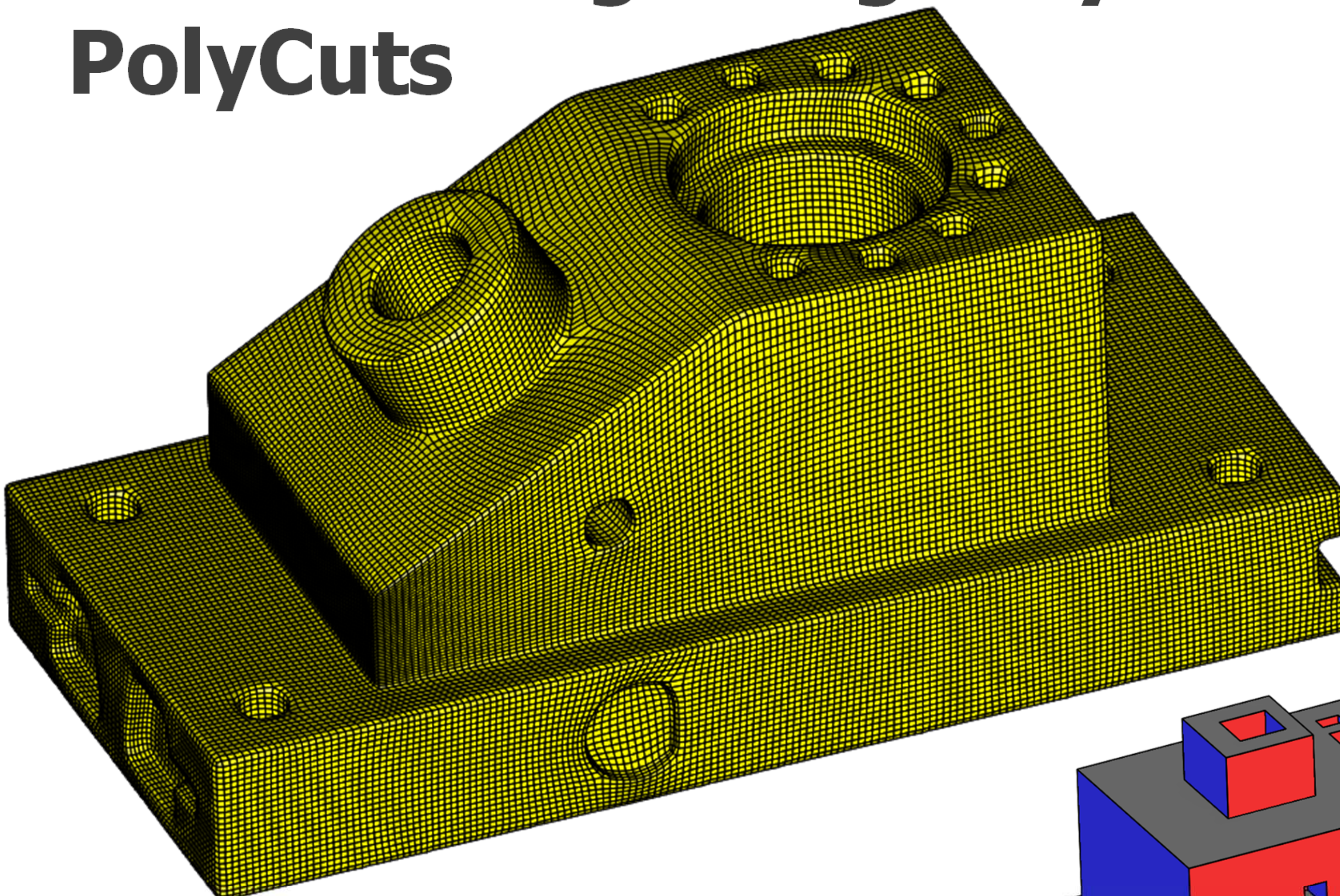
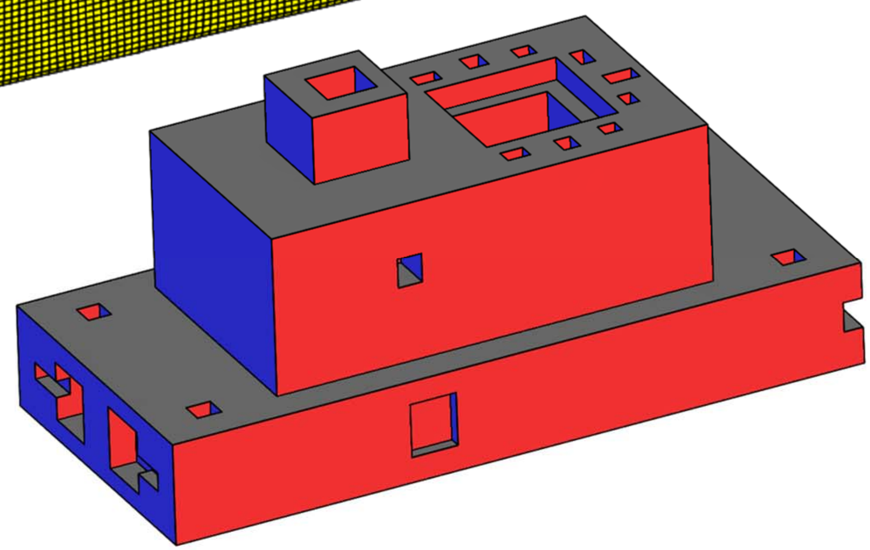
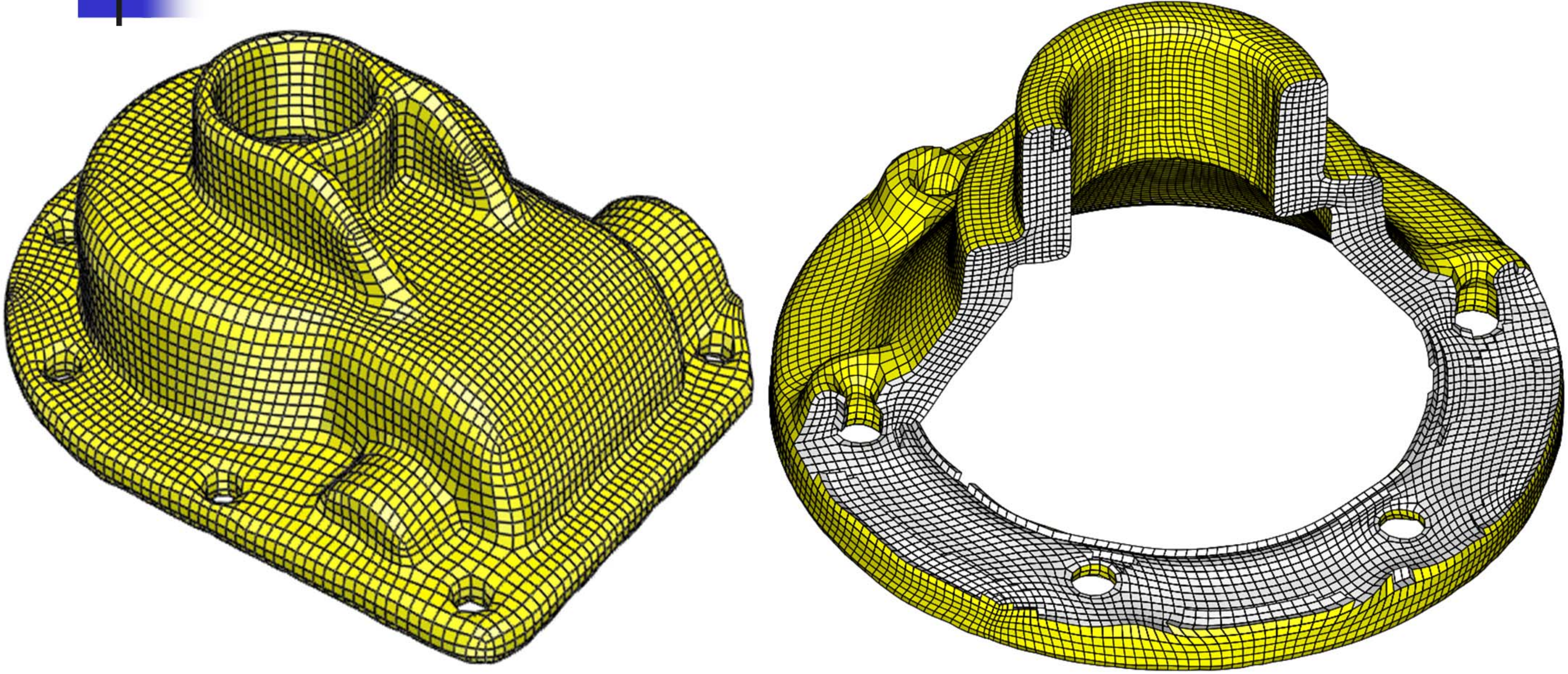
Hex meshing using PolyCubes and PolyCuts



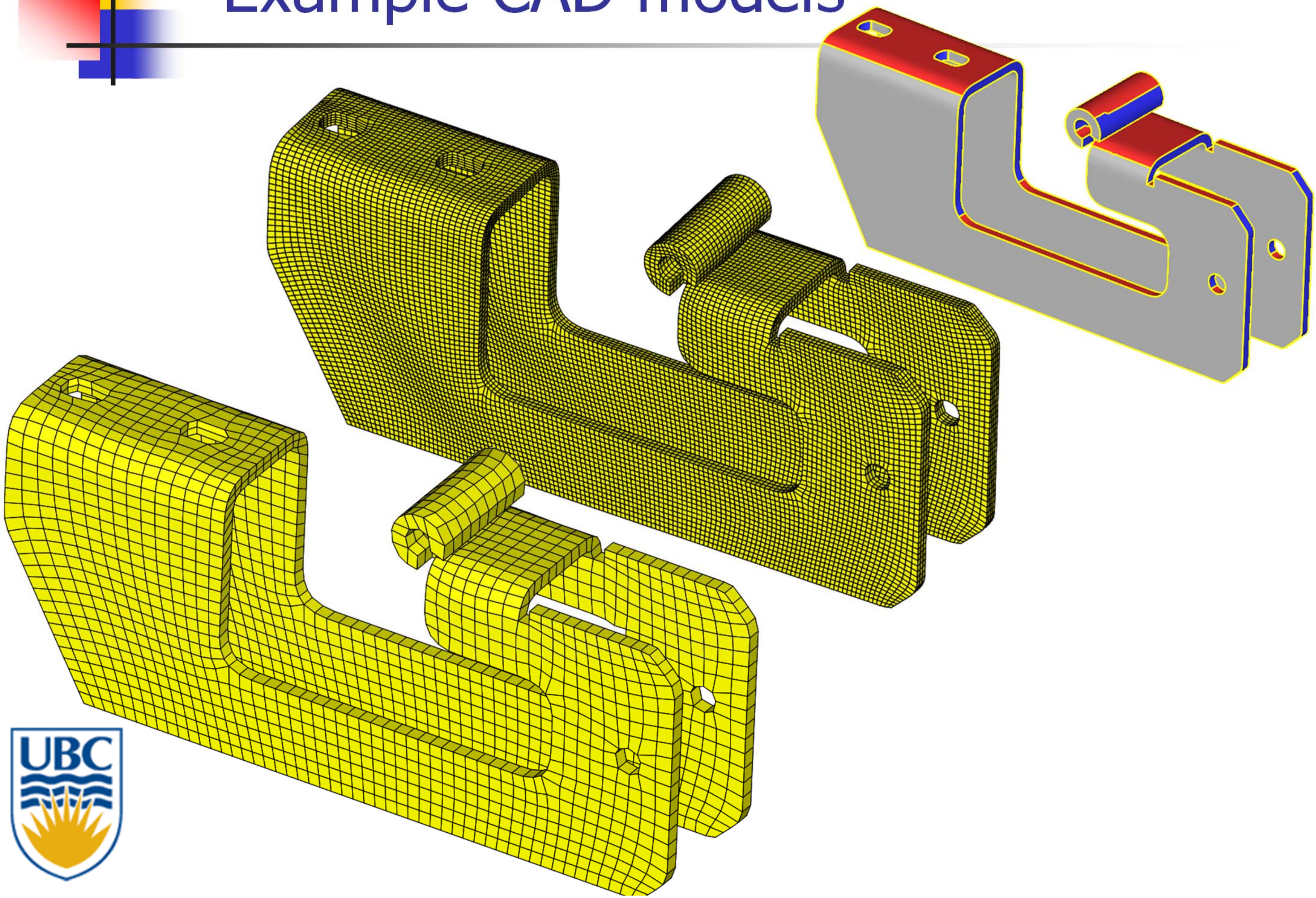


Automatic Hex Meshing

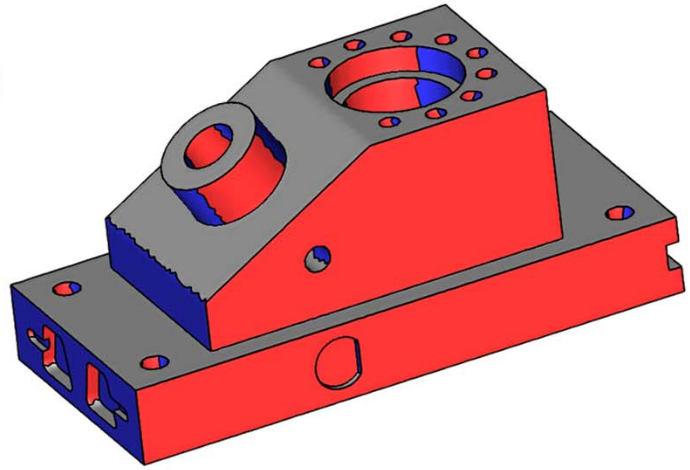


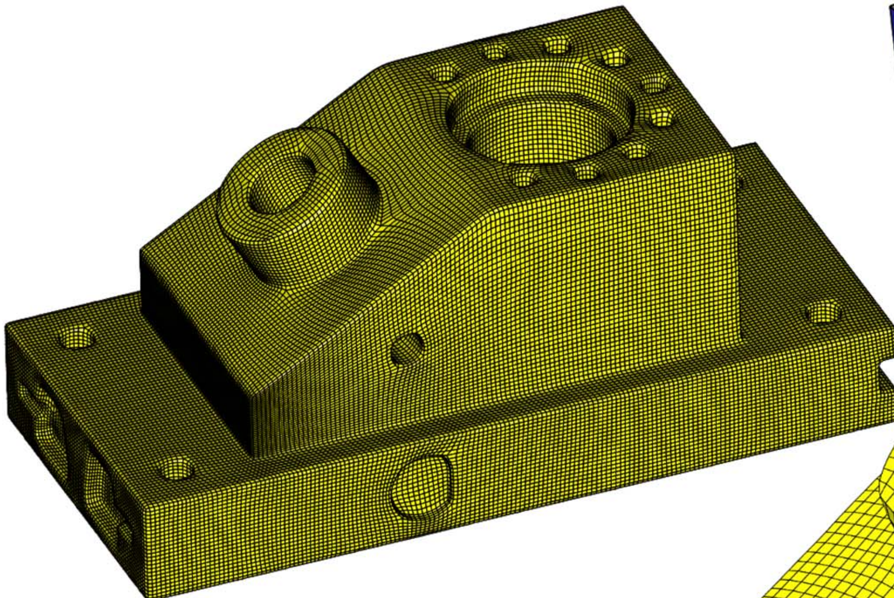
- All-hex elements
- High minimum/average quality
- (Near-)Regular connectivity

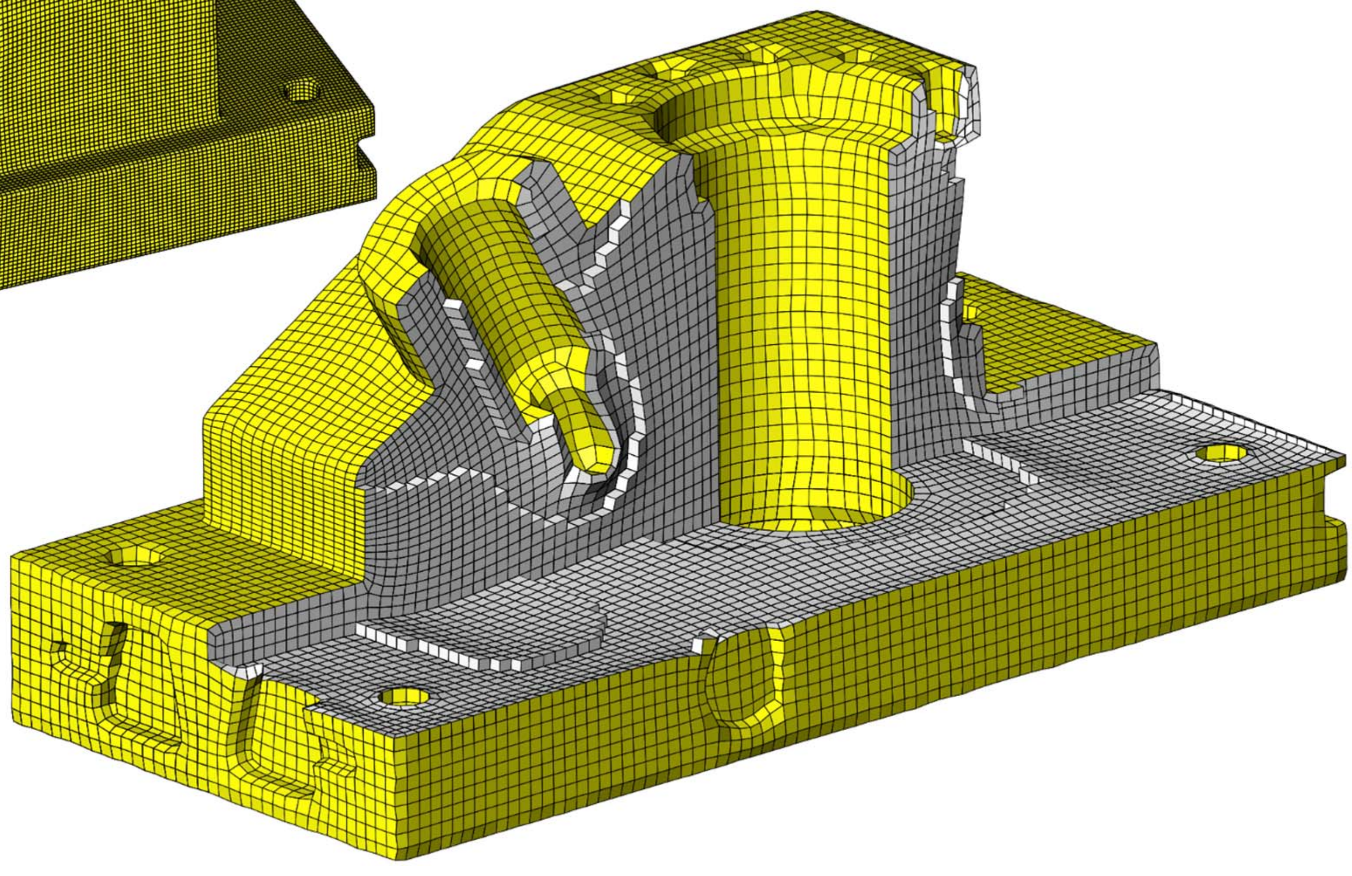
Example CAD models



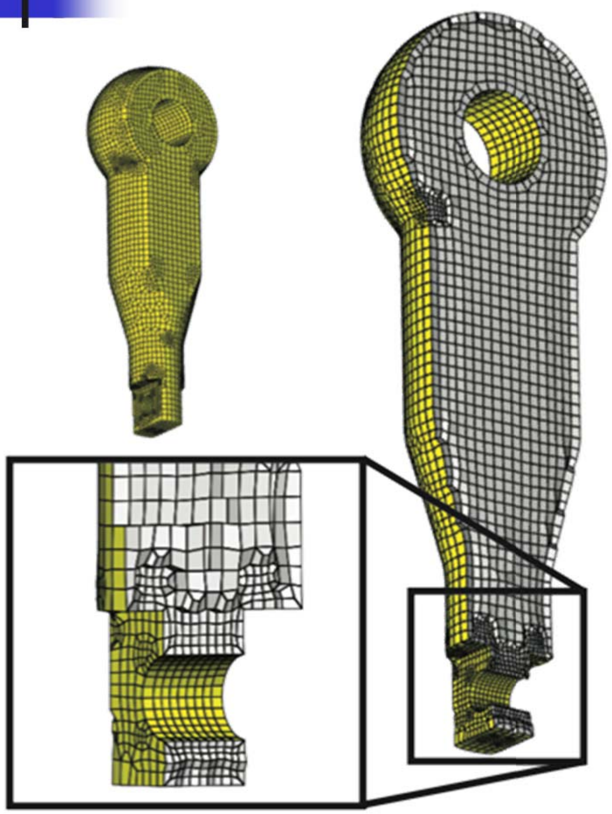
Traditional CAD



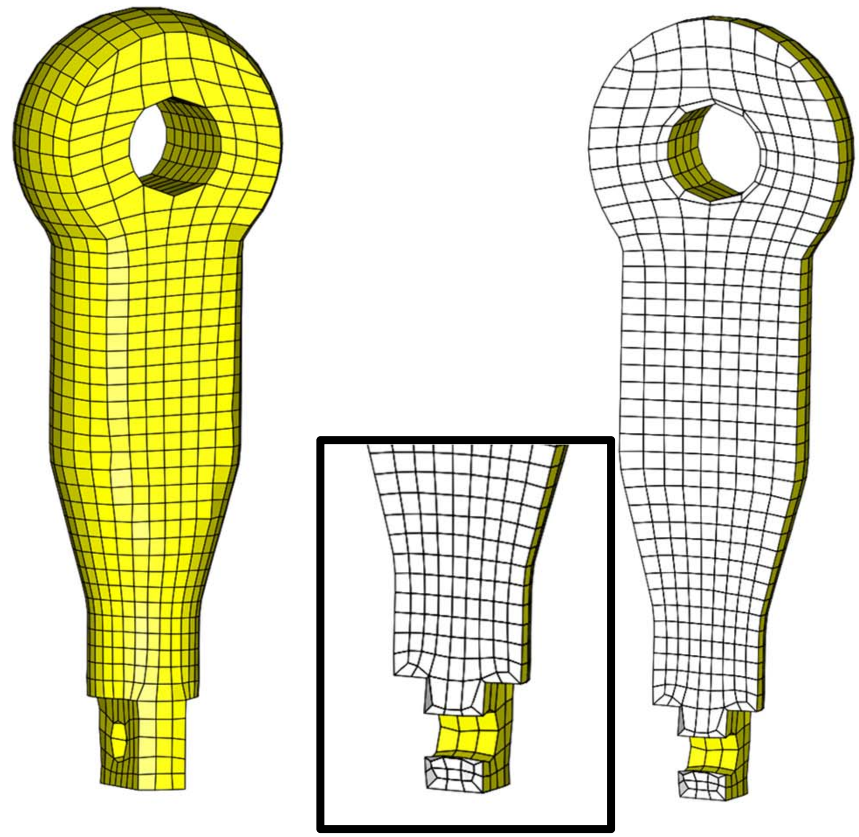




Comparison to Industry State of the Art



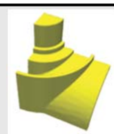
Hexotic/Marechal 2009



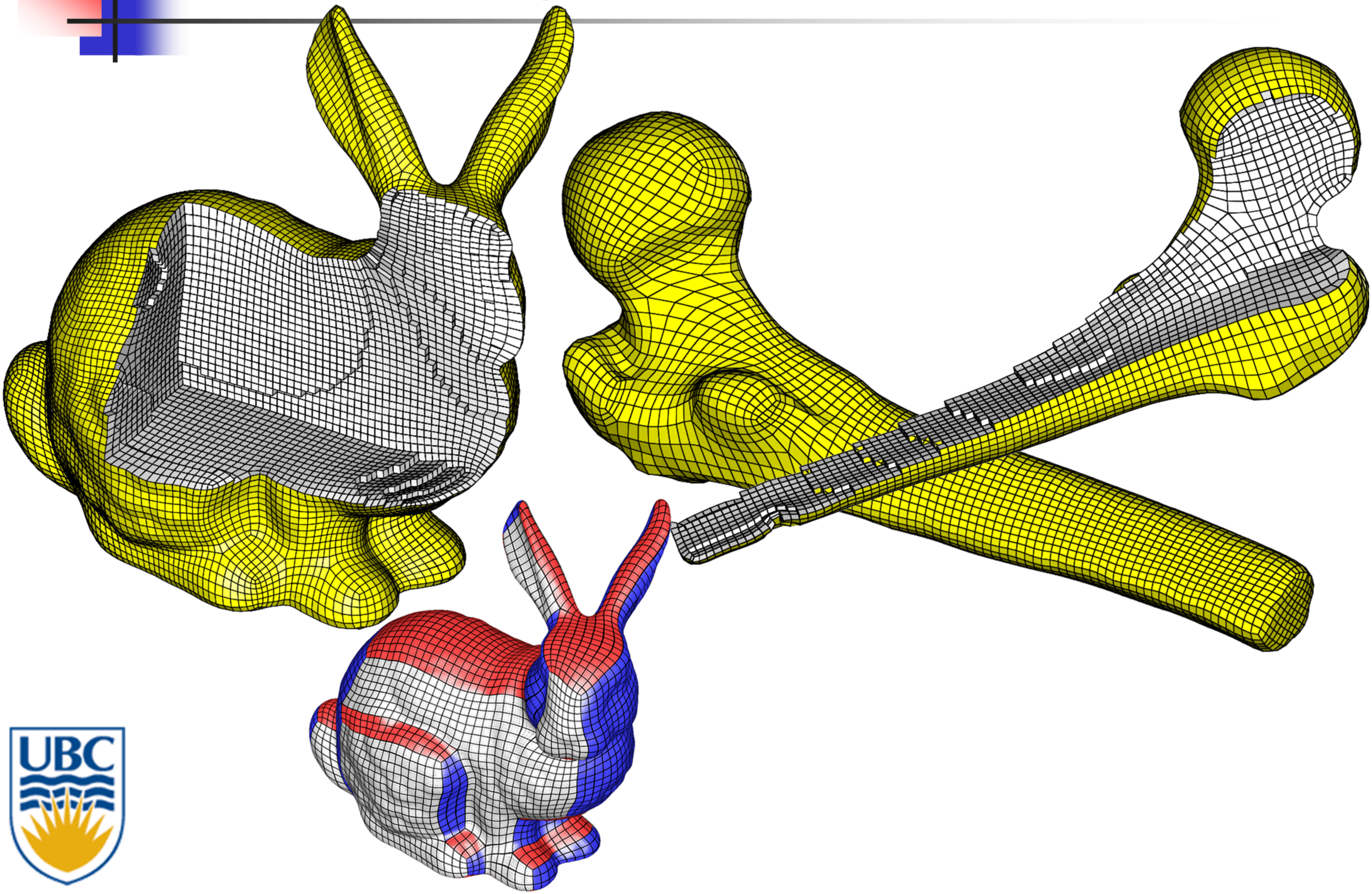
PolyCut

- Improves both element quality & mesh regularity

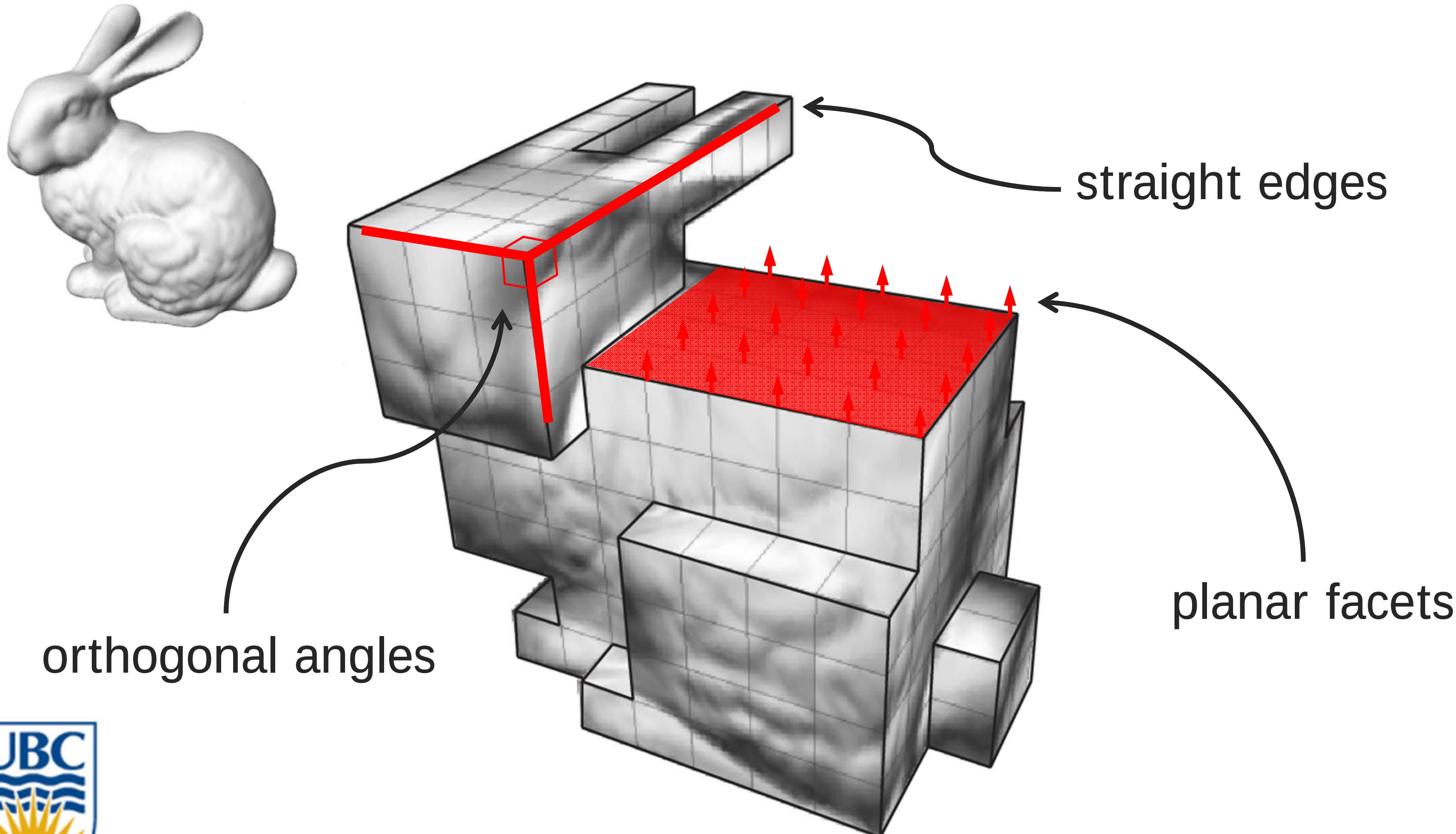
Mesh Quality: Min. Scaled Jacobian

Model	Hexotic	PolyCut
	0.005	0.20
	0.016	0.23
	0.018	0.23
	0.056	0.31
	0.017	0.26

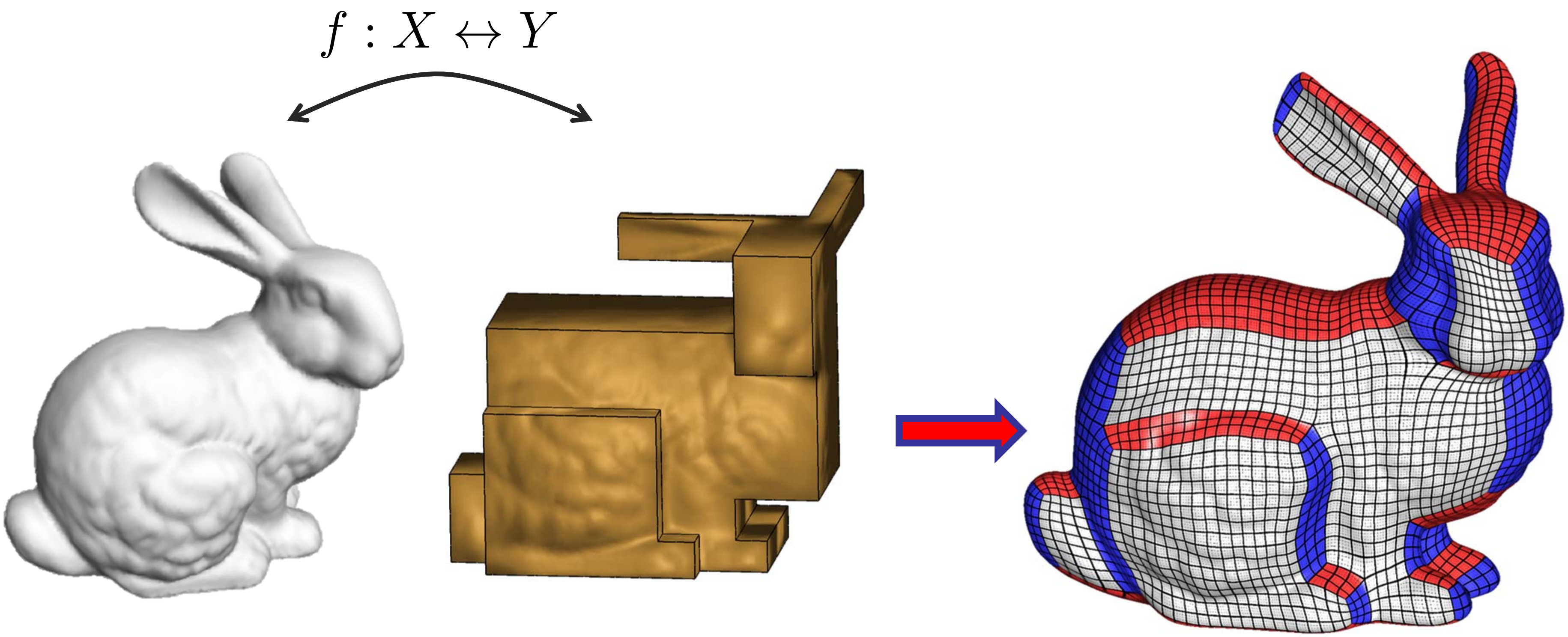
Natural Shapes



PolyCubes

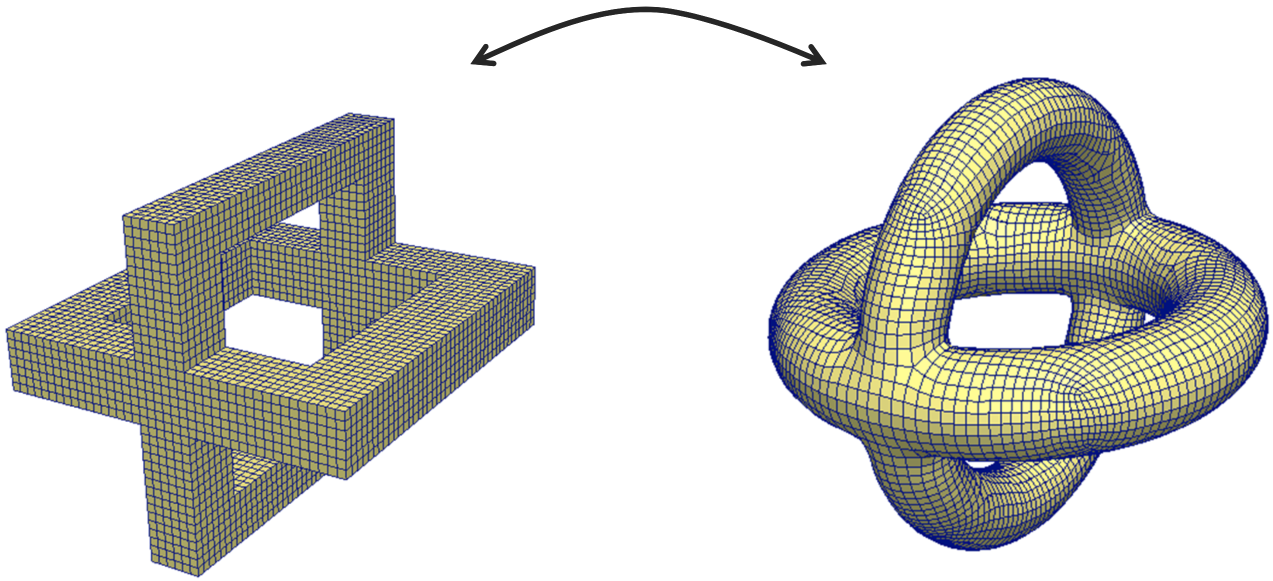


PolyCube Volumetric Mappings



Hex Meshing

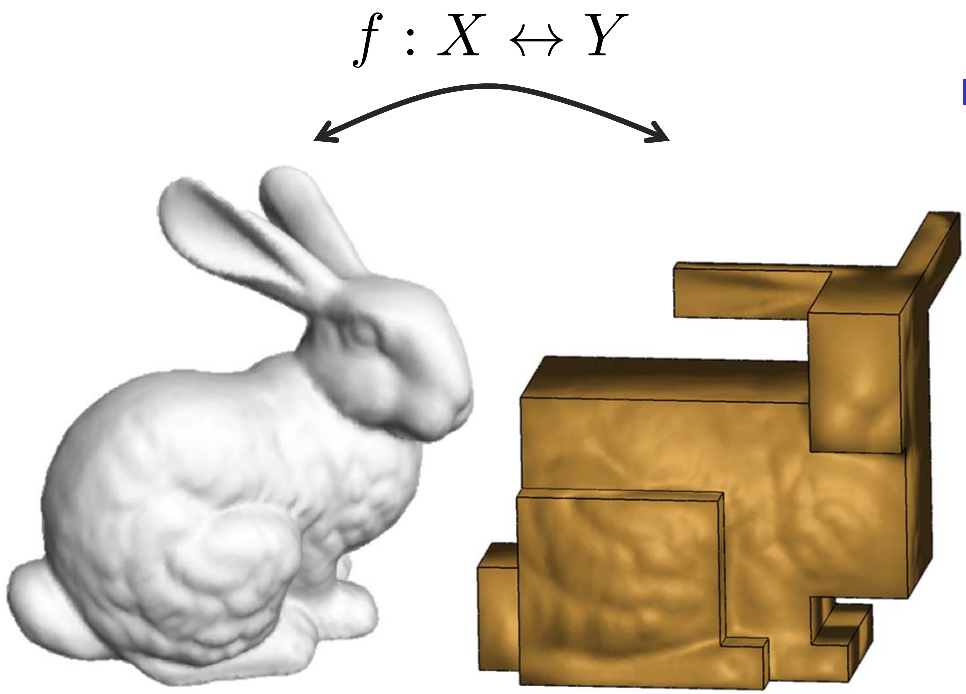
$$f : X \leftrightarrow Y$$



uniformly meshed

mapped & optimized mesh

Mapping Quality

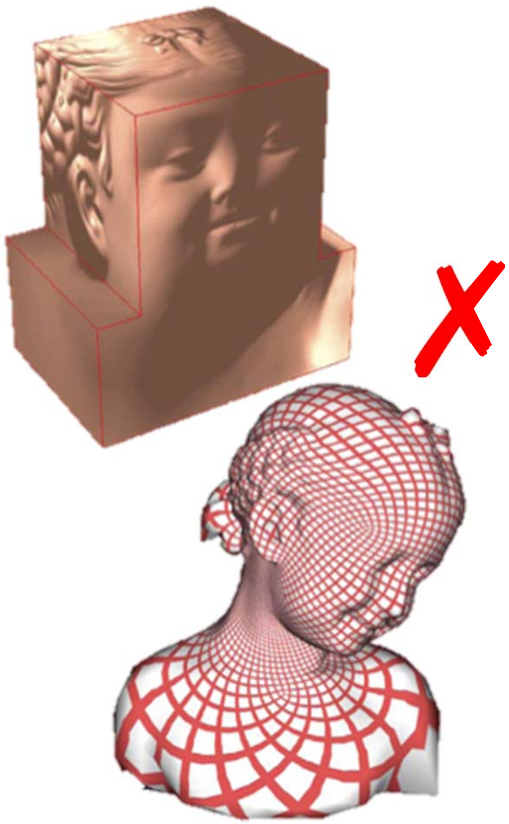


- PolyCube suitability for hex meshing depends on **mapping quality**

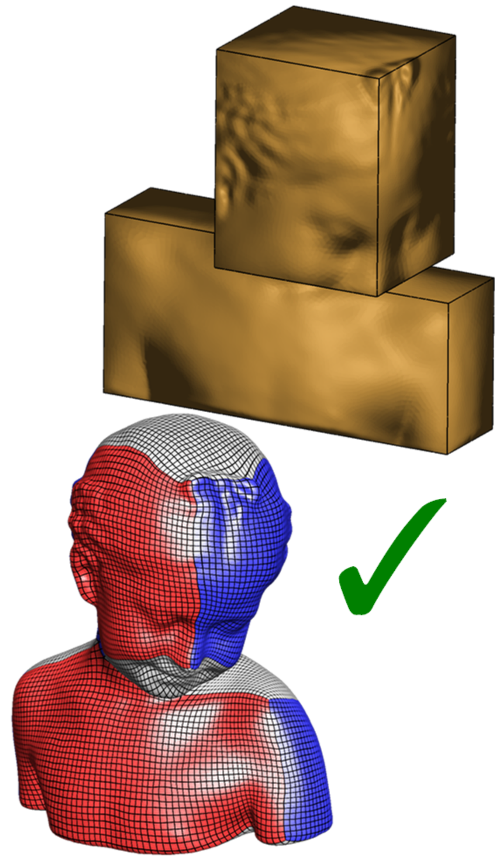
Quality:

- Low distortion everywhere
- Compactness: Small singularity count

Mapping Quality: distortion



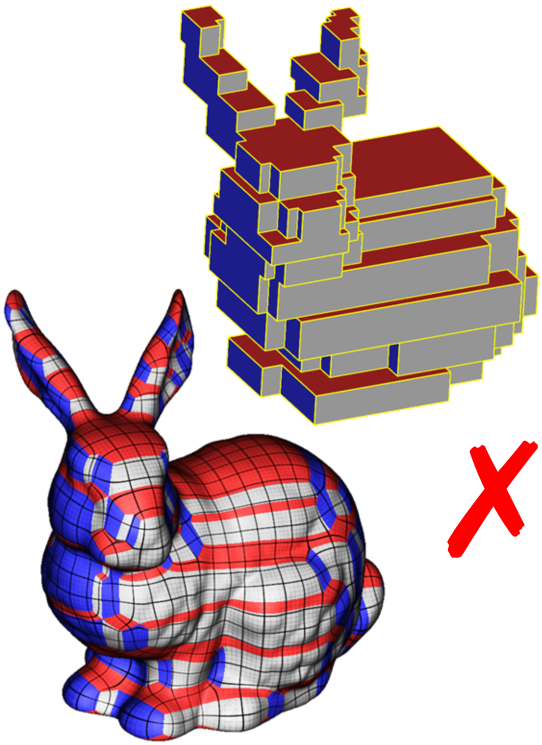
[Wang et al, 2007]



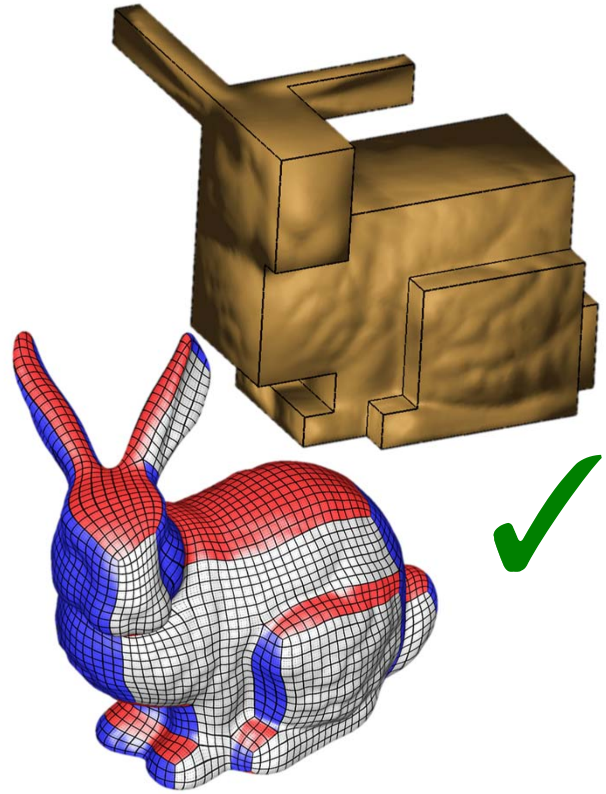
High distortion

Low distortion

Mapping Quality: compactness

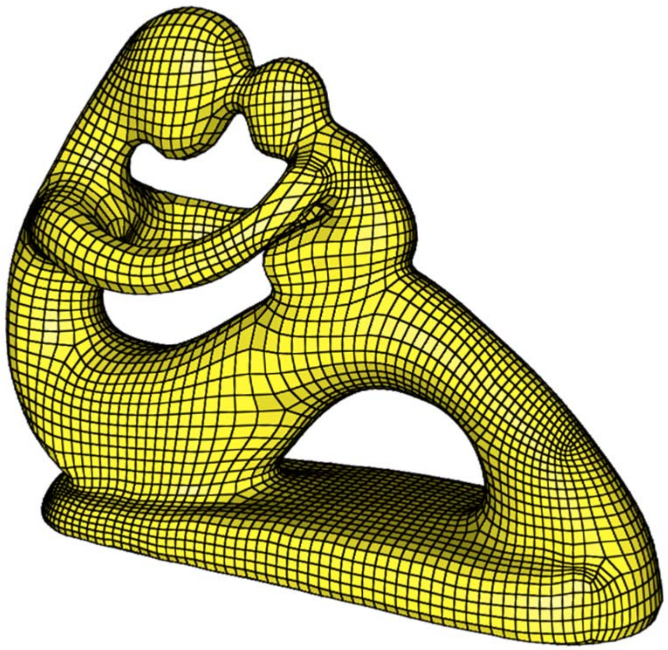


[He et al, 2009]

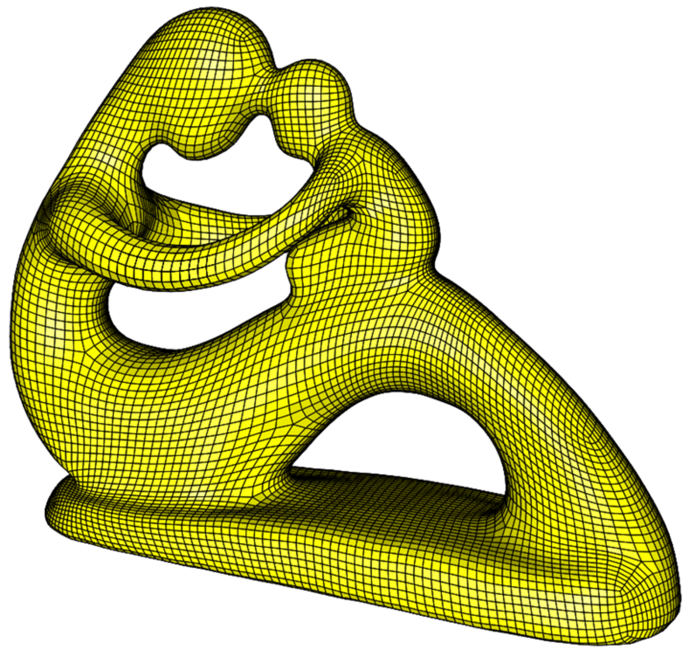


High Chart / Corner Count Low chart / corner count

Impact of PolyCube Quality



[Gregson'11]



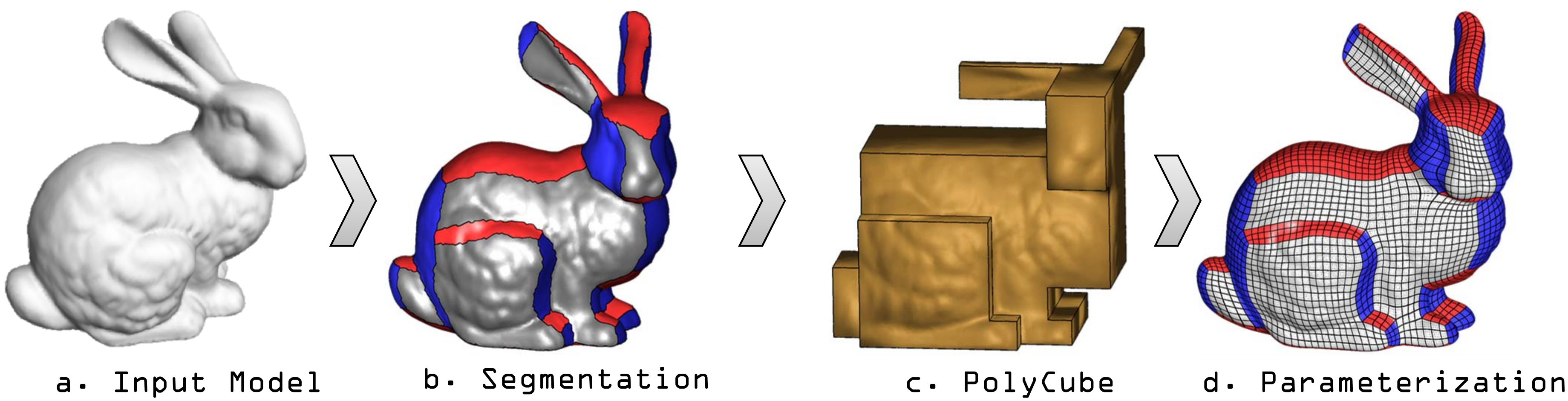
[Livesu'13]

	[Gregson'11]	[Livesu'13]
AVG SCALED JACOBIAN	0.911	0.872
MIN SCALED JACOBIAN	0.196 ←	0.259 ←

(the OPTIMAL value for SJ is 1)

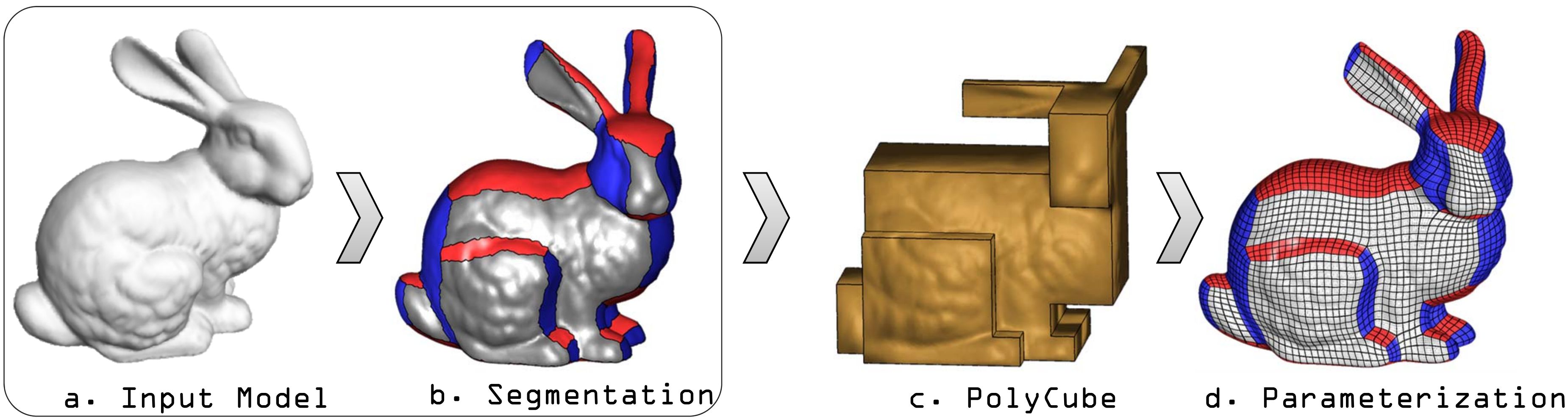
Generating Quality PolyCubes

- PolyCube Segmentation
- PolyCube Construction
- PolyCube Parameterization



Generating Quality PolyCubes

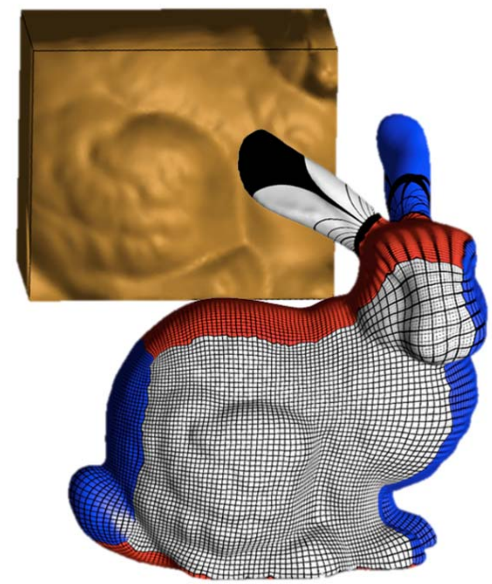
- PolyCube Segmentation
- Base Complex Construction
- PolyCube Parameterization



Defining the structure

GOAL: high quality PolyCube Mappings

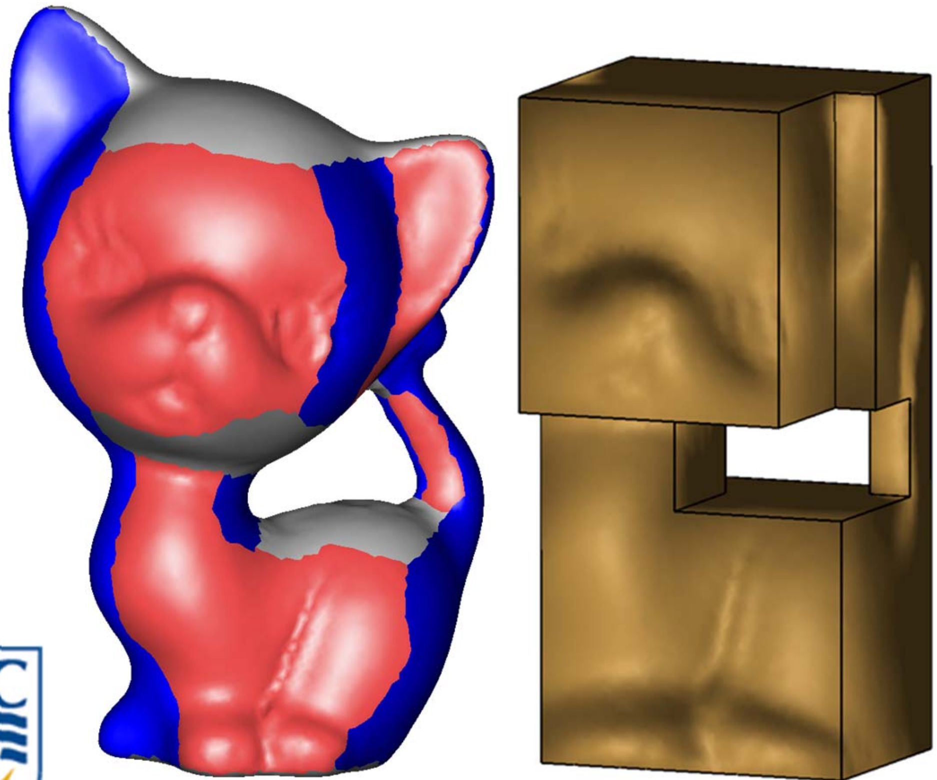
THE STRUCTURE OF THE POLYCUBE COUNTS!



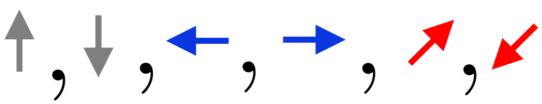
need **proxy** to predict distortion....

Basic idea

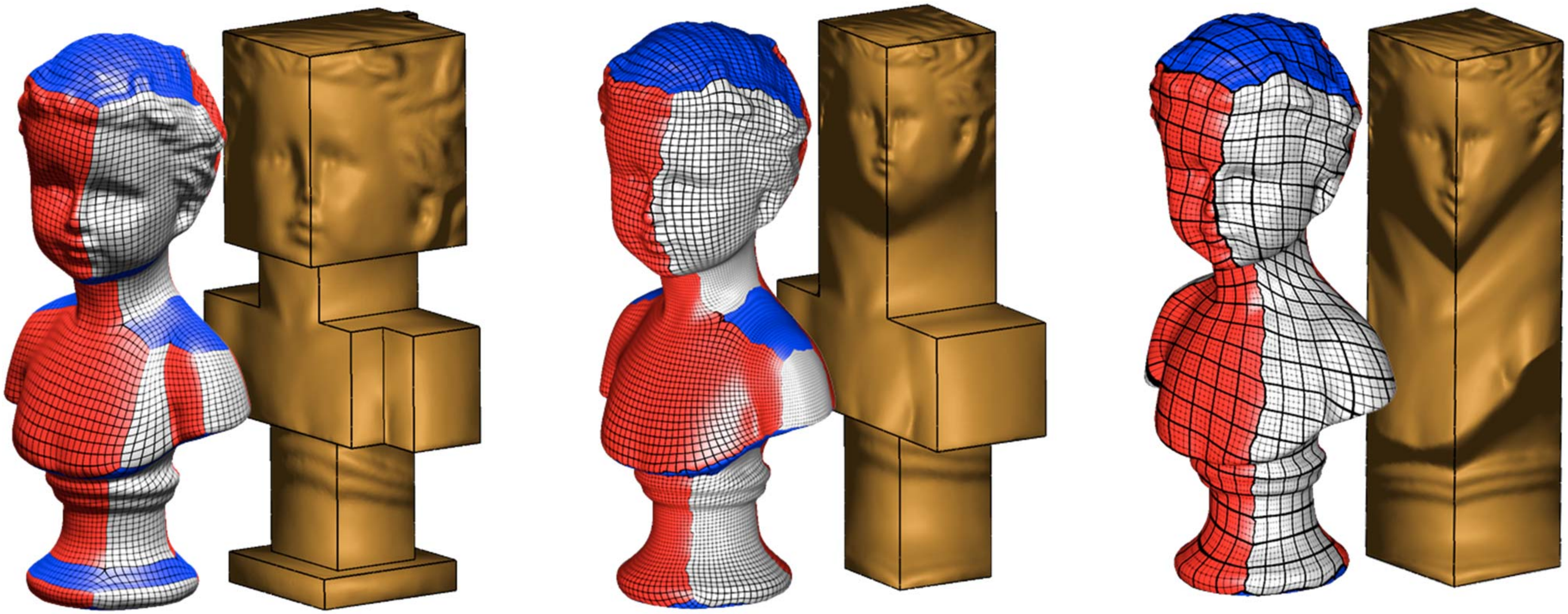
- PolyCube segmentation = **labeling**
- Need to account for compactness+ distortion



Six labels:

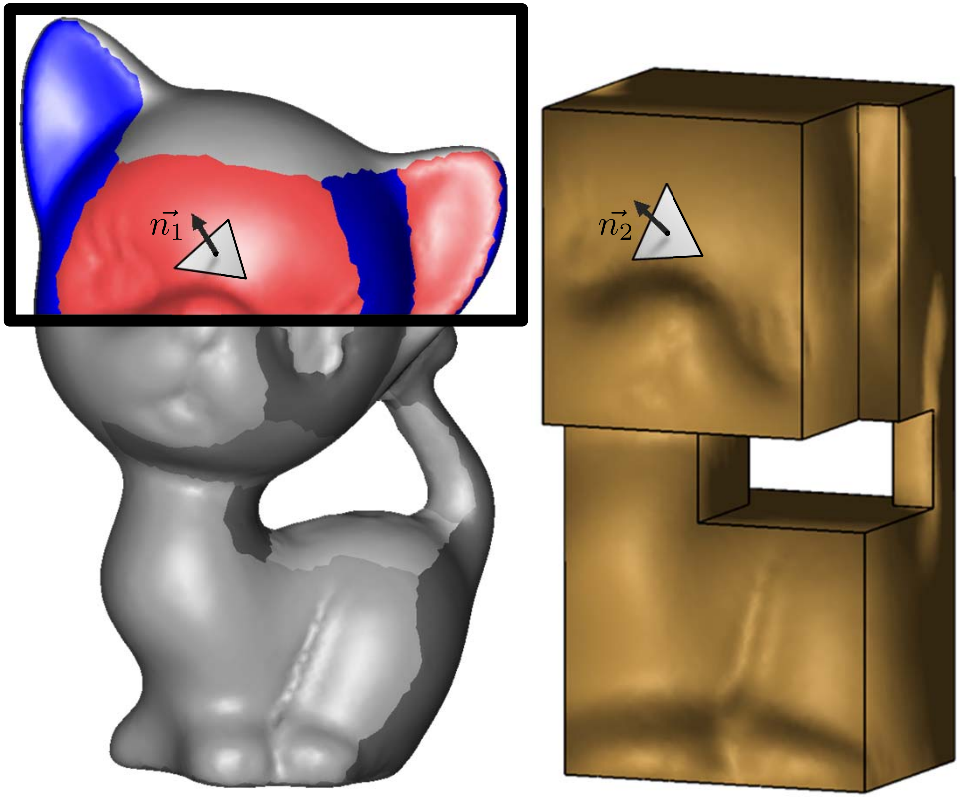


Compactness

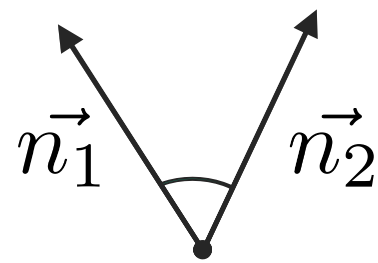


Easy to predict: measure lenght of chart boundaries

Distortion Prediction

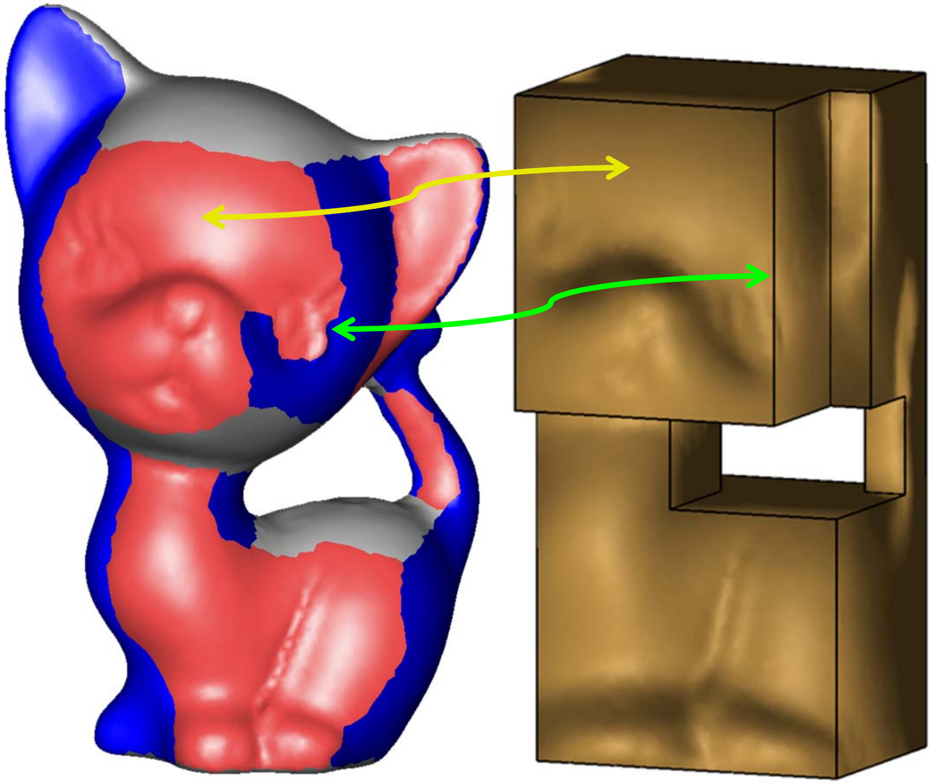


FIDELITY



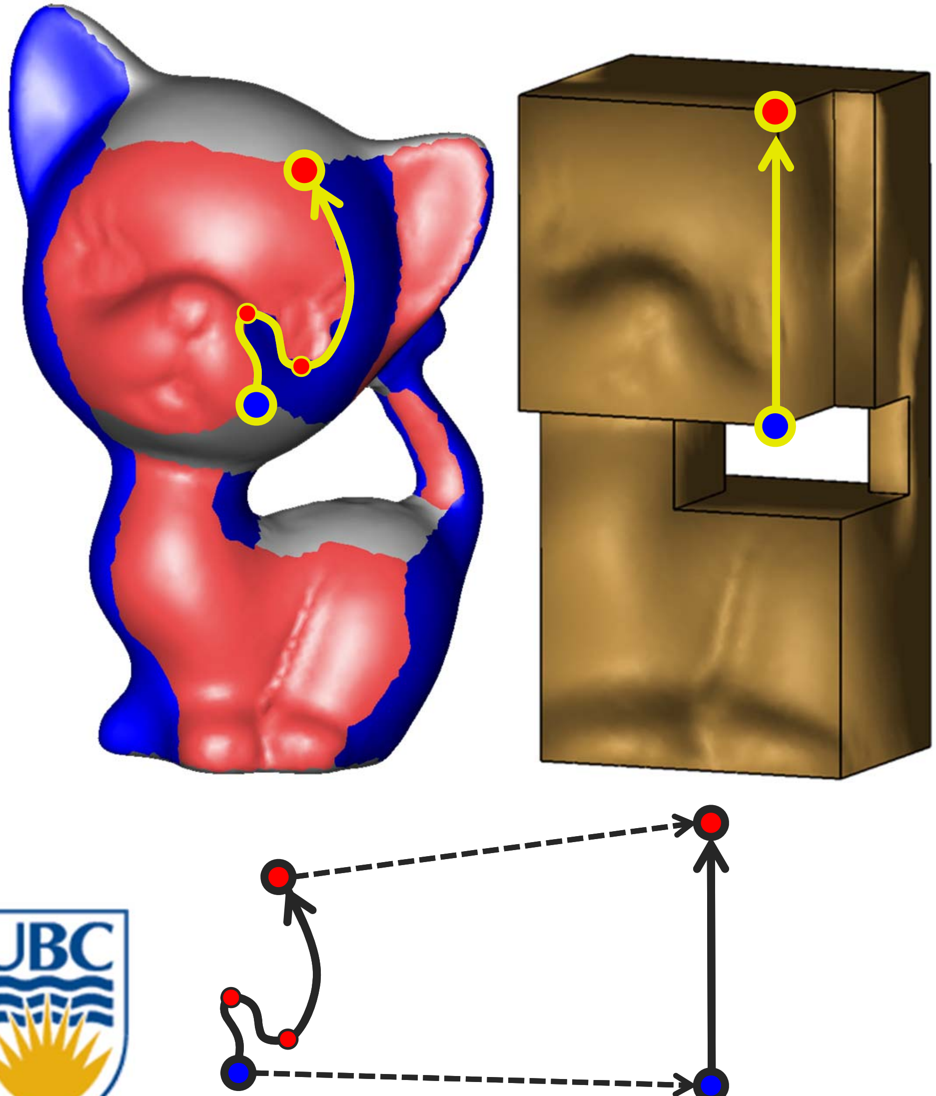
The lower the angular distance, the lower the distortion (less rotation)

One catch...



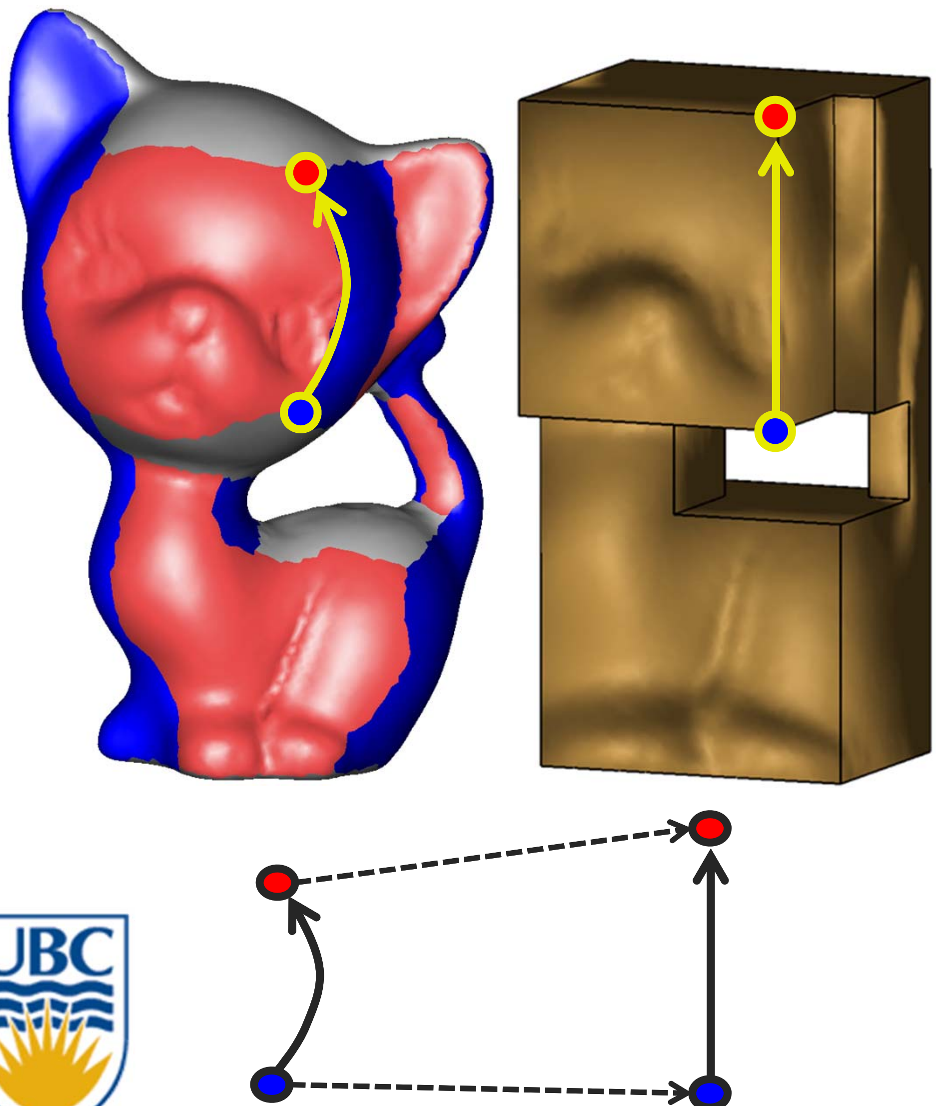
FIDELITY
NOT ENOUGH!

Distortion Prediction



FIDELITY
NOT
ENOUGH!

Distortion Prediction

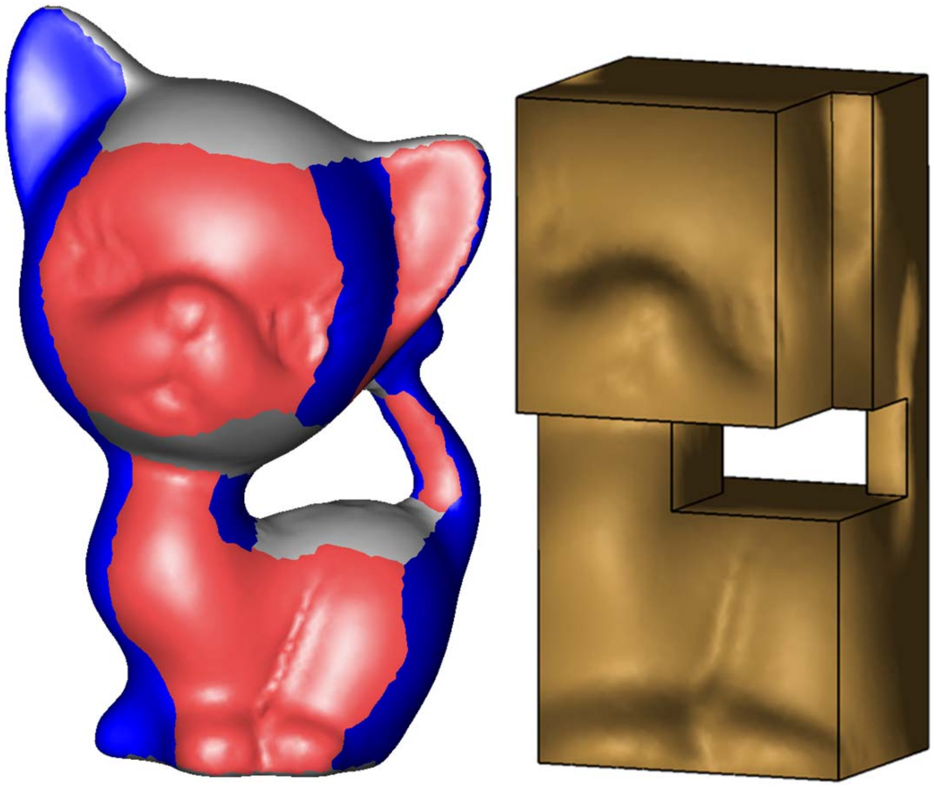


FIDELITY

+

MONOTONICITY

Distortion Prediction



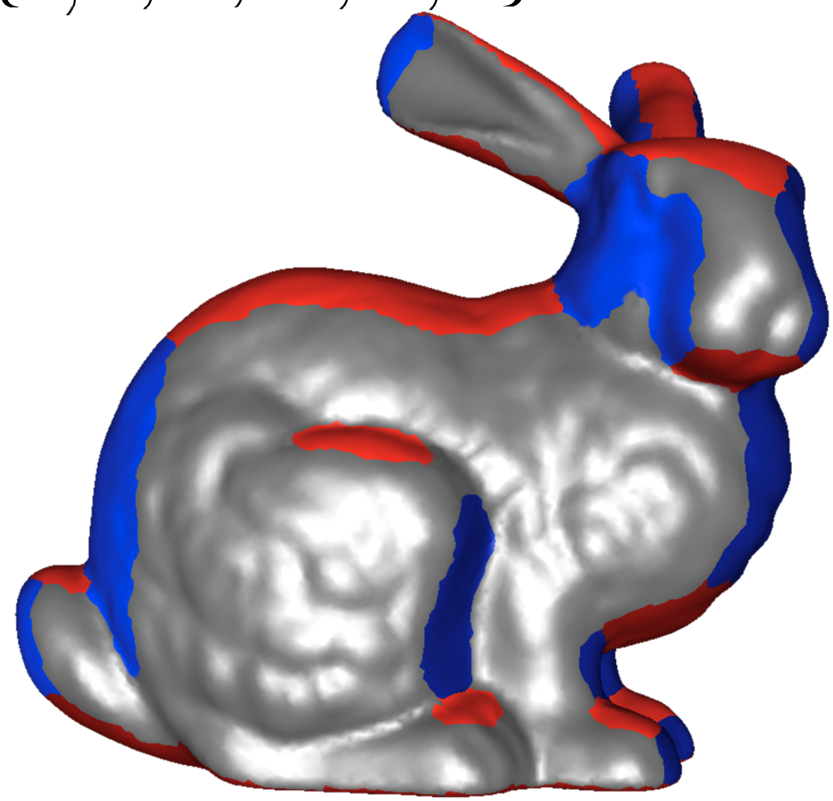
FIDELITY
+
MONOTONICITY

- CANNOT be evaluated prior to segmentation
- But can be evaluated without having an explicit mapping!

Graph Cut

$$E(S) = \sum_{t \in T} F_t(s_t) + c \cdot \sum_{pq \in E} C_{pq}(s_p, s_q)$$

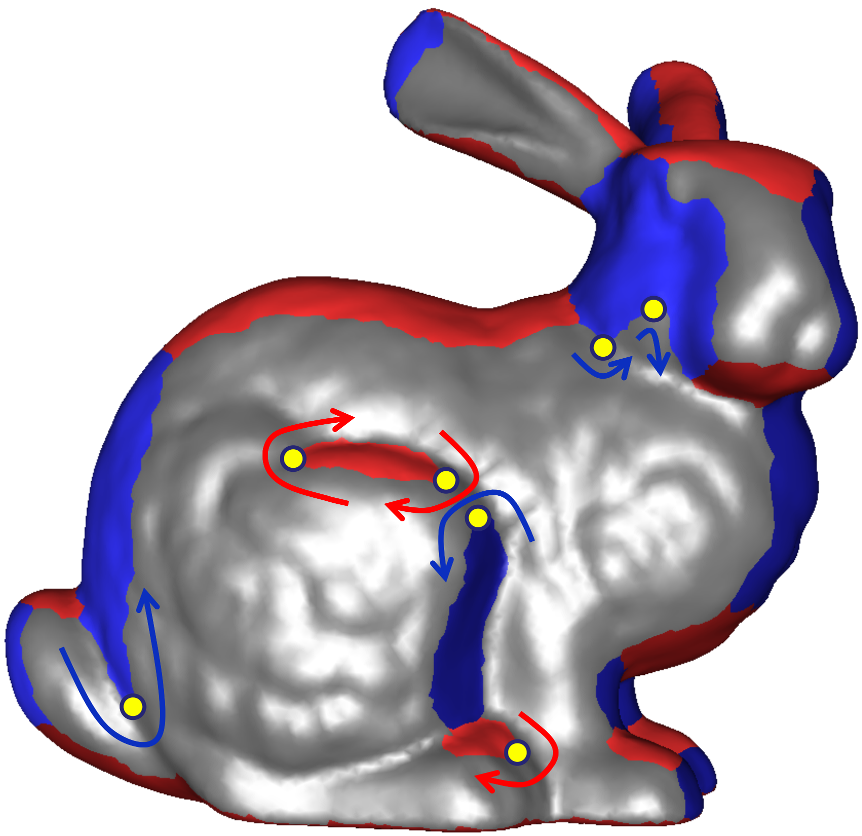
$s \in \{\uparrow, \downarrow, \leftarrow, \rightarrow, \nearrow, \swarrow\}$



- Fidelity+Compactness
- Non-Montone
  - Cannot enforce directly

Graph Cut

$$E(S) = \sum_{t \in T} F_t(s_t) + c \cdot \sum_{pq \in E} C_{pq}(s_p, s_q)$$



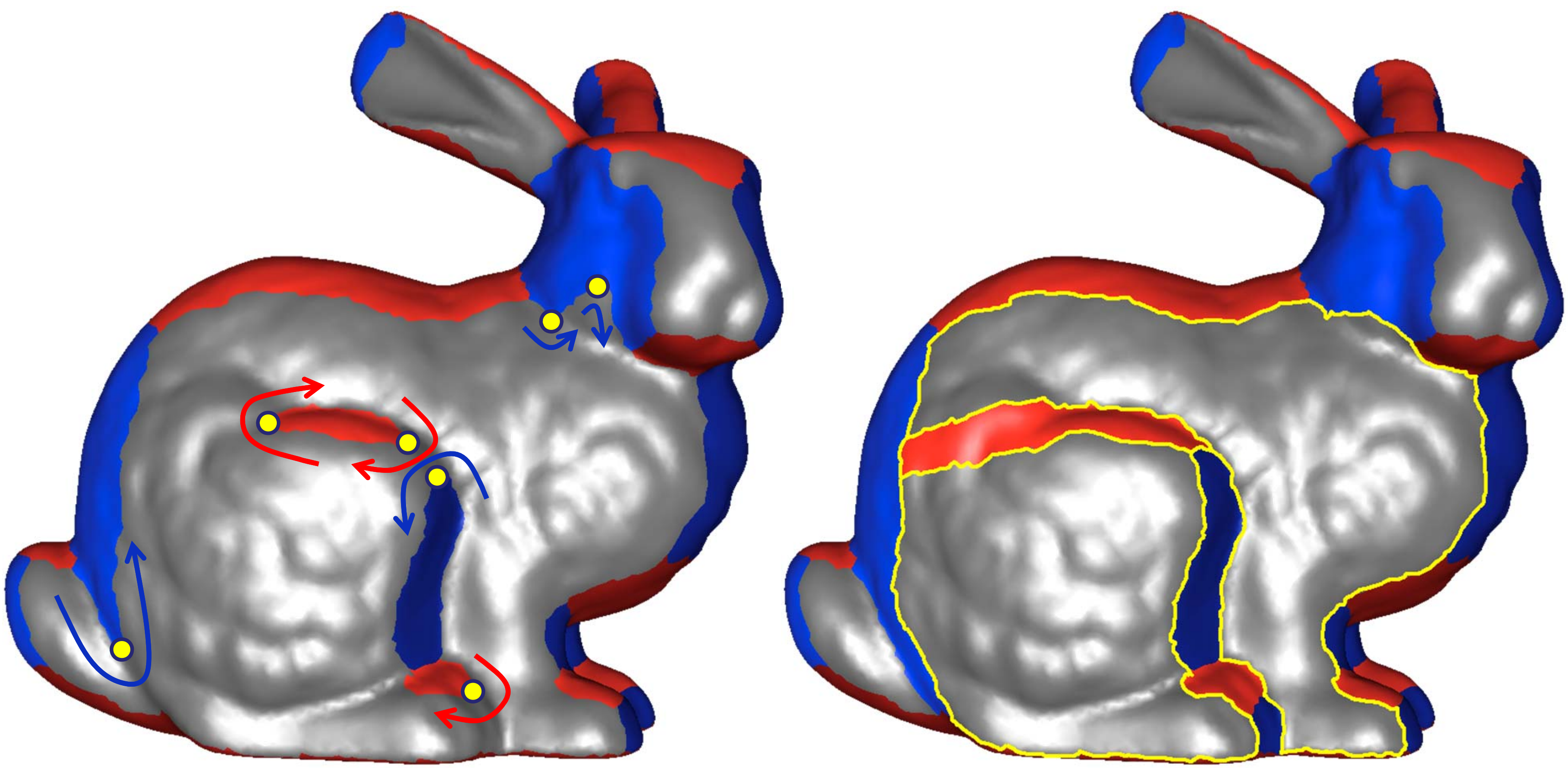
- Fidelity+Compactness

Non-Montone

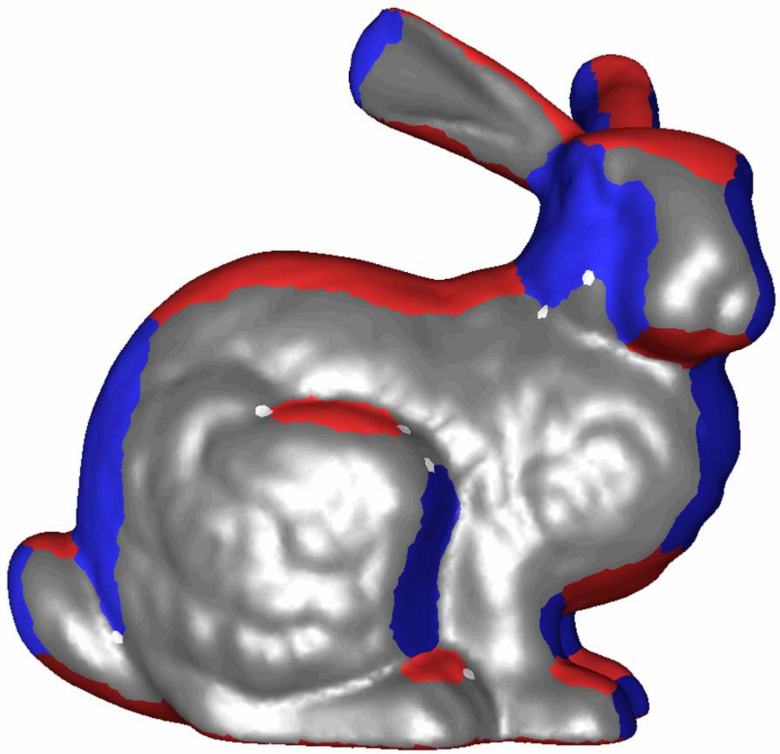
- Cannot enforce directly
- But: *can* evaluate monotonicity of given segmentaiton

Embedding in local search

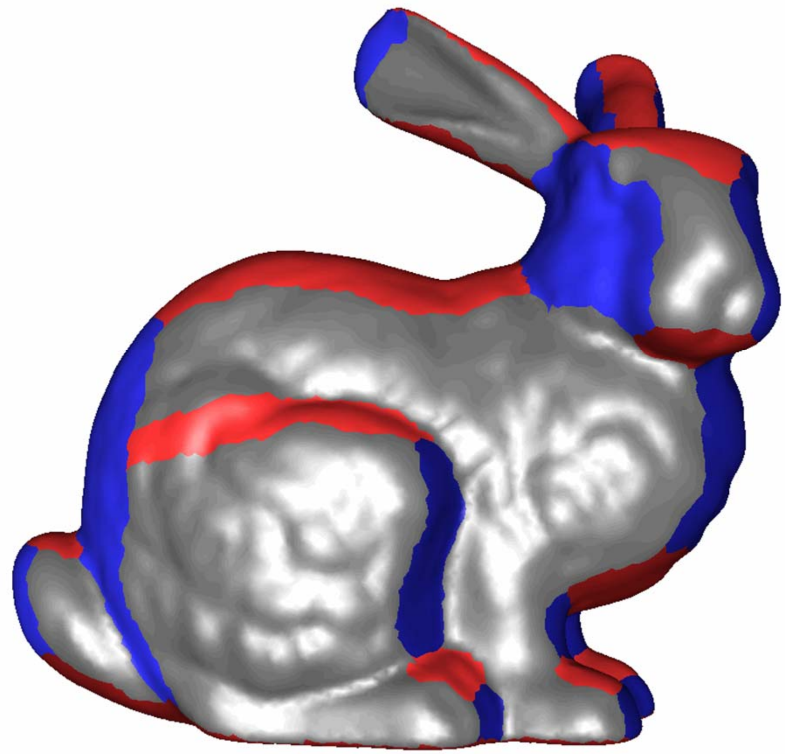
- Use tailored discrete optimization (hill-climbing) to locate all-monotone **nearby** solution



Hill Climbing

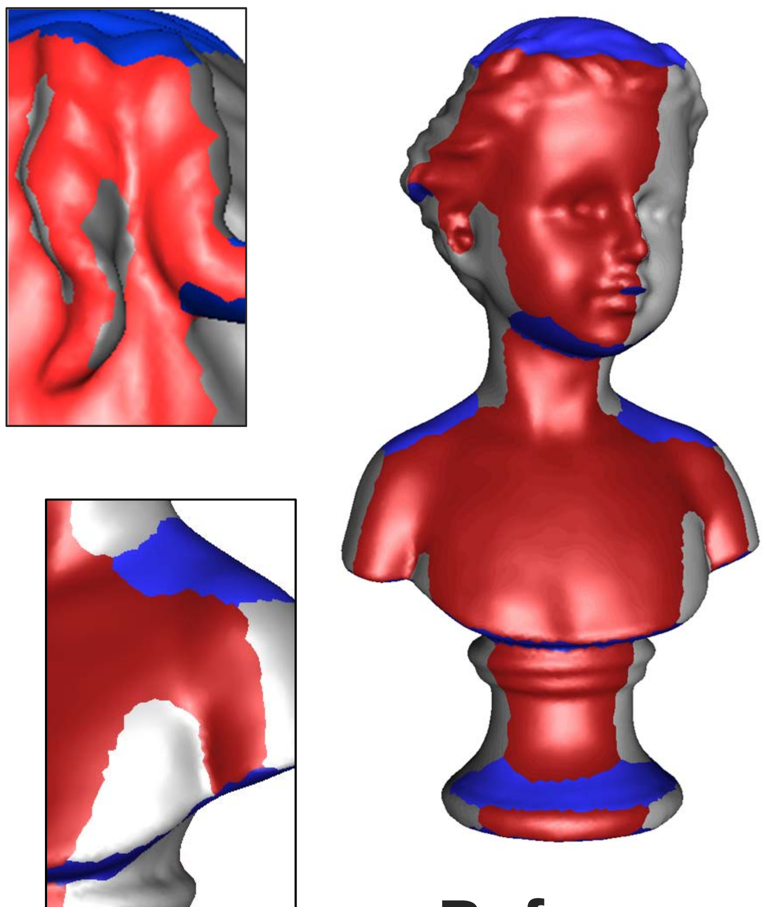


Before Hill Climbing

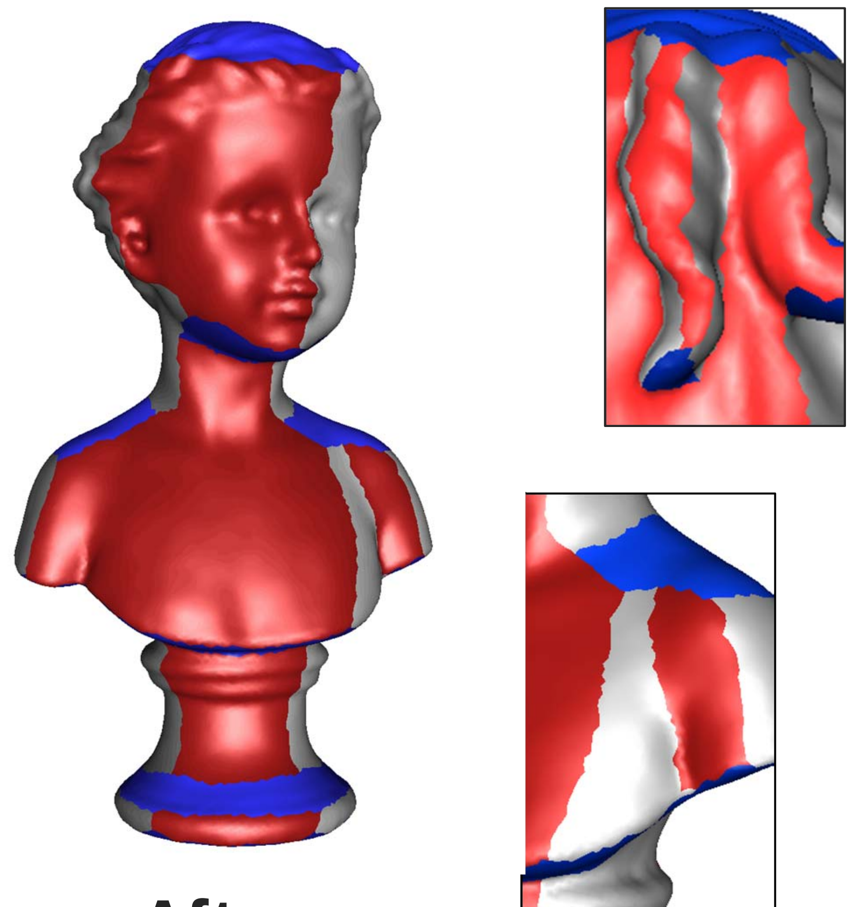


After Hill Climbing

Hill Climbing



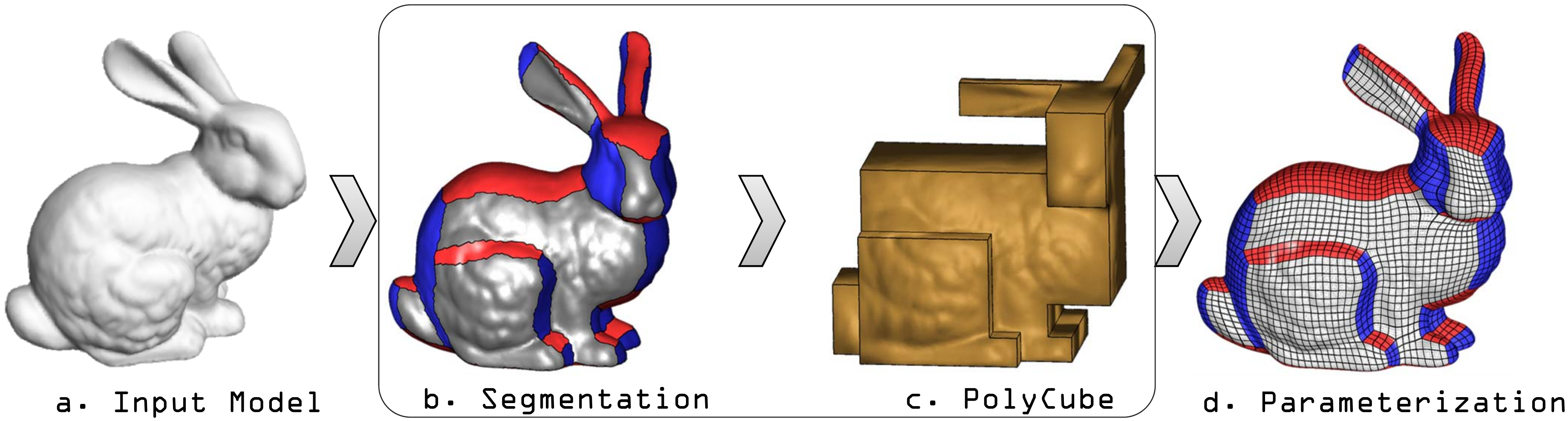
Before
Hill Climbing



After
Hill Climbing

PolyCube Deformation

- PolyCube Construction
  - Minimize mapping distortion
    - Gregson'11

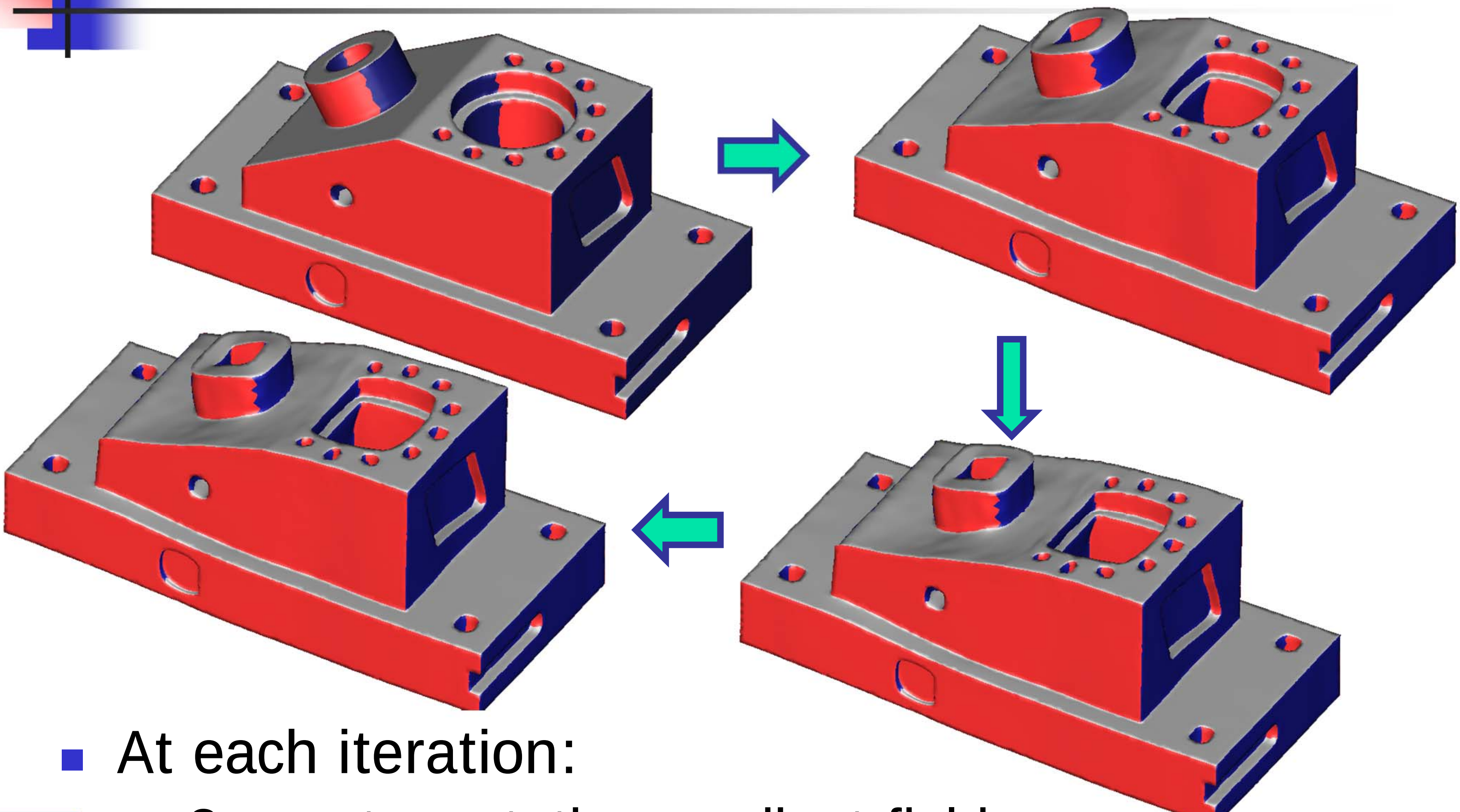


PolyCube Deformation: Basic Ideas

- Rotate surface normals to target directions
  - Twist component unknown
  - Final surface positions not known

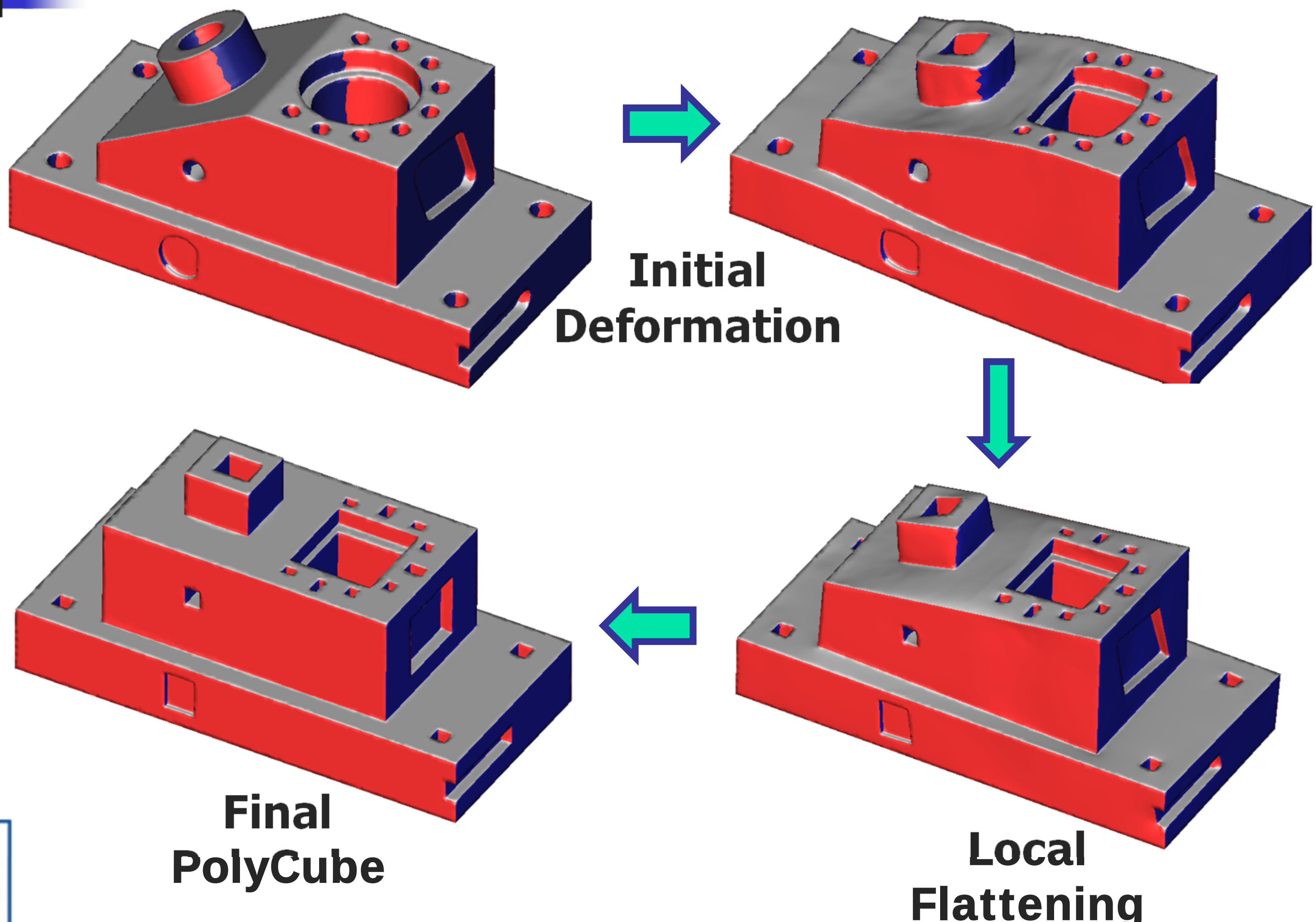
- Use volumetric deformation to communicate between charts/faces
  - Need interior tet mesh
  - Maintain positive volume

Iterative Normal Rotation

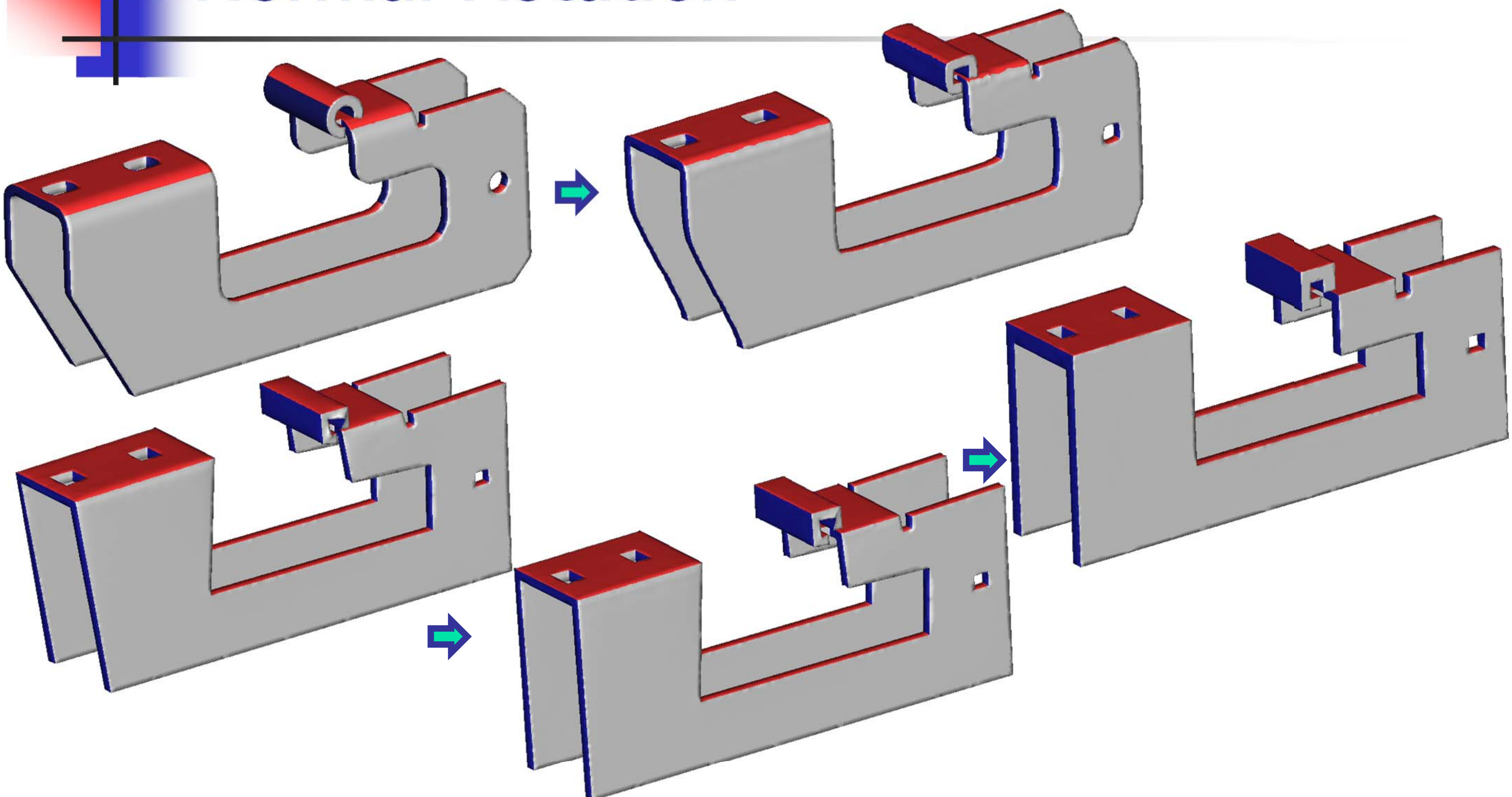


- At each iteration:
  - Computes rotation gradient field
  - Integrates gradient field to deform model

PolyCube deformation



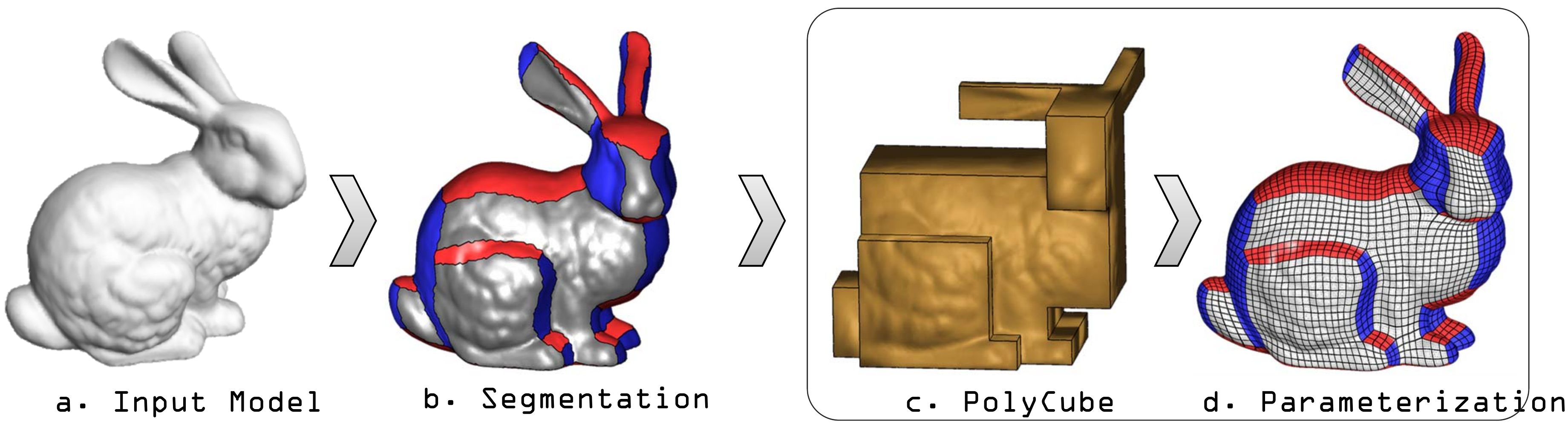
Normal Rotation



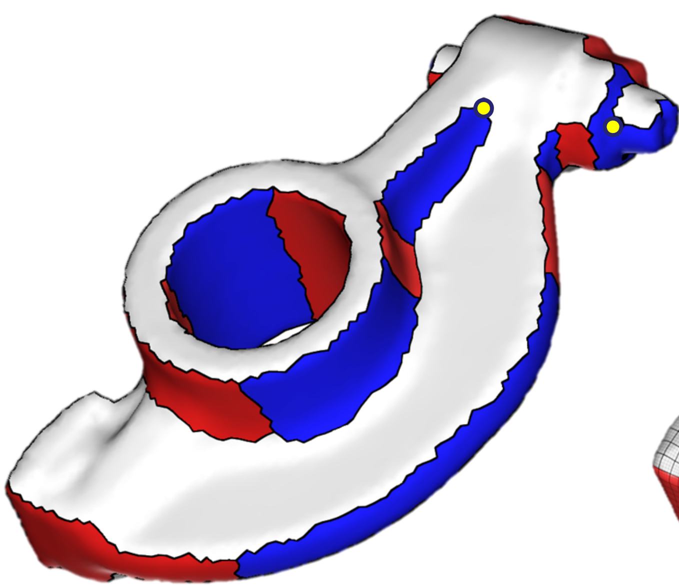
- Iterative gradual rotation
- Last step: combine with planarity constraints + interval setting

PolyCube Mapping

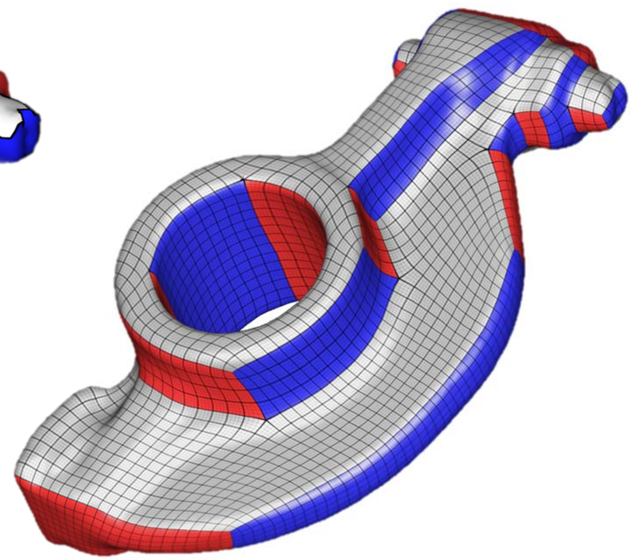
- PolyCube Segmentation/Base Complex Construction
- PolyCube Parameterization



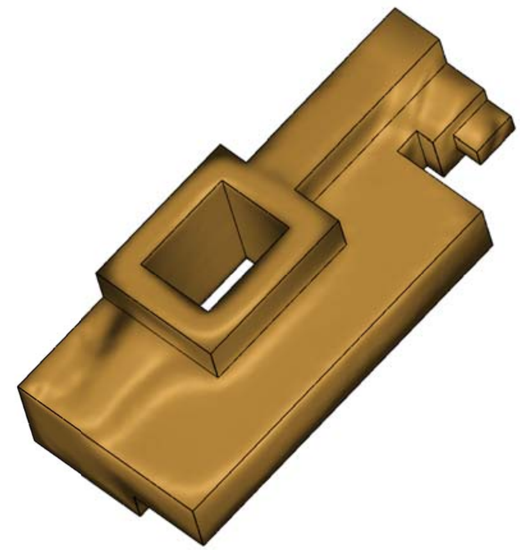
Rocker Arm



Initial Labeling
(6 turning points)



Final Labeling

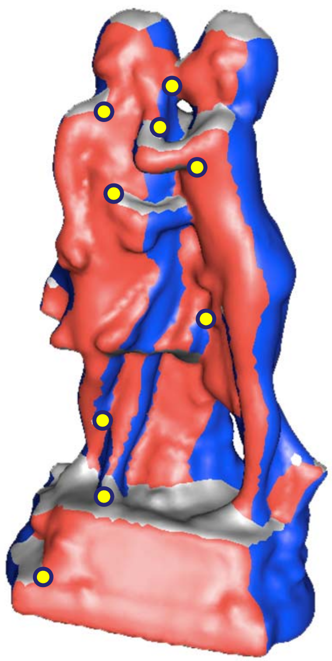


PolyCube

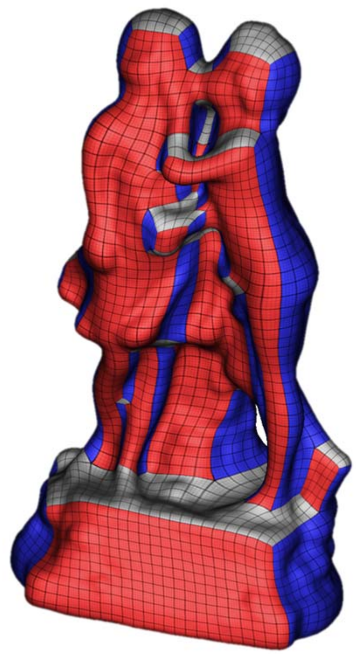
CORNERS	62
CHARTS	34
STRETCH	0.857

(the OPTIMAL value for STRETCH is 1)

Kiss Statue



Initial Labeling
(19 turning points)



Final Labeling

PolyCube

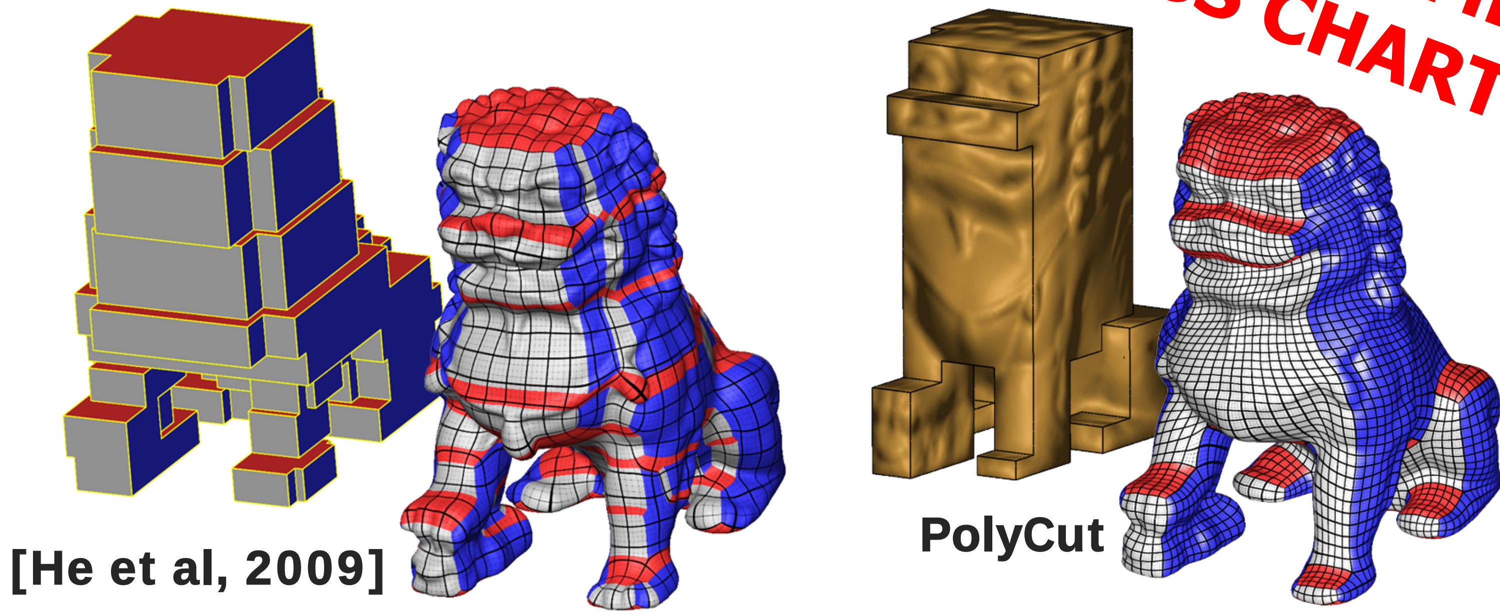
CORNERS	120
CHARTS	56
STRETCH	0.871

(the OPTIMAL value for STRETCH is 1)

Comparisons

Comparisons vs. He et al. (2009)

FOUR TIMES LESS CHARTS!

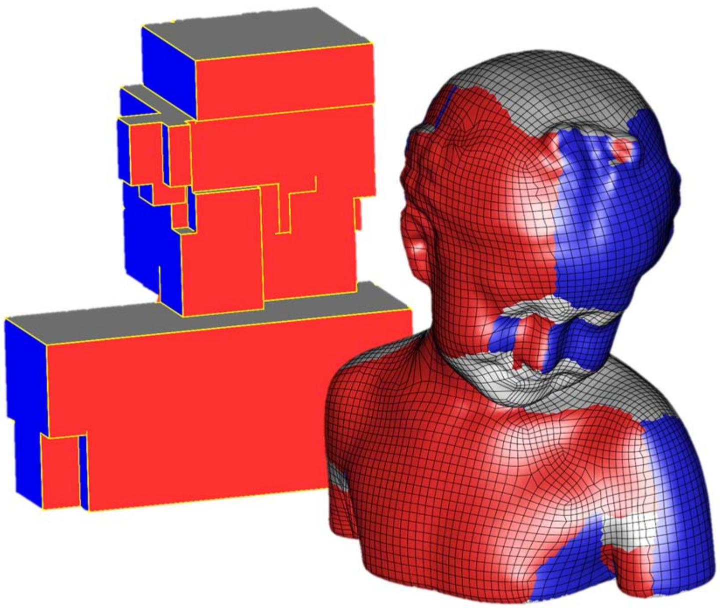


	[He et al, 2009]	PolyCut
CORNERS	285	74
CHARTS	151 ←	39 ←
STRETCH	0.804 ←	0.831 ←

(the OPTIMAL value for STRETCH is 1)

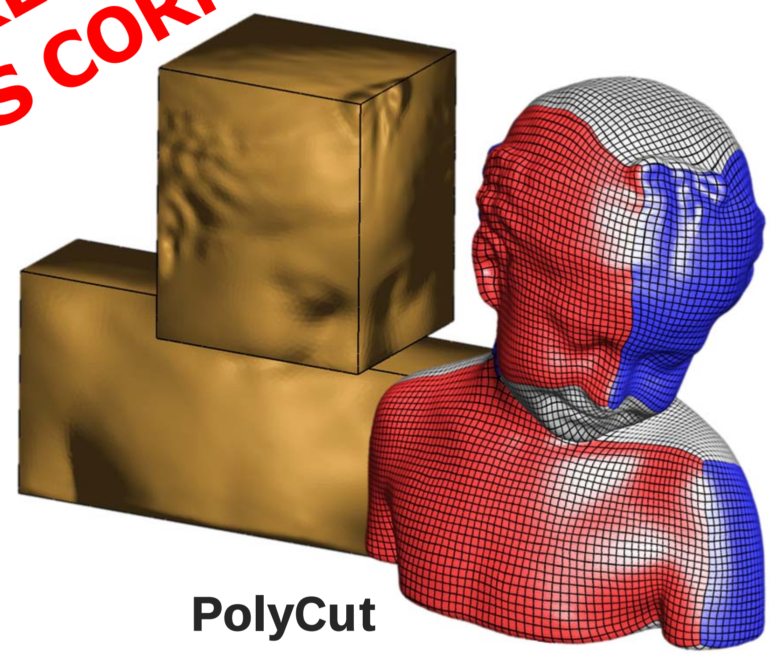
Comparisons vs. Gregson' 2011

THREE TIMES LESS CORNERS!



[Gregson et al, 2011]

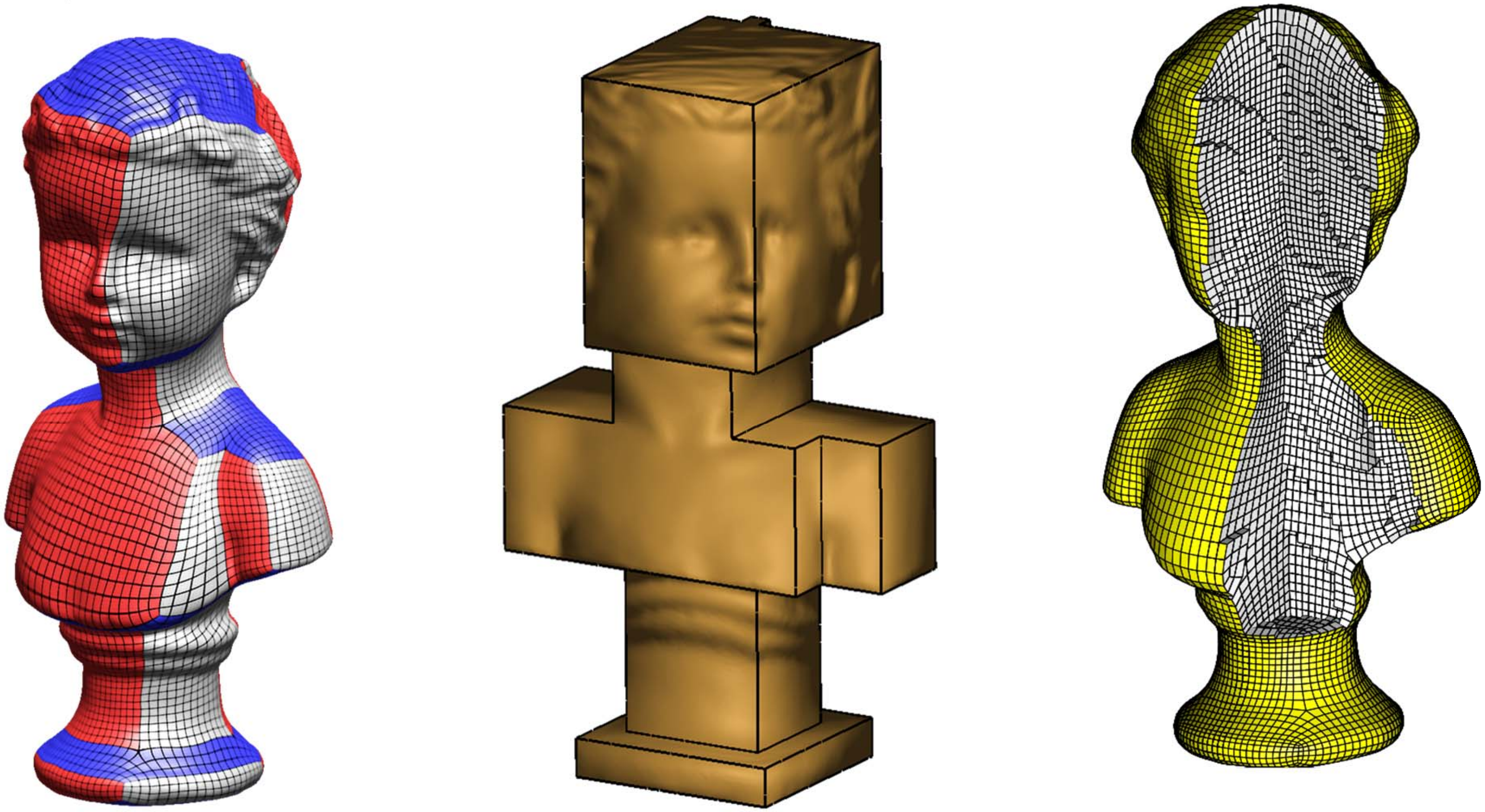
CORNERS	115 ←
CHARTS	61
STRETCH	0.712 ←

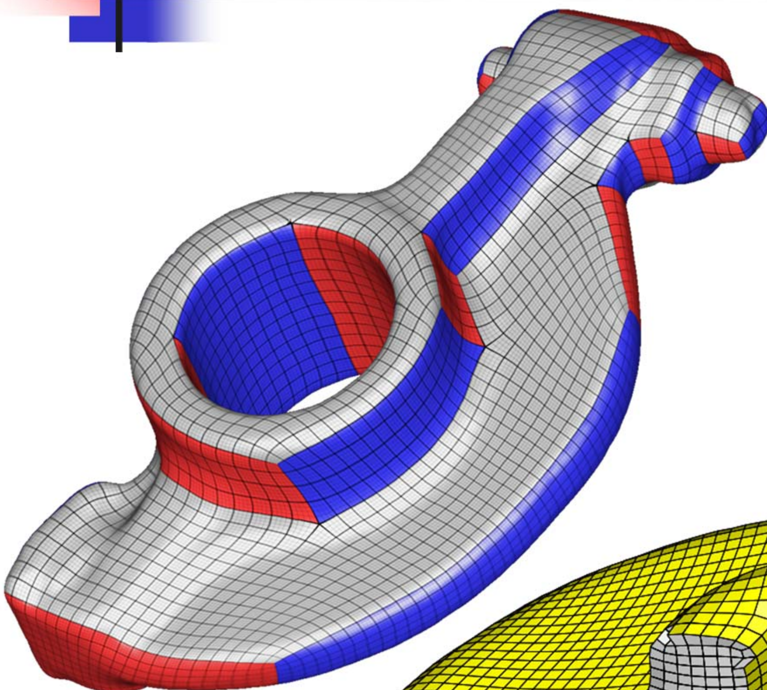


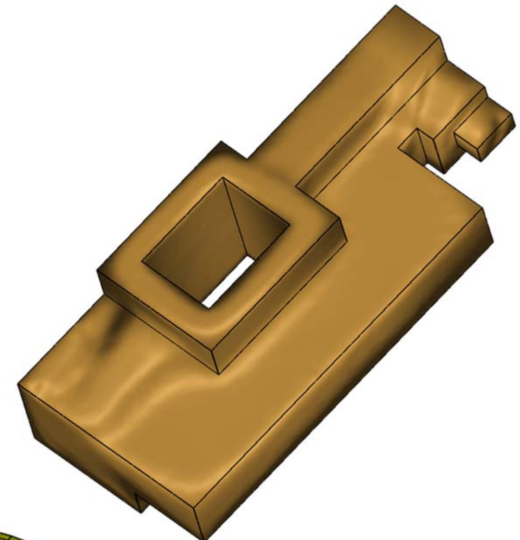
PolyCut

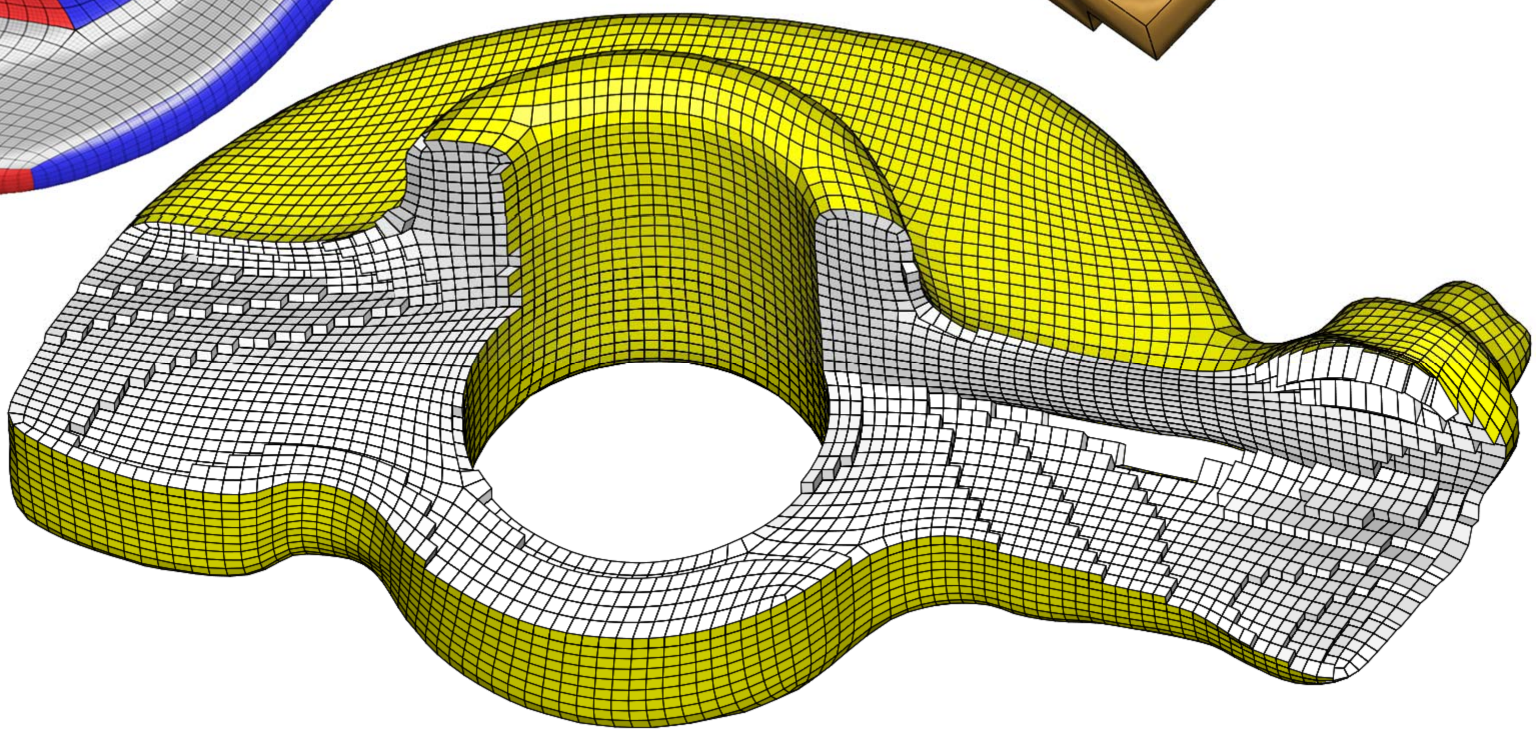
CORNERS	30 ←
CHARTS	17
STRETCH	0.843 ←

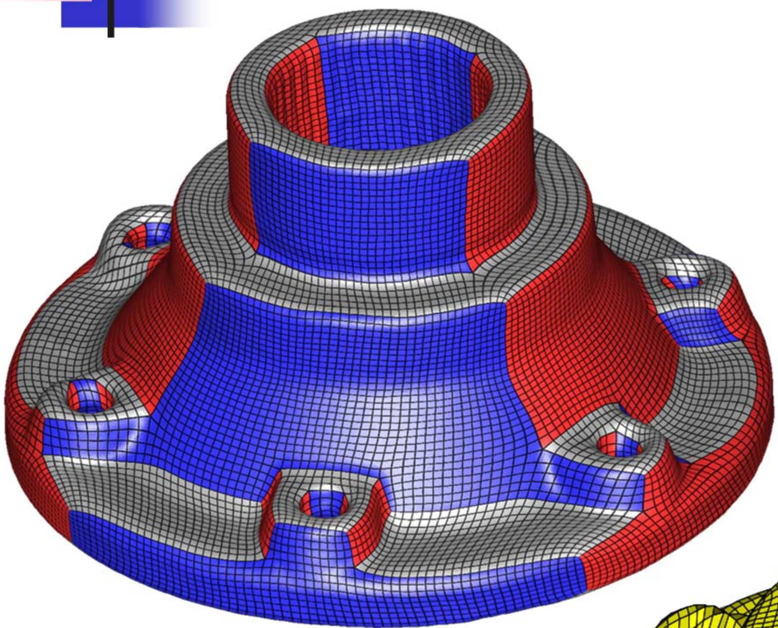
Hex Meshing

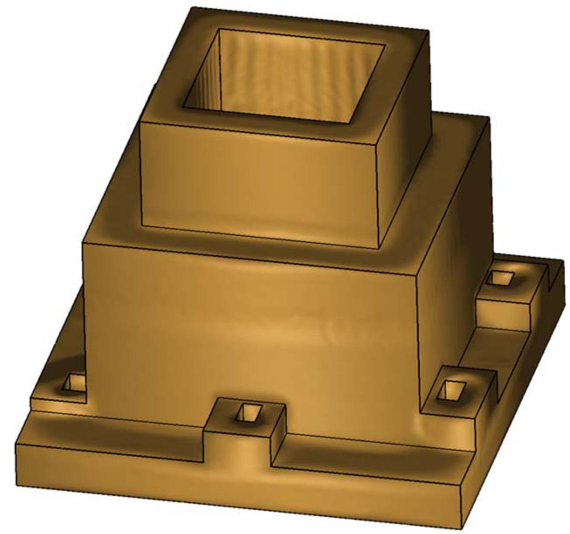


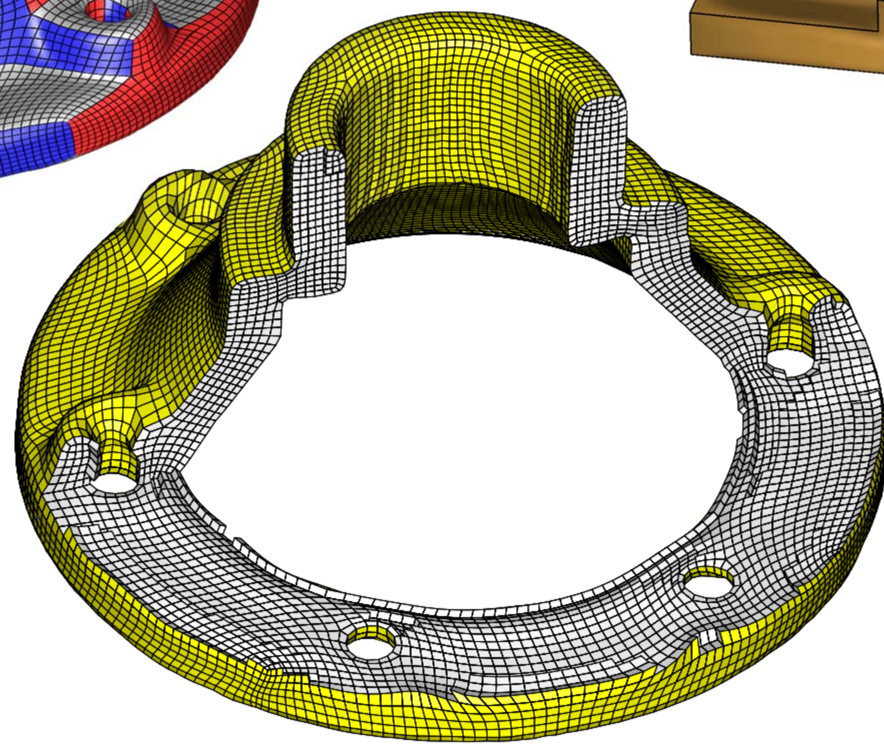




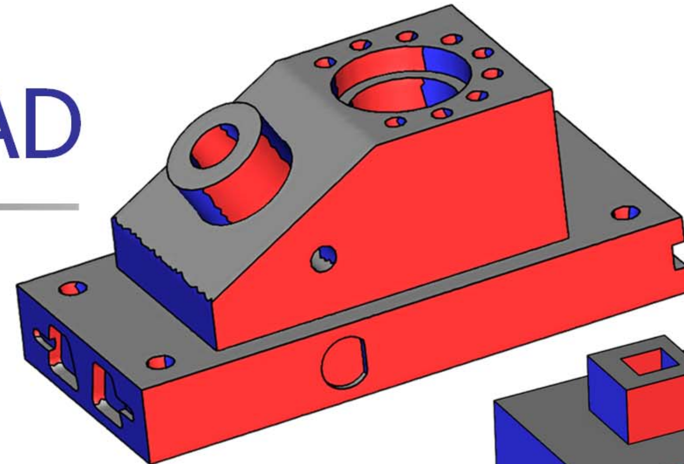


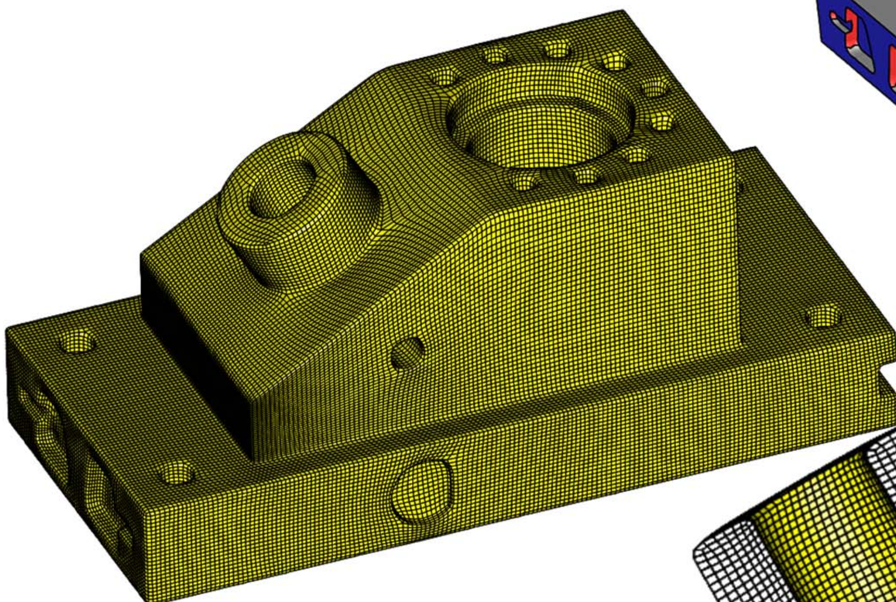


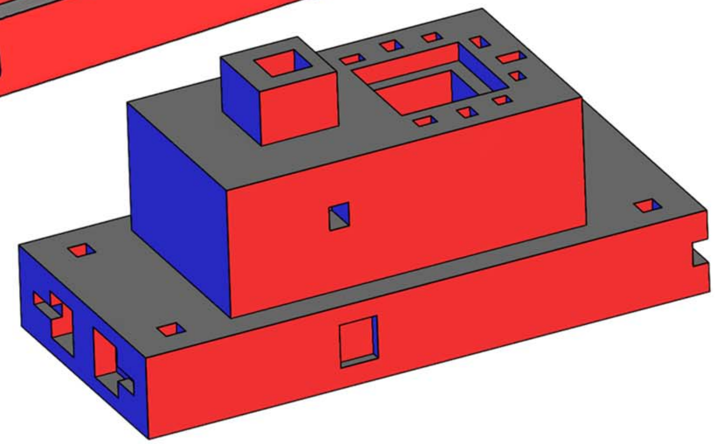


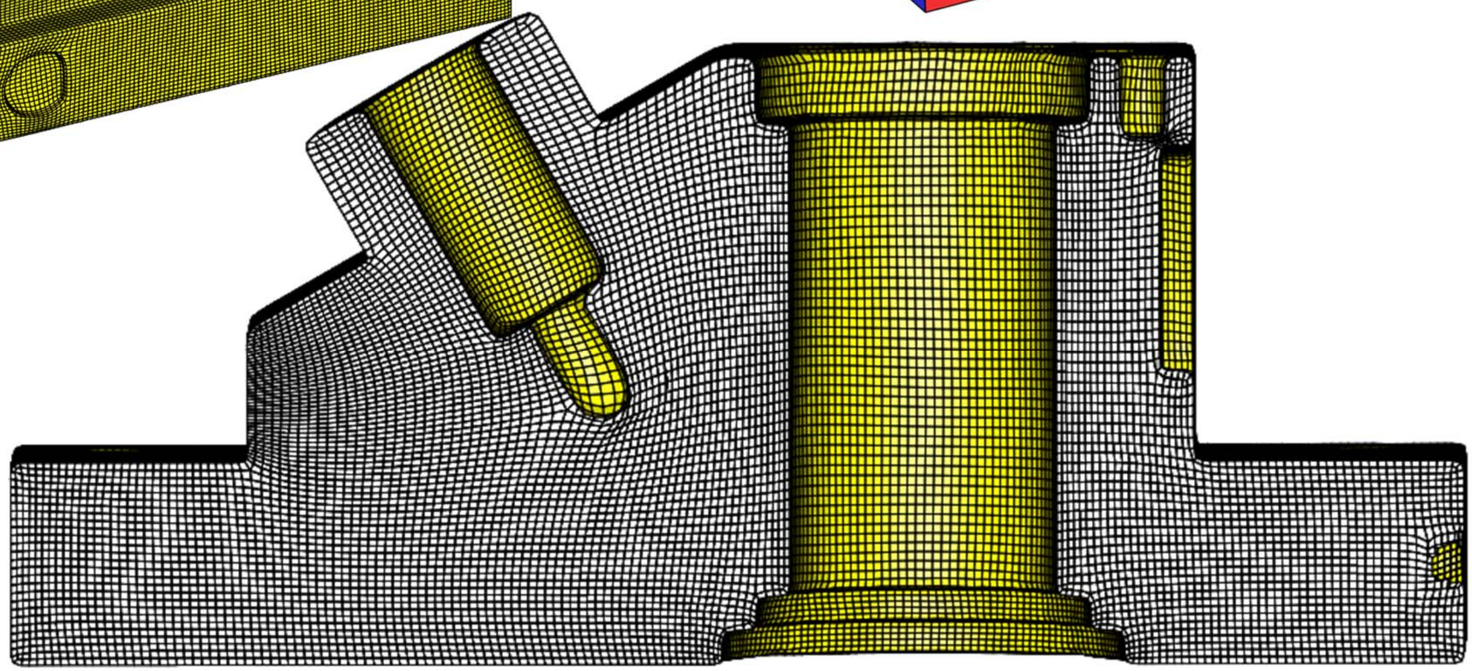


Traditional CAD

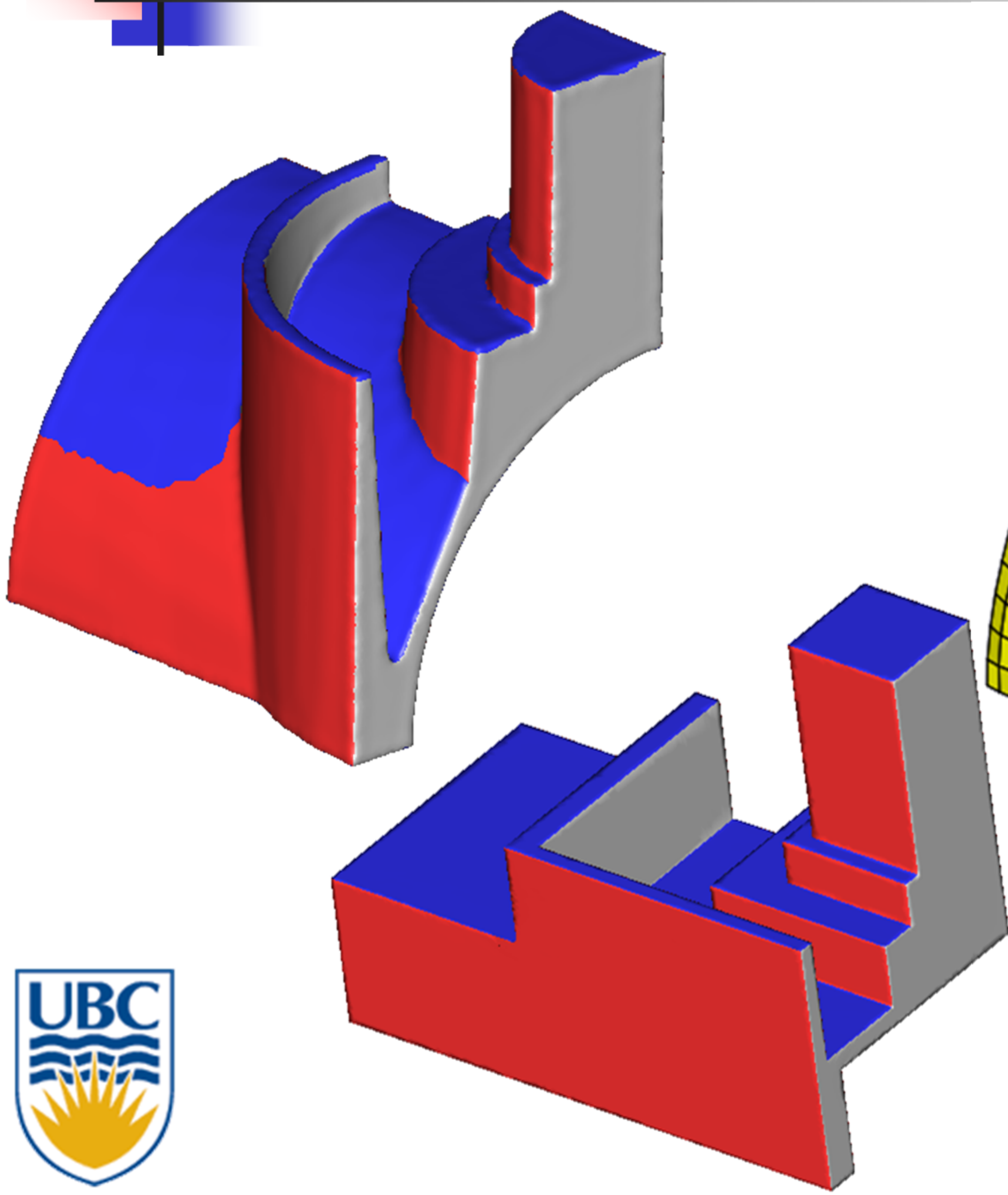


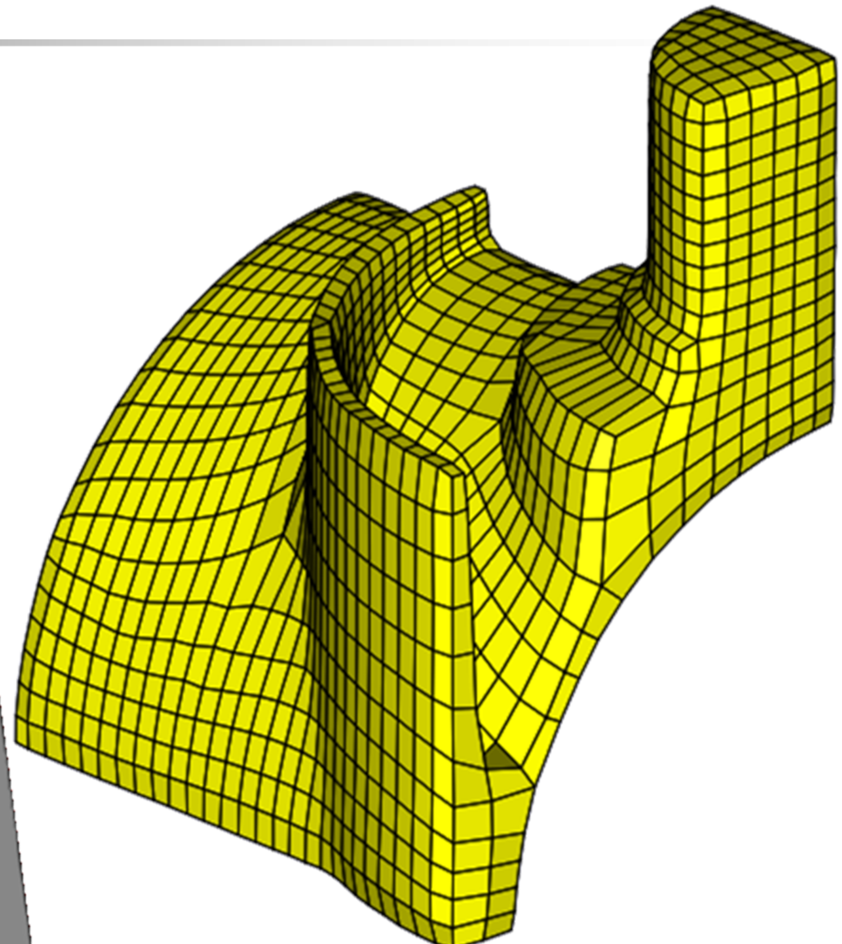






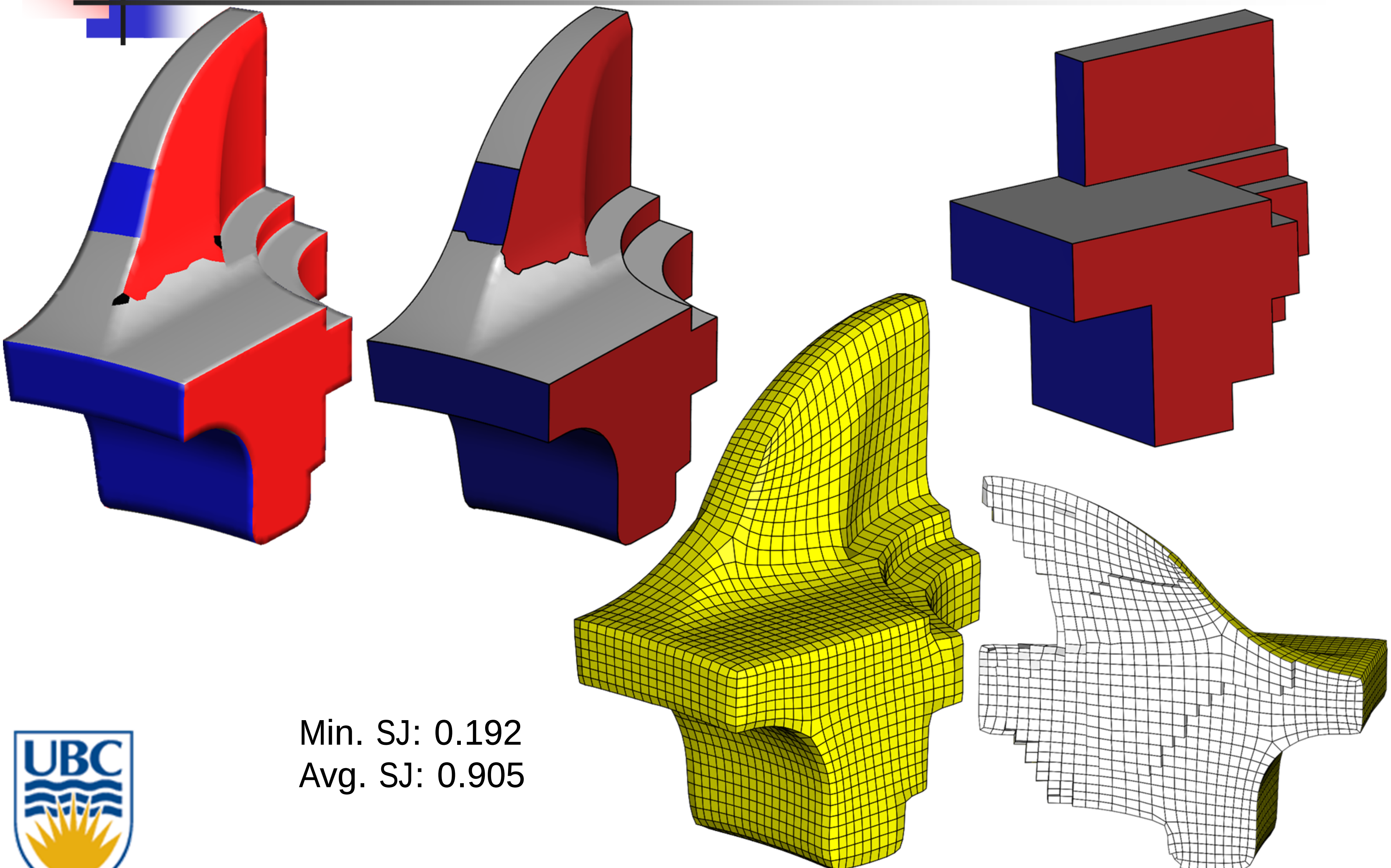
Traditional CAD Models





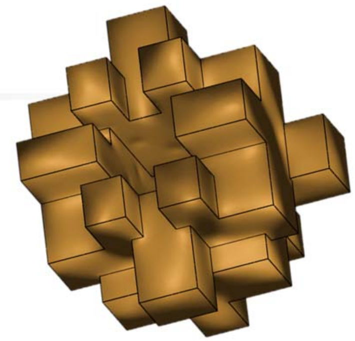
Min. SJ: 0.262
Avg. SJ: 0.667

Traditional CAD

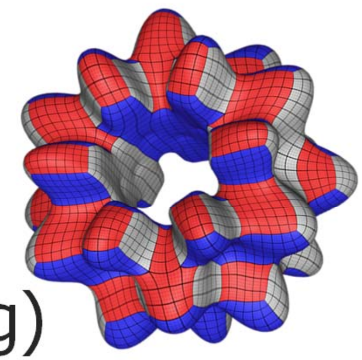


Min. SJ: 0.192
Avg. SJ: 0.905

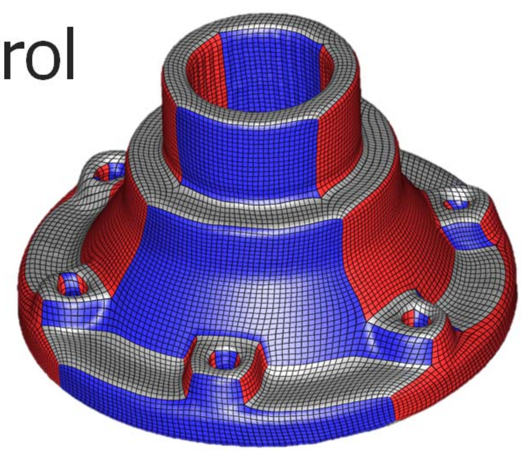
Conclusions

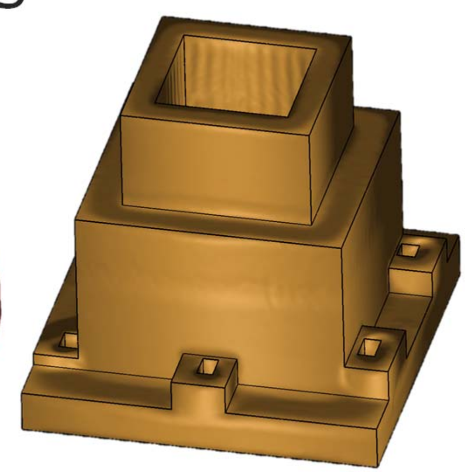


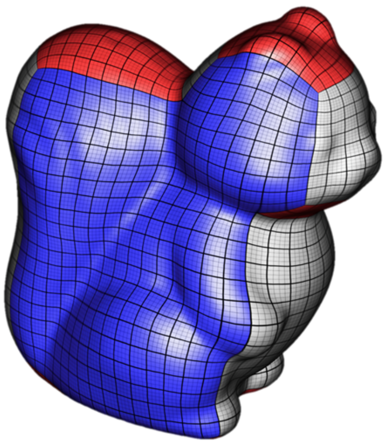
- Automatic all-hex meshing algorithm
  - highly regular
  - high quality (min. & avg.)



- **Key:** PolyCube construction
- PolyCut
  - Local search framework (Hill Climbing)
  - Low distortion PolyCube mappings
  - Singularity control







Questions?