

CPSC 436D

Video Game Programming



Curves (basics)



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Curves



Mathematical representations:

- Explicit functions:
- Parametric functions
- Implicit functions

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Explicit functions

- $y = f(x)$
- E.g. $y = a x + b$
- Single y value for each x
- Useful for?
 - *Terrain*
 - “*height field*” *geometry*

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Parametric Functions

- 2D: x and y are functions of a parameter value t
- 3D: x , y , and z are functions of a parameter value t

$$C(t) := \begin{pmatrix} P_y^0 \\ P_x^0 \end{pmatrix} t + \begin{pmatrix} P_y^1 \\ P_x^1 \end{pmatrix} (1-t)$$

Line (segment)

$$C(t) := \begin{pmatrix} \cos t \\ \sin t \end{pmatrix}$$

Circle (arc)

- Depends on parameter range $t_1 < t < t_2$

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Implicit Function

- Curve (2D) or Surface (3D) defined by zero set (roots) of function
- E.g:

$$S(x, y) : x^2 + y^2 - 1 = 0$$

$$S(x, y, z) : x^2 + y^2 + z^2 - 1 = 0$$

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Lines & Segments

Segment Γ_1 **from** $P_0 = (x_0^1, y_0^1)$ **to** $P_1 = (x_1^1, y_1^1)$

A diagram showing a straight line segment connecting two points, P_0 and P_1 . The point P_0 is labeled (x_0^1, y_0^1) with a red dot at its second coordinate. The point P_1 is labeled (x_1^1, y_1^1) . Above the segment, the label Γ_1 is written.

$$G_1 = \begin{cases} x^1(t) = x_0^1 + (x_1^1 - x_0^1)t \\ y^1(t) = y_0^1 + (y_1^1 - y_0^1)t \end{cases} t \in [0,1]$$

Line through $P_0 = (x_0^1, y_0^1)$ **and** $P_1 = (x_1^1, y_1^1)$

- Parametric $G_1(t), t \in (-\infty, \infty)$
- Implicit $Ax+By+C=0$
 - Solve 2 equations in 2 unknowns (set $A^2+B^2=1$)

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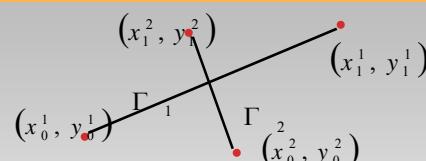


Interpolation

- Animation
 - Move object from position 1 to position 2
 - Rotate object from orientation 1 to orientation 2
- Modeling
 - Polygon = union of line segments
 - How to know if point inside?

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Line-Line Intersection



$$G_1 = \begin{cases} x^1(t) = x_0^1 + (x_1^1 - x_0^1)t \\ y^1(t) = y_0^1 + (y_1^1 - y_0^1)t \end{cases} \quad t \in [0,1] \quad G_2 = \begin{cases} x^2(r) = x_0^2 + (x_1^2 - x_0^2)r \\ y^2(r) = y_0^2 + (y_1^2 - y_0^2)r \end{cases} \quad r \in [0,1]$$

Intersection: x & y values equal in both representations -
two linear equations in two unknowns (r, t)

$$x_0^1 + (x_1^1 - x_0^1)t = x_0^2 + (x_1^2 - x_0^2)r$$

$$y_0^1 + (y_1^1 - y_0^1)t = y_0^2 + (y_1^2 - y_0^2)r$$

Question: What is the meaning of $r, t < 0$ or $r, t > 1$?

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