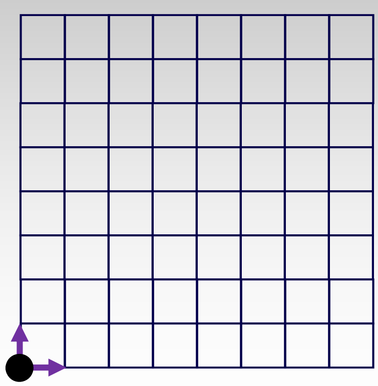
CPSC 436D
Video Game Programming

Transformations

COORDINATE SYSTEMS

Coordinate system = Origin + Basis Vectors



COORDINATE SYSTEMS

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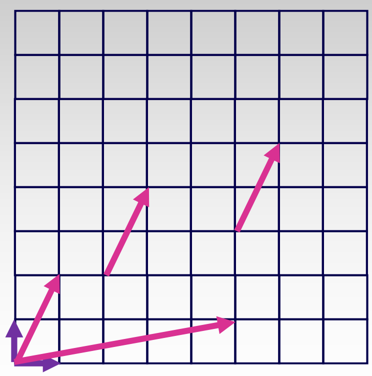
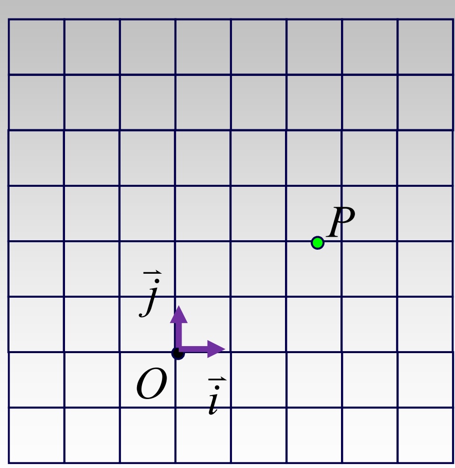


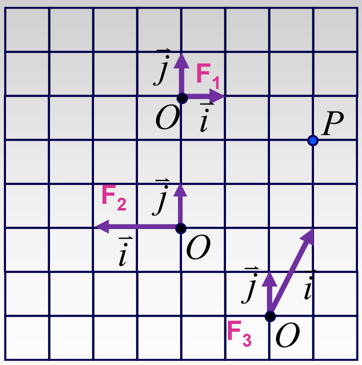
COORDINATE SYSTEMS



$$P = O + x\vec{i} + y\vec{j}$$

equivalent: ***P* = (*x*, *y*)**

COORDINATE SYSTEMS



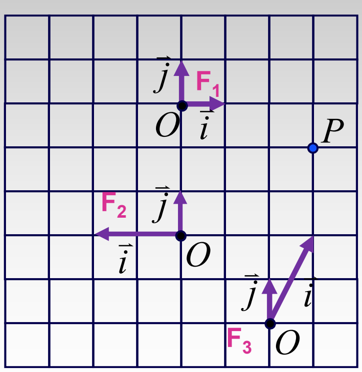
$$P = O + x\vec{i} + y\vec{j}$$

F_1

F_2

F_3

COORDINATE SYSTEMS



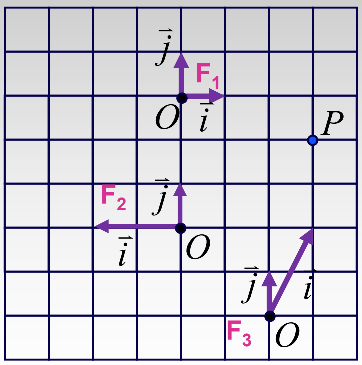
$$P = O + x\vec{i} + y\vec{j}$$

F_1 **P(3,-1)**

F_2

F_3

COORDINATE SYSTEMS



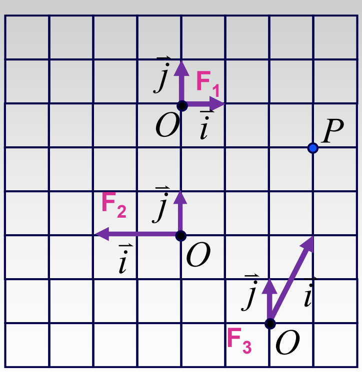
$$P = O + x\vec{i} + y\vec{j}$$

F_1 **P(3,-1)**

F_2 **P(-1.5,2)**

F_3

COORDINATE SYSTEMS



$$P = O + x\vec{i} + y\vec{j}$$

F_1 **P(3,-1)**

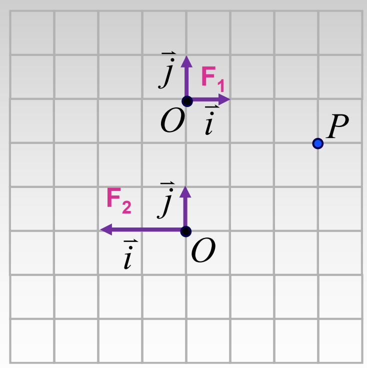
F_2 **P(-1.5,2)**

F_3 **P(1,2)**

Transformations

Transformations as a change of frame

$$P = O + x\vec{i} + y\vec{j}$$



$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}_1 + x_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix}_1 + y_1 \begin{bmatrix} 0 \\ 1 \end{bmatrix}_1$$

$$\begin{bmatrix} x \\ y \end{bmatrix}_2 = \begin{bmatrix} 0 \\ 3 \end{bmatrix}_2 + x_1 \begin{bmatrix} -0.5 \\ 0 \end{bmatrix}_2 + y_1 \begin{bmatrix} 0 \\ 1 \end{bmatrix}_2$$

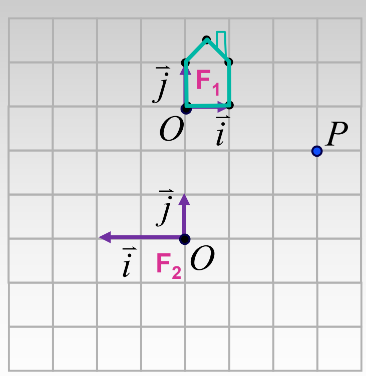
$$\begin{bmatrix} x \\ y \end{bmatrix}_2 = \begin{bmatrix} -0.5 & 0 & 0 \\ 0 & 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}_1$$

check: $P_1(3,-1)$ becomes $P_2(-1.5,2)$

$$P_2 = MP_1$$

TRANSFORMATIONS

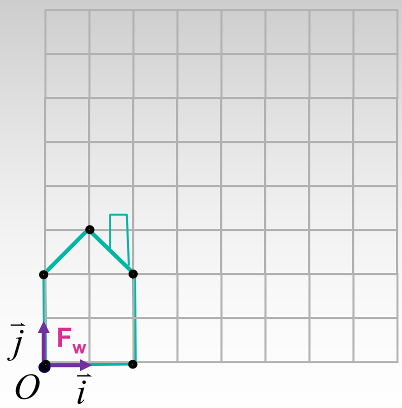
change of basis expressed using a matrix



$$\begin{bmatrix} x \\ y \end{bmatrix}_2 = \begin{bmatrix} -0.5 & 0 & 0 \\ 0 & 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}_1$$

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}_2 = \begin{bmatrix} -0.5 & 0 & 0 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}_1$$

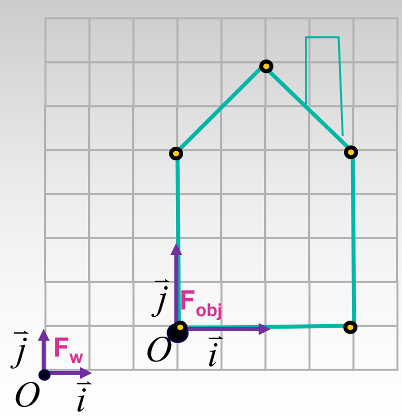
Usage of Transformations



set up the modeling matrix M

for each vertex v
 v' = Mv

Usage of Transformations



$$P = O + x\vec{i} + y\vec{j}$$

$$\begin{bmatrix} x_{obj} \\ y_{obj} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}_{obj} + x_{obj} \begin{bmatrix} 1 \\ 0 \end{bmatrix}_{obj} + y_{obj} \begin{bmatrix} 0 \\ 1 \end{bmatrix}_{obj}$$

$$\begin{bmatrix} x \\ y \end{bmatrix}_w = \begin{bmatrix} 3 \\ 1 \end{bmatrix}_w + x_{obj} \begin{bmatrix} 2 \\ 0 \end{bmatrix}_w + y_{obj} \begin{bmatrix} 0 \\ 2 \end{bmatrix}_w$$

$$\begin{bmatrix} x \\ y \end{bmatrix}_w = \begin{bmatrix} 2 & 0 & 3 \\ 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}_{obj}$$

Using Transformations

$$\begin{bmatrix} x \\ y \end{bmatrix}_2 = \begin{bmatrix} 2 & 0 & 3 \\ 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}_1$$

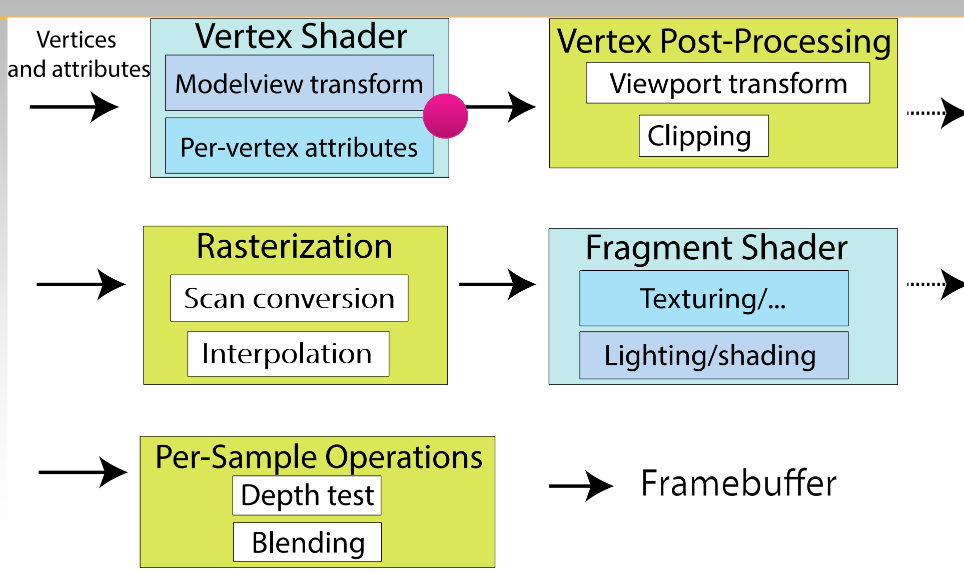
2D

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}_w = \begin{bmatrix} 2 & 0 & 3 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}_{obj}$$

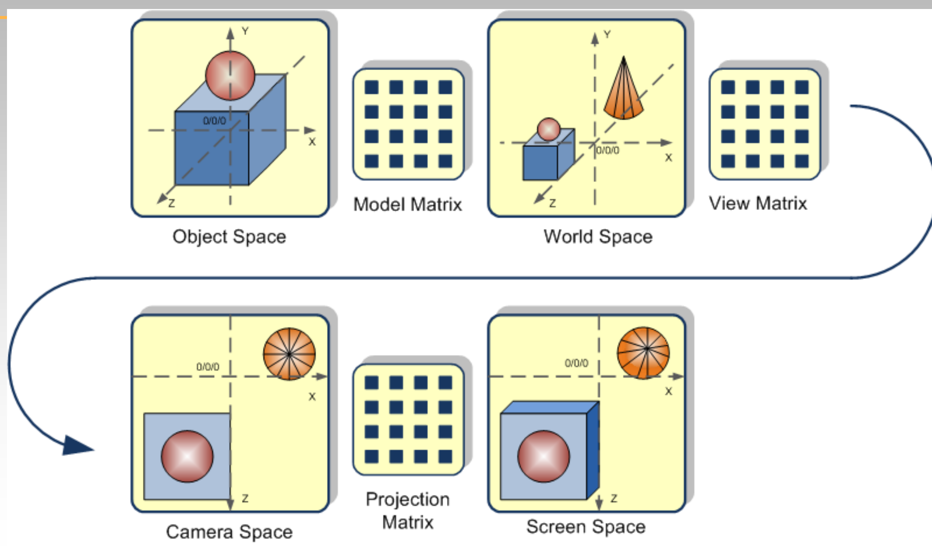
3D

$$\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}_w = \begin{bmatrix} 2 & 0 & 0 & 3 \\ 0 & 2 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}_{obj}$$

PIPELINE



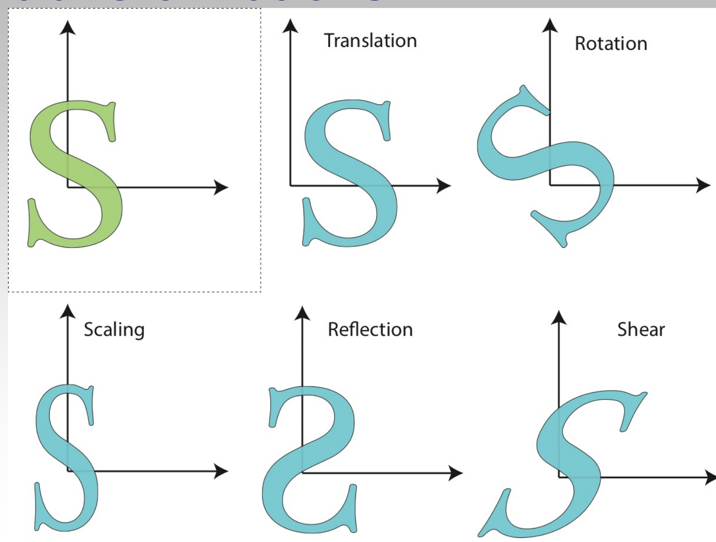
coordinate systems



HOW TO TRANSFORM COORDINATES

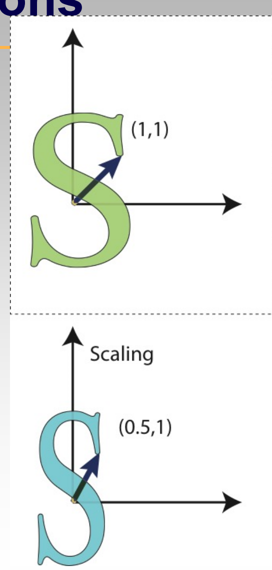
- Between coordinate frames
- Or animate objects

linear transformations



Matrix representations

Scale:



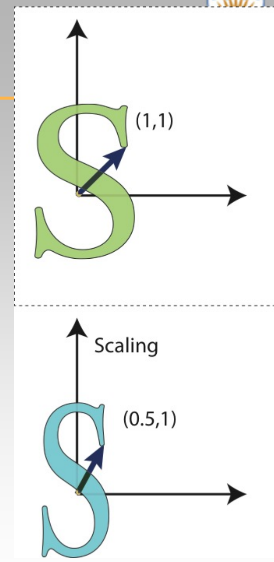
Matrix representations

Scale:

$$M = \begin{bmatrix} \alpha & 0 \\ 0 & \beta \end{bmatrix}$$

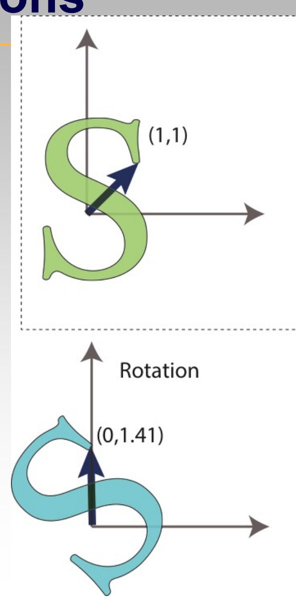
Example:

$$\begin{bmatrix} \alpha & 0 \\ 0 & \beta \end{bmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} \alpha \\ 2\beta \end{pmatrix}$$



Matrix representations

Rotation



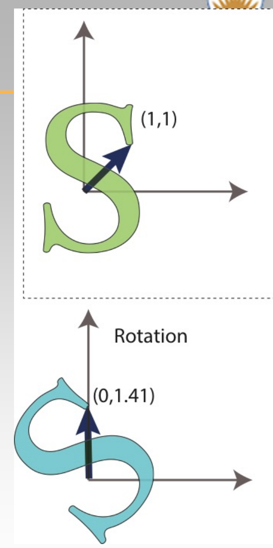
Matrix representations

Rotation

$$R(\alpha) = \begin{bmatrix} \cos(\alpha) & -\sin(\alpha) \\ \sin(\alpha) & \cos(\alpha) \end{bmatrix}$$

Example:

$$\begin{bmatrix} \cos(\alpha) & -\sin(\alpha) \\ \sin(\alpha) & \cos(\alpha) \end{bmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} \cos(\alpha) - \sin(\alpha) \\ \cos(\alpha) + \sin(\alpha) \end{pmatrix}$$



What does this 2D transformation do?

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

A. Rotates by 90 deg
B. Scales by a factor of 2
C. Rotates by -90 deg
D. Nothing

What does this 2D transformation do?

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix},$$

A. Rotates by 90 deg
B. Reflects the object
C. Rotates by -90 deg
D. Scales the object

TRANSLATION

There's a minor glitch.

- Translation: can't be represented as 2x2 matrix multiplication

general transformations

We need to represent all the linear transformations + translation.

Ideas?

$$T(\boldsymbol{v}) = M\boldsymbol{v} + \boldsymbol{b}$$

AUGMENTED MATRIX

$$\mathrm{M}_{2x2}\begin{bmatrix}\mathrm{x}\\ \mathrm{y}\end{bmatrix}=\begin{bmatrix}\mathrm{x}'\\ \mathrm{y}'\end{bmatrix}$$

$$\begin{bmatrix}\mathrm{M}_{2x2} & & 0\\ & & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\mathrm{x}\\ \mathrm{y}\\ 1\end{bmatrix}=\begin{bmatrix}\mathrm{x}'\\ \mathrm{y}'\\ 1\end{bmatrix}$$

Haven't changed much, have we?

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AUGMENTED MATRIX

$$\begin{bmatrix}M_{2x2} & & b_x\\ & & b_y\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}x\\ y\\ 1\end{bmatrix}=\begin{bmatrix}x'+b_x\\ y'+b_y\\ 1\end{bmatrix}=\begin{bmatrix}x'\\ y'\\ 1\end{bmatrix}+\boldsymbol{b}$$

Translation

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Affine transformations

- Linear (rotation, scaling, shear, reflections) + TRANSLATION

Affine transformations

- Linear (rotation, scaling, shear, reflections) + TRANSLATION

- How to convert a linear transformation matrix into affine matrix?

AFFINE Transformations

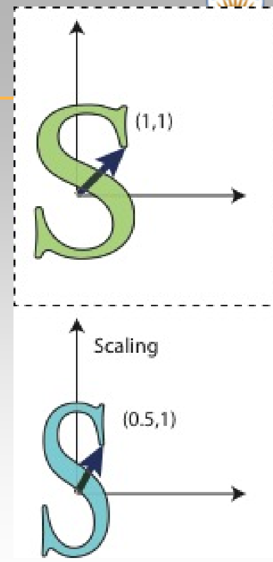
Scale:

$$M = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$$

$$M = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Example:

$$\begin{pmatrix} a \cdot 1 \\ b \cdot 2 \\ 1 \end{pmatrix} = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$$



AFFINE Transformations

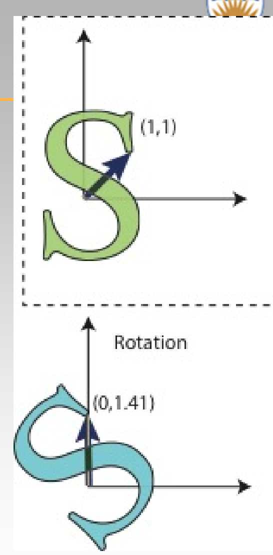
Rotation

$$M = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$$

$$M = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Example:

$$\begin{pmatrix} a \cdot 1 \\ b \cdot 2 \\ 1 \end{pmatrix} = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$$



AFFINE Transformations

Translation

$$M = \begin{bmatrix} 1 & 0 & C_x \\ 0 & 1 & C_y \\ 0 & 0 & 1 \end{bmatrix}$$

Example:

$$\begin{bmatrix} 1 & 0 & C_x \\ 0 & 1 & C_y \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} x + C_x \\ y + C_y \\ 1 \end{pmatrix}$$

TRANSFORMATION COMPOSITION

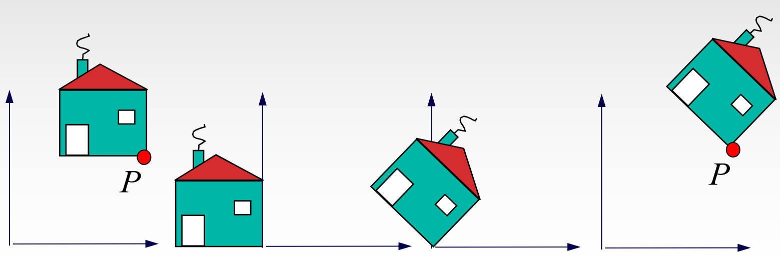
- **What operation rotates XY by θ around** $P = \begin{pmatrix} p_x \\ p_y \end{pmatrix}$ **?**

TRANSFORMATION COMPOSITION

- What operation rotates **XY** by θ around $P = \begin{pmatrix} ?p_x \\ p_y \end{pmatrix}$
- Answer:
  - Translate *P* to origin

TRANSFORMATION COMPOSITION

- What operation rotates **XY** by $\theta < 0$ around $P = \begin{pmatrix} ?p_x \\ p_y \end{pmatrix}$
- Answer:
  - Translate *P* to origin
  - Rotate around origin by θ
  - Translate back



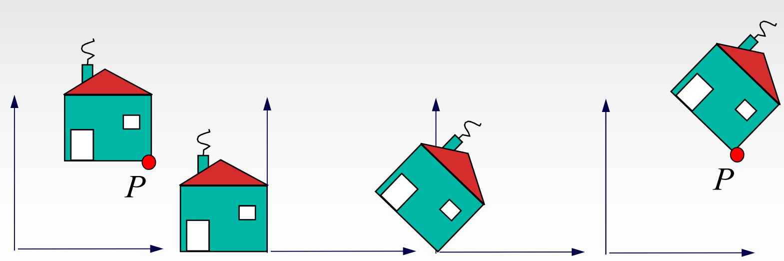
TRANSFORMATION COMPOSITION

$$T^{(p_x,p_y)} R^{\theta} T^{(-p_x,-p_y)}(V)$$

$$= \begin{bmatrix} 1 & 0 & p_x \\ 0 & 1 & p_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -p_x \\ 0 & 1 & -p_y \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} v_x \\ v_y \\ 1 \end{pmatrix}$$

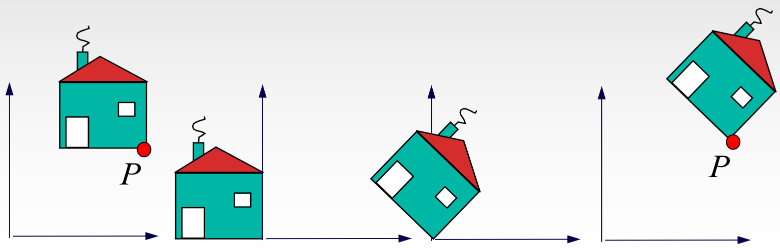
two interpretations

$$\begin{pmatrix} \cos\theta & -\sin\theta & p_x \cdot (1-\cos\theta) + p_y \cdot \sin\theta \\ \sin\theta & \cos\theta & p_y \cdot (1-\cos\theta) + p_x \cdot \sin\theta \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} v_x \\ v_y \\ 1 \end{pmatrix}$$



TRANSFORMING COORDINATES

$$\begin{pmatrix} \cos\theta & -\sin\theta & p_x \cdot (1-\cos\theta) + p_y \cdot \sin\theta \\ \sin\theta & \cos\theta & p_y \cdot (1-\cos\theta) + p_x \cdot \sin\theta \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} v_x \\ v_y \\ 1 \end{pmatrix}$$



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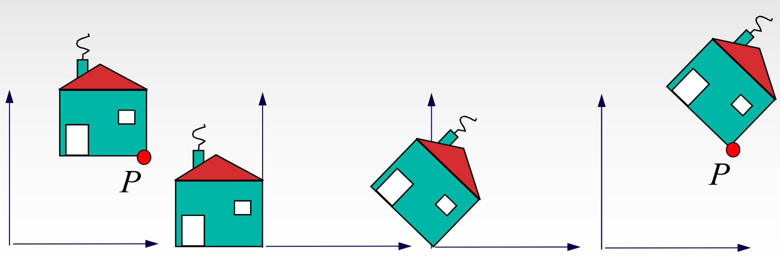
TRANSFORMING COORDINATES

$$\begin{pmatrix} \cos\theta & -\sin\theta & p_x \cdot (1-\cos\theta) + p_y \cdot \sin\theta \\ \sin\theta & \cos\theta & p_y \cdot (1-\cos\theta) + p_x \cdot \sin\theta \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} v_x \\ v_y \\ 1 \end{pmatrix}$$



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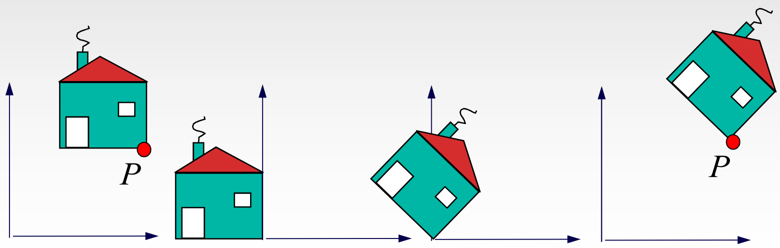
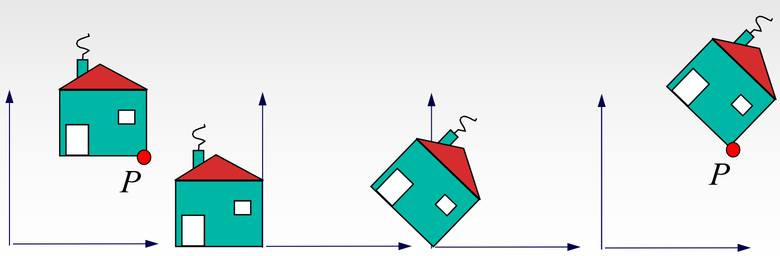
TRANSFORMING COORDINATES

$$\begin{pmatrix} \cos\theta & -\sin\theta & p_x\cdot(1-\cos\theta)+p_y\cdot\sin\theta \\ \sin\theta & \cos\theta & p_y\cdot(1-\cos\theta)+p_x\cdot\sin\theta \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} v_x \\ v_y \\ 1 \end{pmatrix}$$



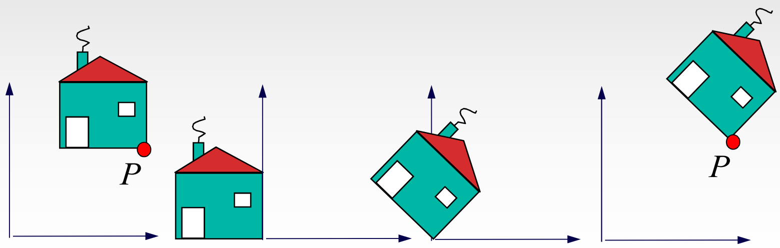
TRANSFORMING COORDINATE FRAME

$$\begin{pmatrix} \cos\theta & -\sin\theta & p_x\cdot(1-\cos\theta)+p_y\cdot\sin\theta \\ \sin\theta & \cos\theta & p_y\cdot(1-\cos\theta)+p_x\cdot\sin\theta \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} v_x \\ v_y \\ 1 \end{pmatrix}$$



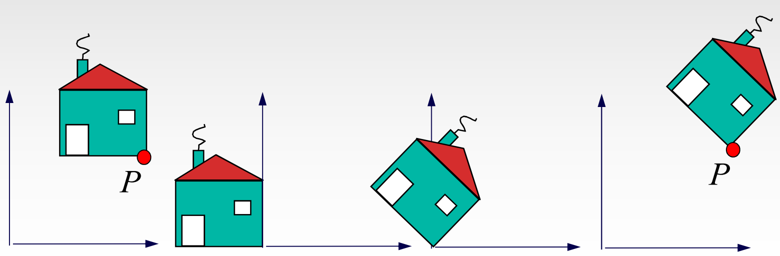
TRANSFORMING COORDINATE FRAME

$$\begin{pmatrix}\cos\theta & -\sin\theta & p_x\cdot(1-\cos\theta)+p_y\cdot\sin\theta\\ \sin\theta & \cos\theta & p_y\cdot(1-\cos\theta)+p_x\cdot\sin\theta\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}v_x\\ v_y\\ 1\end{pmatrix}$$



TRANSFORMING COORDINATE FRAME

$$\begin{pmatrix}\cos\theta & -\sin\theta & p_x\cdot(1-\cos\theta)+p_y\cdot\sin\theta\\ \sin\theta & \cos\theta & p_y\cdot(1-\cos\theta)+p_x\cdot\sin\theta\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}v_x\\ v_y\\ 1\end{pmatrix}$$



TRANSFORMING COORDINATE FRAME

Columns are new basis vectors (and new origin)!

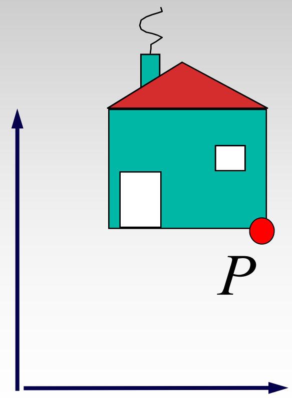
$$\begin{pmatrix} \cos\theta & -\sin\theta & p_x \cdot (1-\cos\theta) + p_y \cdot \sin\theta \\ \sin\theta & \cos\theta & p_y \cdot (1-\cos\theta) + p_x \cdot \sin\theta \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} v_x \\ v_y \\ 1 \end{pmatrix}$$

TRANSFORMING COORDINATE FRAME

$$T^{(p_x, p_y)} R^{\theta} T^{(-p_x, -p_y)} \begin{pmatrix} v_x \\ v_y \\ 1 \end{pmatrix}$$

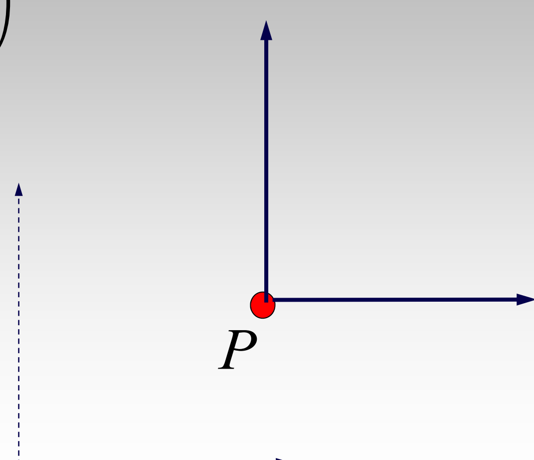
TRANSFORMING COORDINATE FRAME

$$T^{(p_x,p_y)}R^{\theta}T^{(-p_x,-p_y)}\begin{pmatrix}v_x\\v_y\\1\end{pmatrix}$$



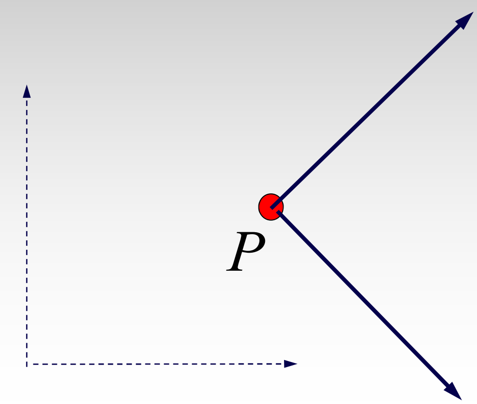
TRANSFORMING COORDINATE FRAME

$$T^{(p_x,p_y)}R^{\theta}T^{(-p_x,-p_y)}\begin{pmatrix}v_x\\v_y\\1\end{pmatrix}$$



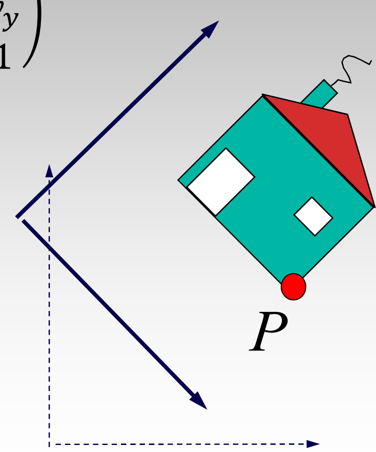
TRANSFORMING COORDINATE FRAME

$$T^{(p_x,p_y)}\boldsymbol{R}^{\boldsymbol{\theta}}T^{(-p_x,-p_y)}\begin{pmatrix}v_x\\v_y\\1\end{pmatrix}$$



TRANSFORMING COORDINATE FRAME

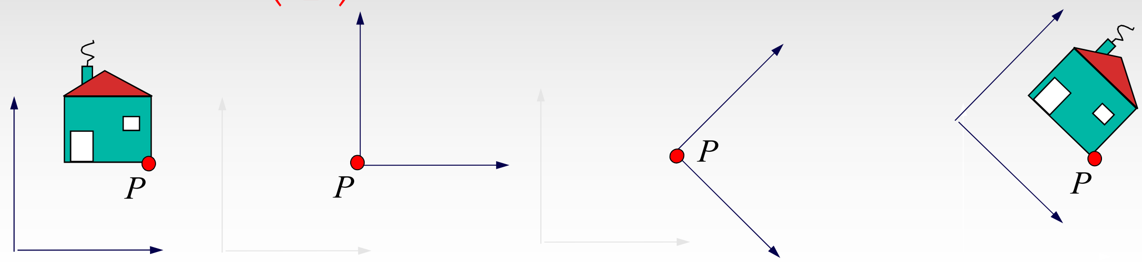
$$T^{(p_x,p_y)}R^{\theta}\boldsymbol{T}^{(-\boldsymbol{p_x},-\boldsymbol{p_y})}\begin{pmatrix}v_x\\v_y\\1\end{pmatrix}$$



TRANSFORMING COORDINATE FRAME

World Coordinate Frame | **Object Coordinate Frame**

$$\begin{pmatrix} v'_x \\ v'_y \\ 1 \end{pmatrix} = T^{(p_x,p_y)} R^{\theta} T^{(-p_x,-p_y)} \begin{pmatrix} v_x \\ v_y \\ 1 \end{pmatrix}$$



TWO INTERPRETATIONS OF COMPOSITE

World Coordinate Frame | **Object Coordinate Frame**

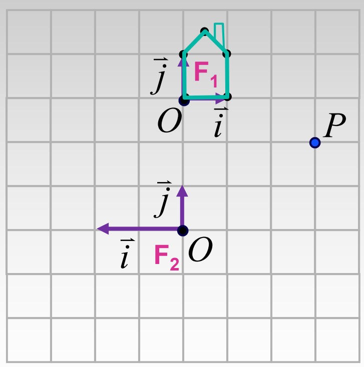
$$\begin{pmatrix} v'_x \\ v'_y \\ 1 \end{pmatrix} = T^{(p_x,p_y)} R^{\theta} T^{(-p_x,-p_y)} \begin{pmatrix} v_x \\ v_y \\ 1 \end{pmatrix}$$

1) *read from inside-out as transformation of object*

2) *read from outside-in as transformation of the coordinate frame*

TRANSFORMATIONS - EXAMPLE

change of basis expressed using a matrix

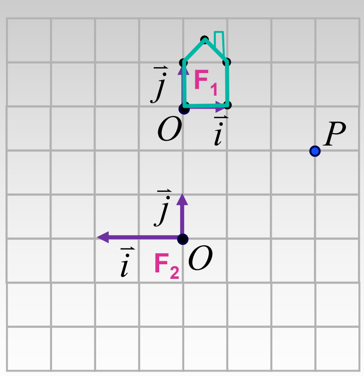


$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}_1 + x_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix}_1 + y_1 \begin{bmatrix} 0 \\ 1 \end{bmatrix}_1$$

$$\begin{bmatrix} x \\ y \end{bmatrix}_2 = \begin{bmatrix} 0 \\ 3 \end{bmatrix}_2 + x_1 \begin{bmatrix} -0.5 \\ 0 \end{bmatrix}_2 + y_1 \begin{bmatrix} 0 \\ 1 \end{bmatrix}_2$$

TRANSFORMATIONS - EXAMPLE

change of basis expressed using a matrix



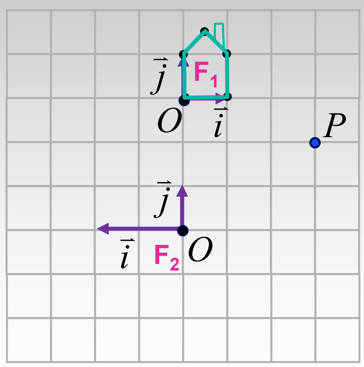
$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}_1 + x_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix}_1 + y_1 \begin{bmatrix} 0 \\ 1 \end{bmatrix}_1$$

$$\begin{bmatrix} x \\ y \end{bmatrix}_2 = \begin{bmatrix} 0 \\ 3 \end{bmatrix}_2 + x_1 \begin{bmatrix} -0.5 \\ 0 \end{bmatrix}_2 + y_1 \begin{bmatrix} 0 \\ 1 \end{bmatrix}_2$$

$$\begin{pmatrix} x_2 \\ y_2 \\ 1 \end{pmatrix} = \begin{pmatrix} -0.5 & 0 & 0 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ y_1 \\ 1 \end{pmatrix}$$

TRANSFORMATIONS - EXAMPLE

change of basis expressed using a matrix



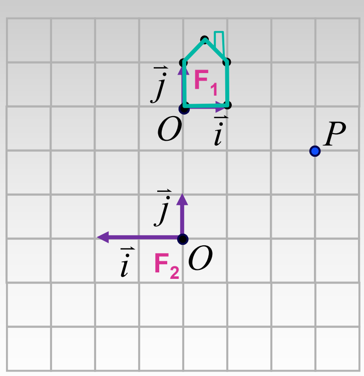
$$\begin{bmatrix}x_1\\y_1\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}_1 + x_1\begin{bmatrix}1\\0\end{bmatrix}_1 + y_1\begin{bmatrix}0\\1\end{bmatrix}_1$$

$$\begin{bmatrix}x\\y\end{bmatrix}_2 = \begin{bmatrix}0\\3\end{bmatrix}_2 + x_1\begin{bmatrix}-0.5\\0\end{bmatrix}_2 + y_1\begin{bmatrix}0\\1\end{bmatrix}_2$$

$$\begin{pmatrix}x_2\\y_2\\1\end{pmatrix} = \begin{pmatrix}-0.5&0&0\\0&1&3\\0&0&1\end{pmatrix}\begin{pmatrix}x_1\\y_1\\1\end{pmatrix}$$

TRANSFORMATIONS - EXAMPLE

change of basis expressed using a matrix



$$\begin{bmatrix}x_1\\y_1\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}_1 + x_1\begin{bmatrix}1\\0\end{bmatrix}_1 + y_1\begin{bmatrix}0\\1\end{bmatrix}_1$$

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$$\begin{pmatrix}x_2\\y_2\\1\end{pmatrix} = \begin{pmatrix}-0.5&0&0\\0&1&3\\0&0&1\end{pmatrix}\begin{pmatrix}x_1\\y_1\\1\end{pmatrix}$$

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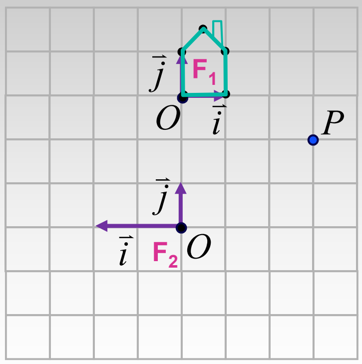
TRANSFORMATIONS - EXAMPLE

change of basis expressed using a matrix

$$\begin{bmatrix}x_1\\y_1\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}_1 + x_1\begin{bmatrix}1\\0\end{bmatrix}_1 + y_1\begin{bmatrix}0\\1\end{bmatrix}_1$$

$$\begin{bmatrix}x\\y\end{bmatrix}_2 = \begin{bmatrix}0\\3\end{bmatrix}_2 + x_1\begin{bmatrix}-0.5\\0\end{bmatrix}_2 + y_1\begin{bmatrix}0\\1\end{bmatrix}_2$$

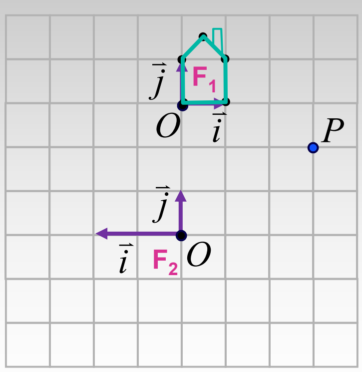


$$\overrightarrow{i_1}\quad \overrightarrow{j_1}\quad O_1$$

$$\begin{pmatrix}x_2\\y_2\\1\end{pmatrix} = \begin{pmatrix}-0.5 & 0 & 0\\0 & 1 & 3\\0 & 0 & 1\end{pmatrix}\begin{pmatrix}x_1\\y_1\\1\end{pmatrix}$$

TRANSFORMATIONS - EXAMPLE

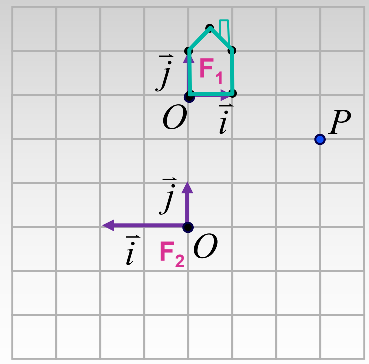
How to transform F_2 into F_1?



TRANSFORMATIONS - EXAMPLE

How to transform F_2 into F_1?

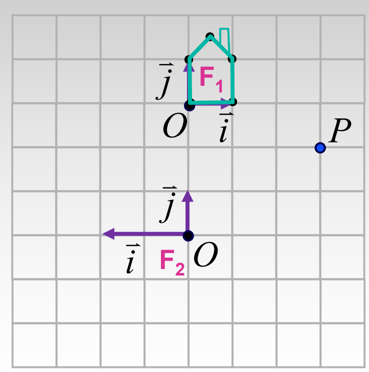
- Scale by (-0.5, 1)



TRANSFORMATIONS - EXAMPLE

How to transform F_2 into F_1?

- Scale by (-0.5, 1)
- Translate by (0,3)

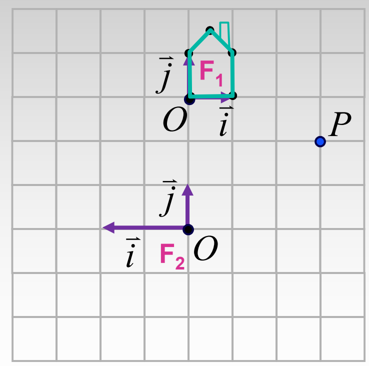


TRANSFORMATIONS - EXAMPLE

How to transform F_2 into F_1?

- Scale by (-0.5, 1)
- Translate by (0,3)

$$M = M_{scale} \cdot M_{translate}$$



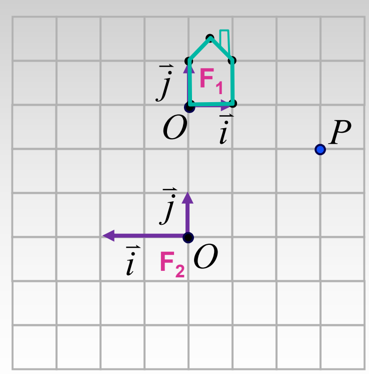
TRANSFORMATIONS - EXAMPLE

How to transform F_2 into F_1?

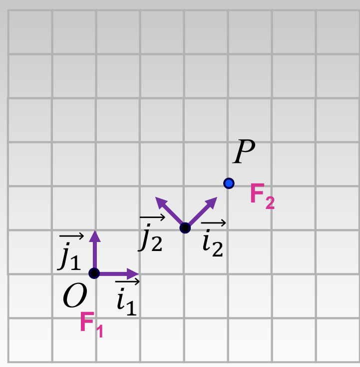
- Scale by (-0.5, 1)
- Translate by (0,3)

$$M = M_{scale} \cdot M_{translate} = \begin{pmatrix} -0.5 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{pmatrix}$$

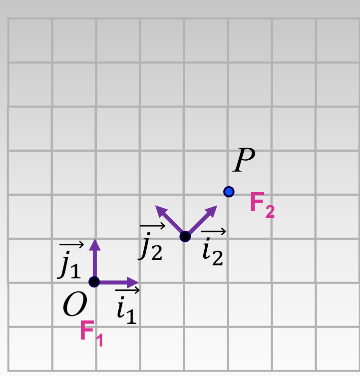
$$= \begin{pmatrix} -0.5 & 0 & 0 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{pmatrix}$$



ANOTHER EXAMPLE

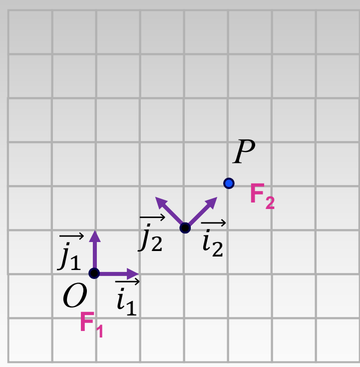


ANOTHER EXAMPLE



$$P = \begin{pmatrix} 3 \\ 2 \end{pmatrix}_1 = \begin{pmatrix} \sqrt{2} \\ 0 \end{pmatrix}_2$$

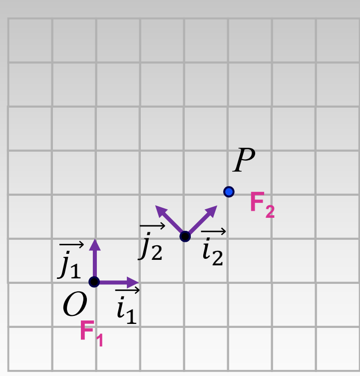
ANOTHER EXAMPLE



$$P = \begin{pmatrix} 3 \\ 2 \end{pmatrix}_1 = \begin{pmatrix} \sqrt{2} \\ 0 \end{pmatrix}_2$$

$$M_{21} = \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 2 \\ 1/\sqrt{2} & 1/\sqrt{2} & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

ANOTHER EXAMPLE



$$P = \begin{pmatrix} 3 \\ 2 \end{pmatrix}_1 = \begin{pmatrix} \sqrt{2} \\ 0 \end{pmatrix}_2$$

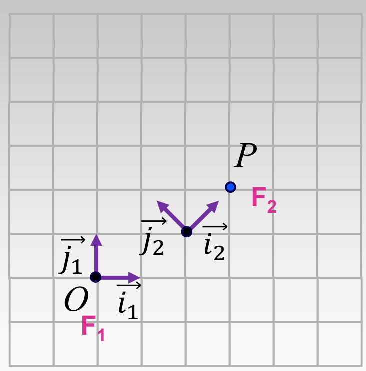
Converts coordinates from 2 into 1

$$M_{21} = \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 2 \\ 1/\sqrt{2} & 1/\sqrt{2} & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$M_{21} \cdot \begin{pmatrix} \sqrt{2} \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$$

ANOTHER EXAMPLE

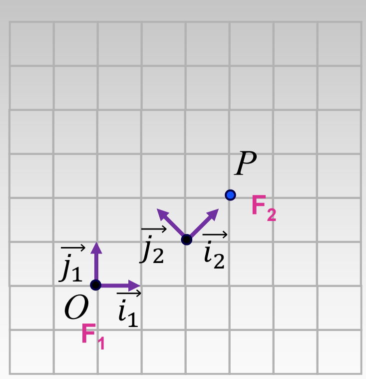
To convert point from 2 into 1, let's think the other way:
how to get to coordinate frame 2 from 1?



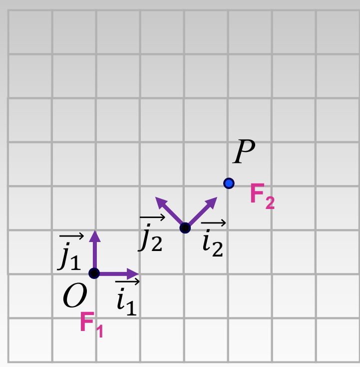
ANOTHER EXAMPLE

To convert point from 2 into 1, let's think the other way:
how to get to coordinate frame 2 from 1?

$$M_{21} = Tr_{(2,1)} \cdot Rot_{\pi/4}$$



ANOTHER EXAMPLE



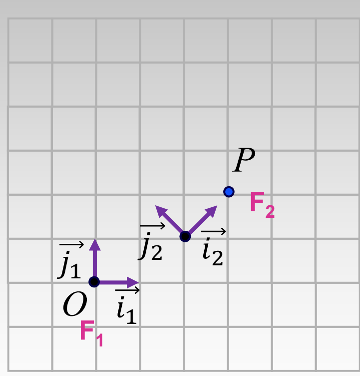
To convert point from 2 into 1, let's think the other way: how to get to coordinate frame 2 from 1?

$$M_{21} = Tr_{(2,1)} \cdot Rot_{\pi/4} =$$

$$\begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \cos\left(\frac{\pi}{4}\right) & -\sin\left(\frac{\pi}{4}\right) & 0 \\ \sin\left(\frac{\pi}{4}\right) & \cos\left(\frac{\pi}{4}\right) & 0 \\ 0 & 0 & 1 \end{pmatrix} =$$

$$\begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 2 \\ 1/\sqrt{2} & 1/\sqrt{2} & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

ANOTHER EXAMPLE



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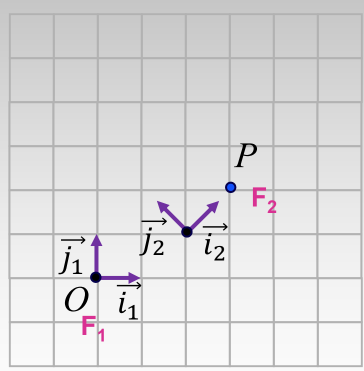
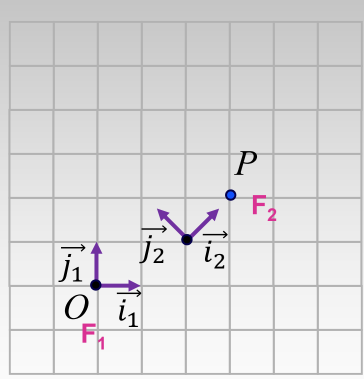
ANOTHER EXAMPLE



Notice that matrix multiplication is not commutative!

$AB \neq BA$

ANOTHER EXAMPLE



Notice that matrix multiplication is not commutative!

$AB \neq BA$

$$M_{21} = Tr_{(2,1)} \cdot Rot_{\pi/4} = \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 2 \\ 1/\sqrt{2} & 1/\sqrt{2} & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$Rot_{\pi/4} \cdot Tr_{(2,1)} = \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} & \mathbf{1/\sqrt{2}} \\ 1/\sqrt{2} & 1/\sqrt{2} & \mathbf{3/\sqrt{2}} \\ 0 & 0 & 1 \end{pmatrix}$$