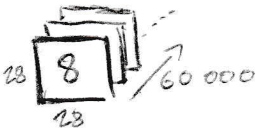
A4 Q2.3

$X \in \{0,1\}^{28 \times 28 \times 60000}$



1. Create Inhomogeneous markov chain for each column.

For each column j:

you need - initial probability p_1^j : probability the top pixel in the colum is a 1 ($p(x_1^j)$)

· estimate using the fraction of images where column j starts with a 1

- transition probabilities for $i = 2:28$

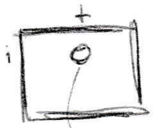
$p(x_i^j = 1 \mid x_{i-1}^j = 1)$: probability pixel i is a 1 given pixel $i-1$ is a 1

$p(x_i^j = 1 \mid x_{i-1}^j = 0)$: probability pixel i is a 1 given pixel $i-1$ is a 0

· again estimate using counts

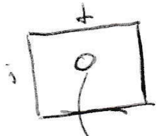
Can store transition probabilities in $28 \times 28 \times 2$ tensor

$P_T(:,:,1)$



$p(x_i^j = 1 \mid x_{i-1}^j = 1)$

$P_T(:,:,2)$



$p(x_i^j = 1 \mid x_{i-1}^j = 0)$

when sampling

if $x_{i-1}^j = 1$

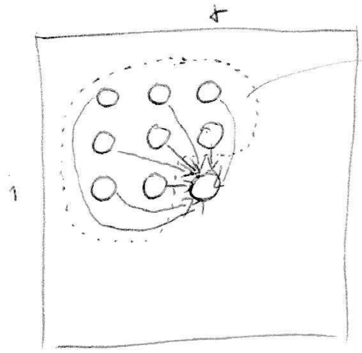
sample x_i^j according to $P_T(i,j,1)$

if $x_{i-1}^j = 0$

sample x_i^j according to $P_T(i,j,2)$

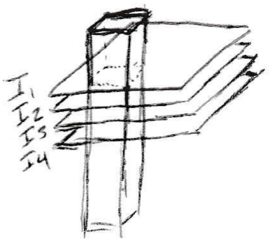
2.

For each (i,j) in each image we create a DAG



Let's call these the parents $P_1 \dots P_N$

for each pixel (i,j) get the set of parents in each image



$$\longrightarrow \tilde{X}_{ij} = \begin{array}{c} \\ I_1 \\ I_2 \\ I_3 \\ \vdots \end{array} \begin{array}{c} P_1 \; P_2 \; \cdots \qquad P_N \\ \begin{bmatrix} 0 & 0 & 1 & 0 & \cdots \\ 0 & 1 & 0 & \cdots & \\ 1 & 0 & \cdots & & \\ & & & & \end{bmatrix} \end{array}$$

and the value of pixel (i,j) in each image

$$Y_{ij} = \begin{array}{c} I_1 \\ I_2 \\ I_3 \\ \vdots \end{array} \begin{bmatrix} 1 \\ 1 \\ 0 \\ \\ \end{bmatrix}$$

models$\{i,j\}$ = binaryTabular$(\tilde{X}_{ij}, Y_{ij}, \alpha = 1)$

When sampling

for each (i,j)

get values of parents

$p = [p_1 \; p_2 \; \cdots \; p_n]$

m = models$\{i,j\}$

$I(i,j) = m$.sample(m, p)

4. Almost identical to 2.

- set & parents changes
- use different encoding for y

ie $y = \begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix}$ instead of $\begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

- sampler returns $\{-1$ or $1\}$ - convert back to $\{0$ or $1\}$ when setting pixel values