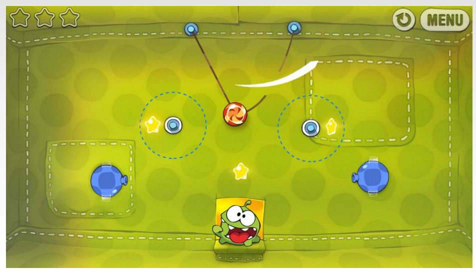
CPSC 436D
Video Game Programming

Curves (basics)



Curves

Mathematical representations:

- Explicit functions:
- Parametric functions
- Implicit functions

Explicit functions

- y = f(x)
- E.g. y = a x + b
- Single y value for each x
- Useful for?
  - *Terrain*
  - *"height field" geometry*

Parametric Functions

- 2D: x and y are functions of a parameter value t
- 3D: x, y, and z are functions of a parameter value t

$$C(t) := \begin{pmatrix} P_y^0 \\ P_x^0 \end{pmatrix} t + \begin{pmatrix} P_y^1 \\ P_x^1 \end{pmatrix} (1-t)$$

Line (segment)

$$C(t) := \begin{pmatrix} \cos t \\ \sin t \end{pmatrix}$$

Circle (arc)

- Depends on parameter range t1 < t < t2

Implicit Function

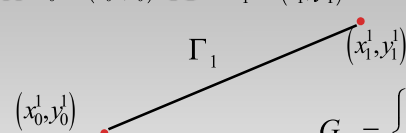
- Curve (2D) or Surface (3D) defined by zero set (roots) of function
- E.g:

$$S(x,y): x^2 + y^2 - 1 = 0$$

$$S(x,y,z): x^2 + y^2 + z^2 - 1 = 0$$

Lines & Segments

Segment Γ_1 ***from*** $P_0 = (x_0^1, y_0^1)$ ***to*** $P_1 = (x_1^1, y_1^1)$



$$G_1 = \begin{cases} x^1(t) = x_0^1 + (x_1^1 - x_0^1)t \\ y^1(t) = y_0^1 + (y_1^1 - y_0^1)t \end{cases} \quad t \in [0,1]$$

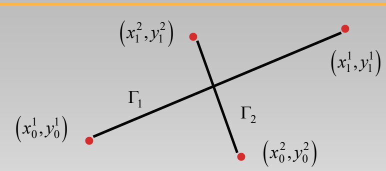
Line through $P_0 = (x_0^1, y_0^1)$ ***and*** $P_1 = (x_1^1, y_1^1)$

- Parametric $G_1(t), t \in (-\infty, -\infty)$
- Implicit $Ax+By+C=0$
  - *Solve 2 equations in 2 unknowns (set $A^2+B^2=1$)*

Interpolation

- Animation
  - *Move object from position 1 to position 2*
  - *Rotate object from orientation 1 to orientation 2*
- Modeling
  - *Polygon = union of line segments*
    - How to know if point inside?

Line-Line Intersection



$$G_1 = \begin{cases} x^1(t) = x_0^1 + \left(x_1^1 - x_0^1\right)t \\ y^1(t) = y_0^1 + \left(y_1^1 - y_0^1\right)t \end{cases} \quad t \in [0,1] \qquad G_2 = \begin{cases} x^2(r) = x_0^2 + \left(x_1^2 - x_0^2\right)r \\ y^2(r) = y_0^2 + \left(y_1^2 - y_0^2\right)r \end{cases} \quad r \in [0,1]$$

Intersection: *x* & *y* values equal in both representations - two linear equations in two unknowns (*r,t*)

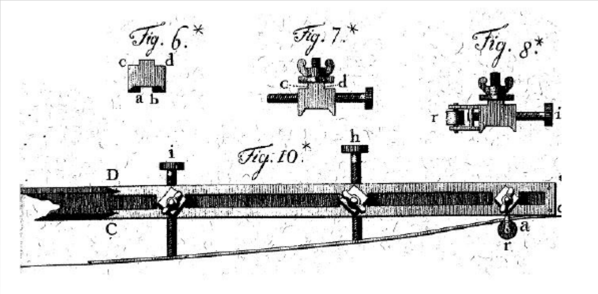
$$x_0^1 + \left(x_1^1 - x_0^1\right)t = x_0^2 + \left(x_1^2 - x_0^2\right)r$$
$$y_0^1 + \left(y_1^1 - y_0^1\right)t = y_0^2 + \left(y_1^2 - y_0^2\right)r$$

Question: What is the meaning of *r,t* < 0 or *r,t* >1 ?

Splines – Free Form Curves

Geometric meaning of coefficients (base)

- Approximate/interpolate set of positions, derivatives, etc..



Will see one example

Splines – Free Form Curves

Usually parametric

- $C(t)=[x(t),y(t)]$ or $C(t)=[x(t),y(t),z(t)]$

Description = basis functions + coefficients

$$C(t)=\sum_{i=0}^{n} P_i B_i(t) = (x(t), y(t))$$

$$x(t)=\sum_{i=0}^{n} P_i^x B_i(t)$$

$$y(t)=\sum_{i=0}^{n} P_i^y B_i(t)$$

- Same basis functions for all coordinates

Hermite Cubic Basis

Geometrically-oriented coefficients

- 2 positions + 2 tangents

Require $C(0)=P_0, C(1)=P_1, C'(0)=T_0, C'(1)=T_1$

Define basis function per requirement

$$C(t) = P_0 h_{00}(t) + P_1 h_{01}(t) + T_0 h_{10}(t) + T_1 h_{11}(t)$$

Hermite Basis Functions

$$C(t) = P_0 h_{00}(t) + P_1 h_{01}(t) + T_0 h_{10}(t) + T_1 h_{11}(t)$$

To enforce $C(0)=P_0, C(1)=P_1, C'(0)=T_0, C'(1)=T_1$ ***basis should satisfy***

$$h_{ij}(t): i, j = 0,1, t \in [0,1]$$

curve	$C(0)$	$C(1)$	$C'(0)$	$C'(1)$
$h_{00}(t)$	1	0	0	0
$h_{01}(t)$	0	1	0	0
$h_{10}(t)$	0	0	1	0
$h_{11}(t)$	0	0	0	1

Hermite Cubic Basis

Can satisfy with cubic polynomials as basis

$$h_{ij}(t) = a_3 t^3 + a_2 t^2 + a_1 t + a_0$$

Obtain - solve 4 linear equations in 4 unknowns for each basis function $h_{ij}(t): i, j = 0,1, t \in [0,1]$

curve	$C(0)$	$C(1)$	$C'(0)$	$C'(1)$
$h_{00}(t)$	1	0	0	0
$h_{01}(t)$	0	1	0	0
$h_{10}(t)$	0	0	1	0
$h_{11}(t)$	0	0	0	1

Hermite Cubic Basis

Four polynomials that satisfy the conditions

$$h_{00}(t) = t^2(2t-3)+1 \qquad h_{01}(t) = -t^2(2t-3)$$
$$h_{10}(t) = t(t-1)^2 \qquad h_{11}(t) = t^2(t-1)$$