

# Model-Independent Online Learning for Influence Maximization

Sharan Vaswani<sup>1</sup>, Branislav Kveton<sup>2</sup>, Zheng Wen<sup>2</sup>, Mohammad Ghavamzadeh<sup>3</sup>,  
Laks Lakshmanan<sup>1</sup>, Mark Schmidt<sup>1</sup>

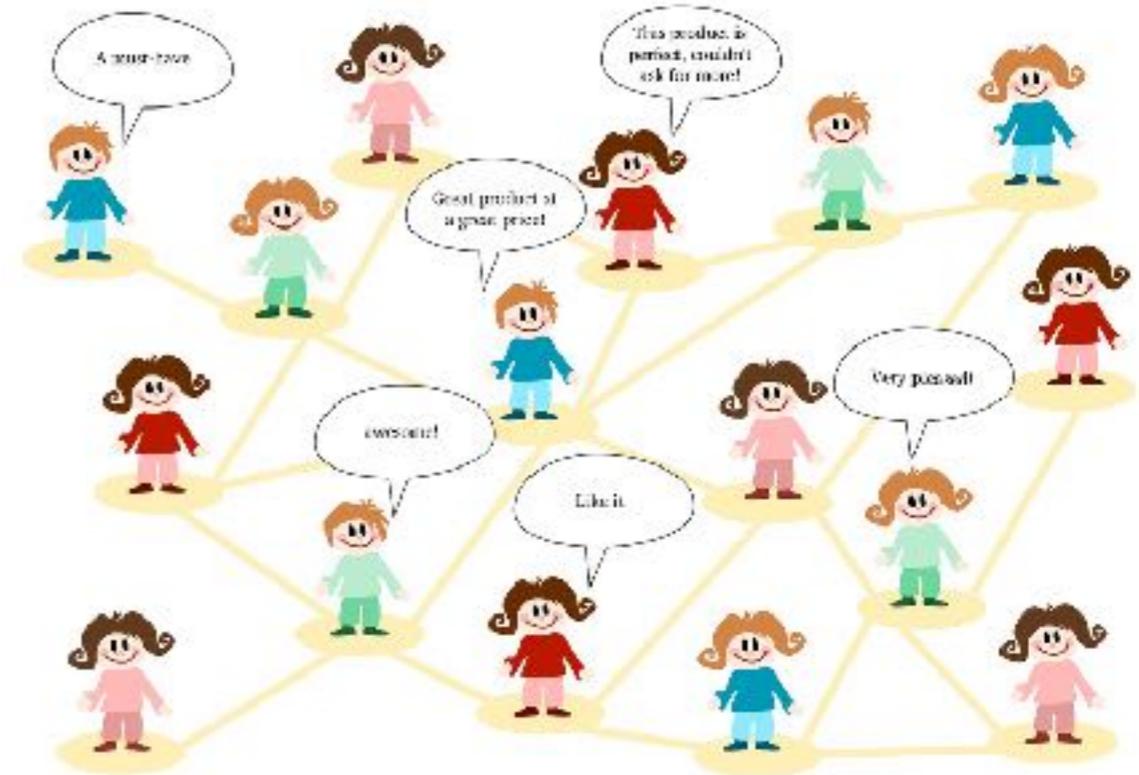
<sup>1</sup> University of British Columbia <sup>2</sup> Adobe Research <sup>3</sup> Deepmind

# Influence Maximization

**Underlying principle:** Influence propagates through ‘word of mouth’ in a social network

**Idea:** Give discounts to ‘influential’ users who will trigger off word-of-mouth epidemics

**Aim:** Find a subset of users (‘**seed set**’) who will **influence** maximum people to become aware of a product



# Influence Maximization

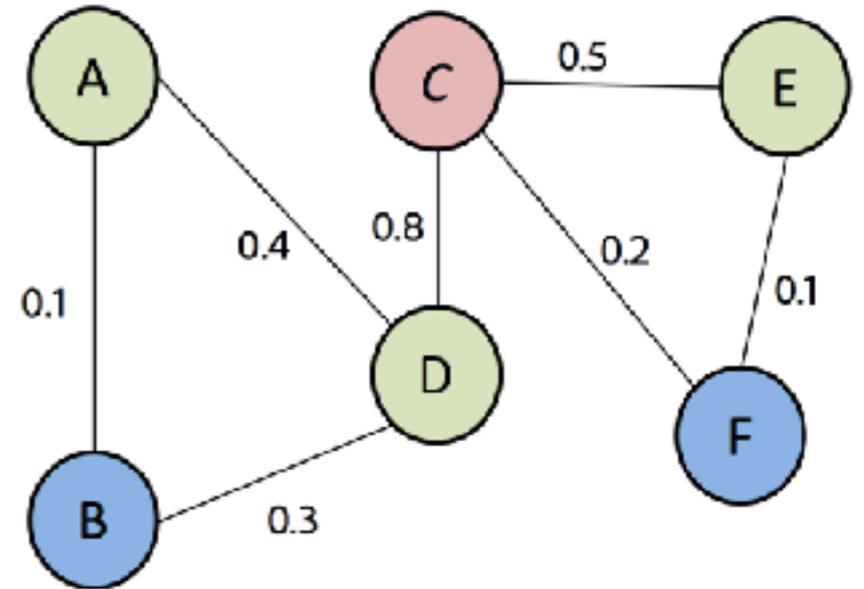
Input:  $(\mathcal{G}, \mathcal{C}, \mathcal{D})$

Weighted Graph

Stochastic diffusion Model

Set of feasible seed sets

E.g:  $\mathcal{C} \subseteq \{S \subseteq \mathcal{V} : |S| \leq K\}$



**Objective:** Find  $\mathcal{S}^* \in \arg \max_{\mathcal{S} \in \mathcal{C}} F(\mathcal{S}) = \sum_{v \in \mathcal{V}} F(\mathcal{S}, v)$

Expected number of nodes influenced by  $\mathcal{S}$

Pr ( $\mathcal{S}$  influences target node  $v$ )

**Key Property:** For common diffusion models,  $F(\mathcal{S})$  is submodular in  $\mathcal{S}$

# Influence Maximization

Challenges to using IM in practice:

- **Challenge 1:** IM is not robust to the choice of the diffusion model [1] nor its model parameters [2].
- **Challenge 2:** Learning model parameters requires considerable data, often unavailable to a new marketer.

[1] Du, Nan, et al. "Influence function learning in information diffusion networks." ICML, 2014

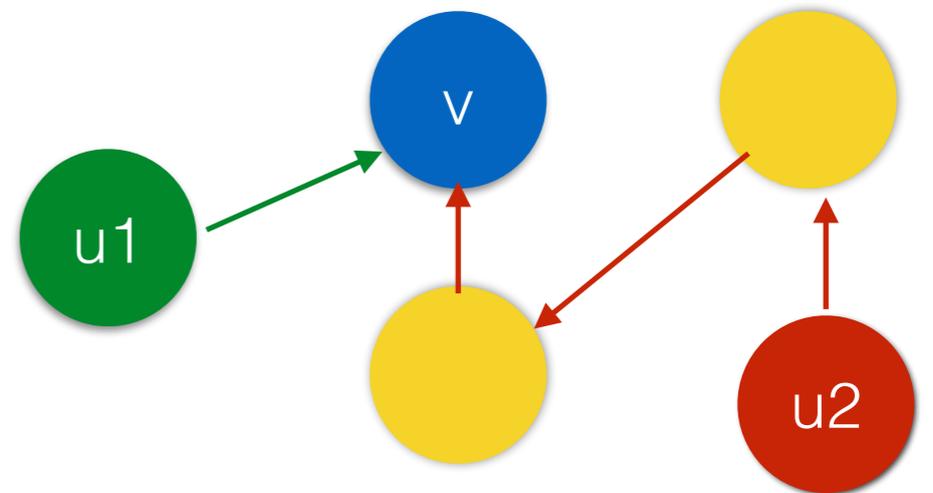
[2] Goyal, Amit, et al. "Learning influence probabilities in social networks." WSDM, 2010

# Model Independent Formulation

**Assumption 1:**  $F(S)$  is monotonic in  $S$ .

**Key Idea:** Parametrize the problem in terms of pairwise reachability probabilities  $p_{u,v}^* = F(\{u\}, v) \leftarrow \text{Pr}(u \text{ influences } v \text{ under a diffusion model})$

**Surrogate Objective:** Find  $\tilde{S} \in \arg \max_{S \in \mathcal{C}} \left[ \sum_{v \in \mathcal{V}} \left( \max_{u \in S} p_{u,v}^* \right) \right]$



## Advantages:

- Common parametrization for all progressive models.
- Guaranteed approximation.
- Surrogate objective  $f(S, p^*)$  is submodular irrespective of the diffusion model.

# Influence Maximization

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# Online Influence Maximization

**Setting:** New marketer who has no past data to learn the reachability probabilities

**Idea:** Perform IM while simultaneously learning  $p_{u,v}^*$  through trial and error across multiple rounds.

## Basic Protocol:

- for**  $t = 1$  **to**  $T$  **do** submodular optimization subroutine  
probability estimates
- Choose  $\mathcal{S}_t \leftarrow \text{ORACLE}(\mathcal{G}, \mathcal{C}, \bar{p})$
  - Diffusion occurs according to an underlying diffusion model.
  - Observe semi-bandit feedback.
- for**  $u \in \mathcal{S}_t$  **do** size n binary vector. each entry = 1 iff that node is influenced by the seed u
- Get pairwise influence feedback  $\mathbf{y}_{u,t}$
- Update parameter estimates  $\bar{p}_{u,v}$

# Online Influence Maximization

**Challenge 1:** Learn  $n^2$  reachability probabilities

**Assumption 2.** For all  $u, v \in \mathcal{V}$ ,  $p_{u,v}^*$  can be “well approximated” by the inner product of  $\theta_u^*$  and  $\mathbf{x}_v$ , i.e.,

$$p_{u,v}^* \approx \langle \theta_u^*, \mathbf{x}_v \rangle \triangleq \mathbf{x}_v^\top \theta_u^*$$

$\mathbf{x}_v$   $d$  dimensional feature describing a target node  
(Eigenbasis features, node2vec [3])

$\theta_u^*$  Vector to be learnt for every source node.

## Advantages:

- Reduces the number of parameters from  $O(n^2)$  to  $O(dn)$ .
- In each round, mean estimates of  $\bar{p}_{u,v}$  can be updated by solving  $K$  regression problems.

[3] Grover, Aditya, et al. "node2vec: Scalable feature learning for networks." KDD, 2016.

# Online Influence Maximization

**Challenge 2:** Trade off exploration and exploitation

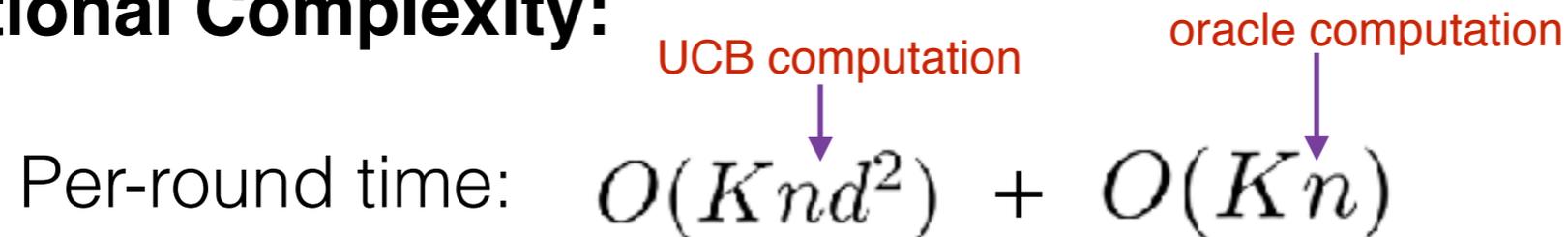
## Basic Idea:

Use the Upper Confidence Bound algorithm i.e. use overestimate (mean + variance) of reachability probabilities as input to the oracle.

## Computational Complexity:

Per-round time:  $O(Knd^2)$  +  $O(Kn)$

UCB computation  
oracle computation

The diagram shows the per-round time complexity as the sum of two terms:  $O(Knd^2)$  and  $O(Kn)$ . Above the first term, the text "UCB computation" is written in red, with a purple arrow pointing down to the  $d^2$  part of the term. Above the second term, the text "oracle computation" is written in red, with a purple arrow pointing down to the  $n$  part of the term.

# Online Influence Maximization

## Performance metric:

$$R^{\kappa}(T) = T \cdot F(\mathcal{S}^*) - \frac{1}{\kappa} \mathbb{E} \left[ \sum_{t=1}^T F(\mathcal{S}_t) \right]$$

↑ Regret after T rounds     
 ↑ Optimal seed set in hindsight     
 ↑ To account for the approximation in ORACLE     
 ↑ Selected seed set

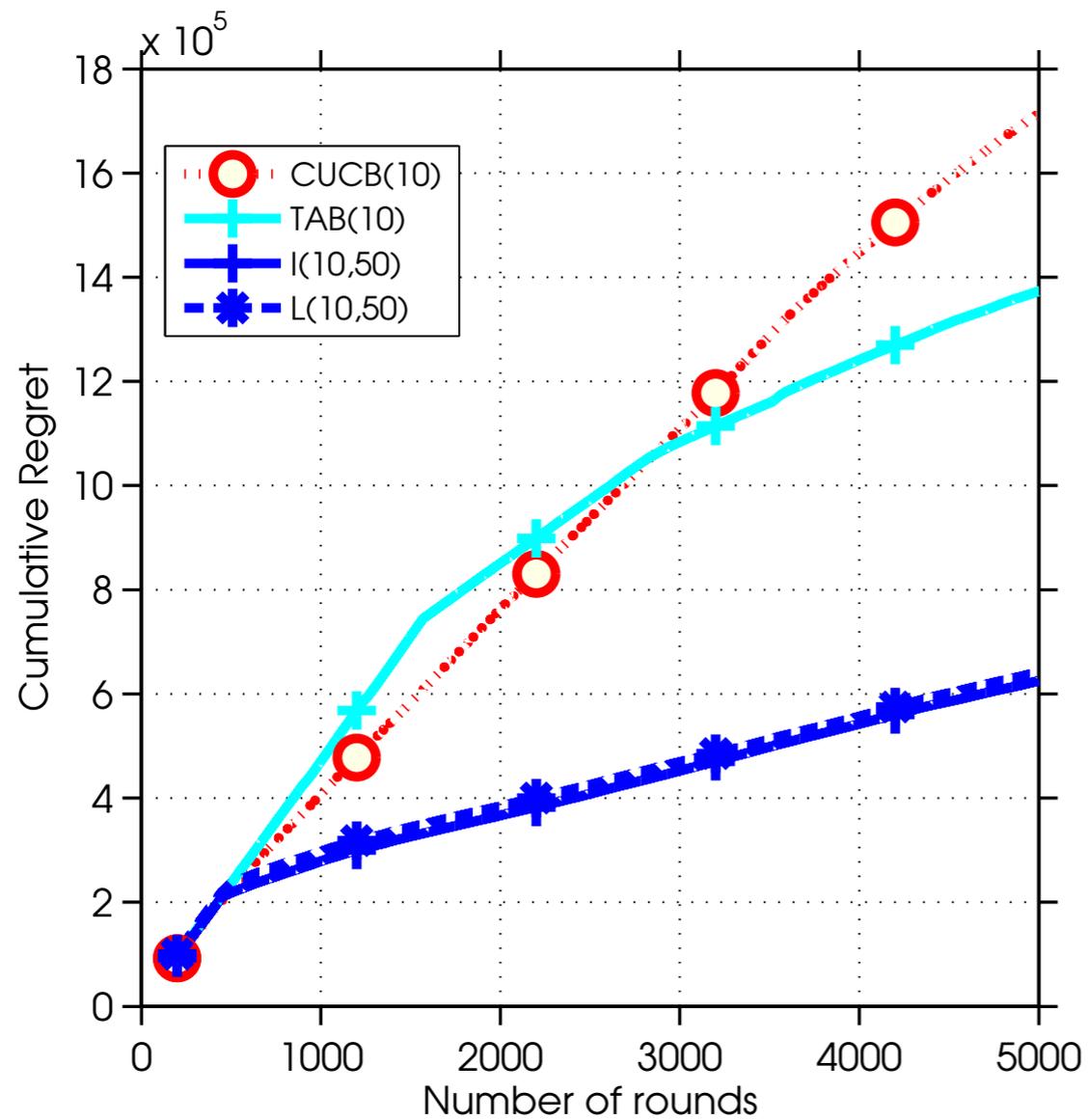
## Regret Bound:

$$R^{\rho\alpha}(T) = \tilde{O}(n^2 d \sqrt{KT} / (\alpha\rho))$$

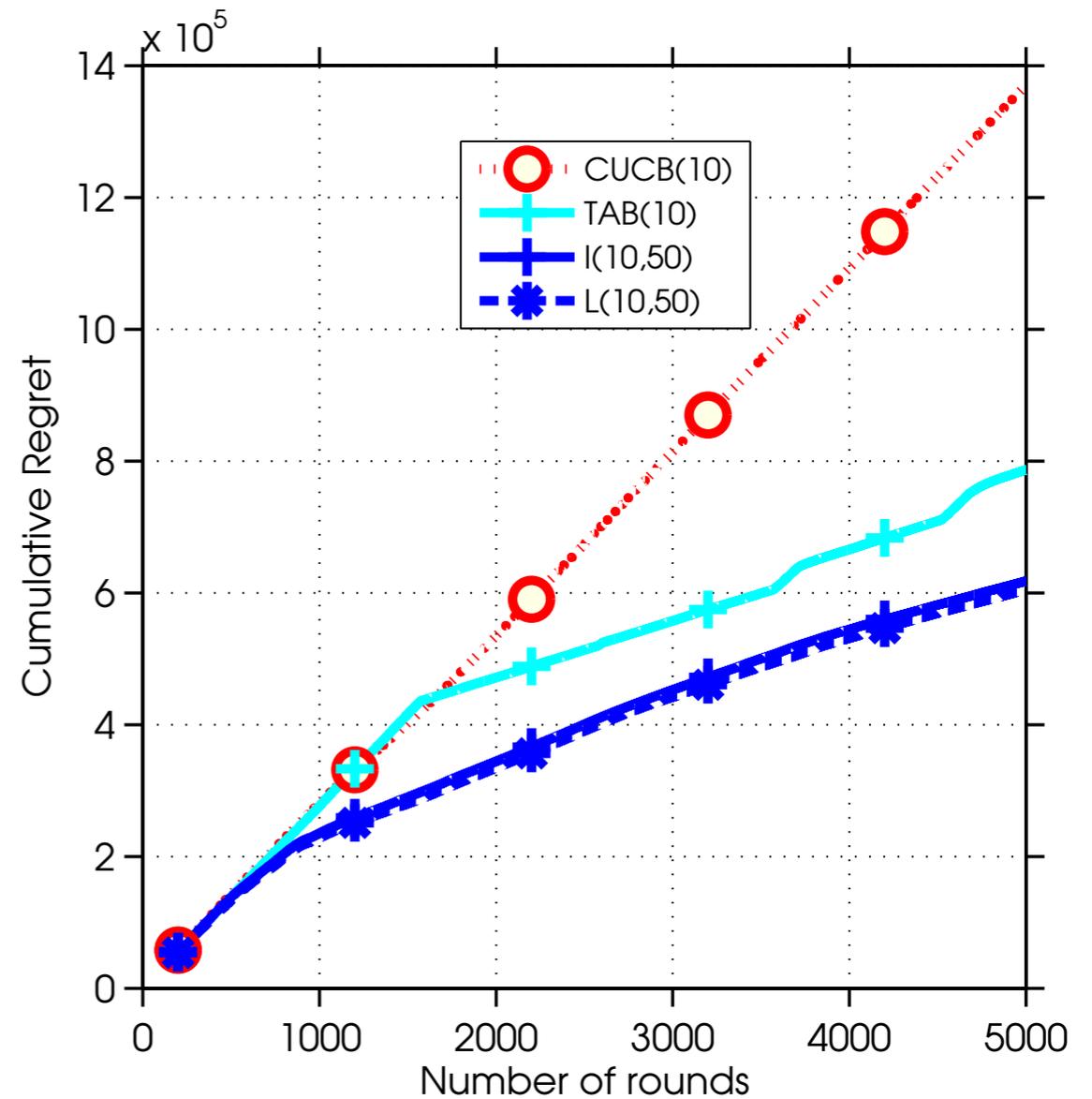
↑ best dependence on network size     
 ↑ standard linear bandit dependence     
 ↑ ORACLE approximation factor     
 ← Surrogate approximation factor

↙ standard combinatorial bandit dependence     
 ↘ near optimal dependence

# Experiments on Facebook dataset



IC model



LT model

# Conclusion

## **Contributions:**

- Developed a model-independent parametrization for IM and proposed a surrogate objective function.
- Proposed and analyzed a UCB based algorithm for model-independent online IM.

## **Future Work:**

- Extend the framework to different feedback models and bandit algorithms.
- Generalization across source nodes for better statistical efficiency.