Non-Uniform Stochastic Average Gradient Method for Training Conditional Random Fields

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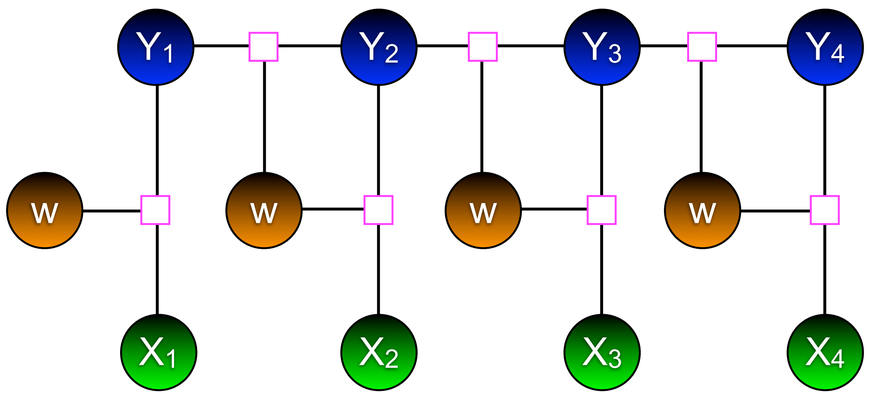
Motivation and Overview of Contribution

- Conditional random fields (CRFs) are a ubiquitous tool for structured prediction:
  - Allow the use of a high-dimensional feature set.
  - Formulated as convex optimization problem.
  - But very slow to train.
- Stochastic average gradient (SAG) methods are a new strategy for convex optimization:
  - Only look at a single training example on each iteration, like stochastic gradient methods.
  - Linear convergence rate similar to methods that process the entire dataset on every iteration.
- Our contribution is applying SAG with non-uniform sampling (NUS) to CRFs:
  - We show how to reduce the memory requirements using structure in the gradients.
  - We propose a practical NUS scheme that substantially improves empirical performance.
  - We analyze the rate of convergence of the SAGA variant under non-uniform sampling.
- SAG with NUS often outperforms existing methods for training CRFs.

Conditional Random Fields (CRFs)



- CRFs model probability of output $y \in \mathcal{Y}$ given input $x \in \mathcal{X}$ and features $F(x,y)$ using

$$p(y|x,w) = \frac{\exp(w^T F(x,y))}{\sum_{y'} \exp(w^T F(x,y'))}.$$

- Given training examples $\{x_i, y_i\}$, standard approach is minimizing ℓ_2-regularized NLL:

$$\min_w f(w) = \frac{1}{n}\sum_{i=1}^{n} -\log p(y_i|x_i,w) + \frac{\lambda}{2}\|w\|^2.$$

- Evaluating each $\log p(y_i|x_i,w)$ is expensive due to sum over y'.

Related Work on Deterministic, Stochastic, and Hybrid Methods

- Deterministic gradient methods like L-BFGS [Wallach 2002, Sha & Pereira, 2003]:
  - Require $O(\log(1/\epsilon))$ iterations but n gradient evaluations per iteration.
- Stochastic gradient methods [Vishwanathan et al., 2006, Finkel et al., 2008]:
  - Require $O(1/\epsilon)$ iterations but only 1 gradient evaluation per iteration.
- Online exponentiated gradient [Collins et al., 2008]:
  - Requires $O(\log(1/\epsilon))$ iterations in terms of dual and 1 dual gradient evaluation per iteration.
- Hybrid deterministic-stochastic [Friedlander & Schmidt, 2012]:
  - Requires $O(\log(1/\epsilon))$ iterations and growing number of gradient evaluations per iteration.

Stochastic Average Gradient (SAG) for CRFs

- Stochastic average gradient [LeRoux, et al. 2012]:
  - Requires $O(\log(1/\epsilon))$ iterations and 1 gradient evaluation per iteration.
- SAG uses the iteration

$$w^{t+1} = w^t - \frac{\alpha}{n}\sum_{i=1}^{n} s_i^t,$$

  where we set $s_i^t = -\nabla \log p(y_i|x_i,w^t) + \lambda w^t$ for one randomly-chosen training example.
- Challenge is the memory required for storing the s_i^t:
  - $\nabla \log p(y_i|x_i,w^t)$ often sparse but depends on number of features
  - λw^t is typically dense.
- Implementation issues for CRFs:
  - Sparse trick 1: to avoid storing λw^t use the exact gradient of the regularizer,

$$w^{t+1} = (1-\alpha\lambda)w^t - \frac{\alpha}{n}\sum_{i=1}^{n} g_i^t,$$

    where we set $g_i^t = -\nabla \log p(y_i|x_i,w^t)$ for one randomly-chosen example.
  - Sparse trick 2: use the representation $w^t = \beta^t v^t$ and 'lazy updates' to avoid dense vector operations.
  - Step size: we set $\alpha = 1/L$ with $L = L_g + \lambda$, and double approximation L_g when

$$f_i(w - (1/L)g_i) > f_i(w) - \frac{1}{2L_g}\|g_i\|^2, \quad (1)$$

    but we multiply L_g by $2^{-1/n}$ after each iteration to slowly increase step size.
  - Convergence: we can stop if $\|\lambda w^t + \frac{1}{n}\sum_{i=1}^{n} g_i^t\|$ is sufficiently small.
  - Reducing the memory: for 'part-based' features, $F_j(x,y) = F(x)\mathbb{I}[y_k = s]$, the gradient has the form

$$\nabla_j \log p(y|x,w) = F(x)(\mathbb{I}[y_k = s] - p(y_k = s|x,w)).$$

    and SAG update depends on differences in gradients,

$$\nabla_j \log p(y|x,w) - \nabla_j \log p(y|x,w_{\text{old}}) = F(x)(p(y_k = s|x,w_{\text{old}}) - p(y_k = s|x,w)),$$

    so we only need to store marginals $p(y_k = s|x,w)$, that are shared across features that depend on $y_k = s$.

SAG with Practical Non-Uniform Sampling (NUS) Strategy

- Assume each gradient has its own Lipschitz constant L_i, a value such that

$$\|\nabla f_i(w) - \nabla f_i(v)\| \leq L_i\|w - v\|, \forall w, v,$$

  bounding how fast gradient i can change.
- Key idea behind NUS: bias sampling probability p_i towards Lipschitz constant L_i:
  - Gradients that can change more quickly get updated more often.
  - Convergence rate depends on $\bar{L}$ =mean(L_i) instead of L =max(L_i).
- Practical 'partially-based' strategy:
  - With probability 1/2 choose i uniformly.
  - With probability 1/2 sample i with probability $L_i/\sum_j L_j$.
  - Use a larger step-size of $\alpha = \frac{1}{2}(1/L + 1/\bar{L})$.
  - Initialize with $L_i = \bar{L}$ the first time an example is chosen.
  - Each time i is chosen, set $L_i = 0.9L_i$ then double it while (1) holds.
  - If (1) holds ξ times (without backtracking), do not change L_i for $2^{\xi-1}$ next times i is sampled.
  - Code: `http://www.cs.ubc.ca/~schmidtm/Software/SAG4CRF.html`.

Convergence Analysis for SAGA with Non-Uniform Sampling

- We analyze a NUS extension of SAGA, which has similar performance but easier analysis.
- Let the sequences $\{w^t\}$ and $\{s_j^t\}$ be defined by

$$w^{t+1} = w^t - \alpha\left[\frac{1}{np_{j_t}}(\nabla f_{j_t}(w^t) - s_{j_t}^t) + \frac{1}{n}\sum_{i=1}^{n} s_i^t\right],$$

$$s_j^{t+1} = \begin{cases} \nabla f_{r_t}(w^t) & \text{if } j = r_t, \\ s_j^t & \text{otherwise.} \end{cases}$$

  where j_t is chosen with probability p_j.
  - (a) If r_t is set to j_t, then with $\alpha = \frac{np_{\min}}{4L+n\mu}$ we have

$$\mathbb{E}[\|w^t - w^*\|^2] \leq (1-\mu\alpha)^t\left[\|x^0 - x^*\| + C_a\right],$$

    where $p_{\min} = \min_i\{p_i\}$ and

$$C_a = \frac{2p_{\min}}{(4L+n\mu)^2}\sum_{i=1}^{n}\frac{1}{p_i}\|\nabla f_i(x^0) - \nabla f_i(x^*)\|^2.$$

  - (b) If $p_j = \frac{L_j}{\sum_{i=1}^{n} L_i}$ and r_t is chosen uniformly at random, then with $\alpha = \frac{1}{4L}$ we have

$$\mathbb{E}[\|w^t - w^*\|^2] \leq \left(1 - \min\left\{\frac{1}{3n}, \frac{\mu}{8\bar{L}}\right\}\right)^t\left[\|x^0 - x^*\| + C_b\right],$$

    where:

$$C_b = \frac{n}{2\bar{L}}\left[f(x^0) - f(x^*)\right]$$

- (a) SAGA has a linear convergence rate whenever $p_i > 0$ for all i.
- (b) SAGA has a faster rate with p_i proportional to L_i and generating a uniform sample.

Experiment Results

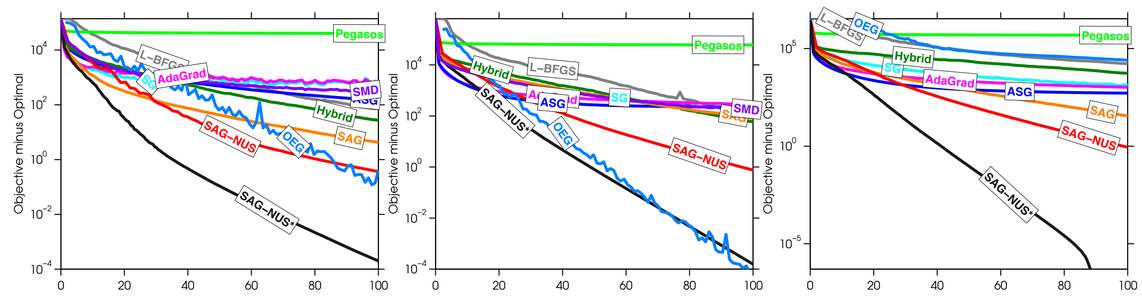


Figure: Training objective sub-optimality against effective number of passes for OCR, CONLL-2000, POS.

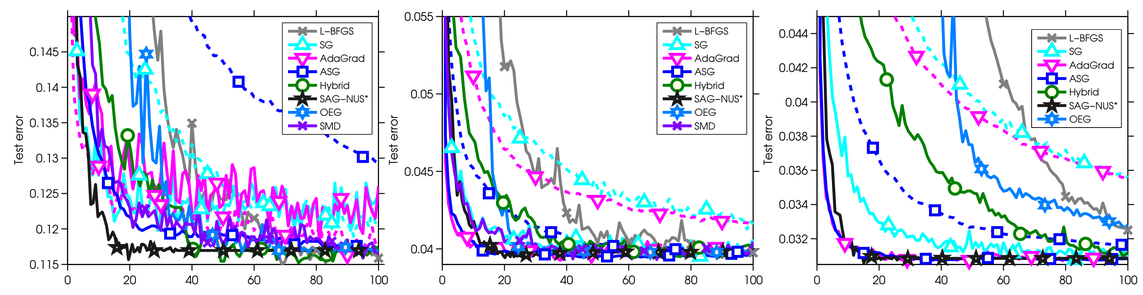


Figure: Test error against effective number of passes, for OCR, CONLL-2000, POS. (Dashed lines are stochastic method with sub-optimal step-size.

Discussion

- If memory requirements prohibitive, use mini-batches or SVRG [Johnson & Zhang, 2013].
- Could use ℓ_1-regularization with proximal versions [Defazio et al., 2014].
- Algorithms applies to any graph structure and approximate inference could be used.
- Could use multi-core computation and distributed parallel implementations.