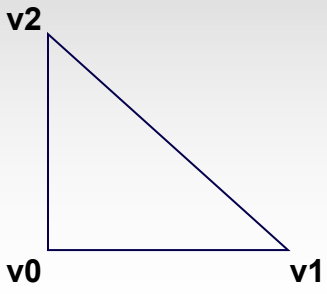
Triangles & Differential Surface Properties

Triangles

Parameterization

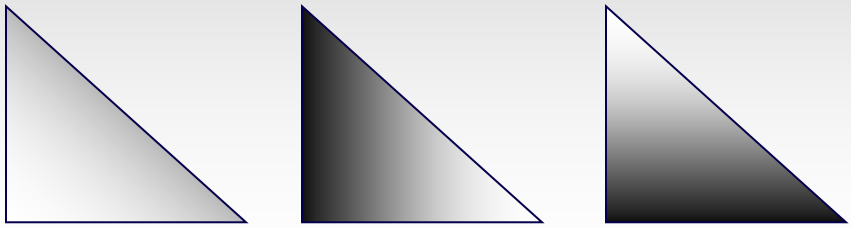
- Searching for
  - *Coordinate system over triangle*
  - *Should be symmetric in all vertices*



Triangles

Barycentric Coordinates:

$$\mathbf{p} = \alpha\mathbf{v}_0 + \beta\mathbf{v}_1 + \gamma\mathbf{v}_2; \alpha + \beta + \gamma = 1$$



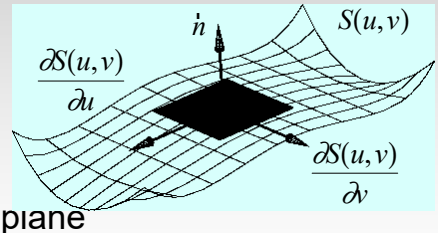
Surfaces – differential geometry

Tangent plane to surface S(u,v) is spanned by two partials of S:

$$\frac{\partial S(u,v)}{\partial u} \quad \frac{\partial S(u,v)}{\partial v}$$

Normal ***to surface***

$$\vec{n} = \frac{\partial S}{\partial u} \times \frac{\partial S}{\partial v}$$



- perpendicular to tangent plane

Any vector in tangent plane is tangential to S(u,v)

Curvature

Normal curvature ***of surface is defined for each tangential direction***

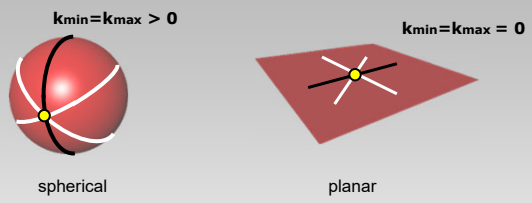
Principal curvatures **Kmin & Kmax**: ***maximum and minimum of normal curvature***

- Correspond to two **orthogonal** tangent directions
  - *Principal directions*
- Not necessarily partial derivative directions
- Independent of parameterization

3D Curvature

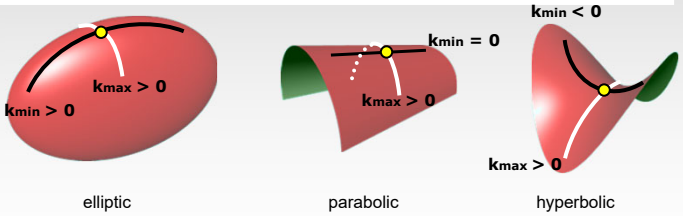
Isotropic

Equal in all directions

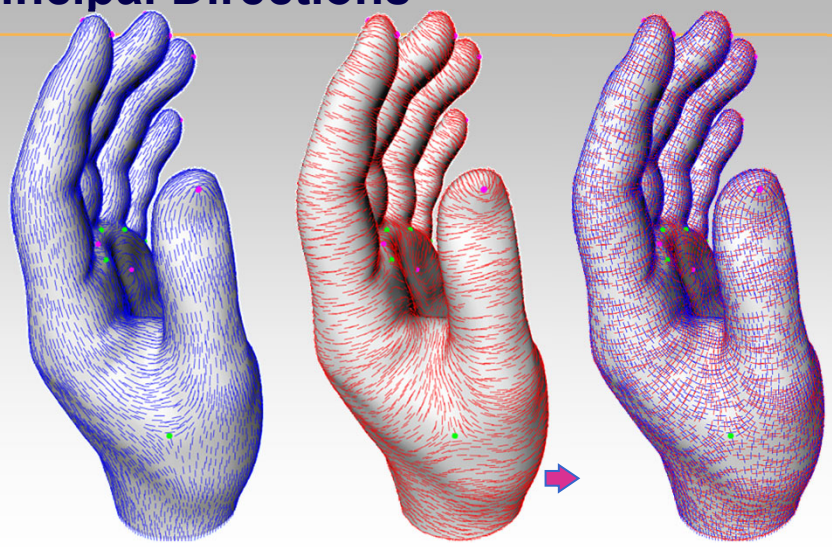


Anisotropic

2 distinct principal directions



Principal Directions



min curvature max curvature

Curvature

Typical measures:

- ***Gaussian*** curvature

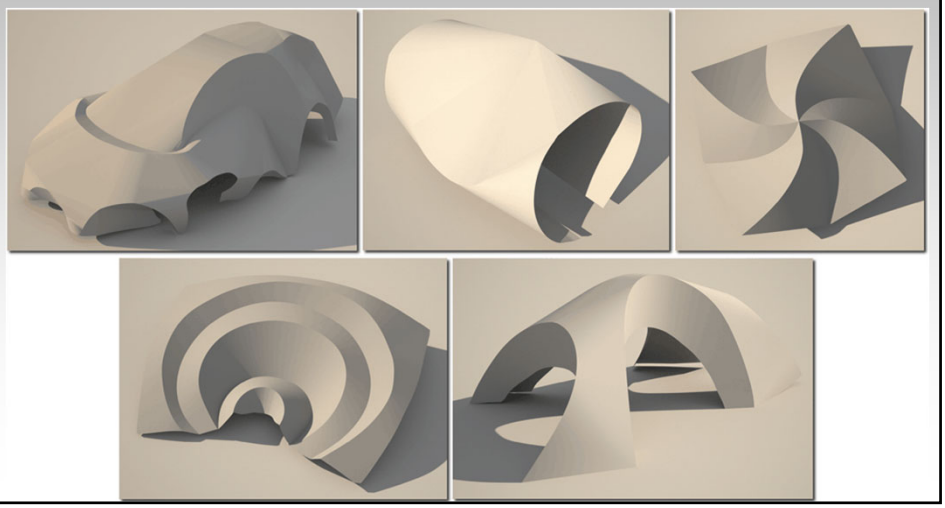
$$K = k_{\min} k_{\max}$$

- ***Mean*** curvature

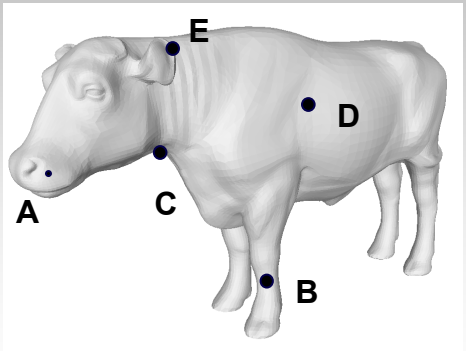
$$H = \frac{k_{\min} + k_{\max}}{2}$$

Developable Surfaces

Surfaces with Gaussian curvature K= 0



Curvature questions



Curvature Questions

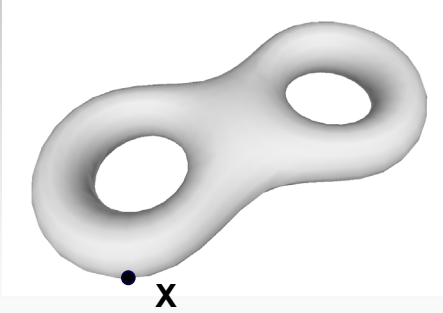
Which type of surface locally is point X?

A. Parabolic

B. Hyperbolic

C. Elliptic (non-isotropic)

D. Isotropic



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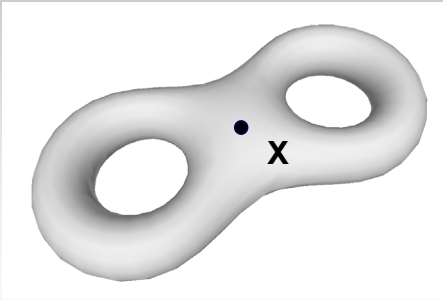

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