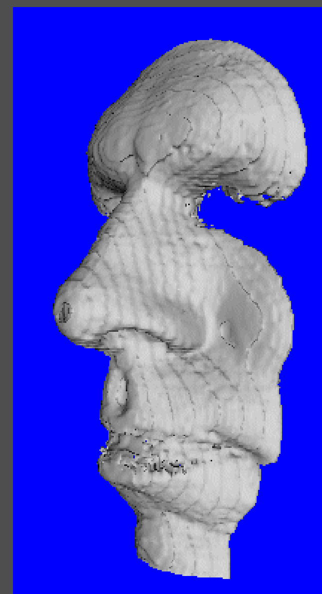
Marching Cubes

(Lorensen and Cline)



Overview

- Marching cubes: method for approximating surface defined by isovalue α, given by grid data
- Input:
  - Grid data (set of 2D images)
  - Threshold value (isovalue) α
- Output:
  - Triangulated surface that matches isovalue surface of α

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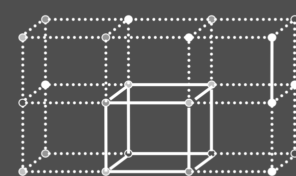
Voxels

- Voxel – cube with values at eight corners
  - Each value is above or below isovalue α
  - Method processes one voxel at a time
- $2^8=256$ possible configurations (per voxel)
  - reduced to 15 (symmetry and rotations)
- Each voxel is either:
  - Entirely inside isosurface
  - Entirely outside isosurface
  - Intersected by isosurface



imager 3

Algorithm

- First pass
  - Identify voxels which intersect isovalue
- Second pass
  - Examine those voxels
  - For each voxel produce set of triangles
    - approximate surface inside voxel

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Configurations

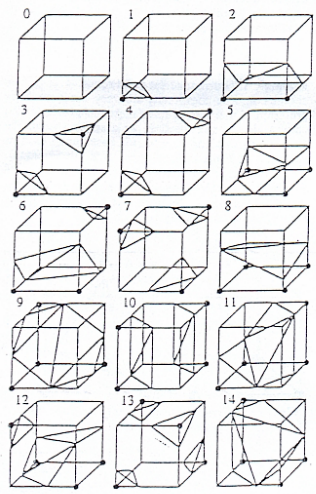
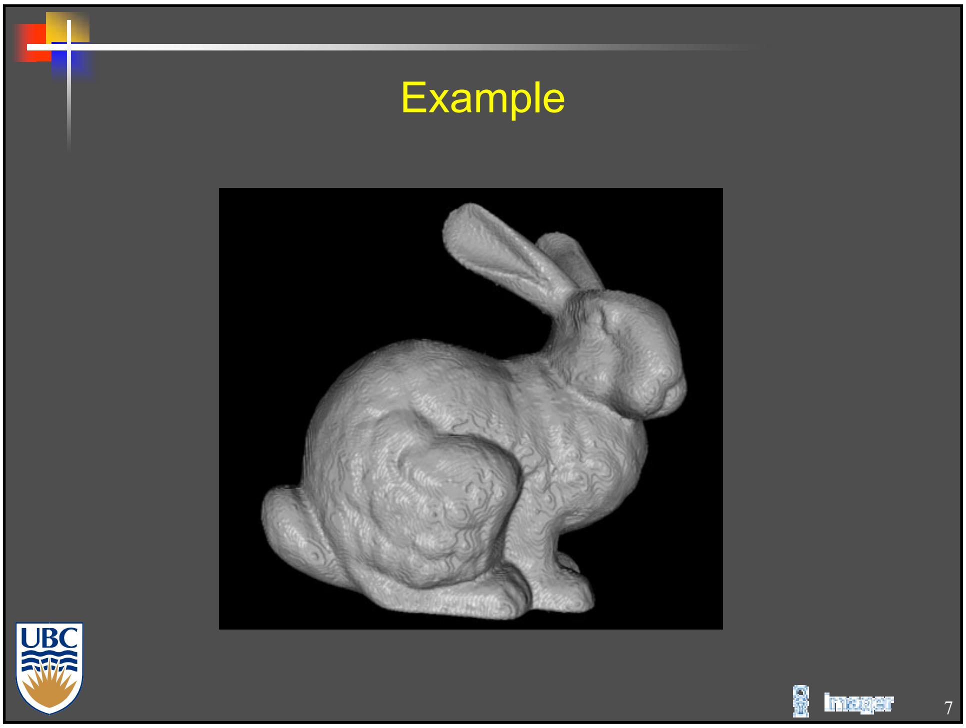


Figure 2. Configurations.

Configurations

- For each configuration add 1-4 triangles to isosurface
- Isosurface vertices computed by:
  - Interpolation along edges (according to pixel values)
    - better shading, smoother surfaces
  - Default – mid-edges


Example
UBC
imager
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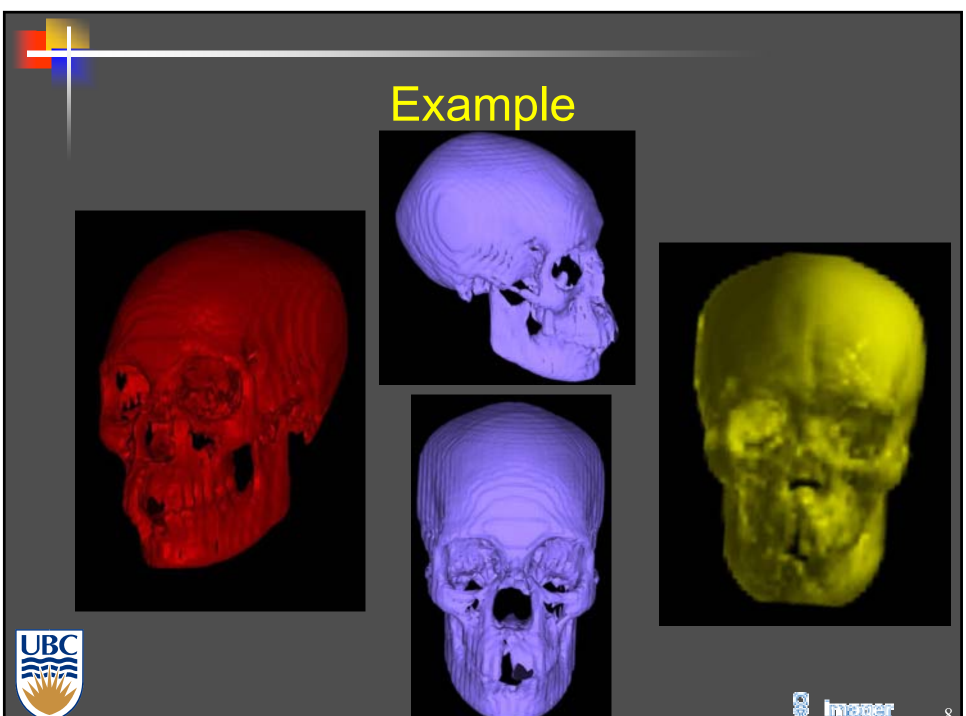
Example
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MC Problem

- Marching Cubes method can produce erroneous results
  - E.g. isovalue surfaces with “holes”
- Example:
  - voxel with configuration 6 that shares face with complement of configuration 3:

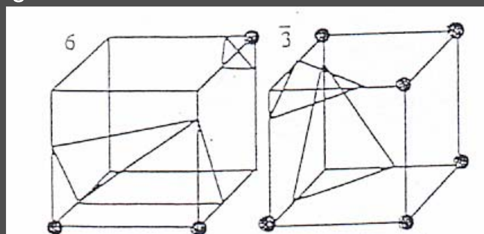

Figure 3. An example illustrating the flaw in the marching cubes method.

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Solution

- Use different triangulations
- For each problematic configuration have more than one triangulation
- Distinguish different cases by choosing pairwise connections of four vertices on common face

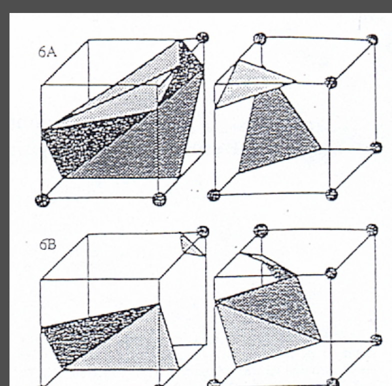

Figure 4. Two possible triangulations which yield a topologically correct isovalue surface.

2.0 Asymptotic Decider

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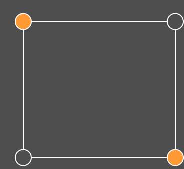
Ambiguous Face

- **Ambiguous Face**: face containing two diagonally opposite marked grid points and two unmarked ones



- Source of the problems in MC method

Solution by Consistency

- Problem:
  - Connection of isosurface points on common face done one way on one face & another way on the other
- Need consistency → use different triangulations
- If choices are consistent get topologically correct surface

Asymptotic Decider

- **Asymptotic Decider**: technique for choosing which vertices to connect on ambiguous face
- Use bilinear interpolation over ambiguous face

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Bilinear Interpolation

- Bilinear interpolation over face - natural extension of linear interpolation along an edge
- Consider face as unit square

$$B(s,t) = \begin{pmatrix} 1-s & s \end{pmatrix} \begin{pmatrix} B_{00} & B_{01} \\ B_{10} & B_{11} \end{pmatrix} \begin{pmatrix} 1-t \\ t \end{pmatrix}$$

$$\{(s,t): 0 \le s \le 1, \quad 0 \le t \le 1\}$$

- B_{ij} - values of four face corners

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Bilinear Interpolation (cont.)

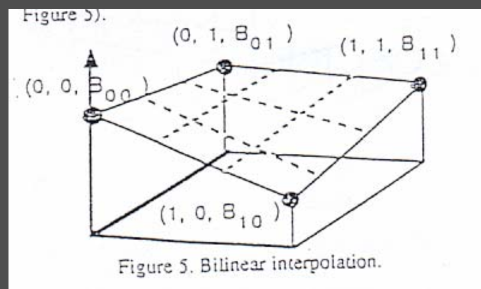

Figure 5. Bilinear interpolation.

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Hyperbolas-Domain relation

- Contour curves of B create hyperbolas

$$\{(s,t) : B(s,t) = \alpha\}$$

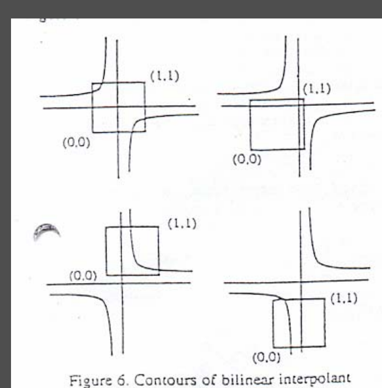

Figure 6. Contours of bilinear interpolant

Relation between hyperbolas and face

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Asymptotic Decider Test

- Ambiguous case: both components of hyperbola intersect the domain
- Criterion for connecting the vertices based upon whether they are joined by a component of the hyperbolic arc
- Selection determined by comparing contour value α

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Asymptotic Decider Test (cont).

- If $\alpha > B(S_\alpha, T_\alpha)$
  - connect $(S_1,1)$-$(1,T_1)$ & $(S_0,0)$-$(0,T_0)$
- else
  - connect $(S_1,1)$-$(0,T_0)$ and $(S_0,0)$-$(1,T_1)$

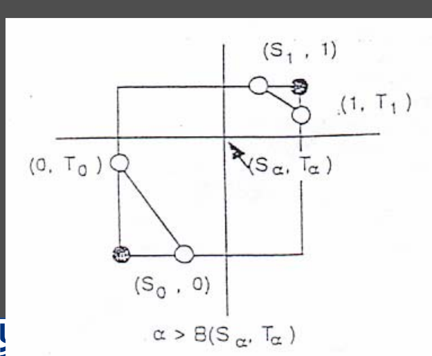

Figure 7a. Notation

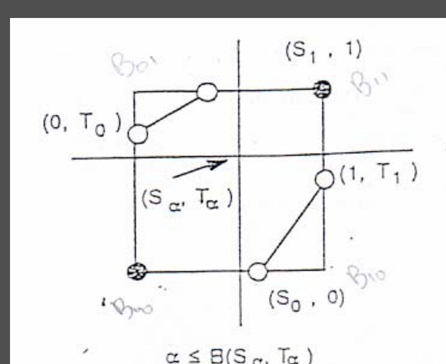

Figure 7b. Notation

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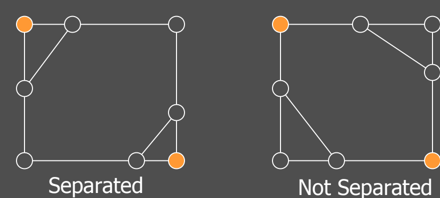
Separation

- Face is **separated** if asymptotic decider implies separation of 2 marked vertices by isovalue surface
- Otherwise, face is said to be **not separated**



Various Cases

- Configurations 0, 1, 2, 4, 5, 8, 9, 11 and 14 have no ambiguous faces → no modifications
- Other configurations need modifications according to number of ambiguous faces

Configuration 3+6

- Exactly one ambiguous face
- Two possible ways to connect vertices
  - two resulting triangulations
- Several different (valid) triangulations

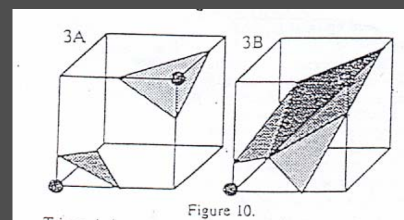


Figure 10.

Configuration 12

- Two ambiguous faces→ $2^2 = 4$ boundary polygons

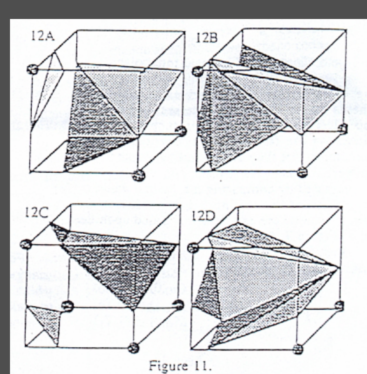


Figure 11.

Configuration 10

- As in configuration 12 - two ambiguous faces
- When both faces are separated (10A) or not separated (10C) there are two components for the isovalue surface

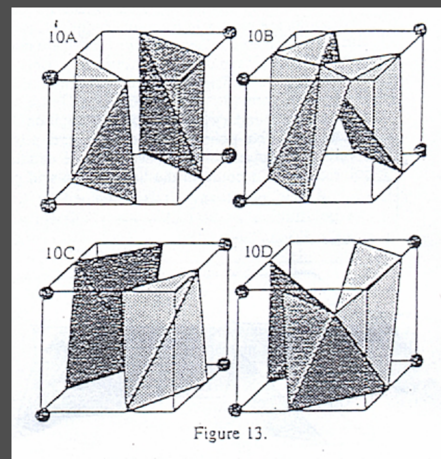


Figure 13.

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Configuration 7

- Three ambiguous faces → 2^3=8 possibilities
- Some are equivalent → only 4 triangulations

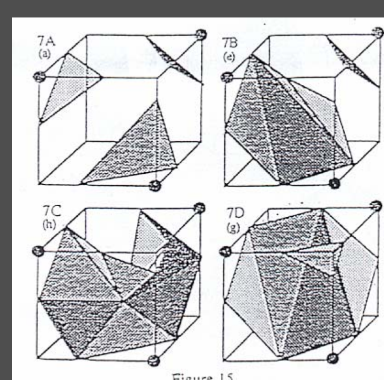

Figure 15.

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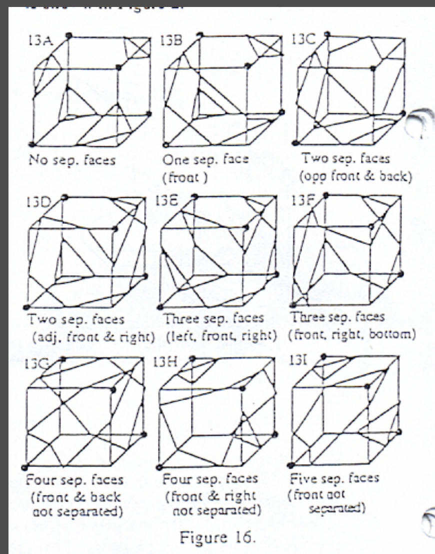
Configuration 13



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Remarks

- Modifications add considerable complexity to MC
- No significant impact on running time or total number of triangles produced
- New configurations occur in real data sets
  - But not very often

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Examples and Remarks (cont)

Config.	Example 1	Example 2	Example 3
0	263,519	285,074	110,993
1	7,705	1,912	1,673
2	8,710	2,065	2,421
3A	60	0	6
3B	46	0	6
4	28	0	0
5	5,611	1,228	1,143
6A	20	0	0
6B	47	0	0
7A	3	0	0
7B,D	3	0	0
7C	3	0	0
8	4,637	906	1,146
9	1,003	304	261
10A,C	13	0	0
10B,D	1	0	0
11	36	0	0
12A,C	7	0	0
12B,D	4	0	0
13	0	0	0
14	69	0	0

Table 1. Frequency of configurations

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