Particle Filtering: An Overview

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SFU Machine Learning Reading Group

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What's in a name?

Particle Filtering, a.k.a:

- Sequential Monte Carlo (SMC)
- Bootstrap filtering
- Condensation
- Interacting Particle Approximations
- Survival of the Fittest
- Genetic Algorithms(?)

Introduction to Particle Filtering

To the videos...

Outline

1. Introduction: What is particle filtering?

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2. Particle Filters and Hidden Markov Models

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4. Re-sampling Particles
5. Particle Filters & Genetic Algorithms

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Setting Up Particle Filtering

Notations:

- Hidden States $\mathbf{x} \in \mathcal{X}$
- Observations $\mathbf{z} \in \mathcal{Z}$

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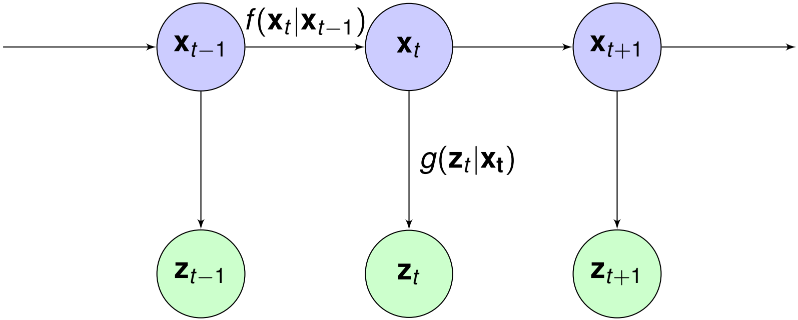
Find:

- Full trajectory distribution $p(\mathbf{x}_{0:n}|\mathbf{z}_{1:n})$

Assuming a Hidden Markov Model



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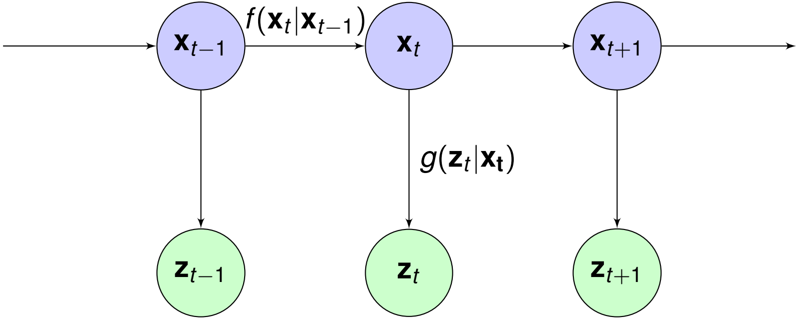

- States (in blue) are hidden
- Observations (in green) are known
- Relationships are “roughly” known

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Remember our goal:

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The joint probabilities can be written in conditional form:

$$\begin{aligned} p(\mathbf{x}_{0:n}, \mathbf{z}_{1:n}) =& p(\mathbf{x}_{0:n-1}, \mathbf{z}_{1:n-1}) f(\mathbf{x}_n|\mathbf{x}_{n-1}) g(\mathbf{z}_n|\mathbf{x}_n) \\ p(\mathbf{z}_{1:n}) =& p(\mathbf{z}_n|\mathbf{z}_{1:n-1}) p(\mathbf{z}_{1:n-1}) \end{aligned}$$

Bayes Rule just keeps giving

Plugging in known $f(\cdot)$ and $g(\cdot)$:

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The $p(\mathbf{z}_{1:n-1})$ distributions cancel out...

Wrapping up Bayes Rule

Thus:

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We know from normalization rules that:

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Let C_n be the integral above. Then...

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...we get:

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- Distributions $f(\cdot)$ and $g(\cdot)$ are also known
- We have everything we need to solve this problem!

What's stopping us?

In some cases, we can get $p(\mathbf{x}_{0:n}|\mathbf{z}_{1:n})$ analytically:

- e.g. Kalman filters
  [$p(\mathbf{x}_0)$ is Gaussian, $f(\mathbf{x}_t|\mathbf{x}_{t-1})$ and $g(\mathbf{z}_t|\mathbf{x}_t)$ are linear]

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The Solution:

- Estimate $p(\mathbf{x}_{0:n}|\mathbf{z}_{1:n})$ from samples (i.e. particles).

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Monte Carlo Estimates

Traditional Monte Carlo:

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- Base case $p(\mathbf{x}_0)$ may not be exactly known

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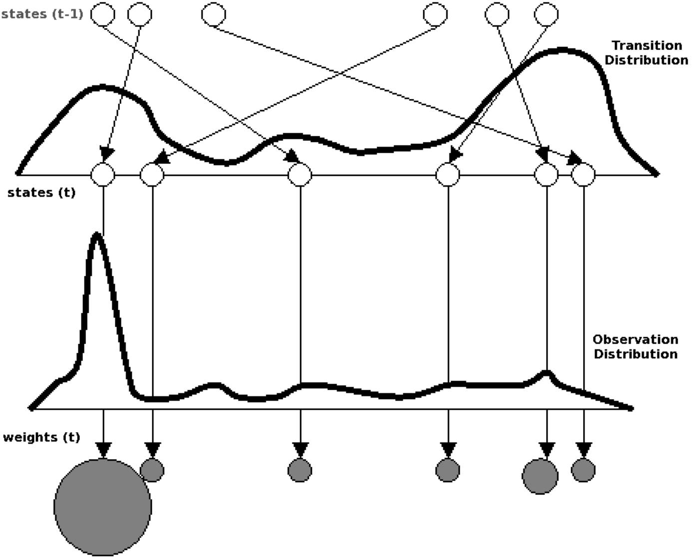
SIS in Image Form



Particle Filtering using SIS

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The "gotchas":

- Doesn't work well in high-dimensions (need large N).

Use SIS with Caution

The “gotchas”:

- Doesn’t work well in high-dimensions (need large *N*).
- Degeneracy: After a few timesteps, all but one particle will have negligible weight

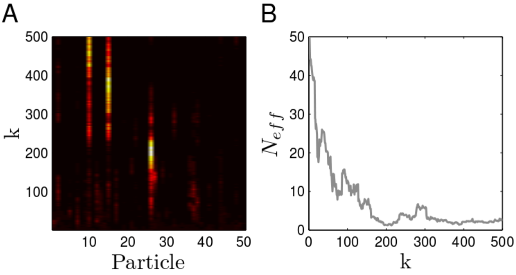

Figure 2: (A) The weights of all 50 particles (x-axis) at each time step k (y-axis). Hotter colors represent larger weights. (B) The effective sample size N_{eff} as a function of time step k.

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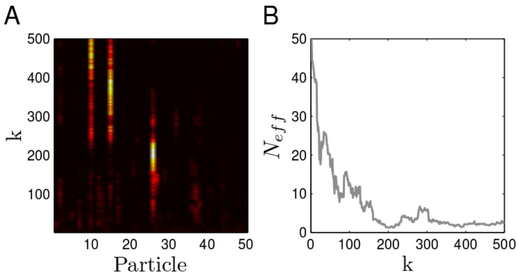
Addressing Degeneracy

Detecting degeneracy:

- At each timestep t, measure effective sample size

$$N_{eff} = \frac{1}{\sum_{i=1}^{N}(w^i)^2}$$



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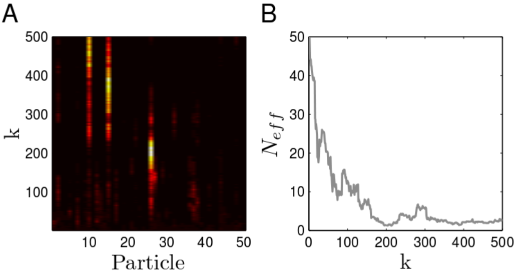
Resampling Particles

If $N_{eff} < \tau$ at timestep t:

- Sample N new particles $\{x_t^1, \cdots, x_t^N\}$ from $\hat{p}(\mathbf{x}_{0:t}|\mathbf{z}_{1:t})$
- Reset weights: $w^i = \frac{1}{N}$



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Sequential Importance Resampling (SIR)

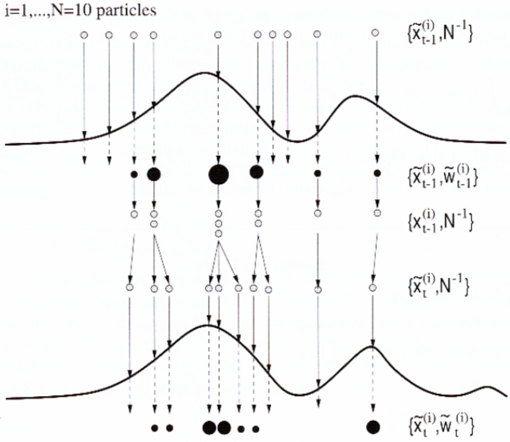
unweighted measure

compute importance weights $\Rightarrow p(x_{t-1}|z_{1:t-1})$

resampling

move particles

predict $p(x_t|z_{1:t-1})$



Pros and Cons of SIR Particle Filtering

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- No assumptions on distributions
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Cons:

- Degeneracy possible
- Good estimates may need large N
- Computationally expensive

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More General Particle Filtering

Consider optimization problems:

$$x^* = \arg\min_x E(x)$$

Often solved with Euler-Lagrange

- Introduce artificial timestep t
- Compute $\frac{\partial x}{\partial t}$
- Update using gradient descent $x_t = x_{t-1} - \delta t \frac{\partial x}{\partial t}$

Looks similar...

Particle Filtering Optimization

Particle Filtering Optimization in 7 Basic Steps:

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7. Select x_n^i with largest weight w^i as x^*.

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Particle filtering is for more than HMM problems.