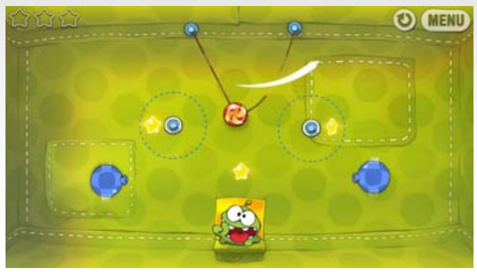
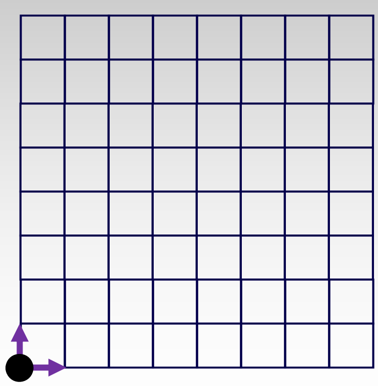
CPSC 436D
Video Game Programming

Transformations



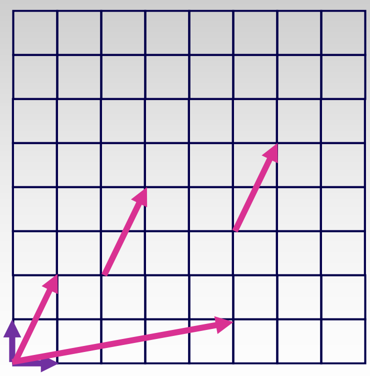
COORDINATE SYSTEMS

Coordinate system = Origin + Basis Vectors

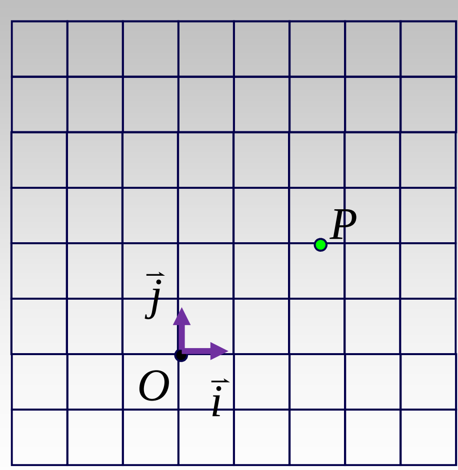


COORDINATE SYSTEMS

Coordinate system = Origin + Basis Vectors



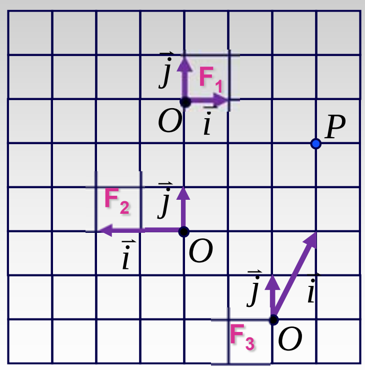
COORDINATE SYSTEMS



$$P = O + x\vec{i} + y\vec{j}$$

equivalent: $P = (x, y)$

COORDINATE SYSTEMS



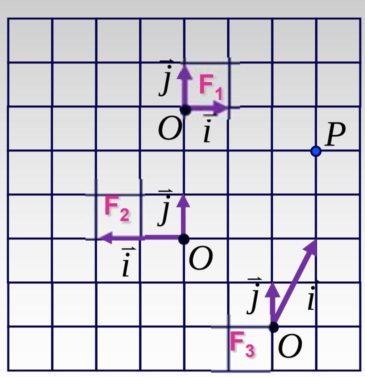
$$P = O + x\vec{i} + y\vec{j}$$

F_1

F_2

F_3

COORDINATE SYSTEMS



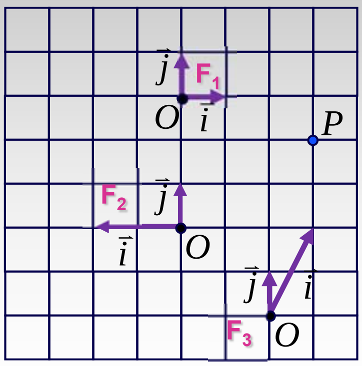
$$P = O + x\vec{i} + y\vec{j}$$

F_1 **P(3,-1)**

F_2

F_3

COORDINATE SYSTEMS



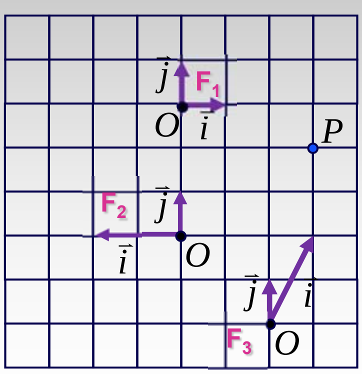
$$P = O + x\vec{i} + y\vec{j}$$

F_1 P(3,-1)

F_2 P(-1.5,2)

F_3

COORDINATE SYSTEMS



$$P = O + x\vec{i} + y\vec{j}$$

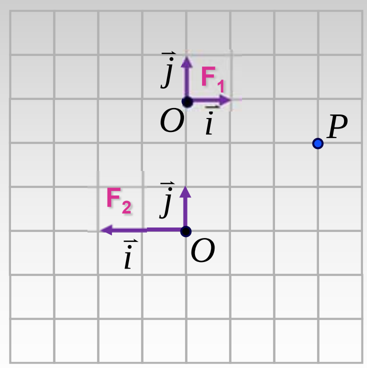
F_1 P(3,-1)

F_2 P(-1.5,2)

F_3 P(1,2)

Transformations

Transformations as a change of frame



$$P = O + x\vec{i} + y\vec{j}$$

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}_1 + x_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix}_1 + y_1 \begin{bmatrix} 0 \\ 1 \end{bmatrix}_1$$

$$\begin{bmatrix} x \\ y \end{bmatrix}_2 = \begin{bmatrix} 0 \\ 3 \end{bmatrix}_2 + x_1 \begin{bmatrix} -0.5 \\ 0 \end{bmatrix}_2 + y_1 \begin{bmatrix} 0 \\ 1 \end{bmatrix}_2$$

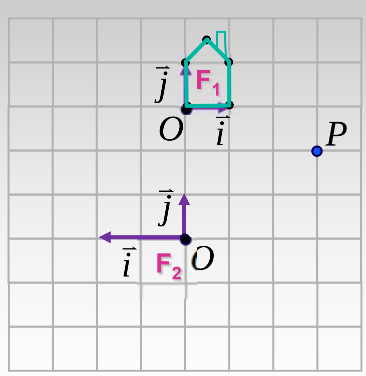
$$\begin{bmatrix} x \\ y \end{bmatrix}_2 = \begin{bmatrix} -0.5 & 0 & 0 \\ 0 & 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}_1$$

check: $P_1(3,-1)$ becomes $P_2(-1.5,2)$

$$P_2 = MP_1$$

TRANSFORMATIONS

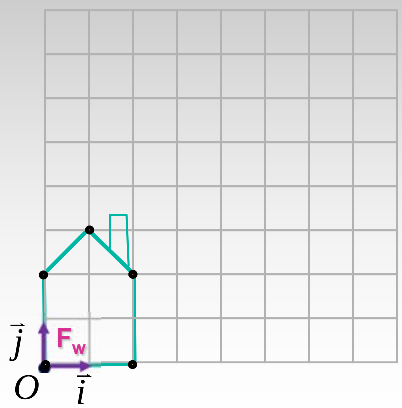
change of basis expressed using a matrix



$$\begin{bmatrix} x \\ y \end{bmatrix}_2 = \begin{bmatrix} -0.5 & 0 & 0 \\ 0 & 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}_1$$

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}_2 = \begin{bmatrix} -0.5 & 0 & 0 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}_1$$

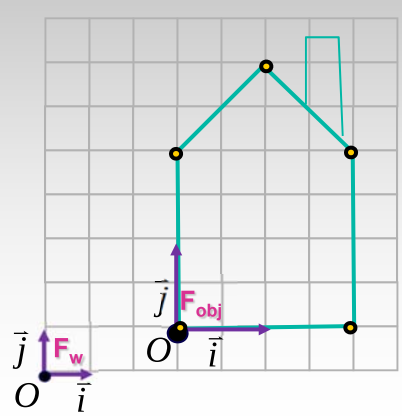
Usage of Transformations



set up the modeling matrix M

for each vertex v
v' = Mv

Usage of Transformations



$$P = O + x\vec{i} + y\vec{j}$$

$$\begin{bmatrix} x_{obj} \\ y_{obj} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}_{obj} + x_{obj} \begin{bmatrix} 1 \\ 0 \end{bmatrix}_{obj} + y_{obj} \begin{bmatrix} 0 \\ 1 \end{bmatrix}_{obj}$$

$$\begin{bmatrix} x \\ y \end{bmatrix}_w = \begin{bmatrix} 3 \\ 1 \end{bmatrix}_w + x_{obj} \begin{bmatrix} 2 \\ 0 \end{bmatrix}_w + y_{obj} \begin{bmatrix} 0 \\ 2 \end{bmatrix}_w$$

$$\begin{bmatrix} x \\ y \end{bmatrix}_w = \begin{bmatrix} 2 & 0 & 3 \\ 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}_{obj}$$

Using Transformations

$$\begin{bmatrix} x \\ y \end{bmatrix}_2 = \begin{bmatrix} 2 & 0 & 3 \\ 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}_1$$

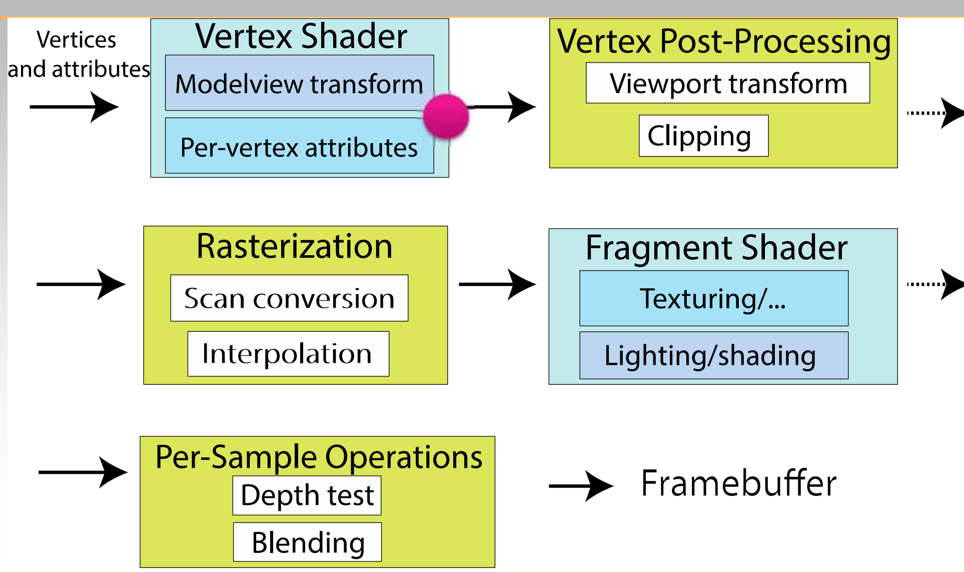
2D

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}_w = \begin{bmatrix} 2 & 0 & 3 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}_{obj}$$

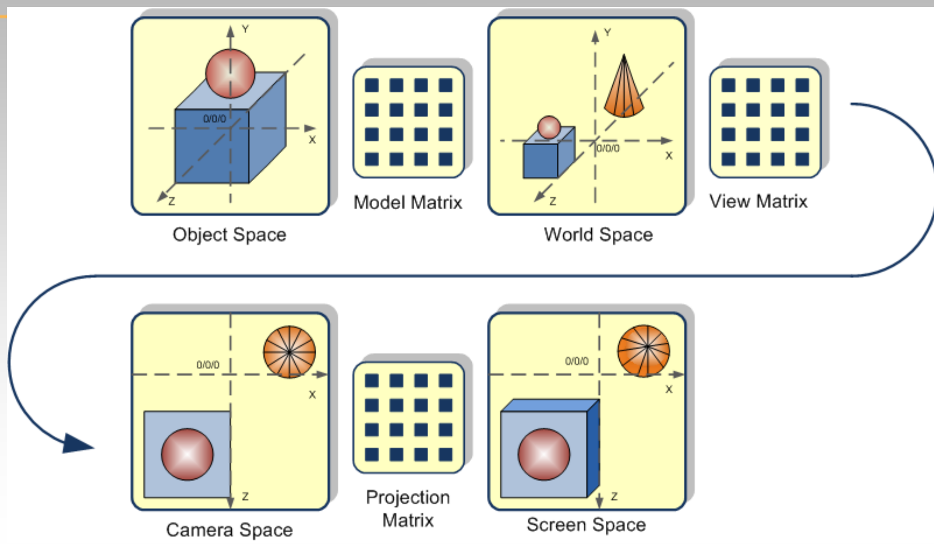
3D

$$\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}_w = \begin{bmatrix} 2 & 0 & 0 & 3 \\ 0 & 2 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}_{obj}$$

PIPELINE



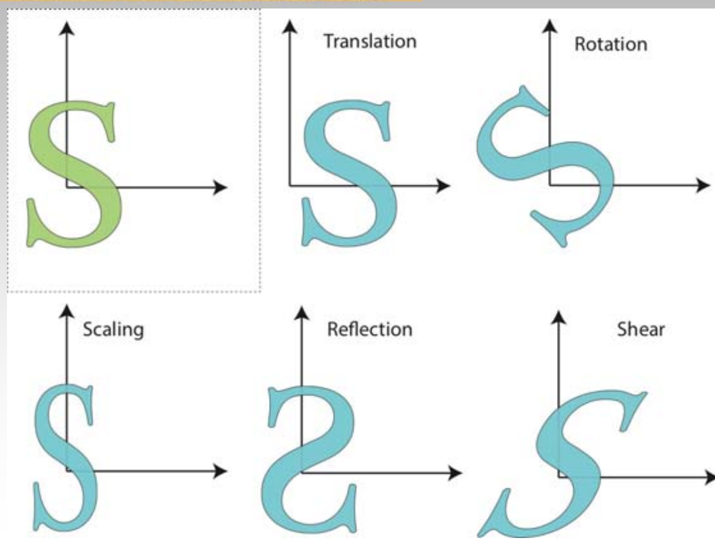
coordinate systems



HOW TO TRANSFORM COORDINATES

- Between coordinate frames
- Or animate objects

linear transformations



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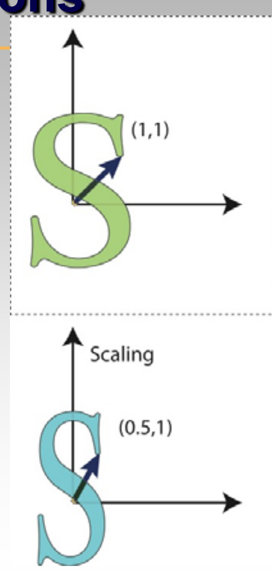
Matrix representations

Scale:



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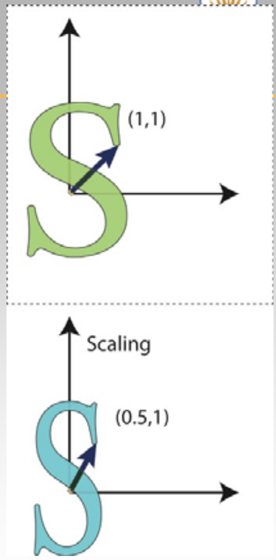
Matrix representations

Scale:

$$M = \begin{bmatrix} \alpha & 0 \\ 0 & \beta \end{bmatrix}$$

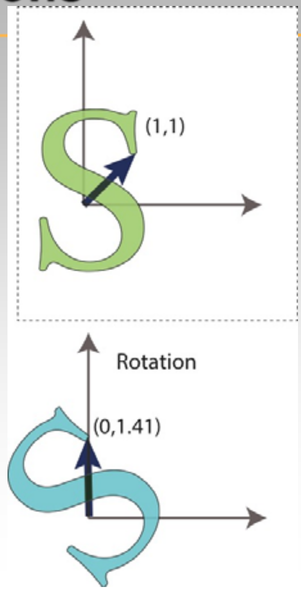
Example:

$$\begin{bmatrix} \alpha & 0 \\ 0 & \beta \end{bmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} \alpha \\ 2\beta \end{pmatrix}$$



Matrix representations

Rotation



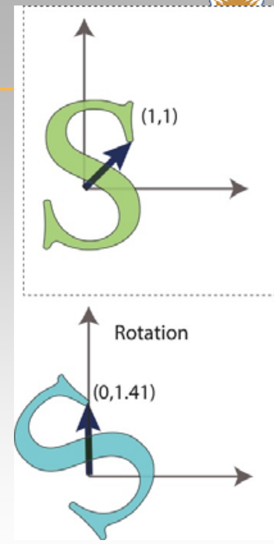
Matrix representations

Rotation

$$R(\alpha) = \begin{bmatrix} \cos(\alpha) & -\sin(\alpha) \\ \sin(\alpha) & \cos(\alpha) \end{bmatrix}$$

Example:

$$\begin{bmatrix} \cos(\alpha) & -\sin(\alpha) \\ \sin(\alpha) & \cos(\alpha) \end{bmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} \cos(\alpha) - \sin(\alpha) \\ \cos(\alpha) + \sin(\alpha) \end{pmatrix}$$



What does this 2D transformation do?

A. Rotates by 90 deg
B. Scales by a factor of 2
C. Rotates by -90 deg
D. Nothing

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

What does this 2D transformation do?

A. Rotates by 90 deg
B. Reflects the object
C. Rotates by -90 deg
D. Scales the object

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix},$$

TRANSLATION

There's a minor glitch.

- Translation: can't be represented as 2x2 matrix multiplication

general transformations

We need to represent all the linear transformations + translation.

Ideas?

$$T(v) = Mv + b$$

Unifying

- We want something more universal:

$$T(v) = Mv$$

- Then we could easily combine different transformations or invert
- Let's extend the vector to have one artificial coordinate: (x,y,1)
- Then what should the matrix be?

AUGMENTED MATRIX

$$M_{2x2} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x' \\ y' \end{bmatrix}$$

$$\begin{bmatrix} M_{2x2} & & 0 \\ & & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$$

Haven't changed much, have we?

AUGMENTED MATRIX

$$\begin{bmatrix} M_{2x2} & \begin{matrix} b_x \\ b_y \end{matrix} \\ 0 \quad 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x' + b_x \\ y' + b_y \\ 1 \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} + \boldsymbol{b}$$

Translation

Affine transformations

- Linear (rotation, scaling, shear, reflections) + TRANSLATION

Affine transformations

- Linear (rotation, scaling, shear, reflections) + TRANSLATION
- How to convert a linear transformation matrix into affine matrix?

AFFINE Transformations

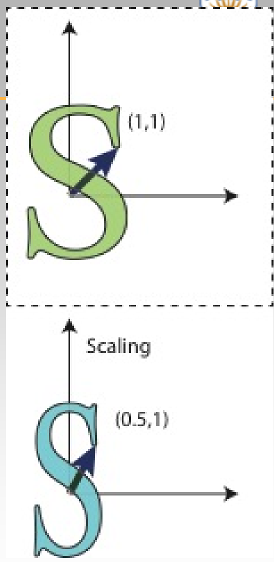
Scale:

$$M = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$$

$$M = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Example:

$$\begin{pmatrix} a \cdot 1 \\ b \cdot 2 \\ 1 \end{pmatrix} = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$$



AFFINE Transformations

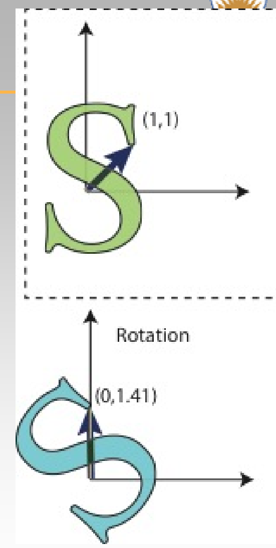
Rotation

$$M = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$$

$$M = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Example:

$$\begin{pmatrix} a \cdot 1 \\ b \cdot 2 \\ 1 \end{pmatrix} = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$$



AFFINE Transformations

Translation

$$M = \begin{bmatrix} 1 & 0 & C_x \\ 0 & 1 & C_y \\ 0 & 0 & 1 \end{bmatrix}$$

Example:

$$\begin{bmatrix} 1 & 0 & C_x \\ 0 & 1 & C_y \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} x + C_x \\ y + C_y \\ 1 \end{pmatrix}$$