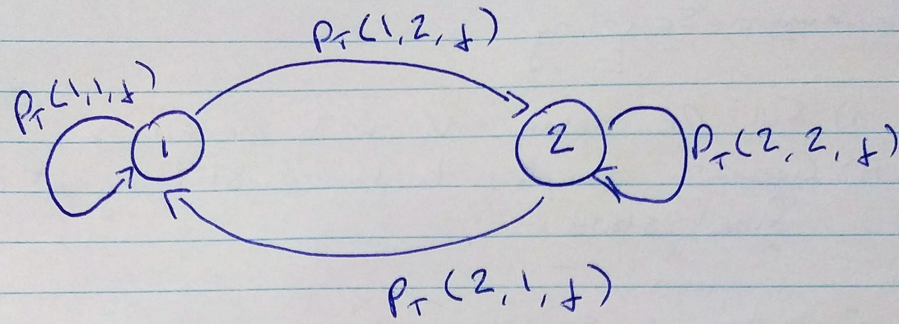
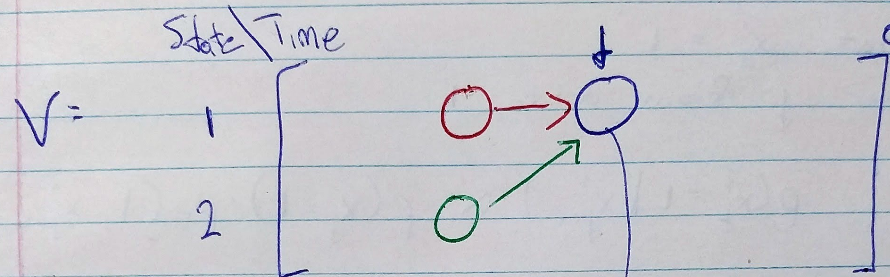




$$p(x_t = 1) = p(x_{t-1} = 1) \cdot p_T(1,1,t-1) + p(x_{t-1} = 2) \cdot p_T(2,1,t-1)$$

Rejection Sampling

$$p(x_t = c \mid x_{t'} = c') = \frac{p(x_t = c,\ x_{t'} = c')}{p(x_{t'} = c')}$$

Generate monte-carlo samples

$p(x_t = c, x_{t'} = c') \leftarrow$ # of samples where $x_t = c$ and $x_{t'} = c'$

$p(x_{t'} = c') \leftarrow$ # of samples where $x_{t'} = c'$
(# of samples accepted)

Backward Sampling

i) fill DP table V with marginals

ii) Generate samples backward starting at conditioned final state

consider a sample x^i

set $x^i_d = 1$

for j from $d-1$ to 1

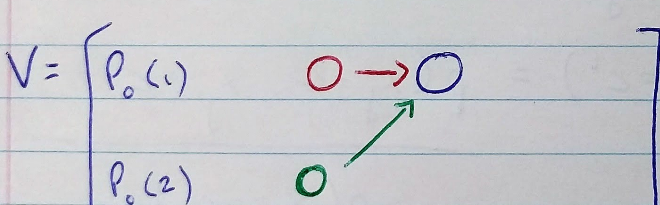
$$p(x^i_j = 1 \mid x^i_{j+1}) \propto p(x_j = 1) \cdot p_T(1, x^i_{j+1}, j)$$

$$p(x^i_j = 2 \mid x^i_{j+1}) \propto p(x_j = 2) \cdot p_T(2, x^i_{j+1}, j)$$

normalize to sum to 1

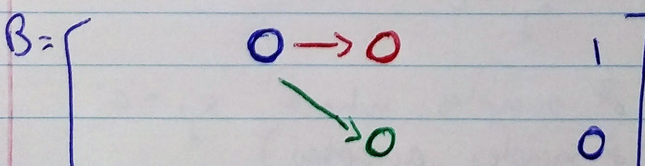
sample from distribution to get x^i_j

forward-Backward



- forward: compute from $j = 2$ to $j = d$

$$p(x_j = 1) = p(x_{j-1} = 1) \cdot p_T(1, 1, j-1) + p(x_{j-1} = 2)\, p_T(2, 1, j-1)$$



- Backward: compute from $j = d-1$ to $j = 1$

$$p(x_j = 1) = p(x_{j+1} = 1) \cdot p_T(1, 1, j) + p(x_{j+1} = 2) \cdot p_T(1, 2, j)$$

$$p(x_j = c \mid x_d) \propto V(c, j) \cdot B(c, j)$$

(Entrywise multiplication)

Normalize so $\sum_c p(x_j = c \mid x_d) = 1$ to get univariate conditionals.

1.3

Thm:

$$x \sim N(\mu_x, \sigma_x^2)$$

$$y \mid x \sim N(Ax + b, \sigma_y^2)$$

$$\Rightarrow \quad y \sim N(A\mu_x + b, \sigma_y^2 + A\sigma_x^2 A^T)$$

Hilroy