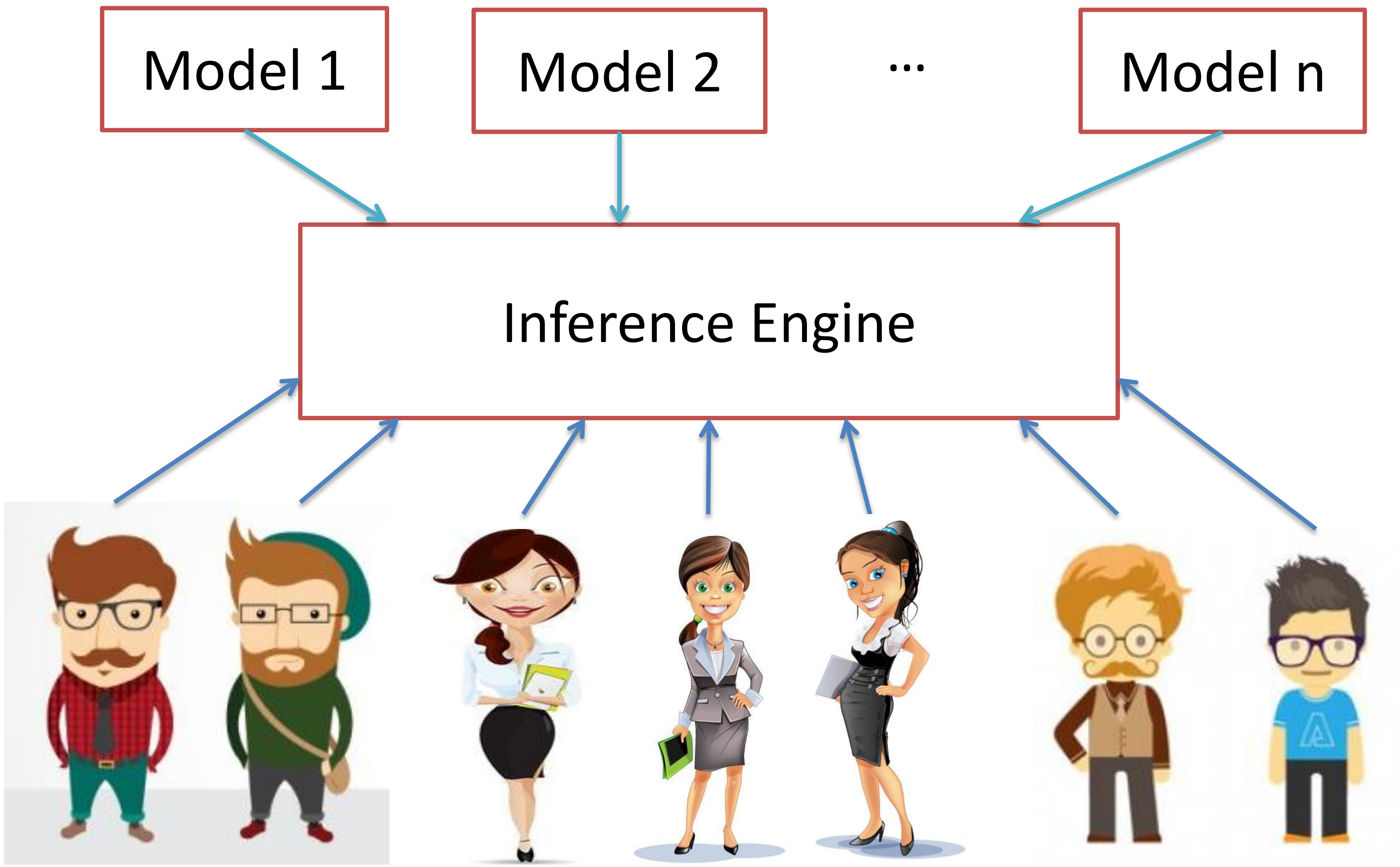
Traditional Works on Inference



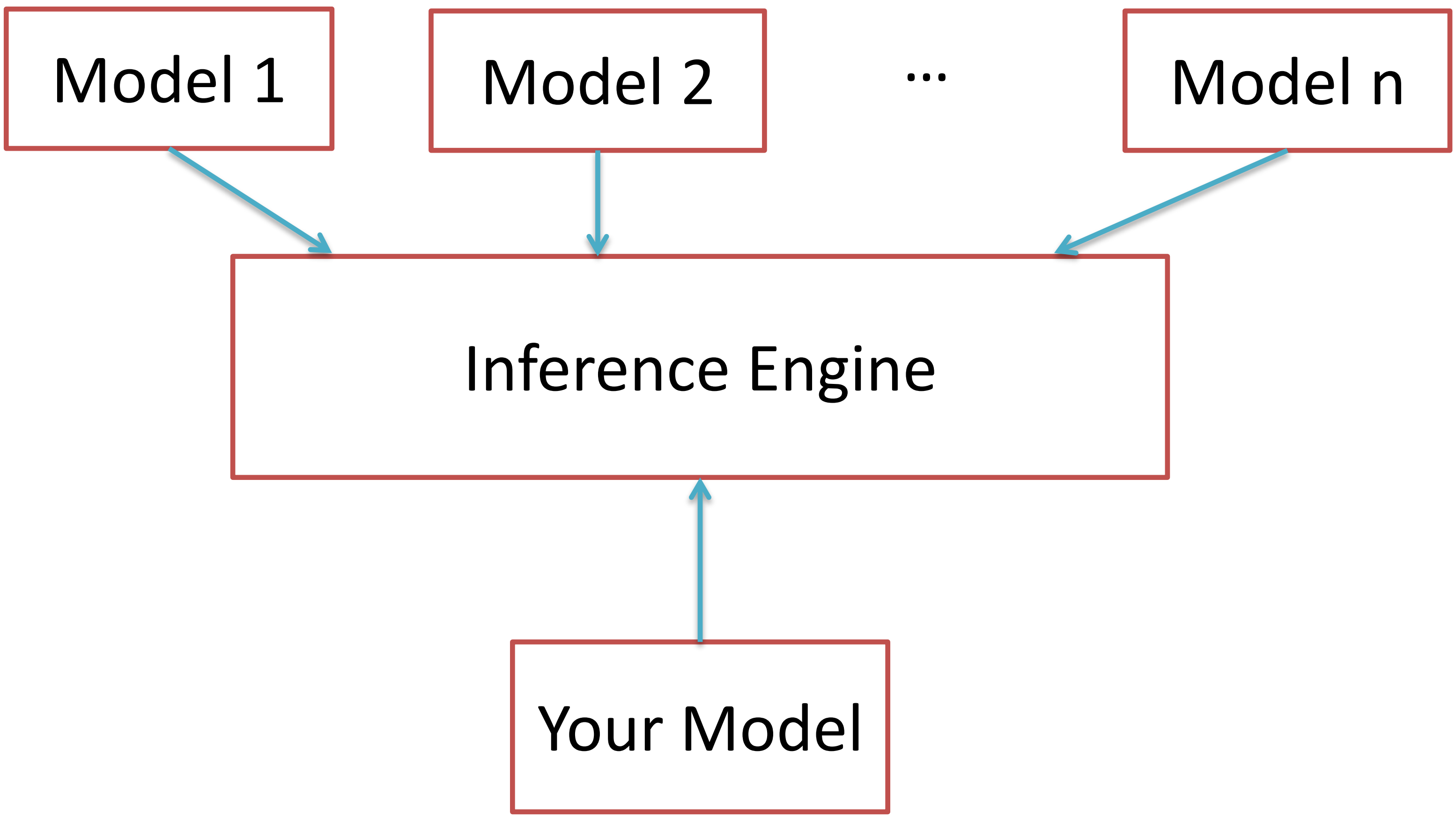
Traditional Works on Inference



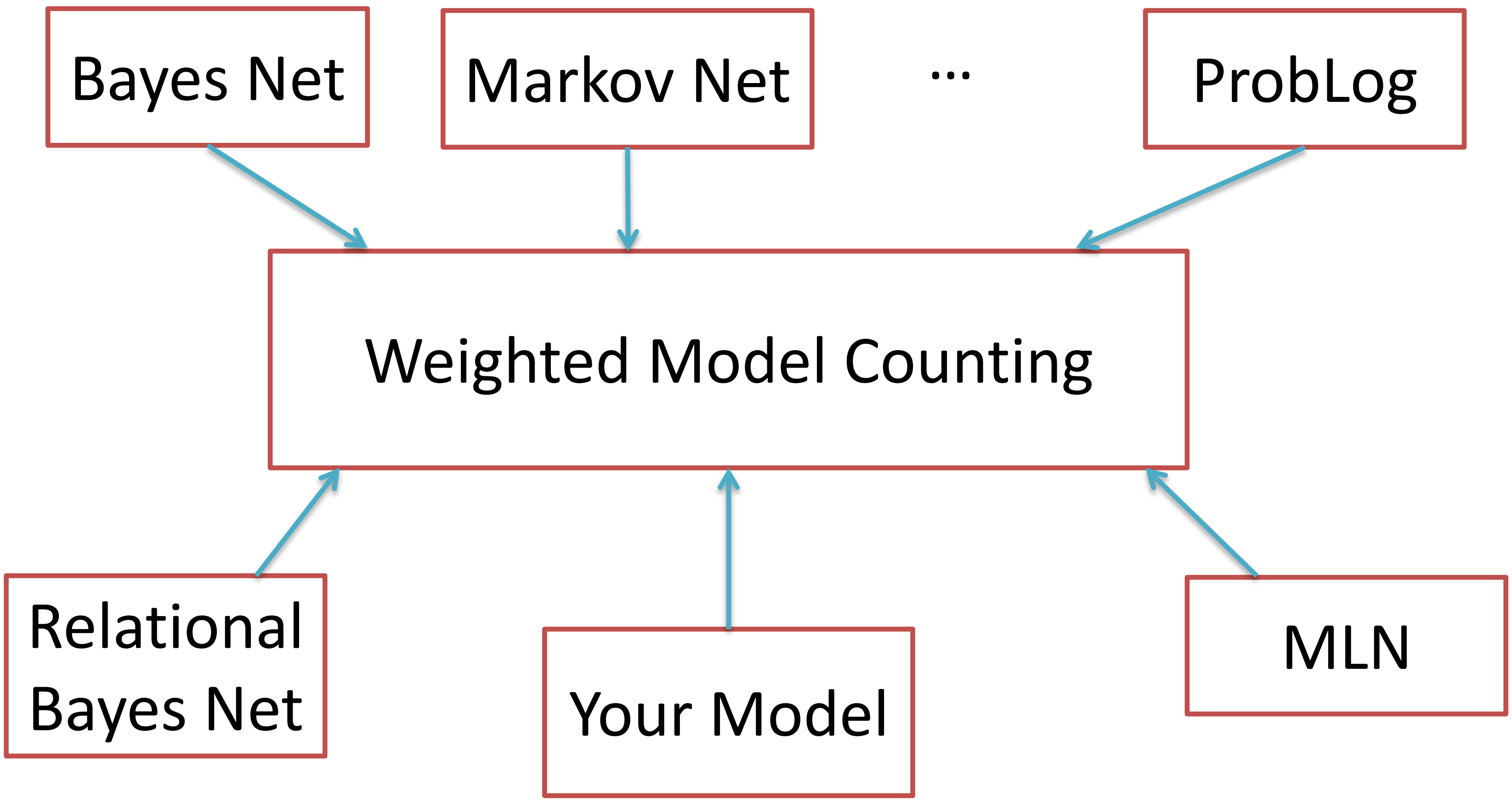
Desired Approach



Desired Approach



WMC is One Such Approach



Model Counting

- $A \vee B$

A	B	Model?
True	True	?
True	False	?
False	True	?
False	False	?

Model Counting

- $A \vee B$

A	B	Model?
True	True	Yes
True	False	Yes
False	True	Yes
False	False	No

3 Models

Weighted Model Counting

- $A \lor B$

A	B	Model?
True	True	Yes
True	False	Yes
False	True	Yes
False	False	No

3 Models

- W(A=True) = 0.2, W(A=False)=0.6
- W(B=True) = 0.1, W(B=False)=1.4

Weighted Model Counting

- $A \lor B$

A	B	Model?	Weight
True	True	Yes	0.2 * 0.1
True	False	Yes	0.2 * 1.4
False	True	Yes	0.6 * 0.1
False	False	No	---

3 Models

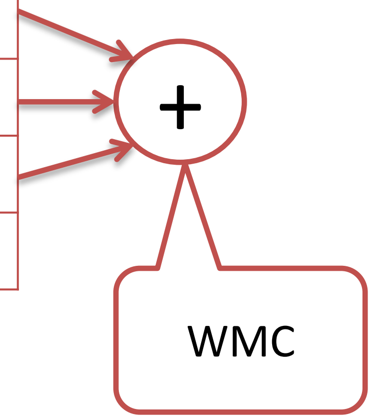
- W(A=True) = 0.2, W(A=False)=0.6
- W(B=True) = 0.1, W(B=False)=1.4

Weighted Model Counting

- $A \lor B$

A	B	Model?	Weight
True	True	Yes	0.2 * 0.1
True	False	Yes	0.2 * 1.4
False	True	Yes	0.6 * 0.1
False	False	No	---

3 Models



- W(A=True) = 0.2, W(A=False)=0.6
- W(B=True) = 0.1, W(B=False)=1.4

Weighted Model Counting

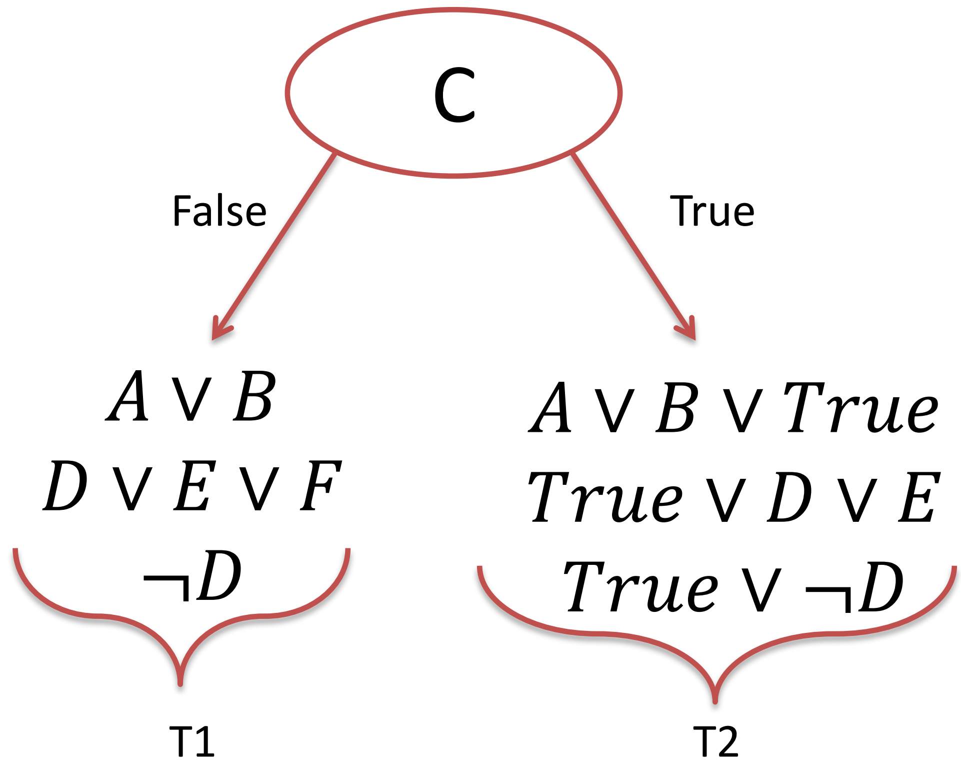
$$A \vee B \vee C$$
$$C \vee D \vee E \vee F$$
$$C \vee \neg D$$

W(A=True) = 0.2
W(A=False) = 0.6
W(B=True) = 1.1
W(B=False) = 1
W(C=True) = 0.1
W(C=False) = 0.7
W(D=True) = 2.2
W(D=False)=1
…

Case Analysis

$$A \vee B \vee C$$
$$C \vee D \vee E \vee F$$
$$C \vee \neg D$$

W(A=True) = 0.2
W(A=False) = 0.6
W(B=True) = 1.1
W(B=False) = 1
W(C=True) = 0.1
W(C=False) = 0.7
W(D=True) = 2.2
W(D=False)=1
…

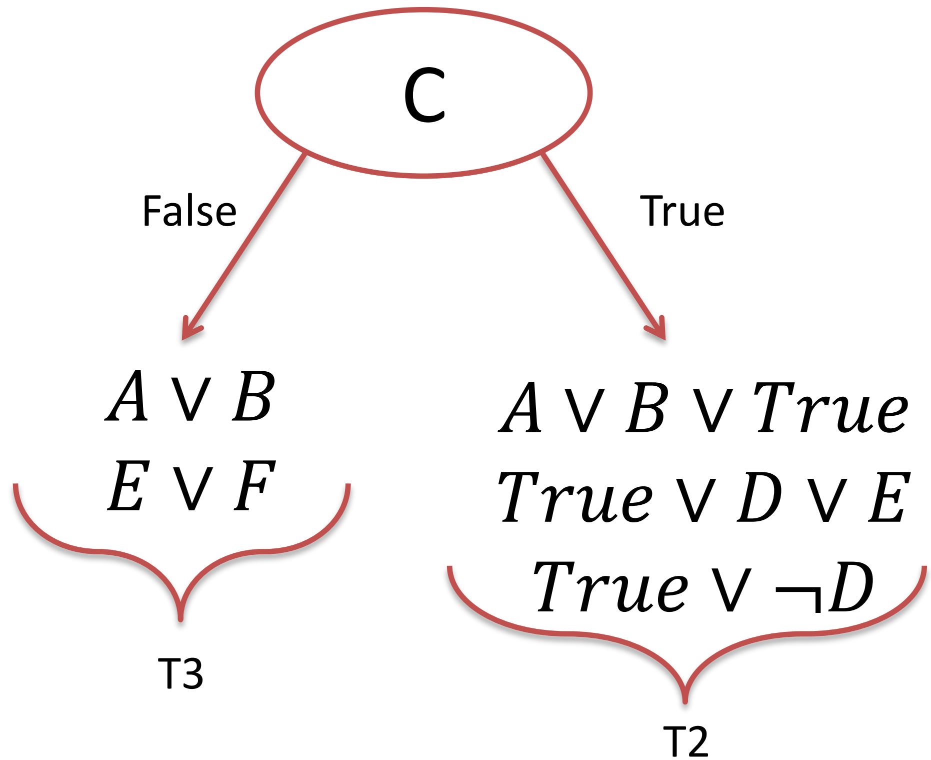


WMC(T)=0.7*WMC(T1) + 0.1*WMC(T2)

Unit Propagation

$$A \vee B \vee C$$
$$C \vee D \vee E \vee F$$
$$C \vee \neg D$$

W(A=True) = 0.2
W(A=False) = 0.6
W(B=True) = 1.1
W(B=False) = 1
W(C=True) = 0.1
W(C=False) = 0.7
W(D=True) = 2.2
W(D=False)=1
…

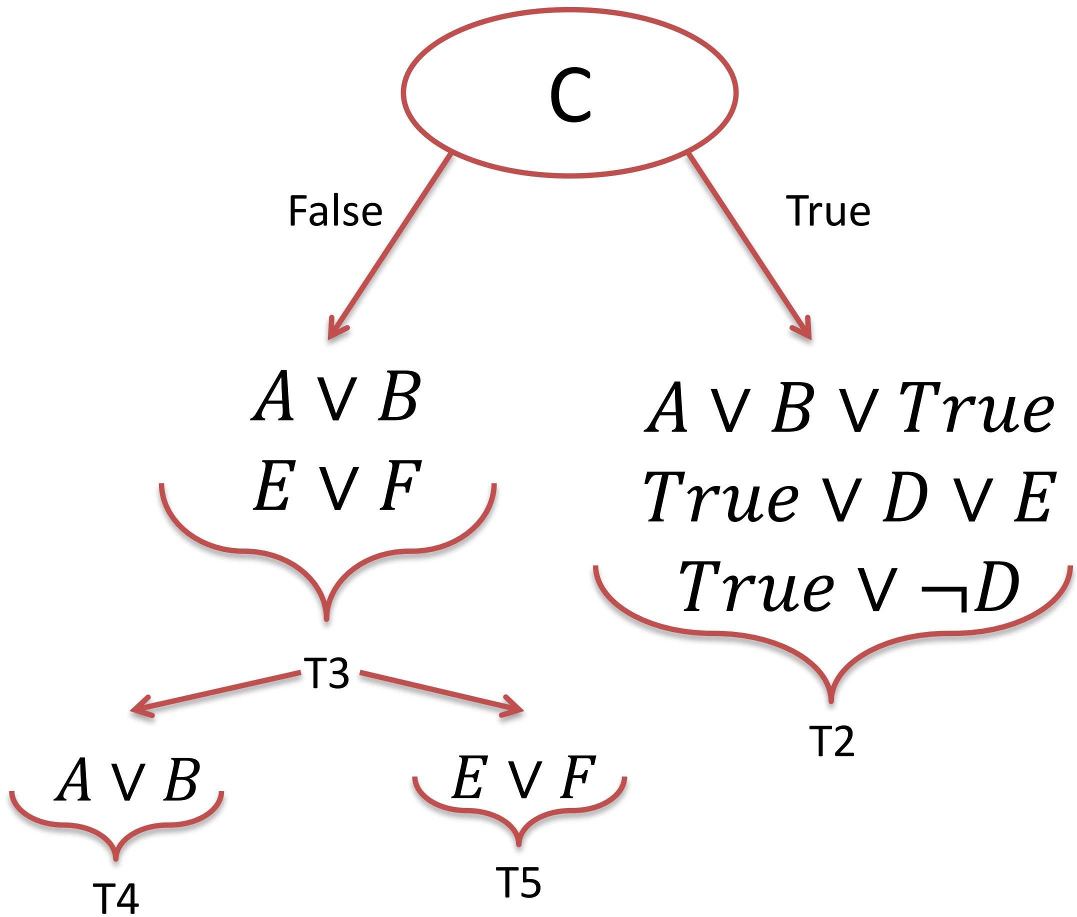


WMC(T1)=1*WMC(T3)

Decomposition

$A \lor B \lor C$
$C \lor D \lor E \lor F$
$C \lor \neg D$

W(A=True) = 0.2
W(A=False) = 0.6
W(B=True) = 1.1
W(B=False) = 1
W(C=True) = 0.1
W(C=False) = 0.7
W(D=True) = 2.2
W(D=False)=1
…

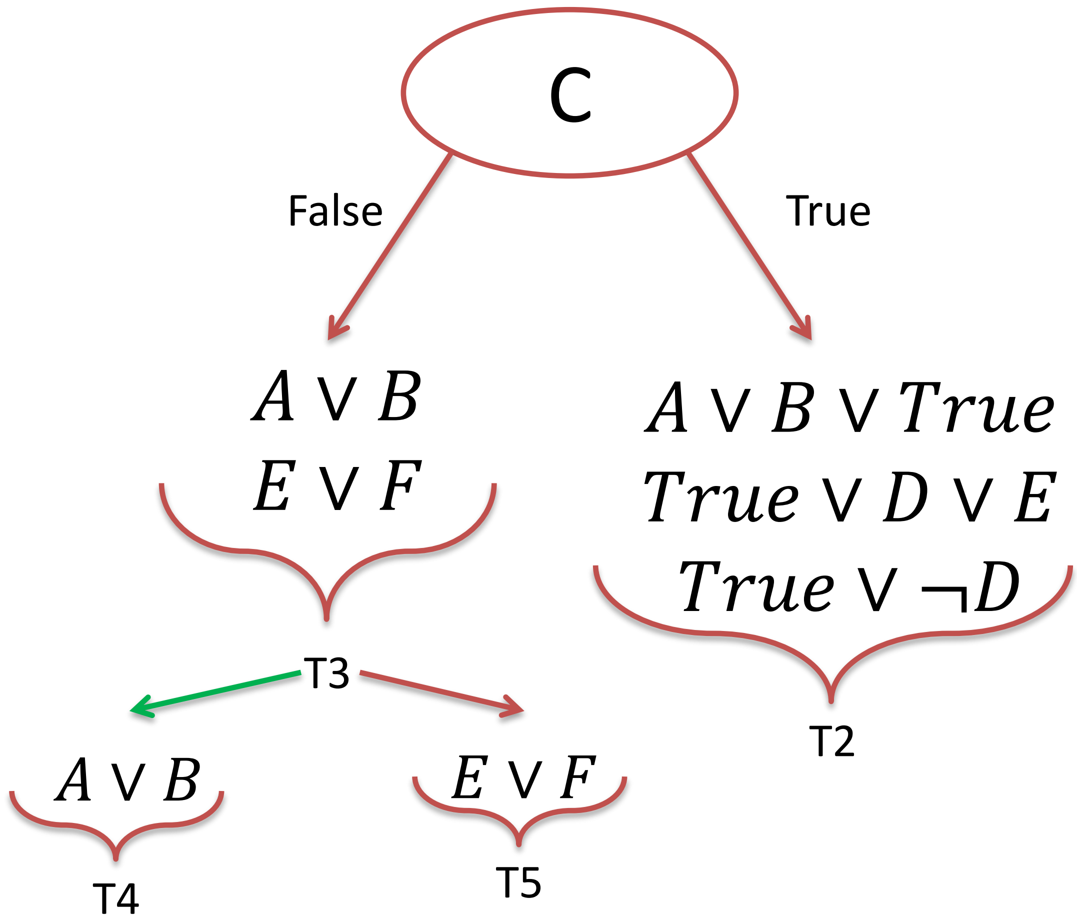


WMC(T3) = WMC(T4) * WMC(T5)

Caching

$$A \lor B \lor C$$
$$C \lor D \lor E \lor F$$
$$C \lor \neg D$$

W(A=True) = 0.2
W(A=False) = 0.6
W(B=True) = 1.1
W(B=False) = 1
W(C=True) = 0.1
W(C=False) = 0.7
W(D=True) = 2.2
W(D=False)=1
…



$\text{Cache}[A \lor B] = \text{WMC(T4)}$

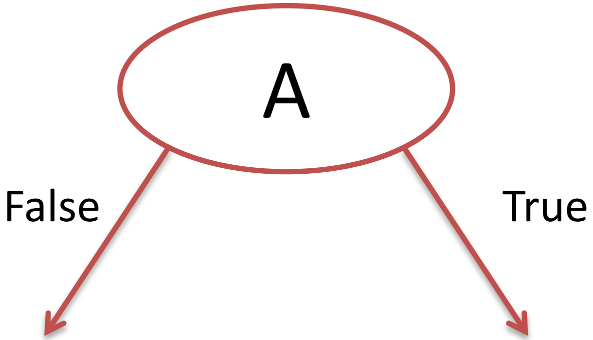
Let’s do an example together

$$A \vee B \vee C$$
$$C \vee D \vee E \vee F$$
$$C \vee \neg D$$

W(A=True) = 0.2
W(A=False) = 0.6
W(B=True) = 1.1
W(B=False) = 1
W(C=True) = 0.1
W(C=False) = 0.7
W(D=True) = 2.2
W(D=False)=1
…

Let's do an example together

$$A \vee B \vee C$$
$$C \vee D \vee E \vee F$$
$$C \vee \neg D$$

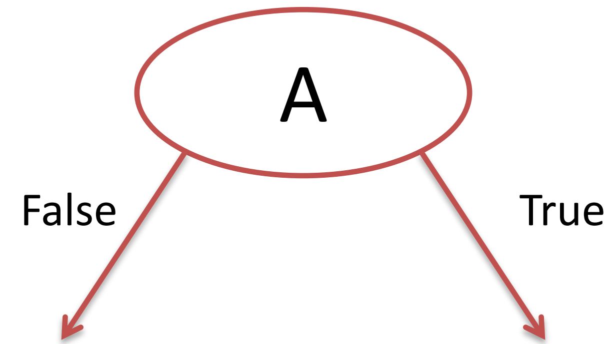


W(A=True) = 0.2
W(A=False) = 0.6
W(B=True) = 1.1
W(B=False) = 1
W(C=True) = 0.1
W(C=False) = 0.7
W(D=True) = 2.2
W(D=False)=1
…

Let’s do an example together

$$A \vee B \vee C$$
$$C \vee D \vee E \vee F$$
$$C \vee \neg D$$

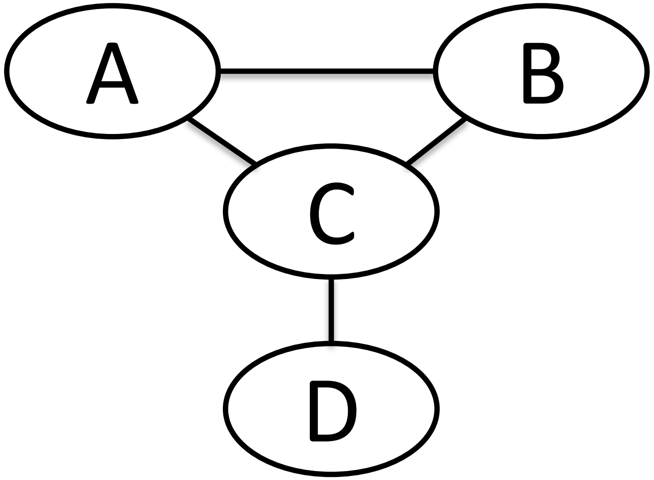
W(A=True) = 0.2
W(A=False) = 0.6
W(B=True) = 1.1
W(B=False) = 1
W(C=True) = 0.1
W(C=False) = 0.7
W(D=True) = 2.2
W(D=False)=1
…



Follow the example on the board

Now we know how to do WMC efficiently

Let's see an example of how we can use it

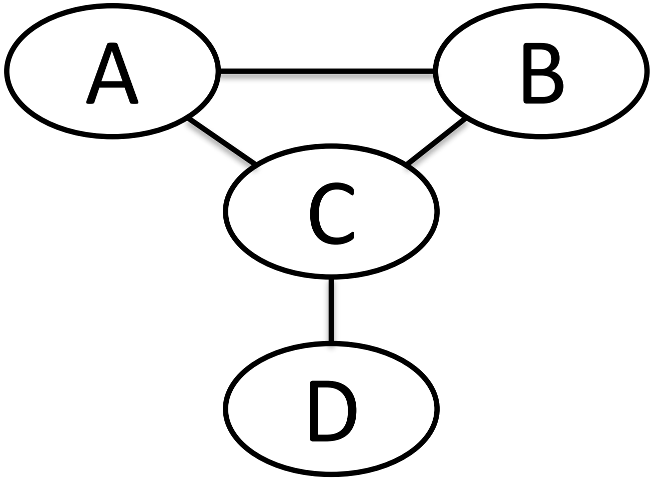


ϕ_1

A	B	C	
T	T	T	1.1
T	T	F	0.2
T	F	T	0
T	F	F	4
F	T	T	0.5
F	T	F	1.5
F	F	T	3.3
F	F	F	3

ϕ_2

C	D	
T	T	0.1
T	F	0.1
F	T	0.2
F	F	0.8



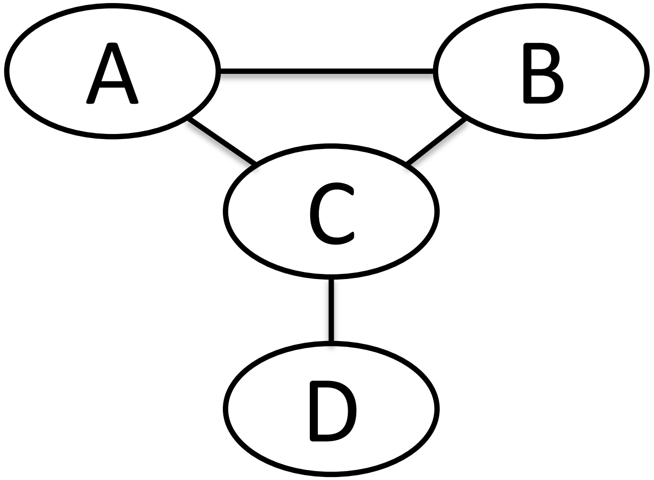
P(A=T, B=T, C=T, D=T)?

ϕ_1

A	B	C	
T	T	T	1.1
T	T	F	0.2
T	F	T	0
T	F	F	4
F	T	T	0.5
F	T	F	1.5
F	F	T	3.3
F	F	F	3

ϕ_2

C	D	
T	T	0.1
T	F	0.1
F	T	0.2
F	F	0.8



P(A=T, B=T, C=T, D=T)?

ϕ_1

A	B	C	
T	T	T	1.1
T	T	F	0.2
T	F	T	0
T	F	F	4
F	T	T	0.5
F	T	F	1.5
F	F	T	3.3
F	F	F	3

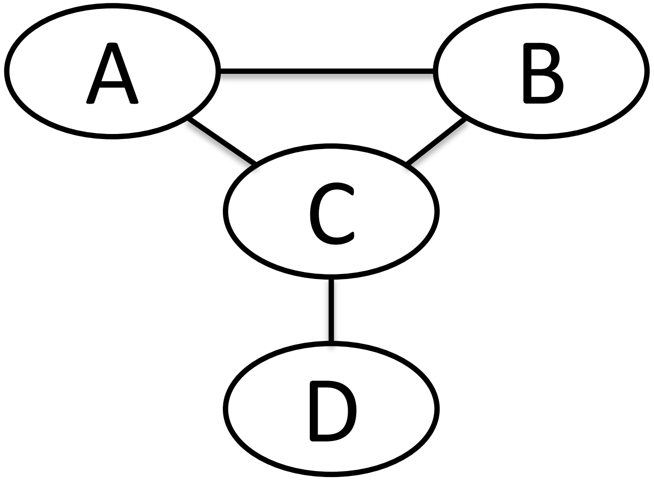
×

ϕ_2

C	D	
T	T	0.1
T	F	0.1
F	T	0.2
F	F	0.8

=

A	B	C	D	
T	T	T	T	0.11
T	T	T	F	0.11
T	T	F	T	0.04
…	…	…	…	
F	T	T	T	0.05
F	T	T	F	0.05
F	T	F	T	0.3
…	…	…	…	
F	F	F	F	2.4



$$P(A{=}T, B{=}T, C{=}T, D{=}T) = \frac{0.11}{Z}$$

ϕ_1

A	B	C	
T	T	T	1.1
T	T	F	0.2
T	F	T	0
T	F	F	4
F	T	T	0.5
F	T	F	1.5
F	F	T	3.3
F	F	F	3

×

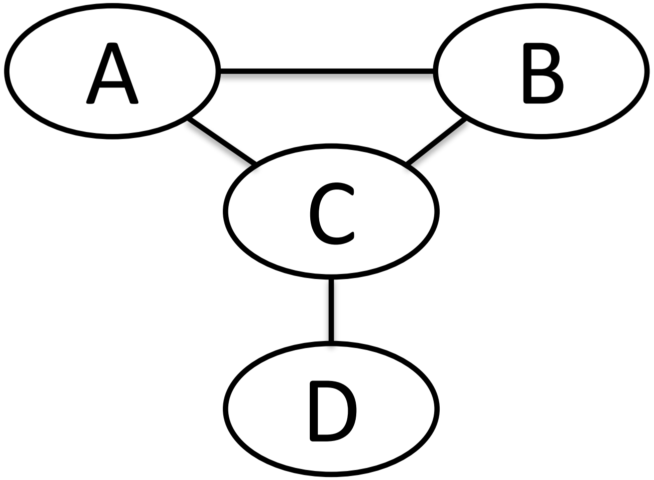
ϕ_2

C	D	
T	T	0.1
T	F	0.1
F	T	0.2
F	F	0.8

=

A	B	C	D	
T	T	T	T	0.11
T	T	T	F	0.11
T	T	F	T	0.04
…	…	…	…	
F	T	T	T	0.05
F	T	T	F	0.05
F	T	F	T	0.3
…	…	…	…	
F	F	F	F	2.4

+ → Z



$$P(A{=}T, B{=}T, C{=}T, D{=}T) = \frac{0.11}{Z}$$

$$P(A{=}T) = \frac{Z_1}{Z}$$

ϕ_1

A	B	C	
T	T	T	1.1
T	T	F	0.2
T	F	T	0
T	F	F	4
F	T	T	0.5
F	T	F	1.5
F	F	T	3.3
F	F	F	3

×

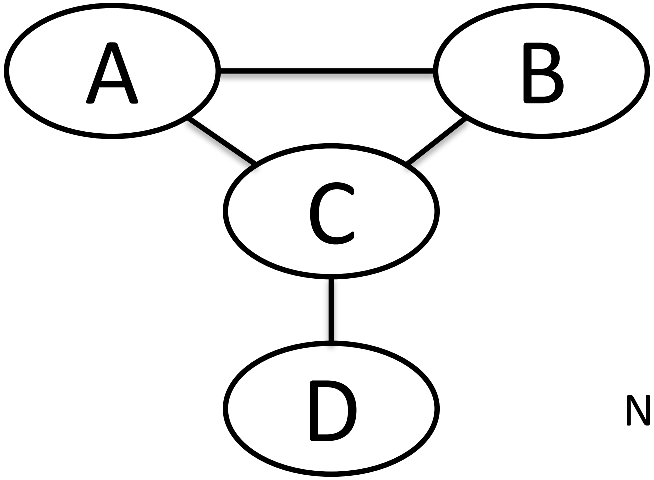
ϕ_2

C	D	
T	T	0.1
T	F	0.1
F	T	0.2
F	F	0.8

=

A	B	C	D	
T	T	T	T	0.11
T	T	T	F	0.11
T	T	F	T	0.04
…	…	…	…	
F	T	T	T	0.05
F	T	T	F	0.05
F	T	F	T	0.3
…	…	…	…	
F	F	F	F	2.4

\+ (sum of the A=T rows) → Z_1



$$P(A{=}T, B{=}T, C{=}T, D{=}T) = \frac{0.11}{Z}$$

$$P(A{=}T) = \frac{Z_1}{Z}$$

Normalization Constant ← $Z = \sum_{A}\sum_{B}\sum_{C}\sum_{D} \phi_1 * \phi_2$

ϕ_1

A	B	C	
T	T	T	1.1
T	T	F	0.2
T	F	T	0
T	F	F	4
F	T	T	0.5
F	T	F	1.5
F	F	T	3.3
F	F	F	3

×

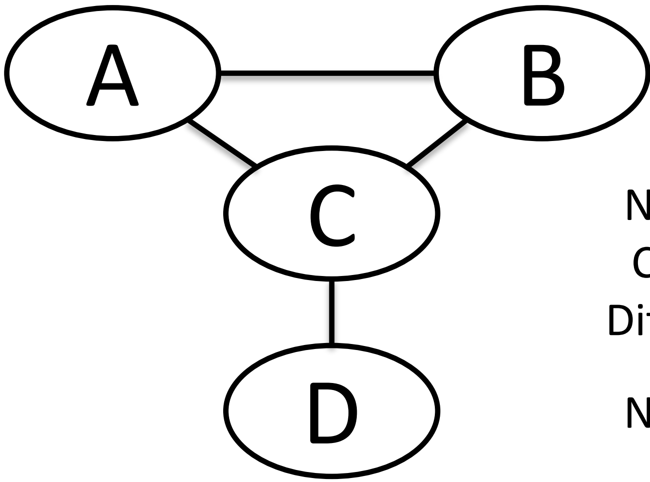
ϕ_2

C	D	
T	T	0.1
T	F	0.1
F	T	0.2
F	F	0.8

=

A	B	C	D	
T	T	T	T	0.11
T	T	T	F	0.11
T	T	F	T	0.04
…	…	…	…	
F	T	T	T	0.05
F	T	T	F	0.05
F	T	F	T	0.3
…	…	…	…	
F	F	F	F	2.4

\+ → Z_1



$$P(A{=}T, B{=}T, C{=}T, D{=}T) = \frac{0.11}{Z}$$

Normalization Constant of a Different Model ← $P(A{=}T) = \frac{Z_1}{Z}$

Normalization Constant ← $Z = \sum_A \sum_B \sum_C \sum_D \phi_1 * \phi_2$

ϕ_1'

A	B	C	
T	T	T	1.1
T	T	F	0.2
T	F	T	0
T	F	F	4
~~F~~	~~T~~	~~T~~	~~0.5~~
~~F~~	~~T~~	~~F~~	~~1.5~~
~~F~~	~~F~~	~~T~~	~~3.3~~
~~F~~	~~F~~	~~F~~	~~3~~

×

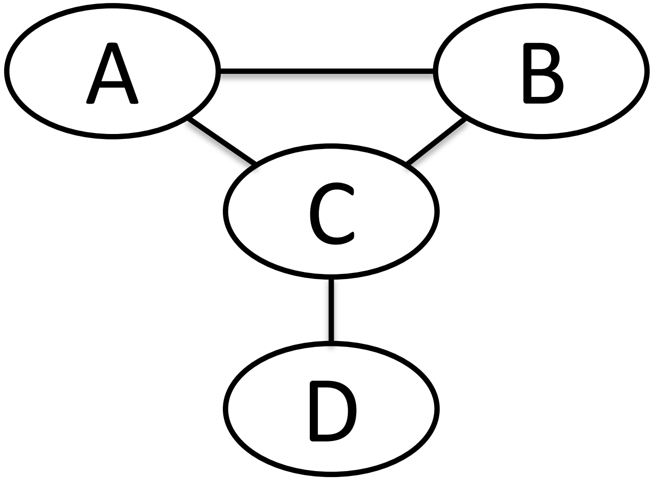
ϕ_2'

C	D	
T	T	0.1
T	F	0.1
F	T	0.2
F	F	0.8

=

A	B	C	D	
T	T	T	T	0.11
T	T	T	F	0.11
T	T	F	T	0.04
…	…	…	…	
~~F~~	~~T~~	~~T~~	~~T~~	~~0.05~~
~~F~~	~~T~~	~~T~~	~~F~~	~~0.05~~
~~F~~	~~T~~	~~F~~	~~T~~	~~0.3~~
~~…~~	~~…~~	~~…~~	~~…~~	
~~F~~	~~F~~	~~F~~	~~F~~	~~2.4~~

+ → Z_1



Normalization constant of the model
=
WMC(the following theory)

ϕ_1

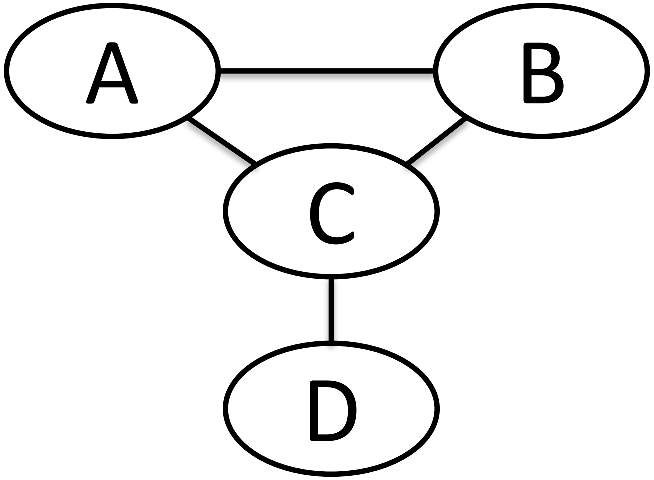
A	B	C	
T	T	T	1.1
T	T	F	0.2
T	F	T	0
T	F	F	4
F	T	T	0.5
F	T	F	1.5
F	F	T	3.3
F	F	F	3

ϕ_2

C	D	
T	T	0.1
T	F	0.1
F	T	0.2
F	F	0.8

$$A \wedge B \wedge C \Leftrightarrow \theta_1$$
$$A \wedge B \wedge \neg C \Leftrightarrow \theta_2$$
$$A \wedge \neg B \wedge C \Leftrightarrow \theta_3$$
…
$$C \wedge D \Leftrightarrow \theta_9$$
$$C \wedge \neg D \Leftrightarrow \theta_{10}$$
…

$$W(A) = 1$$
$$W(\neg A) = 1$$
$$W(B) = 1$$
$$W(\neg B) = 1$$
…
$$W(\theta_1) = 1.1$$
$$W(\neg\theta_1) = 1$$
$$W(\theta_2) = 0.2$$
$$W(\neg\theta_2) = 1$$
…



Normalization constant of the model
=
WMC(the following theory)

ϕ_1

A	B	C	
T	T	T	1.1
T	T	F	0.2
T	F	T	0
T	F	F	4
F	T	T	0.5
F	T	F	1.5
F	F	T	3.3
F	F	F	3

ϕ_2

C	D	
T	T	0.1
T	F	0.1
F	T	0.2
F	F	0.8

$$A \wedge B \wedge C \Leftrightarrow \theta_1$$
$$A \wedge B \wedge \neg C \Leftrightarrow \theta_2$$
~~$A \wedge \neg B \wedge C \Leftrightarrow \theta_3$~~

…

$$C \wedge D \Leftrightarrow \theta_9$$
$$C \wedge \neg D \Leftrightarrow \theta_{10}$$

…

$$\neg A \vee B \vee \neg C$$

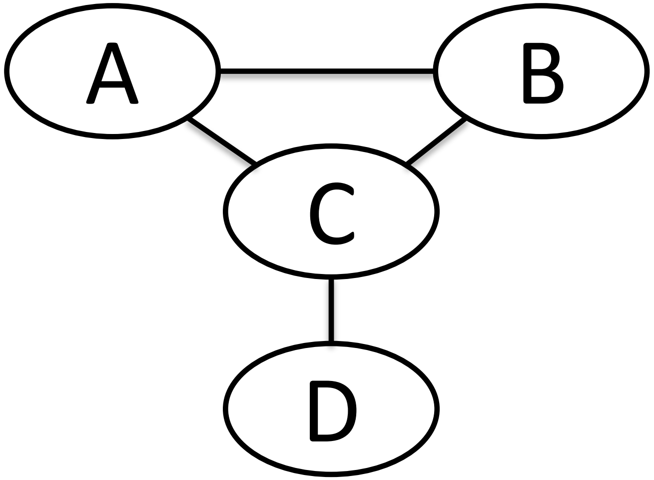
$$W(A) = 1$$
$$W(\neg A) = 1$$
$$W(B) = 1$$
$$W(\neg B) = 1$$

…

$$W(\theta_1) = 1.1$$
$$W(\neg \theta_1) = 1$$
$$W(\theta_2) = 0.2$$
$$W(\neg \theta_2) = 1$$

…

Determinism



Normalization constant of the model

=

WMC(the following theory)

ϕ_1

A	B	C	
T	T	T	1.1
T	T	F	0.2
T	F	T	0
T	F	F	4
F	T	T	0.5
F	T	F	1.5
F	F	T	3.3
F	F	F	3

ϕ_2

C	D	
T	T	0.1
T	F	0.1
F	T	0.2
F	F	0.8

$A \wedge B \wedge C \Leftrightarrow \theta_1$

$A \wedge B \wedge \neg C \Leftrightarrow \theta_2$

~~$A \wedge \neg B \wedge C \Leftrightarrow \theta_3$~~

…

~~$C \wedge D \Leftrightarrow \theta_9$~~

~~$C \wedge \neg D \Leftrightarrow \theta_{10}$~~

…

$\neg A \vee B \vee \neg C$

$C \Leftrightarrow \theta_9$

$W(A) = 1$

$W(\neg A) = 1$

$W(B) = 1$

$W(\neg B) = 1$

…

$W(\theta_1) = 1.1$

$W(\neg\theta_1) = 1$

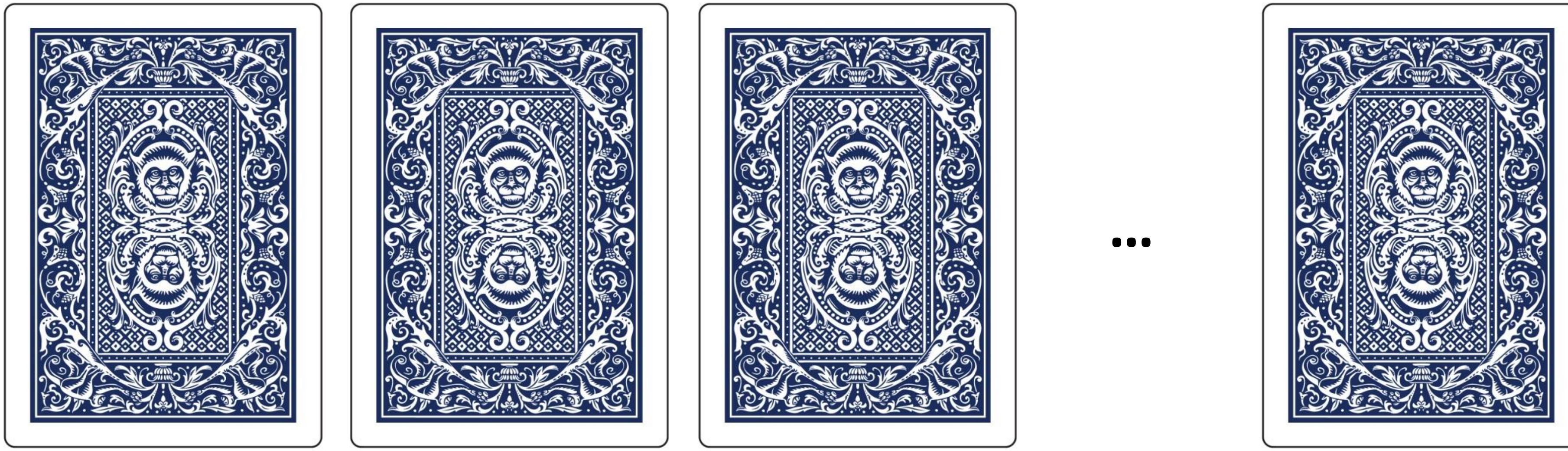
$W(\theta_2) = 0.2$

$W(\neg\theta_2) = 1$

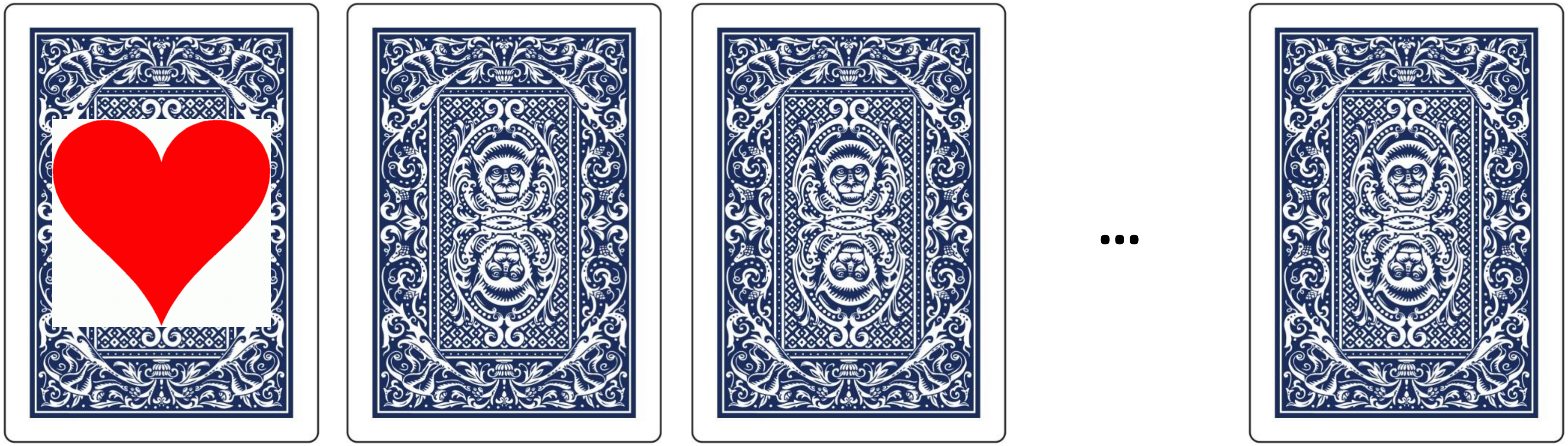
…

Context-Specific Independence

One issue with WMC

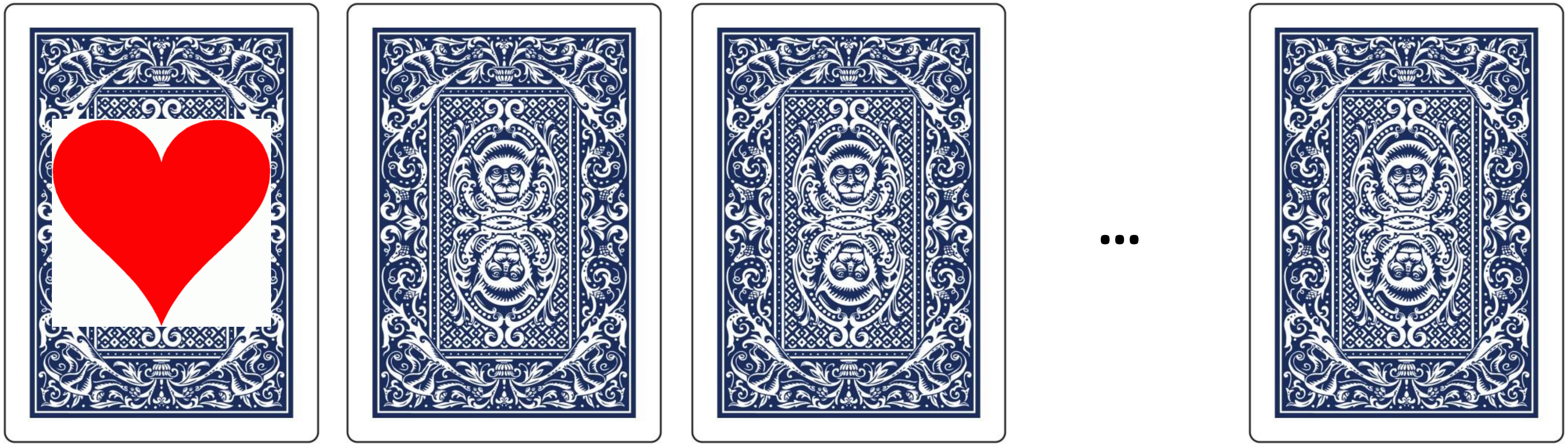


Example Credit: Guy Van den Broeck



Pr(First Card is a ) = ?

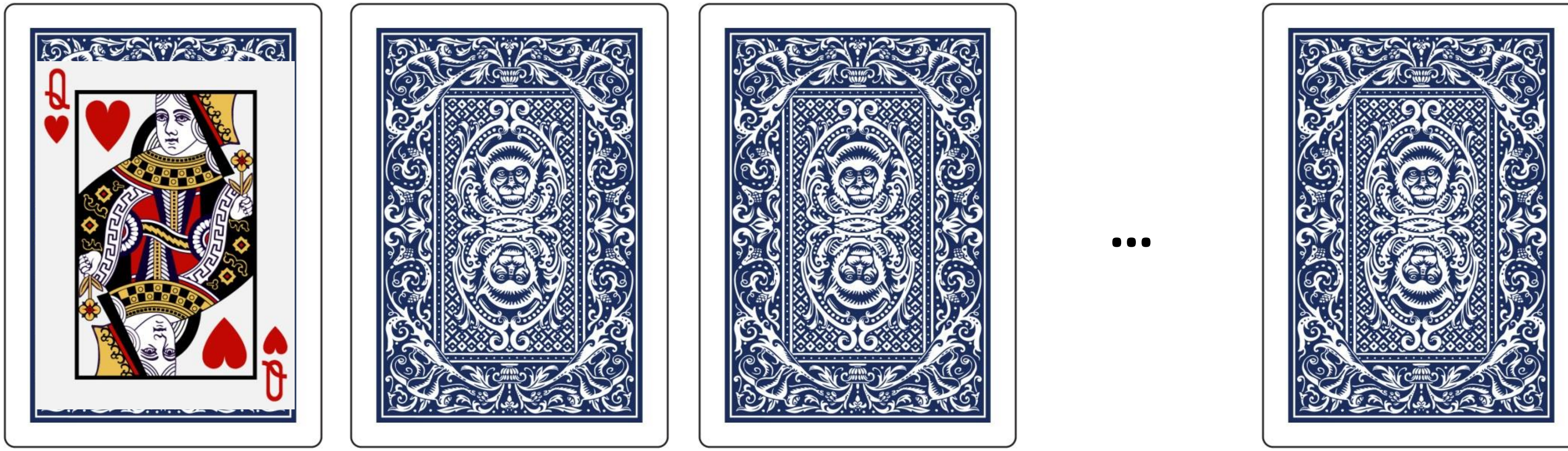
Example Credit: Guy Van den Broeck



Pr(First Card is a ♥) = ?

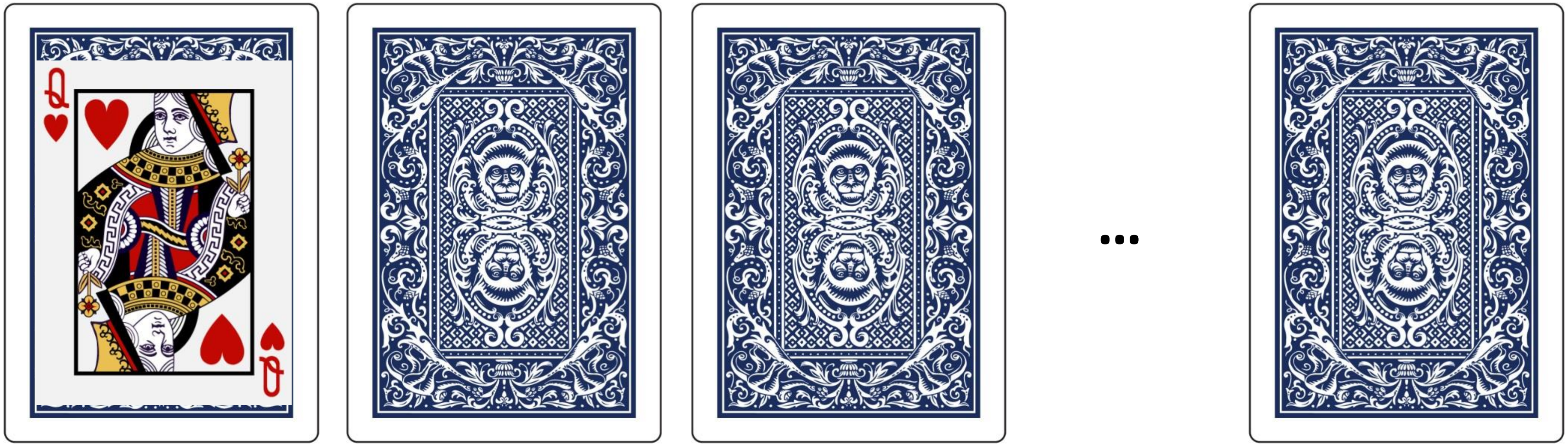
1/4

Example Credit: Guy Van den Broeck



Pr(First Card is Q of ♥) = ?

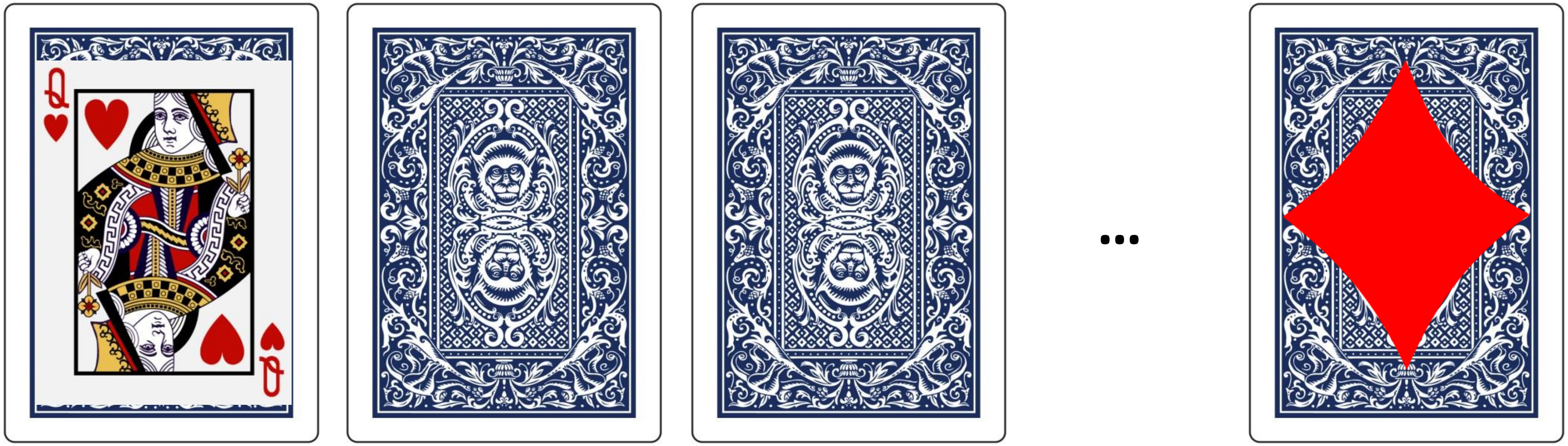
Example Credit: Guy Van den Broeck



Pr(First Card is Q of ♥) = ?

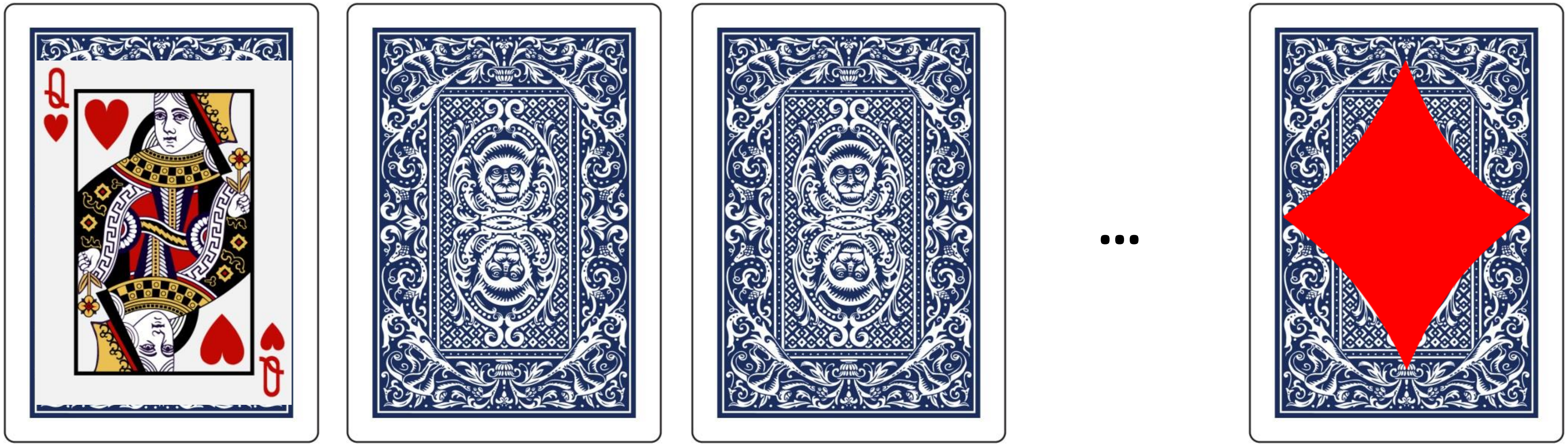
1/52

Example Credit: Guy Van den Broeck



Pr(Last Card is a ♦ | the first card is Q of ♥) = ?

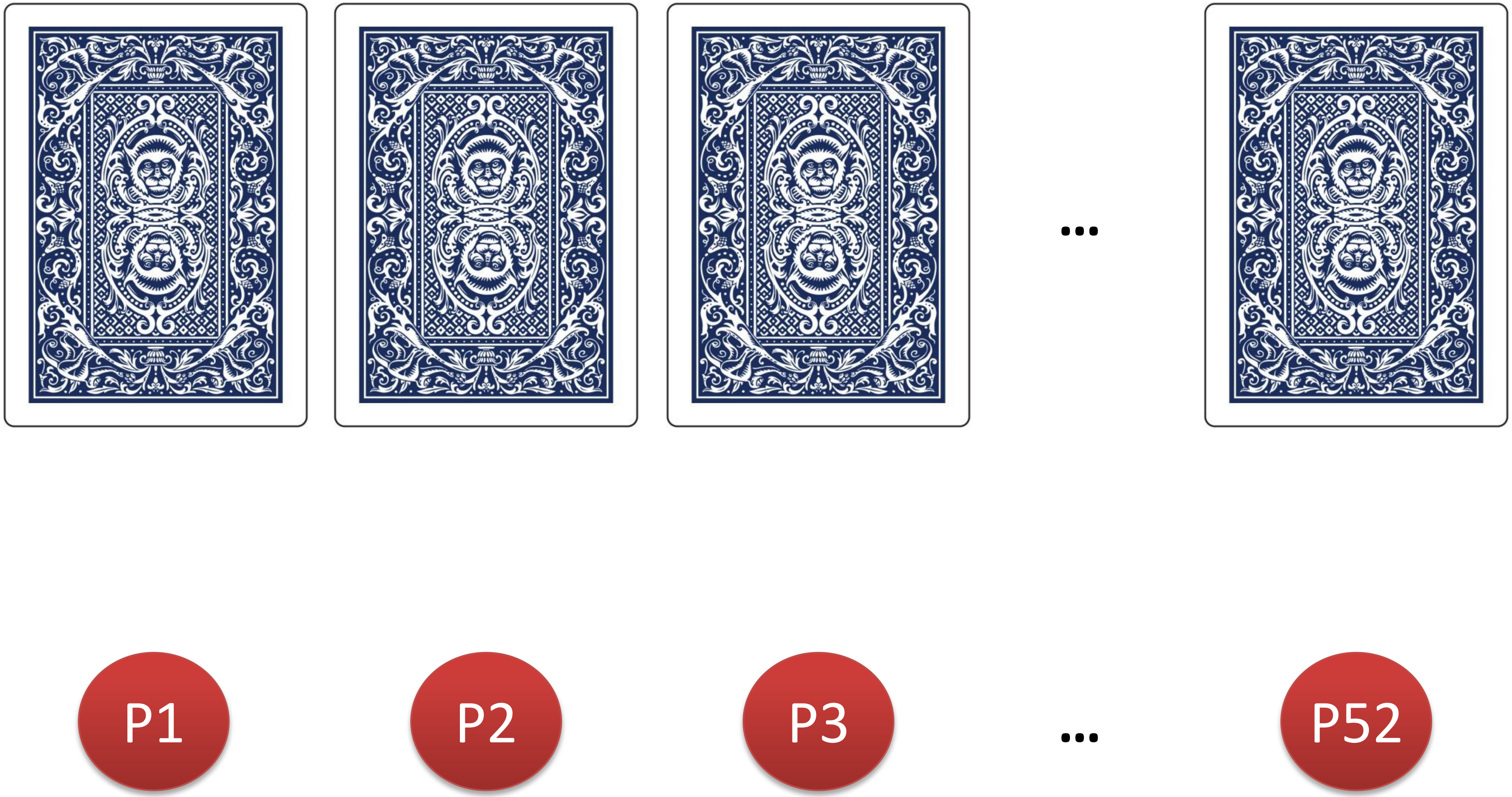
Example Credit: Guy Van den Broeck



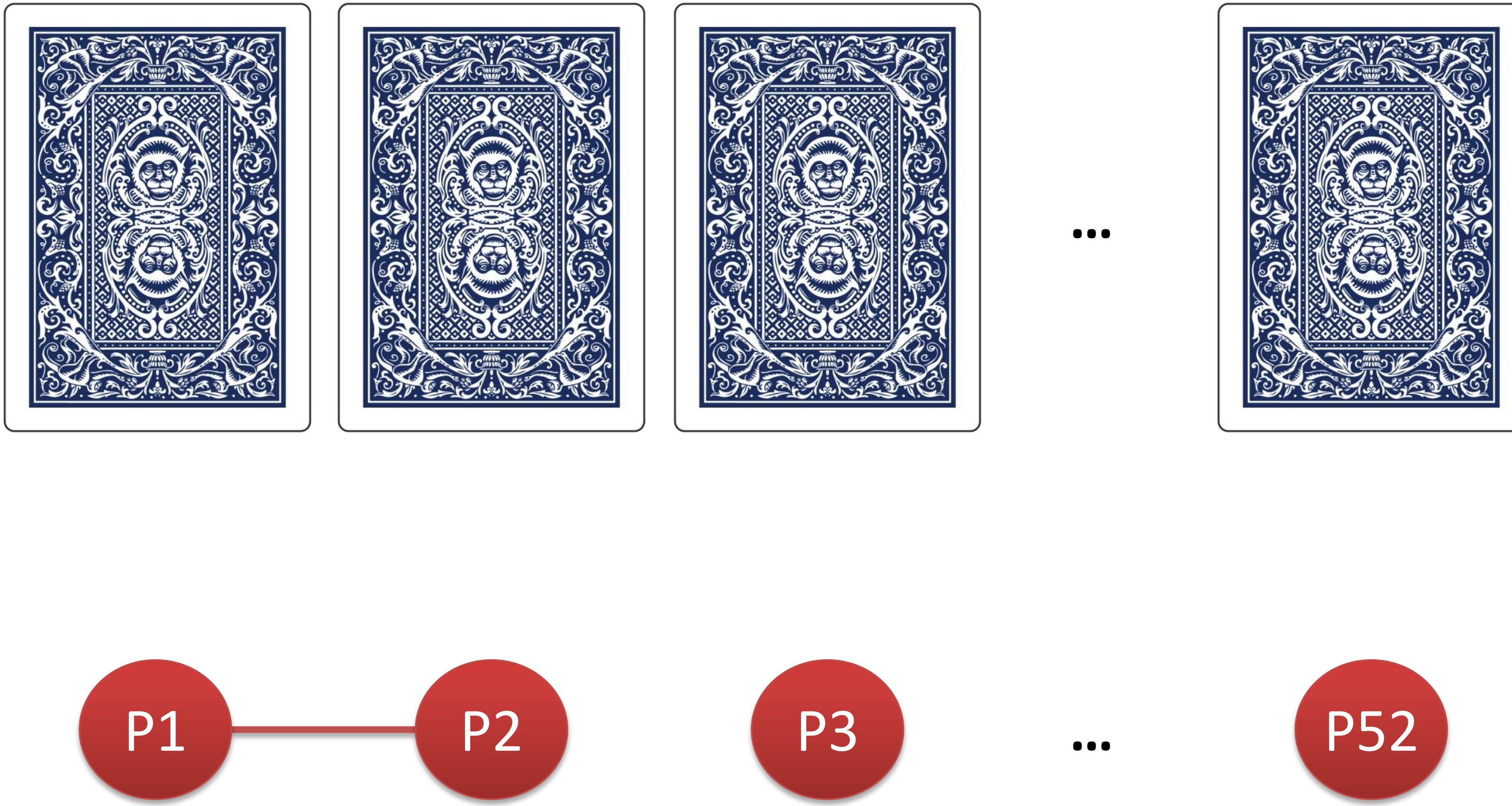
Pr(Last Card is a ♦ | the first card is Q of ♥) = ?

13/51

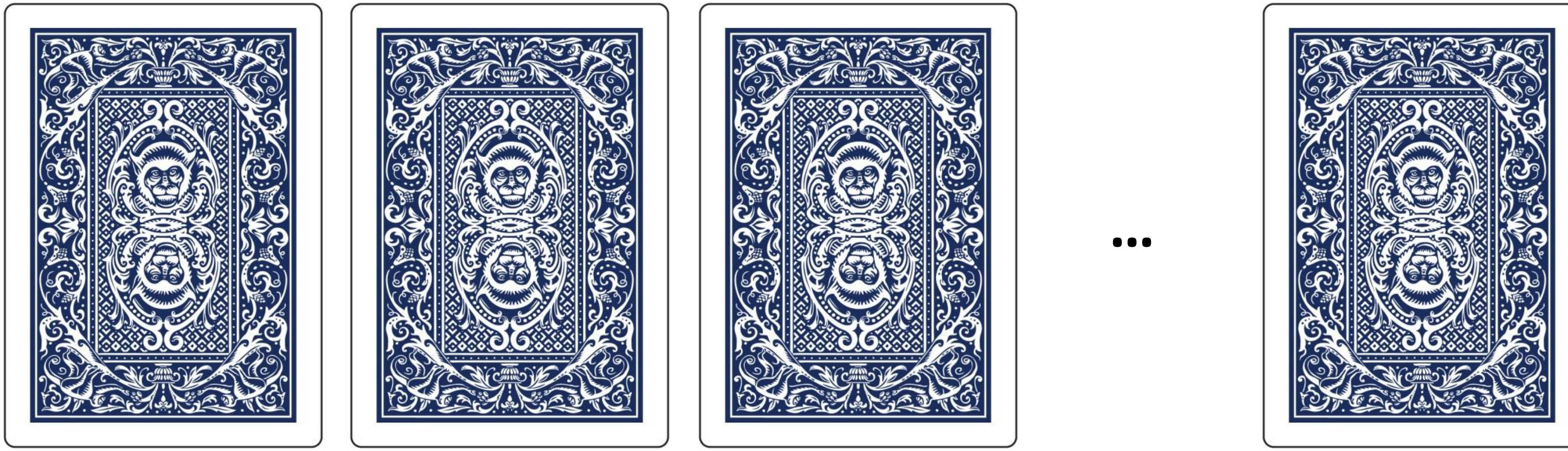
Example Credit: Guy Van den Broeck

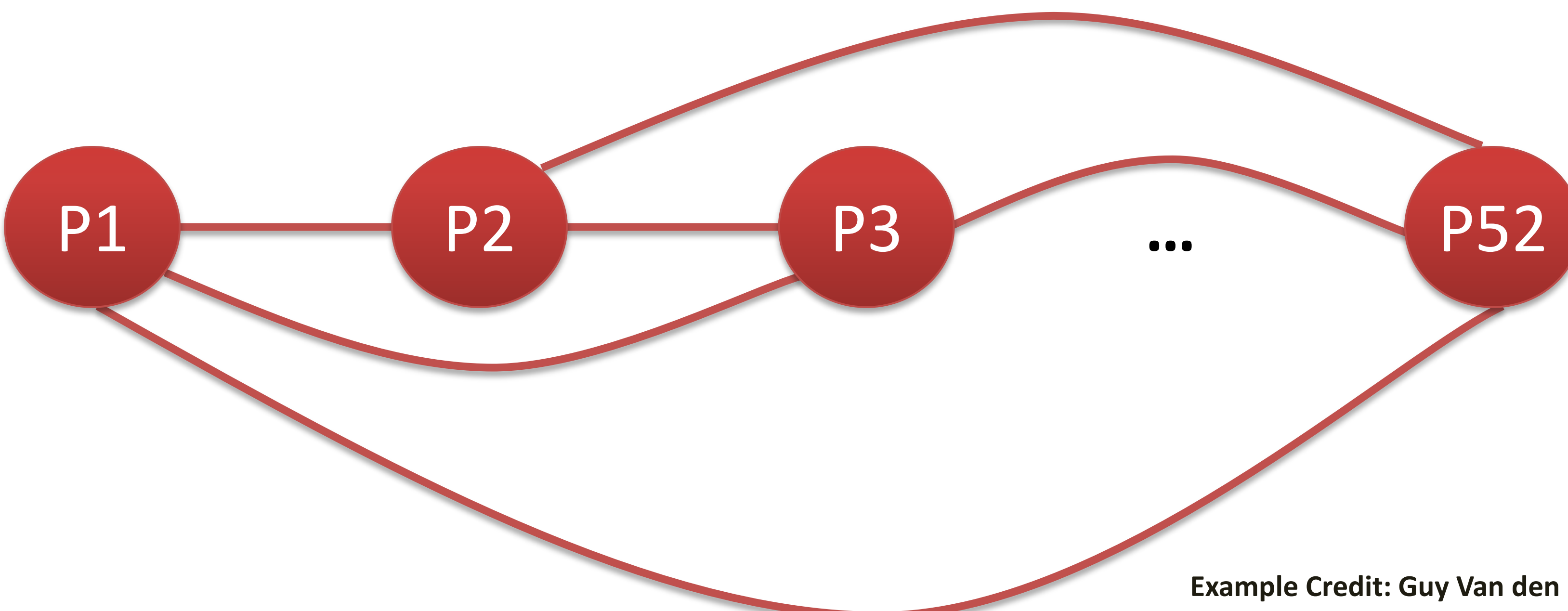


Example Credit: Guy Van den Broeck



Example Credit: Guy Van den Broeck





Example Credit: Guy Van den Broeck

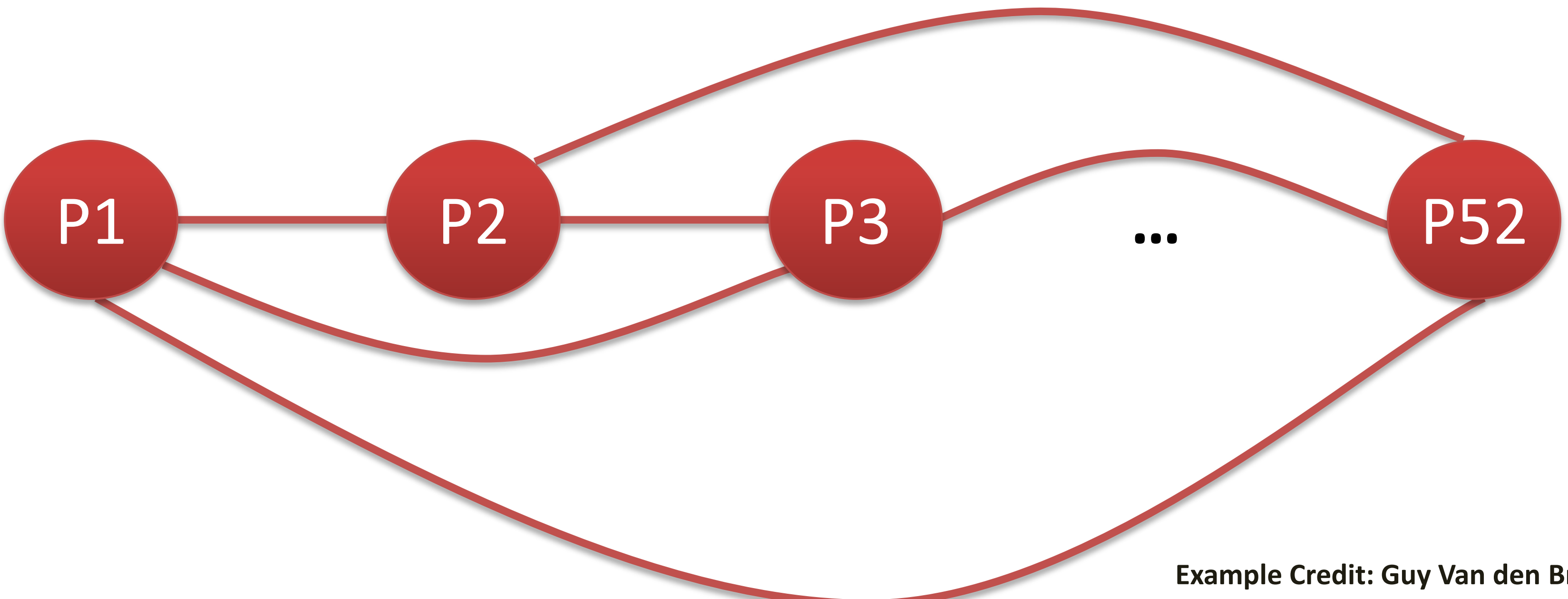
$$P1 = A\diamondsuit \lor P2 = A\diamondsuit \lor \ldots \lor P52 = A\diamondsuit$$

$$P1 = 2\diamondsuit \lor P2 = 2\diamondsuit \lor \ldots \lor P52 = 2\diamondsuit$$

…

$$P1 = A\diamondsuit \Rightarrow \neg P2 = A\diamondsuit \land \ldots \land \neg P52 = A\diamondsuit$$

…



Example Credit: Guy Van den Broeck

$P1 = A$ ♦ ∨ $P2 = A$ ♦ ∨ … ∨ $P52 = A$ ♦

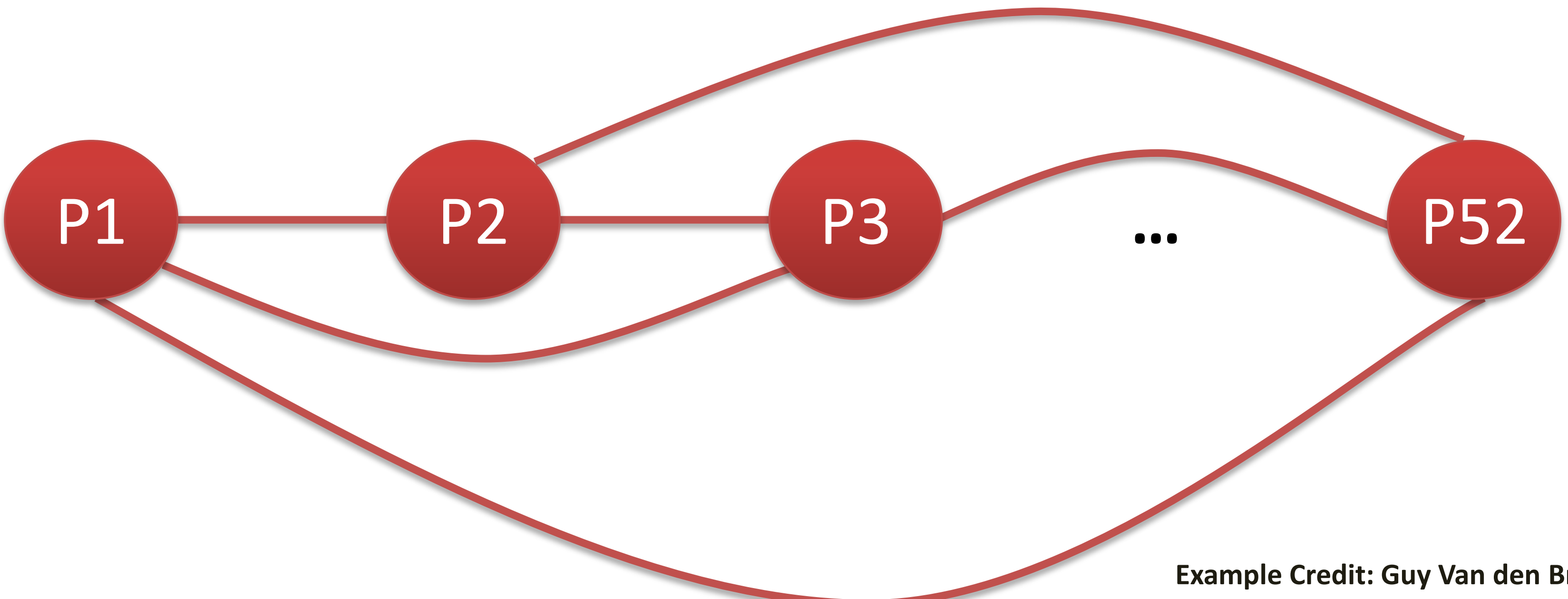
$P1 = 2$ ♦ ∨ $P2 = 2$ ♦ ∨ … ∨ $P52 = 2$ ♦

…

$P1 = A$ ♦ $\Rightarrow \neg P2 = A$ ♦ ∧ … ∧ $\neg P52 = A$ ♦

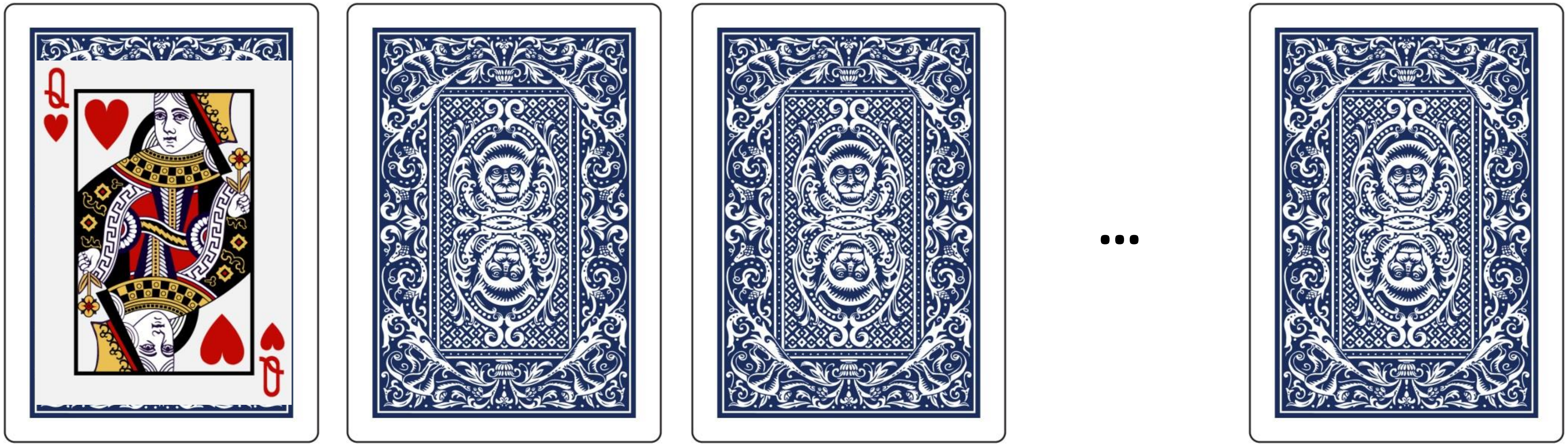
…

WMC is $O(52^{52})$



Example Credit: Guy Van den Broeck

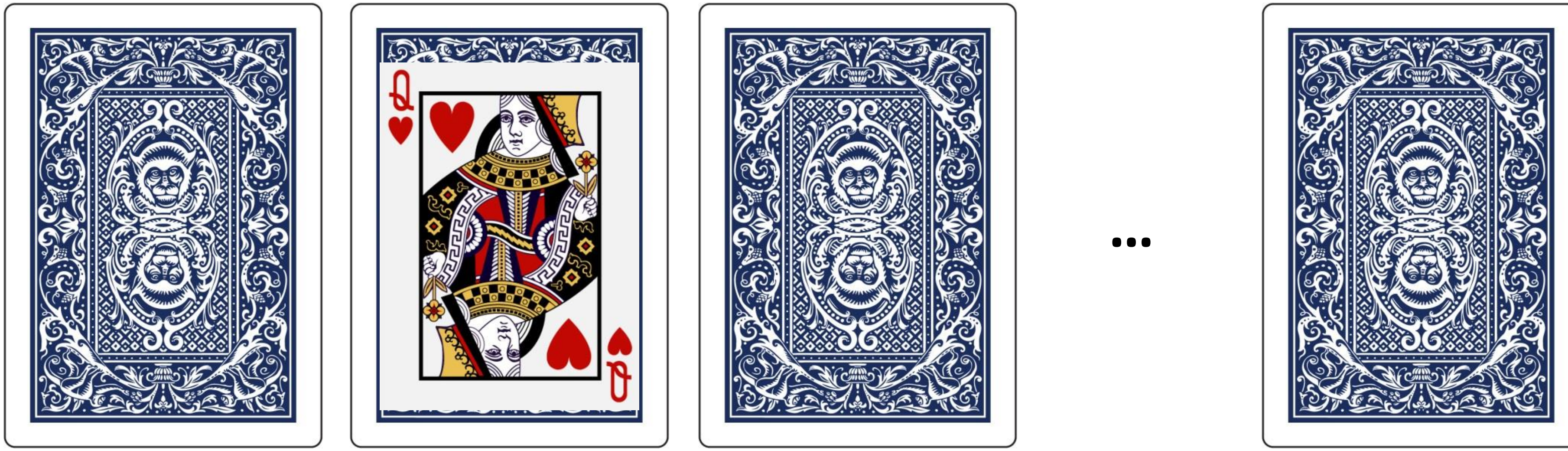
Why are these queries so easy for us
but so difficult for the reasoner?



Pr(First Card is Q of ♥) = ?

1/52

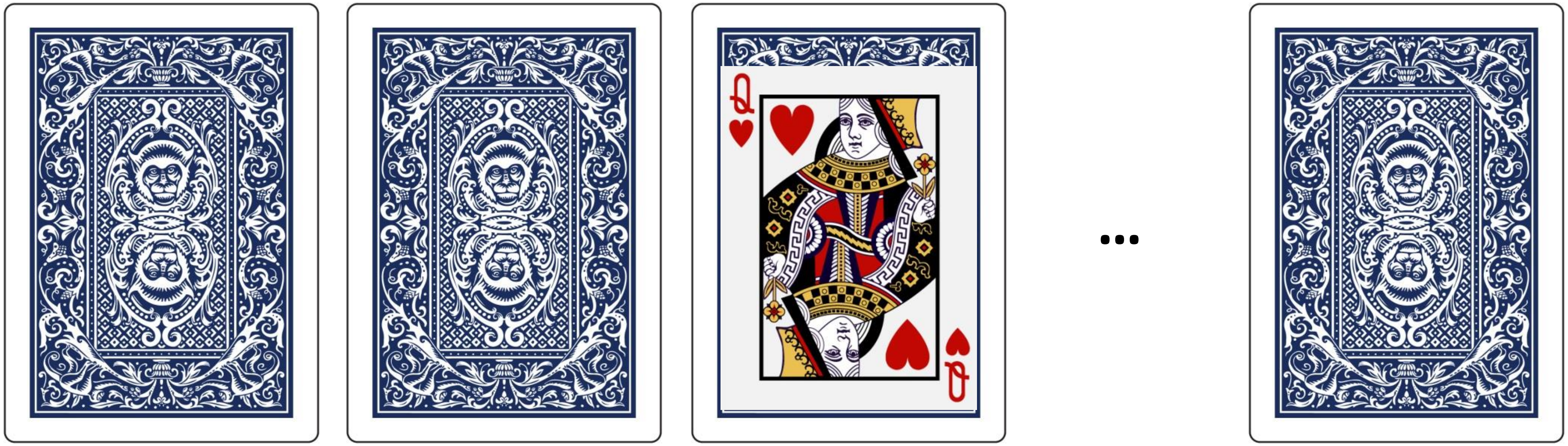
Example Credit: Guy Van den Broeck



Pr(Second Card is Q of ) = ?

1/52

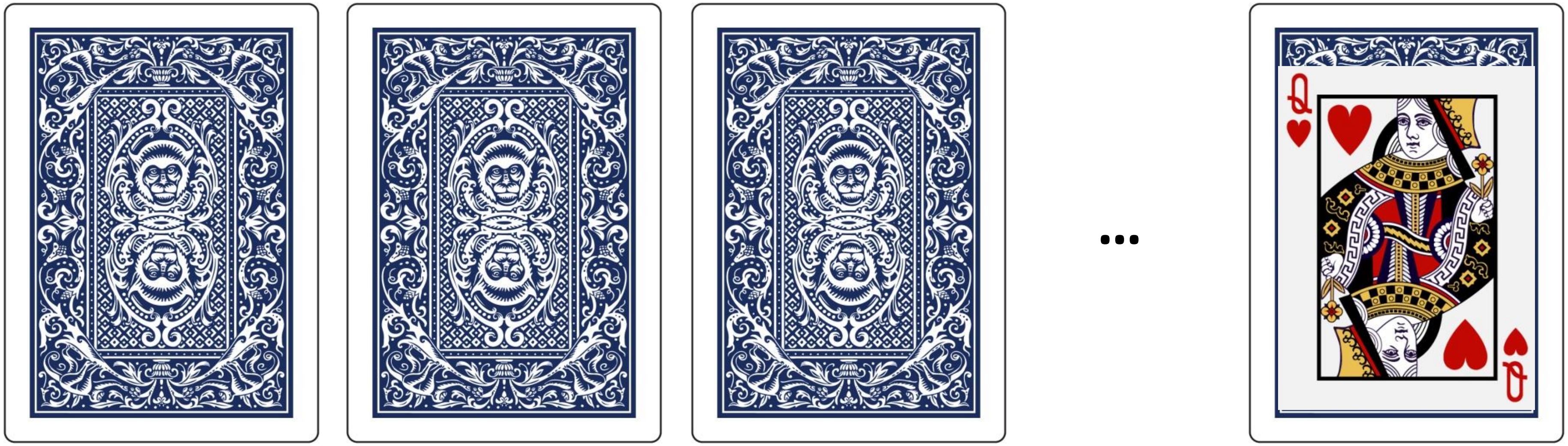
Example Credit: Guy Van den Broeck



Pr(Third Card is Q of ♥) = ?

1/52

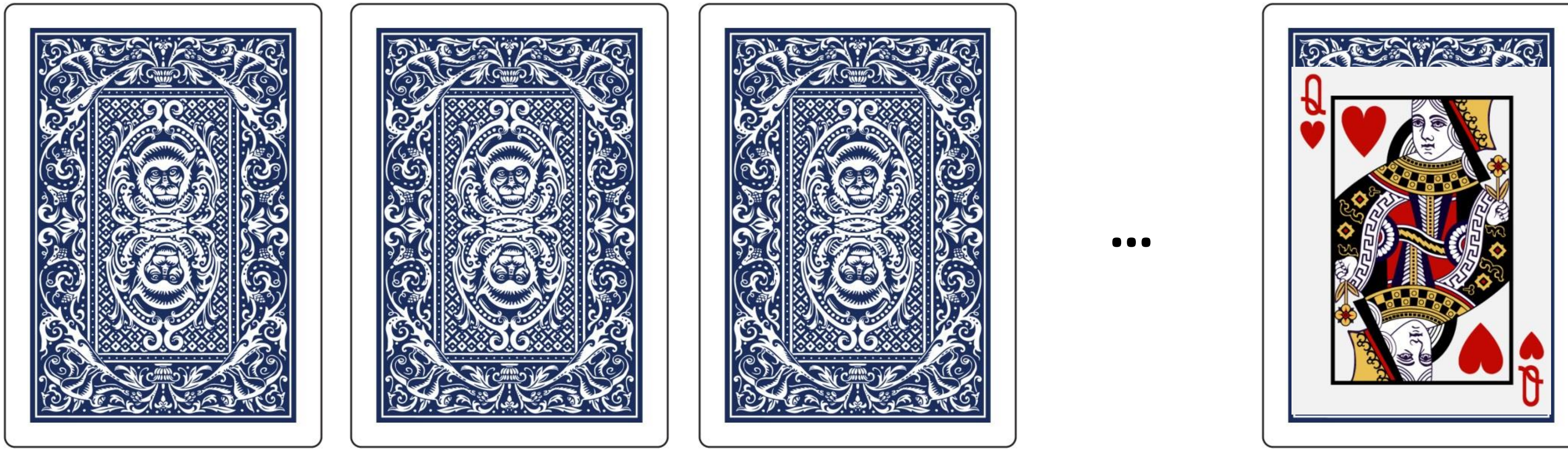
Example Credit: Guy Van den Broeck



Pr(Last Card is Q of ) = ?

1/52

Example Credit: Guy Van den Broeck

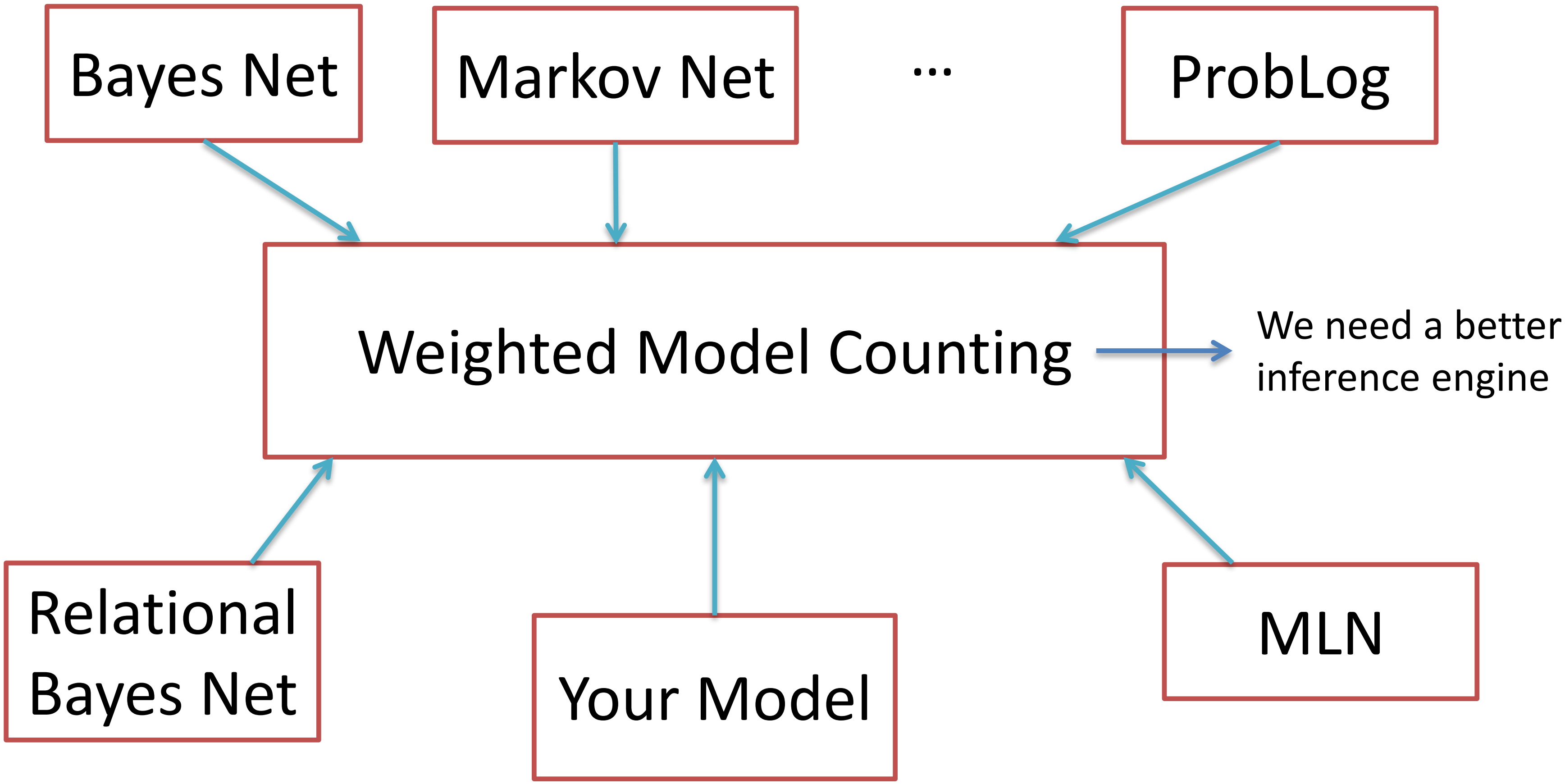


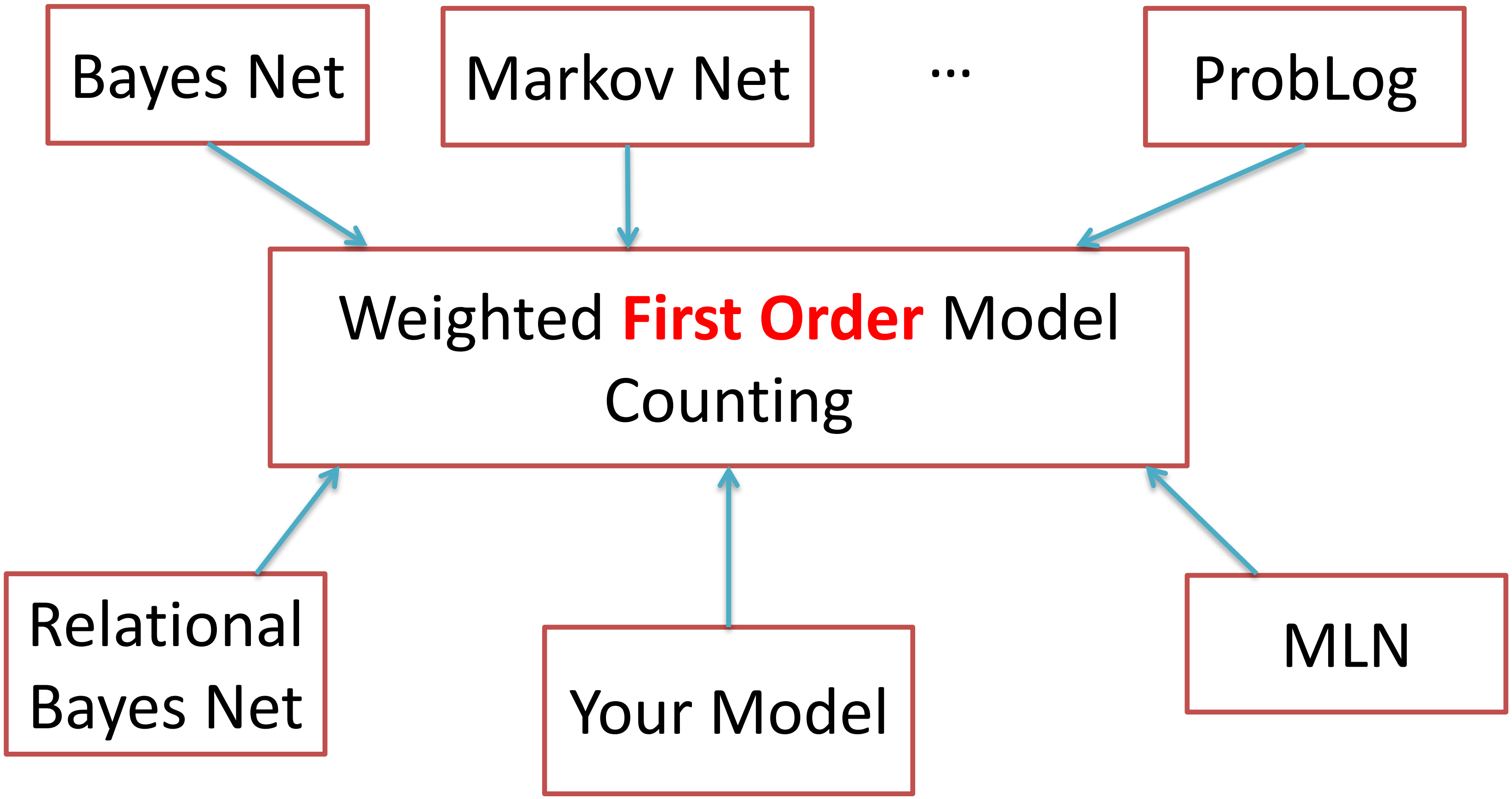
The cards are **exchangeable!**

If the inference engine exploits exchangeability, it can answer these queries in polynomial time

Lifted Inference

Example Credit: Guy Van den Broeck


Bayes Net
Markov Net
…
ProbLog
Weighted Model Counting
We need a better inference engine
Relational Bayes Net
Your Model
MLN



Bayes Net
Markov Net
…
ProbLog
Weighted First Order Model Counting
Relational Bayes Net
Your Model
MLN

First-Order MC (FOMC)

- $\forall x: Smokes(x) \Rightarrow Cancer(x)$ $\quad x \in \{Alice, Bob\}$

First-Order MC (FOMC)

- $\forall x: Smokes(x) \Rightarrow Cancer(x)$ $\quad x \in \{Alice, Bob\}$

S(Alice)	S(Bob)	C(Alice)	C(Bob)	Model?
True	True	True	True	Yes
True	True	True	False	No
…	…	…	…	…
False	False	False	False	Yes

9 Models

Weighted FOMC (WFOMC)

- $\forall x: Smokes(x) \Rightarrow Cancer(x)$ $\quad x \in \{Alice, Bob\}$

S(Alice)	S(Bob)	C(Alice)	C(Bob)	Model?
True	True	True	True	Yes
True	True	True	False	No
…	…	…	…	…
False	False	False	False	Yes

9 Models

- $\forall x: W(Smokes(x)=True)=0.7$
- $\forall x: W(Smokes(x)=False)=0.2$
- $\forall x: W(Cancer(x)=True)=2$
- $\forall x: W(Cancer(x)=False)=1.1$

Weighted FOMC (WFOMC)

- $\forall x: Smokes(x) \Rightarrow Cancer(x)$ $\quad x \in \{Alice, Bob\}$

S(Alice)	S(Bob)	C(Alice)	C(Bob)	Model?	Weight
True	True	True	True	Yes	0.7*0.7*2*2
True	True	True	False	No	---
…	…	…	…	…	…
False	False	False	False	Yes	0.2*0.2*1.1*1.1

…

+

9 Models

WFOMC

- $\forall x$: *W(Smokes(x)=True)=0.7*
- $\forall x$: *W(Smokes(x)=False)=0.2*
- $\forall x$: *W(Cancer(x)=True)=2*
- $\forall x$: *W(Cancer(x)=False)=1.1*

WFOMC

- WFOMC uses all rules from WMC
  - Case analysis
  - Unit propagation
  - Decomposition
  - Caching
- Plus three more rules:
  - Lifted decomposition
  - Lifted case analysis
  - Domain recursion

Lifted Decomposition

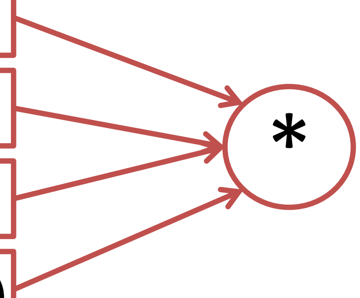
- $\forall x: Smokes(x) \Rightarrow Cancer(x)$ $\quad x \in \{A, B, C, D\}$

Lifted Decomposition

- $\forall x: Smokes(x) \Rightarrow Cancer(x)$ $\quad x \in \{A, B, C, D\}$
  - $Smokes(A) \Rightarrow Cancer(A)$
  - $Smokes(B) \Rightarrow Cancer(B)$
  - $Smokes(C) \Rightarrow Cancer(C)$
  - $Smokes(D) \Rightarrow Cancer(D)$

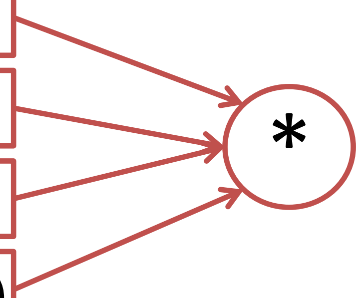
Lifted Decomposition

- $\forall x: Smokes(x) \Rightarrow Cancer(x)$ $\quad\quad x \in \{A, B, C, D\}$
  - $Smokes(A) \Rightarrow Cancer(A)$
  - $Smokes(B) \Rightarrow Cancer(B)$
  - $Smokes(C) \Rightarrow Cancer(C)$
  - $Smokes(D) \Rightarrow Cancer(D)$



Lifted Decomposition

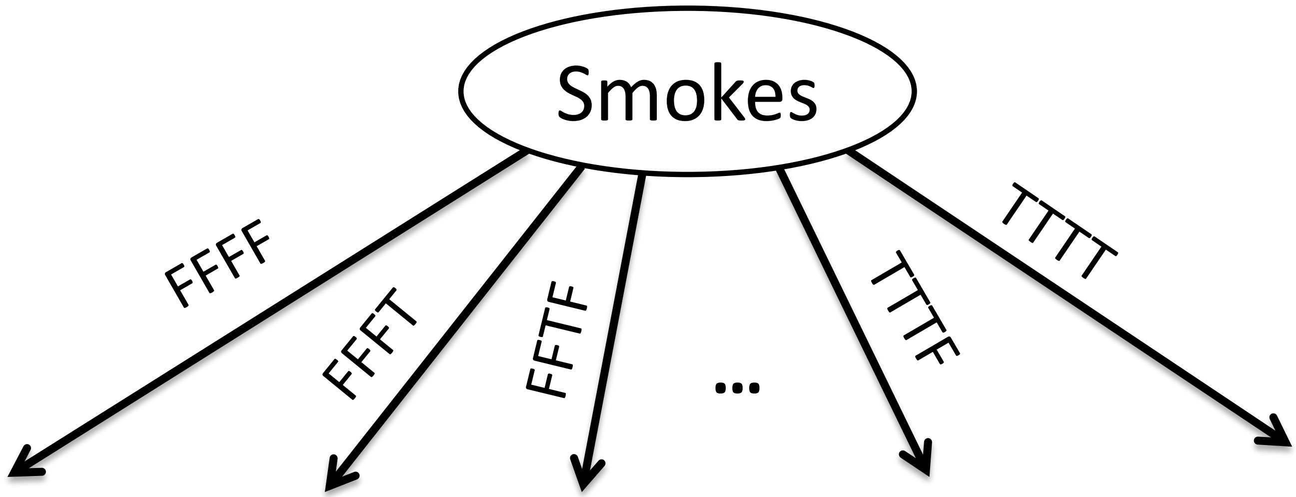
- $\forall x\colon Smokes(x) \Rightarrow Cancer(x)$ $\qquad x \in \{A, B, C, D\}$
  - $Smokes(A) \Rightarrow Cancer(A)$
  - $Smokes(B) \Rightarrow Cancer(B)$
  - $Smokes(C) \Rightarrow Cancer(C)$
  - $Smokes(D) \Rightarrow Cancer(D)$



$$WFOMC(Smokes(A) \Rightarrow Cancer(A))^4$$

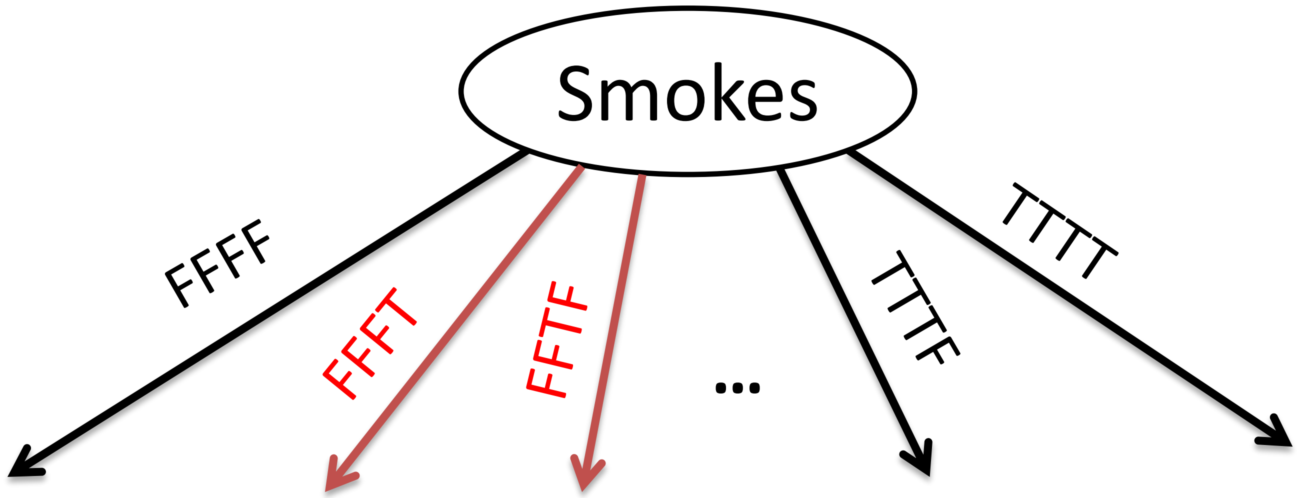
Lifted Case Analysis

» $\forall x, y: Smokes(x) \wedge Friend(x, y) \Rightarrow Smokes(y) \qquad x, y \in \{A, B, C, D\}$



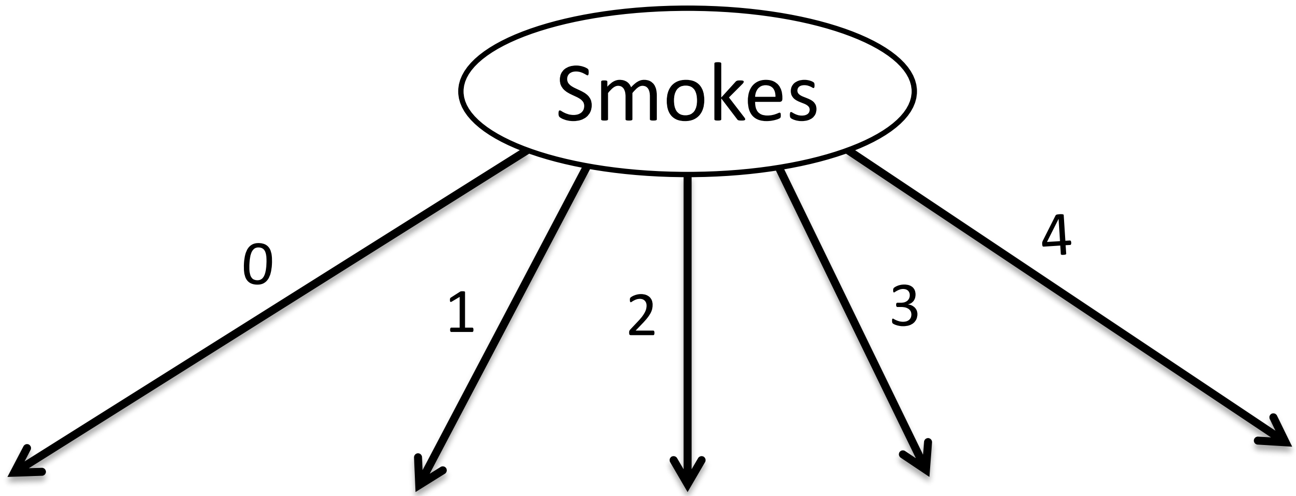
Lifted Case Analysis

» $\forall x, y: Smokes(x) \wedge Friend(x, y) \Rightarrow Smokes(y)$ $\quad x, y \in \{A, B, C, D\}$



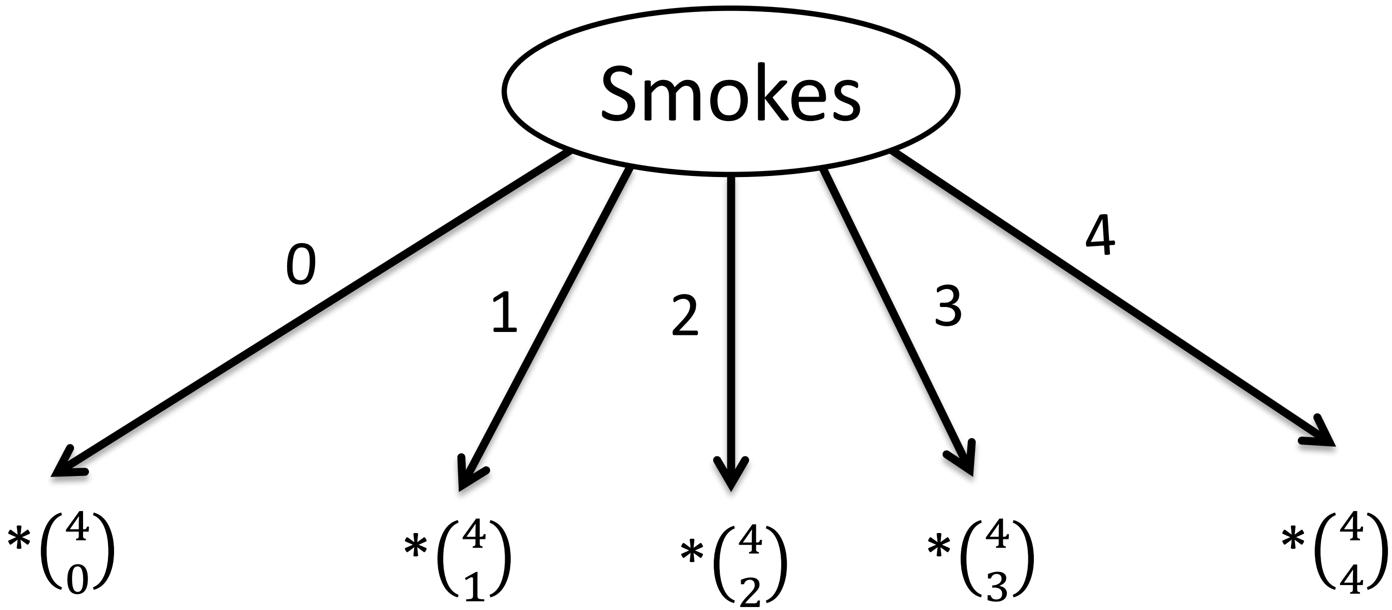
Lifted Case Analysis

» $\forall x, y: Smokes(x) \land Friend(x, y) \Rightarrow Smokes(y) \qquad x, y \in \{A, B, C, D\}$



Lifted Case Analysis

» $\forall x, y: Smokes(x) \wedge Friend(x, y) \Rightarrow Smokes(y)$ $\quad x, y \in \{A, B, C, D\}$



Domain Recursion

- $\forall x, y \neq x: Friend(x, y) \Rightarrow Friend(y, x)$ x, y ∈ {A,B,…,Z}

Domain Recursion

- $\forall x, y \neq x: Friend(x, y) \Rightarrow Friend(y, x)$ x, y ∈ {A,B,…,Z}

- $\forall y': Friend(A, y') \Rightarrow Friend(y', A)$ x’, y’ ∈ {B,…,Z}
- $\forall x': Friend(x', A) \Rightarrow Friend(A, x')$
- $\forall x', y' \neq x': Friend(x', y') \Rightarrow Friend(y', x')$

Domain Recursion

- $\forall x, y \neq x: Friend(x, y) \Rightarrow Friend(y, x)$ x, y ∈ {A,B,…,Z}

- $\forall y': Friend(A, y') \Rightarrow Friend(y', A)$ x', y' ∈ {B,…,Z}
- $\forall x': Friend(x', A) \Rightarrow Friend(A, x')$
- $\forall x', y' \neq x': Friend(x', y') \Rightarrow Friend(y', x')$

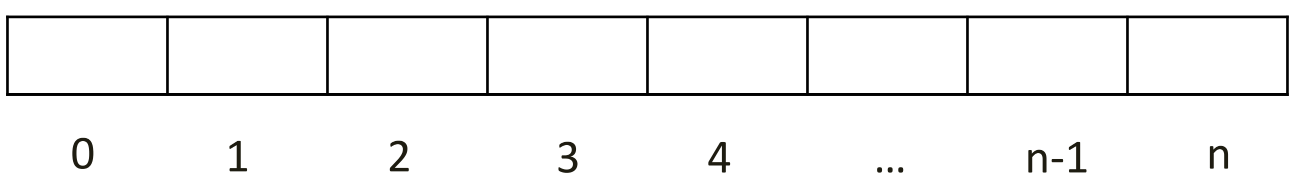
- $\forall x', y' \neq x': Friend(x', y') \Rightarrow Friend(y', x')$ x', y' ∈ {B,…,Z}

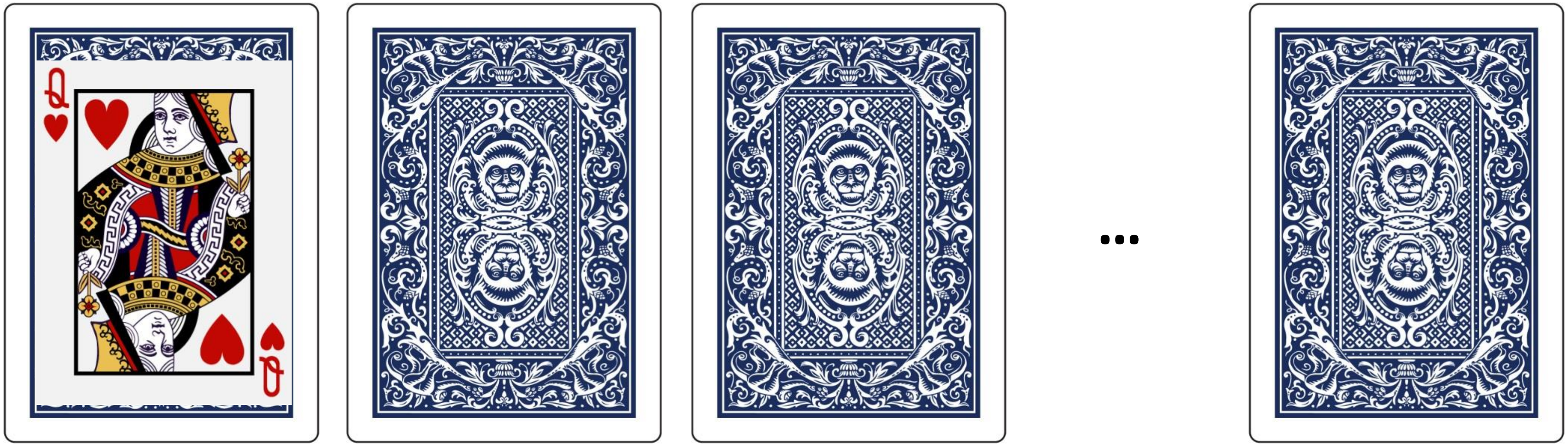
Domain Recursion

- $\forall x, y \neq x: Friend(x, y) \Rightarrow Friend(y, x)$ x, y ∈ {A,B,…,Z}

- $\forall y': Friend(A, y') \Rightarrow Friend(y', A)$ x’, y’ ∈ {B,…,Z}
- $\forall x': Friend(x', A) \Rightarrow Friend(A, x')$
- $\forall x', y' \neq x': Friend(x', y') \Rightarrow Friend(y', x')$

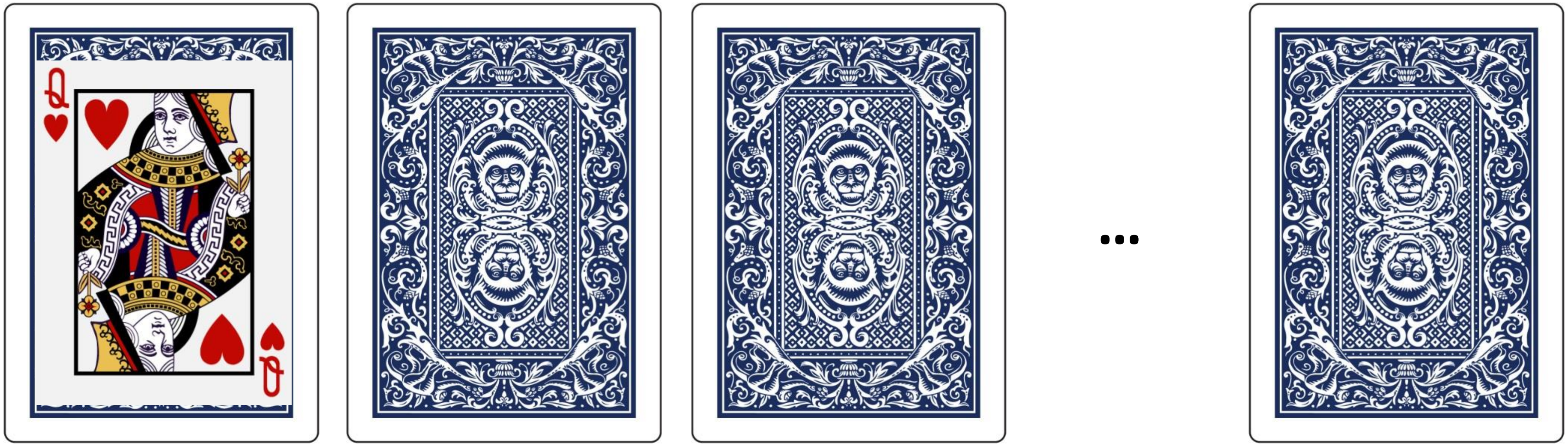
- $\forall x', y' \neq x': Friend(x', y') \Rightarrow Friend(y', x')$ x’, y’ ∈ {B,…,Z}





$$\forall pos, \exists card: In(card, pos)$$
$$\forall card, \exists pos: In(card, pos)$$
$$\forall pos, card1, card2: In(card1, pos) \land In(card2, pos) \Rightarrow card1 = card2$$

Example Credit: Guy Van den Broeck



$$\forall pos, \exists card: In(card, pos)$$
$$\forall card, \exists pos: In(card, pos)$$
$$\forall pos, card1, card2: In(card1, pos) \wedge In(card2, pos) \Rightarrow card1 = card2$$

$$WFOMC = \sum_{k=0}^{n} \binom{n}{k} \sum_{l}^{n} \binom{n}{l} (l+1)^k (-1)^{2n-k-l}$$

Polynomial in n

Example Credit: Guy Van den Broeck

Now we know how to do WFOMC efficiently

Let’s see an example of how we can use it

MLN to WFOMC

$w_1 : Smokes(x) \land Friends(x, y) \Rightarrow Smokes(y)$
$w_2 : Smokes(x) \Rightarrow Cancer(x)$

MLN to WFOMC

$w_1 : Smokes(x) \wedge Friends(x,y) \Rightarrow Smokes(y)$
$w_2 : Smokes(x) \Rightarrow Cancer(x)$

$\theta_1(x,y) \Leftrightarrow \big(Smokes(x) \wedge Friends(x,y) \Rightarrow Smokes(y)\big)$
$\theta_2(x) \Leftrightarrow (Smokes(x) \Rightarrow Cancer(x))$
$W(\theta_1) = \exp(w_1)$
$W(\theta_2) = \exp(w_2)$
$W(any\ other\ predicate) = 1$

MLN to WFOMC

$w_1 : Smokes(x) \wedge Friends(x, y) \Rightarrow Smokes(y)$
$w_2 : Smokes(x) \Rightarrow Cancer(x)$

$\theta_1(x, y) \Leftrightarrow \big(Smokes(x) \wedge Friends(x, y) \Rightarrow Smokes(y)\big)$
$\theta_2(x) \Leftrightarrow (Smokes(x) \Rightarrow Cancer(x))$
$W(\theta_1) = \exp(w_1)$
$W(\theta_2) = \exp(w_2)$
$W(any\ other\ predicate) = 1$

Normalization constant (Z) of the MLN

=

WFOMC of the above theory

Domain Liftability

- A theory T is domain liftable if we can compute its WFOMC in time polynomial in the number of objects.

$$\forall pos, \exists card: In(card, pos)$$
$$\forall card, \exists pos: In(card, pos)$$
$$\forall pos, card1, card2: In(card1, pos) \wedge In(card2, pos) \Rightarrow card1 = card2$$

$$WFOMC = \sum_{k=0}^{n} \binom{n}{k} \sum_{l}^{n} \binom{n}{l} (l+1)^k (-1)^{2n-k-l}$$

Polynomial in n

FO2

$$\forall x, y: Smokes(x) \land Friends(x, y) \Rightarrow Smokes(y)$$
$$\forall x: Smokes(x) \Rightarrow Cancer(x)$$

FO^i is the class of theories where each sentence has at most ***i*** logical variables


FO2
S²FO²

S^2FO^2

A theory T in FO^2

$\wedge$

$\forall x, m_1, m_2: S(x, m_1) \vee S(x, m_2)$
$\forall x, m_1, m_2: \neg S(x, m_1) \vee S(x, m_2)$
$\forall x_1, x_2, m: S(x_1, m) \vee S(x_2, m)$
$\forall x_1, x_2, m_1, m_2: S(x_1, m_1) \vee S(x_2, m_2)$
…

For any sentence $c \in T$, if $S(x, m) \in c$, all other atoms in c have at most one variable.

$\boldsymbol{S^2FO^2}$ Example

$$\forall j, v: InvolvesGas(j) \wedge Smokes(v) \Rightarrow \neg Assigned(j, v)$$

$$\forall v1, v2: Smokes(v1) \wedge Friends(v1, v2) \Rightarrow Smokes(v2)$$

S^2FO^2 Example

$\forall j, v: InvolvesGas(j) \wedge Smokes(v) \Rightarrow \neg Assigned(j, v)$

$\forall v1, v2: Smokes(v1) \wedge Friends(v1, v2) \Rightarrow Smokes(v2)$

This theory is in FO^2

$\boldsymbol{S^2FO^2}$ Example

$$\forall j, v: InvolvesGas(j) \wedge Smokes(v) \Rightarrow \neg Assigned(j, v)$$

$$\forall v1, v2: Smokes(v1) \wedge Friends(v1, v2) \Rightarrow Smokes(v2)$$

$$\forall v1, v2, j: \neg Assigned(j, v1) \vee \neg Assigned(j, v2)$$

$$\forall v, j1, j2: \neg Assigned(j1, v) \vee \neg Assigned(j2, v)$$

This theory is NOT in FO^2

$\boldsymbol{S^2FO^2}$ Example

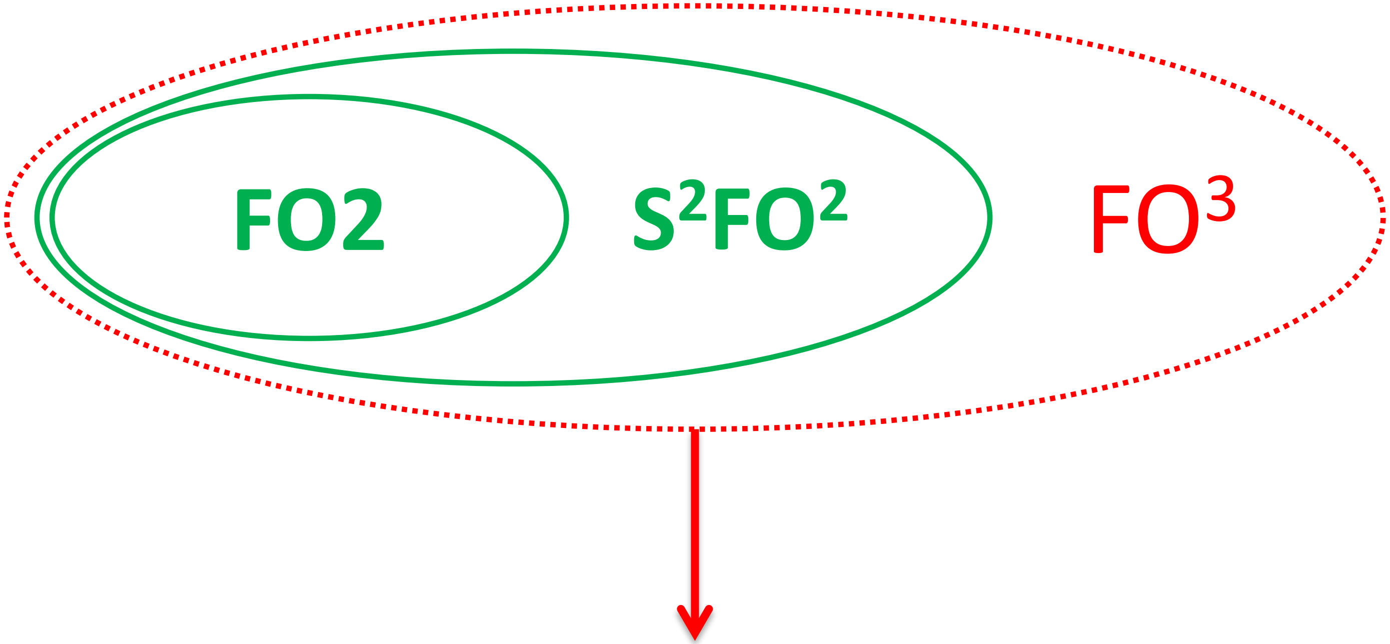
$$\forall j, v: InvolvesGas(j) \wedge Smokes(v) \Rightarrow \neg Assigned(j, v)$$

$$\forall v1, v2: Smokes(v1) \wedge Friends(v1, v2) \Rightarrow Smokes(v2)$$

$$\forall v1, v2, j: \neg Assigned(j, v1) \vee \neg Assigned(j, v2)$$

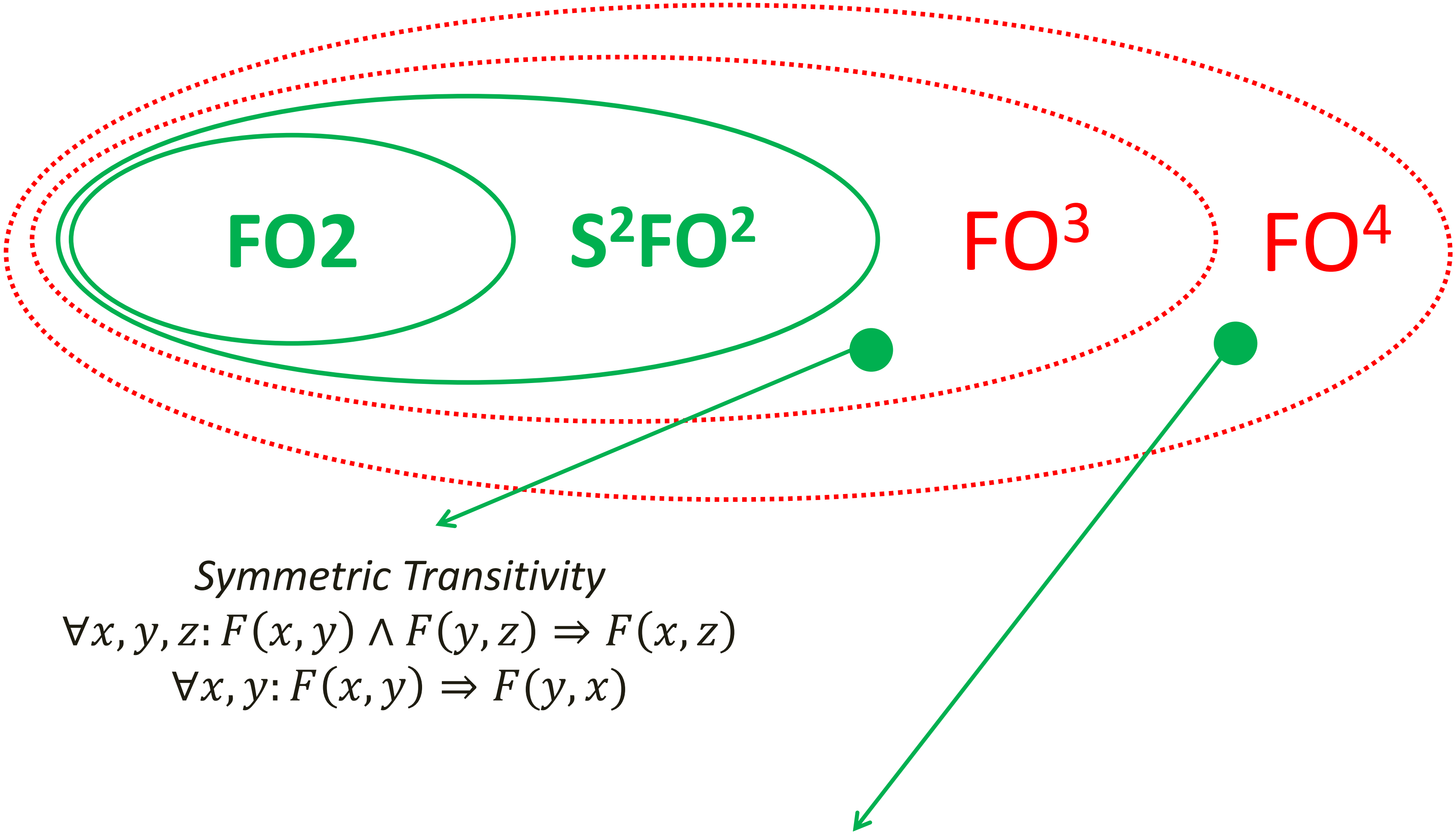
$$\forall v, j1, j2: \neg Assigned(j1, v) \vee \neg Assigned(j2, v)$$

But it is in S^2FO^2



We can't do polynomial time inference for all of them

FO^i is the class of theories where each sentence has at most ***i*** logical variables



Symmetric Transitivity

$$\forall x, y, z: F(x,y) \land F(y,z) \Rightarrow F(x,z)$$

$$\forall x, y: F(x,y) \Rightarrow F(y,x)$$

S4 Clause

$$\forall x_1, x_2, y_1, y_2: S(x_1,y_1) \lor \neg S(x_1,y_2) \lor S(x_2,y_2) \lor \neg S(x_2,y_1)$$

FO^i is the class of theories where each sentence has at most ***i*** logical variables

Software

- **L2C**
  - **https://github.com/Mehran-k/L2C**
- **WFOMC**
  - **https://dtai.cs.kuleuven.be/software/wfomc**
- **PTP**
  - **https://code.google.com/archive/p/alchemy-2/**

Thank you