CPSC 436D
Video Game Programming

Curves (basics)

Curves

Mathematical representations:

- Explicit functions:
- Parametric functions
- Implicit functions

Explicit functions

- y = f(x)
- E.g. y = a x + b
- Single y value for each x
- Useful for?
  - *Terrain*
  - *"height field" geometry*

Parametric Functions

- 2D: x and y are functions of a parameter value t
- 3D: x, y, and z are functions of a parameter value t

$$C(t) := \begin{pmatrix} P_y^0 \\ P_x^0 \end{pmatrix} t + \begin{pmatrix} P_y^1 \\ P_x^1 \end{pmatrix} (1-t)$$

Line (segment)

$$C(t) := \begin{pmatrix} \cos t \\ \sin t \end{pmatrix}$$

Circle (arc)

- Depends on parameter range t1 < t < t2

Implicit Function

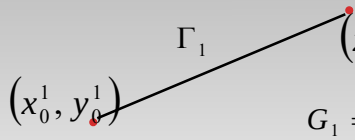
- Curve (2D) or Surface (3D) defined by zero set (roots) of function
- E.g:

$$S(x,y): x^2 + y^2 - 1 = 0$$

$$S(x,y,z): x^2 + y^2 + z^2 - 1 = 0$$

Lines & Segments

Segment Γ_1 ***from*** $P_0 = (x_0^1, y_0^1)$ ***to*** $P_1 = (x_1^1, y_1^1)$



$$G_1 = \begin{cases} x^1(t) = x_0^1 + (x_1^1 - x_0^1)t \\ y^1(t) = y_0^1 + (y_1^1 - y_0^1)t \end{cases} \quad t \in [0,1]$$

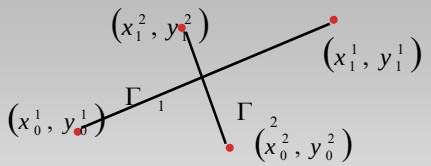
Line through $P_0 = (x_0^1, y_0^1)$ ***and*** $P_1 = (x_1^1, y_1^1)$

- Parametric $\quad G_1(t), t \in (-\infty, -\infty)$
- Implicit $Ax+By+C=0$
  - *Solve 2 equations in 2 unknowns (set $A^2+B^2=1$)*

Interpolation

- Animation
  - *Move object from position 1 to position 2*
  - *Rotate object from orientation 1 to orientation 2*
- Modeling
  - *Polygon = union of line segments*
    - How to know if point inside?

Line-Line Intersection



$$G_1 = \begin{cases} x^1(t) = x_0^1 + (x_1^1 - x_0^1)t \\ y^1(t) = y_0^1 + (y_1^1 - y_0^1)t \end{cases} t \in [0,1] \quad G_2 = \begin{cases} x^2(r) = x_0^2 + (x_1^2 - x_0^2)r \\ y^2(r) = y_0^2 + (y_1^2 - y_0^2)r \end{cases} r \in [$$

Intersection: *x* & *y* values equal in both representations - two linear equations in two unknowns (*r,t*)

$$x_0^1 + (x_1^1 - x_0^1)t = x_0^2 + (x_1^2 - x_0^2)r$$

$$y_0^1 + (y_1^1 - y_0^1)t = y_0^2 + (y_1^2 - y_0^2)r$$

Question: What is the meaning of *r,t*<0 or *r,t*>1 ?