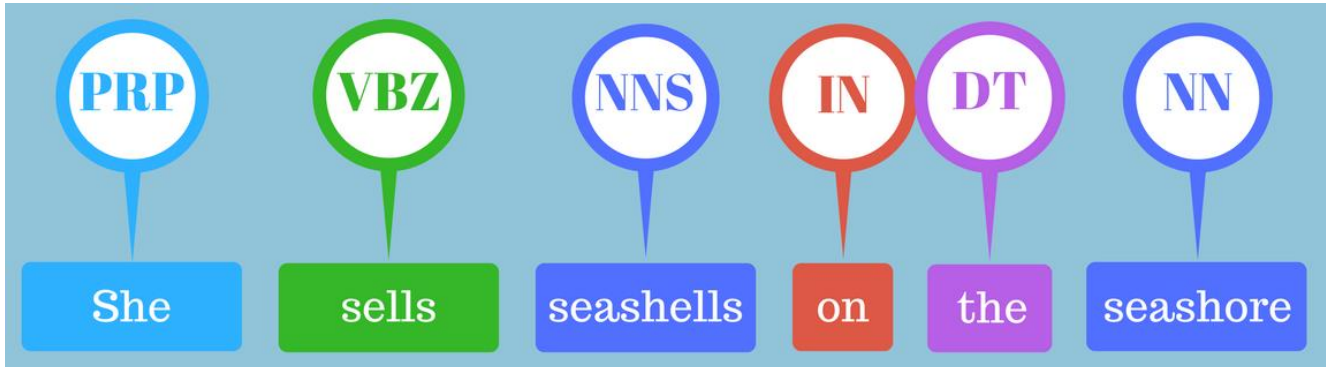
Next Topic: Recurrent Neural Networks

Motivation: Part of Speech (POS) Tagging

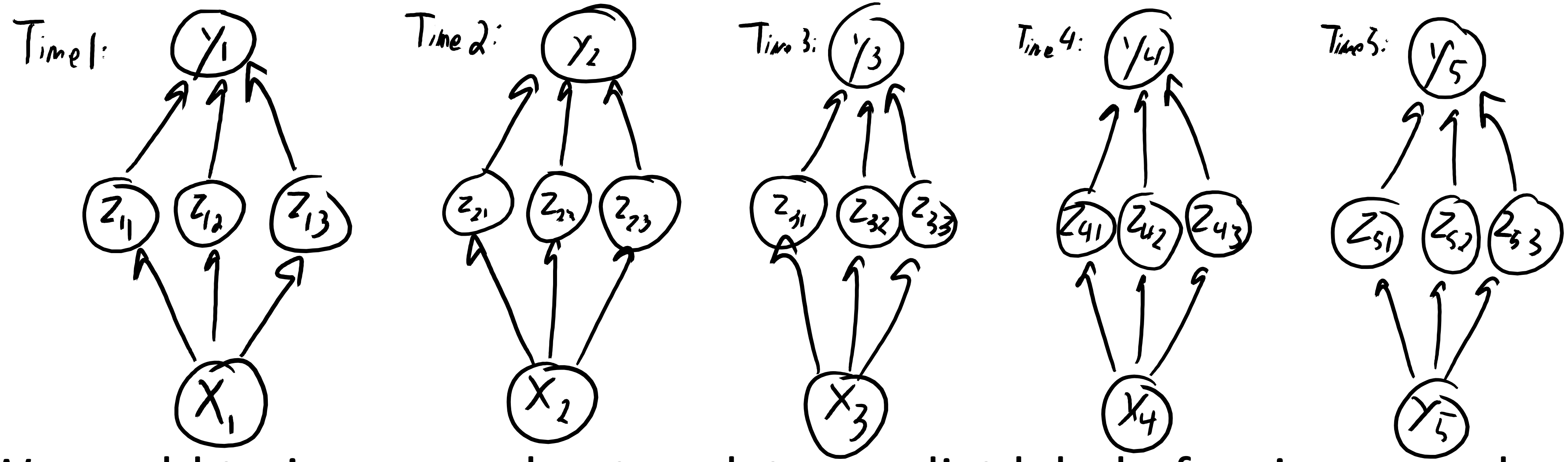
- Consider predicting part of speech for each word in a sentence:



- Input is a sequence of words.
  - Could be represented as "1 of k" or using continuous vectors like word2vec.
- Output is a categorical label for each word.
  - In English there are more than 40 categories.
    - And there are some dependencies in labels (like "only 1 verb in the sentence").
- General problem: sequence labeling.
  - Biological sequences, various language tasks, sound processing.

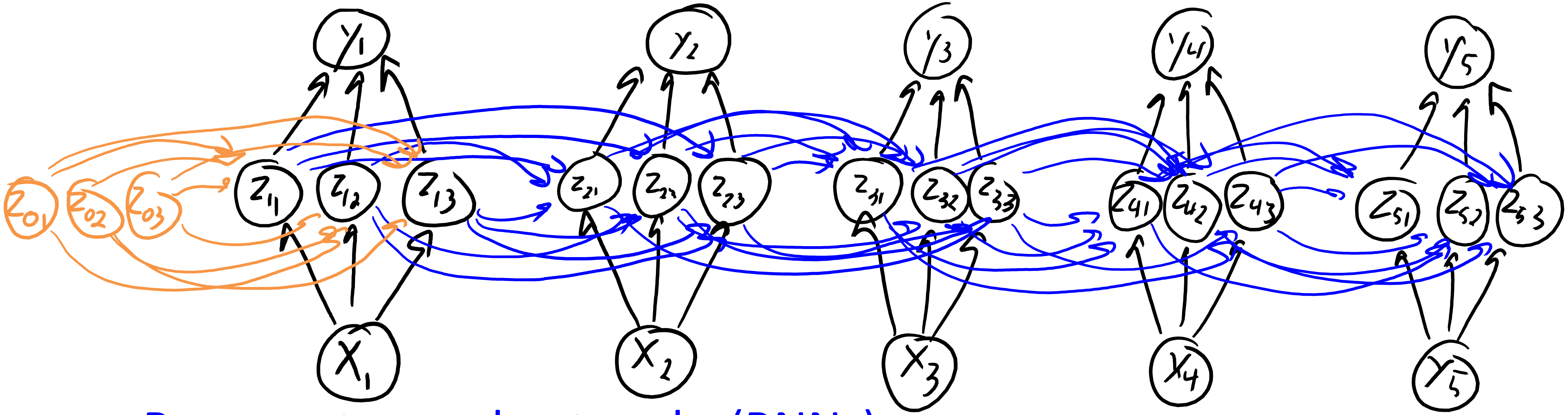
https://medium.com/analytics-vidhya/pos-tagging-using-conditional-random-fields-92077e5eaa31

Individual-Word Neural Network Classifier



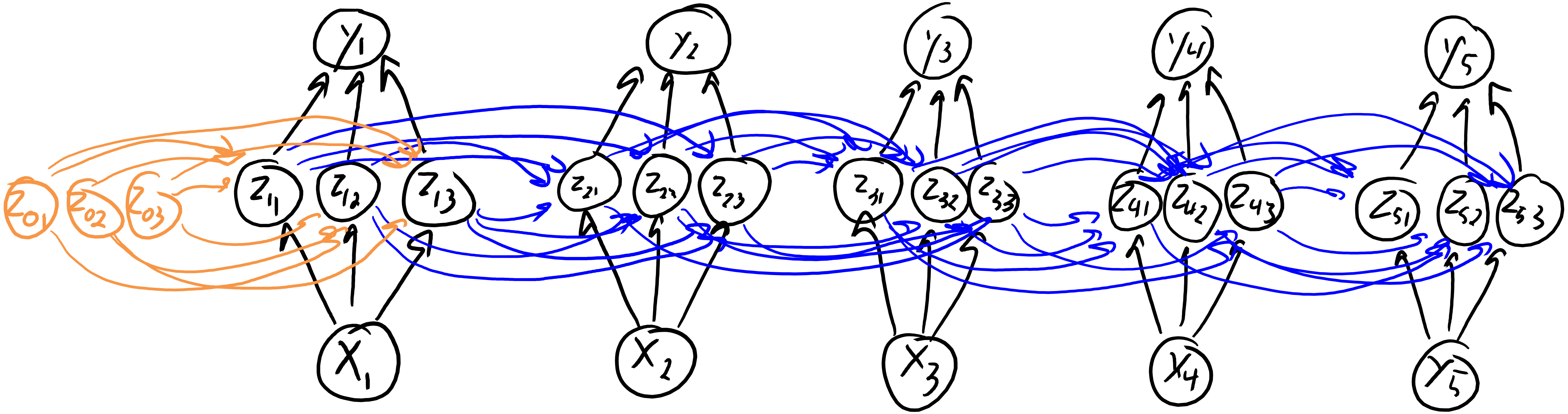
- We could train a neural network to predict label of a given word.
  - Picture has 1 input feature for each time.
    - But each time might have multiple features (if we use something like word2vec).
  - We are also not showing the non-linear transform or bias variables.
- But this type of model would not capture dependencies.
  - Information from earlier in sentence does influence prediction.
    - The word “desert” could be a noun or a verb depending on context.

Recurrent Neural Network for Sequence Labeling



- Recurrent neural networks (RNNs):
  - Add connections between adjacent different times to model dependencies.
  - Add an initial hidden state.
  - Use the same parameters across time.
- Repeating parameters in different places is called parameter tieing.
  - We previously saw convolutions, which use parameter tieing across space.
  - By tieing parameters across time, RNNs can label sequences of different lengths.

Recurrent Neural Network for Sequence Labeling



$$\hat{y}_t = \underbrace{V}h(z_t)$$

We have a matrix 'V' because we are doing multi-class

Use $\hat{y}_t$ vector in softmax at each time

$$z_t = Wx_t + \underbrace{U}\,\underbrace{h(z_{t-1})}$$

weights on temporal connections

hidden units at previous time

Parameters: W, V, U (and possibly z_0)

(Notice that we use the same "tied" matrices $\{W, V, U\}$ for all times 't'.)

Recurrent Neural Network Inference

$$\underbrace{\hat{y}_t}_{k\times 1} = \underbrace{V}_{k\times m} h(\underbrace{z_t}_{m\times 1}) \qquad \underbrace{z_t}_{m\times 1} = \underbrace{W}_{m\times d}\underbrace{x_t}_{d\times 1} + \underbrace{U}_{m\times m} h(\underbrace{z_{t-1}}_{m\times 1})$$

- Assume we have:
  - ‘k’ different classes that each $\hat{y}_t$ can take.
  - ‘m’ hidden units at each time.
  - ‘T’ times (length of sequene).
- Cost to compute all $\hat{y}_t$ if each time has ‘m’ units and we have ‘T’ times:
  - We need to do an O(md) operations ‘T’ times to compute Wx_t for all ‘t’.
  - We need to do an O(km) operation ‘T’ times to compute $\hat{y}_t$ for all ‘t’.
  - We also need to do a $O(m^2)$ operation ‘T’ times to compute each z_t (‘U’ multiplications).
  - Total cost: $O(tmd + tkm + tm^2)$.
- For the likelihood, we could use an independent softmax for each time.
  - $p(y_{1:T} \mid x_{1:T}, W, V, U) = p(y_1 \mid x_1, W, V, U)p(y_2 \mid x_{1:2}, W, V, U)\cdots p(y_T \mid x_{1:T}, W, V, U)$.
    - Where each p(yt | x1:T, W, V, U) is given by softmax over $\hat{y}_t$ values.

RNN Learning

- The objective function we use to train RNNs is the NLL:

$$f(W,V,U) = -\sum_{i=1}^{n}\sum_{t=1}^{T} \log p(y_t^i \mid x_{1:T}^i, W, V, U)$$

  - In the above I assume all sequences have the same length 'T'.
    - But in practice you will often have sequences of different lengths.
- Computing gradient called "backpropagation through time" (BTT).
  - Equations are the same as usual backpropagation/chain-rule.
    - If you do it by hand, make sure to add all terms for tied parameters.
  - Automatic differentiation is commonly used.
- Usually trained with SGD.
  - Sample an example 'i' on each iteration, do BTT, update all parameters.
  - which has usual challenges.

RNN Learning – Extra Challenges

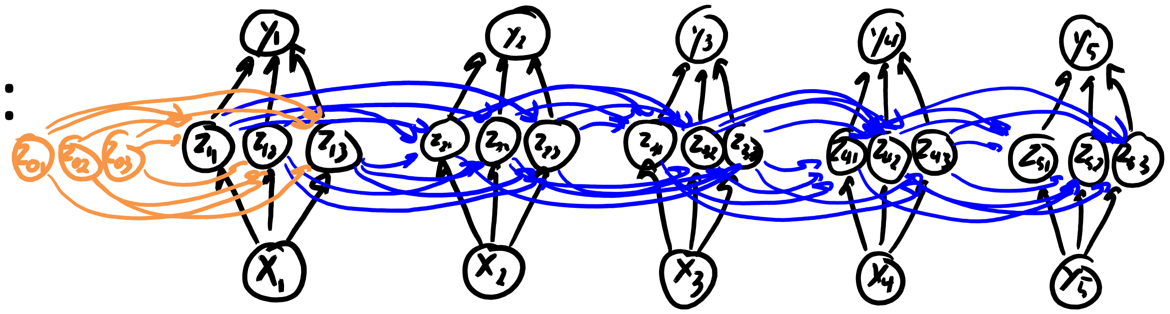
- Unfortunately, training RNNs presents some extra challenges:
  - Computing gradient requires a lot of memory for long sequences.
    - There are a lot intermediate calculations.
    - Make sure AD package handles matrix multiplication.
  - Parameter tieing often leads to vanishing/exploding gradient problems.
    - Consider a linear RNN and just consider the temporal ‘U’ updates:
      - $z^L = U*U*U*\cdots*U*z_0 = U^L z_0$.
      - For typical z_0, the quantity z_L either diverges exponentially or converges to zero exponentially.
        - » If largest singular value of ‘U’ is > 1, $||z_L||$ increases exponentially with ‘L’.
        - » If largest singular value of ‘U’ is < 1, $||z_L||$ converges to zero exponentially with ‘L’.
  - Usual SGD methods tend not to work well.
    - Often need to use optimizers like Adam or use gradient clipping:
      - If norm of gradient is larger than some threshold, “shrink” norm to threshold:
    - People are trying to explore initialization/keeping ‘U’ orthogonal.
      - So that all singular values are 1 (some positive and negative results on this).

$$\text{if } \|g\| > u$$
$$g \leftarrow \frac{gu}{\|g\|}$$

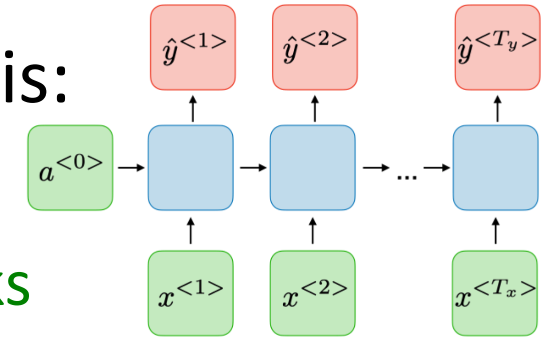
https://towardsdatascience.com/neural-network-optimization-7ca72d4db3e0

Deep RNNs

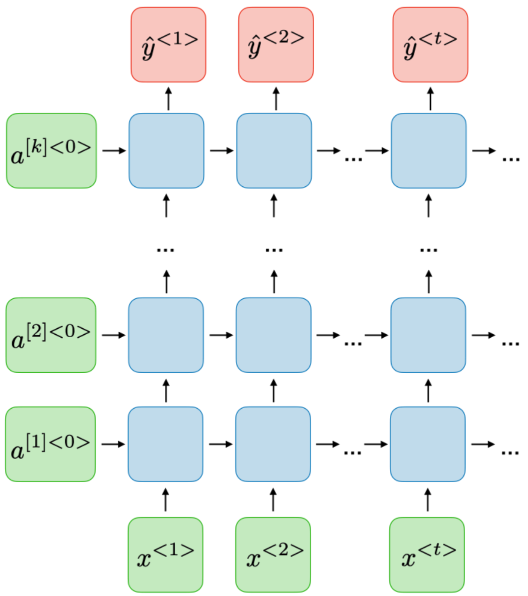
- Instead of drawing this:



- We often use diagrams like this:
  - Up to some notation changes.
  - We connect everything in blocks connected by arrows.



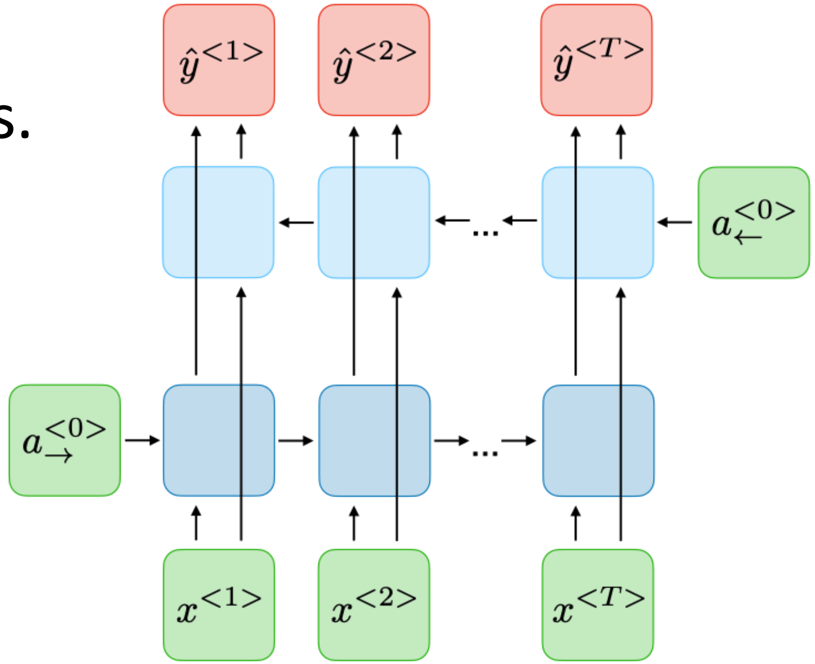
- Deep RNNs add multiple hidden layers at each time:



https://stanford.edu/~shervine/teaching/cs-230/cheatsheet-recurrent-neural-networks

Bi-Directional RNNs

- Sometimes later information later changes meaning:
  - "I've had a perfectly wonderful evening, but this wasn't it."
    - "Paraprosdokian".
- Bi-directional RNNs have hidden layers running in both directions:
  - Use different parameters for the forward and backward directions.



Next Topic: Sequence to Sequence RNNs

Motivating Problem: Machine Translation

- Consider the problem of machine translation:
  - Input is text from one language.
  - Output is text from another language with the same meaning.

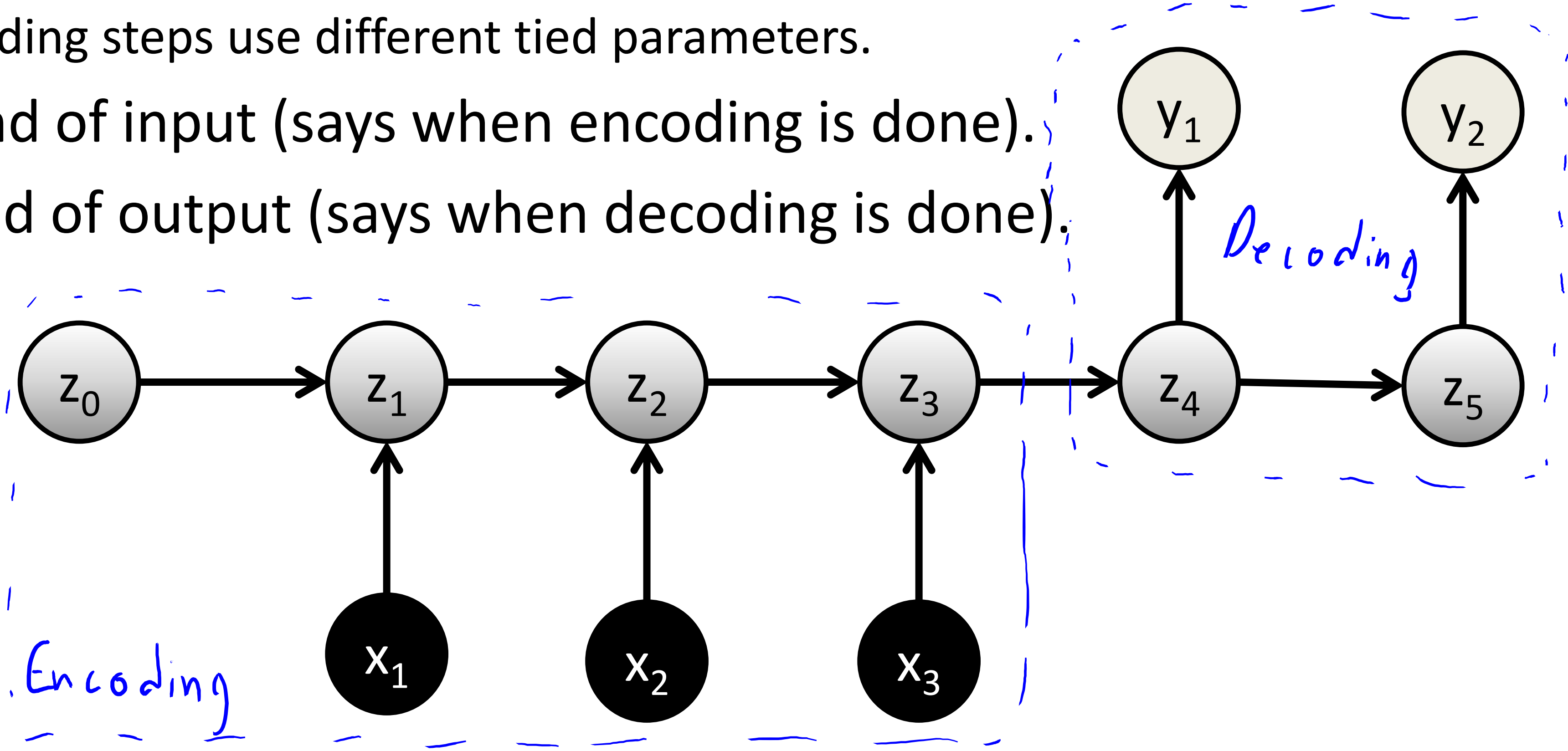
This course is intended as a second or third university-level course on machine learning, a field that focuses on using automated data analysis for tasks like pattern recognition and prediction.

Ce cours est conçu comme un cours de deuxième ou troisième niveau universitaire sur l'apprentissage automatique, un domaine qui se concentre sur l'utilisation de l'analyse de données automatisée pour des tâches telles que la reconnaissance de formes et la prédiction.

- A key difference with pixel labeling:
  - Input and output sequences may have different lengths.
    - We do not just “find the French word corresponding to the English word”.
  - We may not know the output length.

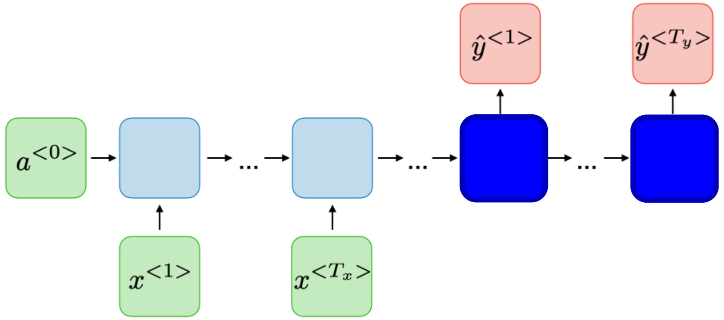
Sequence-to-Sequence RNNs

- Sequence-to-sequence RNNs encode and decode sequences:
  - Each encoding step has one word as input and no output.
  - Each decoding step outputs one word and has no input.
    - Encoding and decoding steps use different tied parameters.
  - Special "BOS" at end of input (says when encoding is done).
  - Speical "EOS" at end of output (says when decoding is done).



Discussion: Sequence-to-Sequence Models

- Representing input and outputs:
  - Could use lexicographic or word2vec representations.
  - Could just have a single character at each time.
    - Could make more sense for some languages.
    - May be able to better handle slang or typos.



- Loss function assuming independent labels given hidden states:

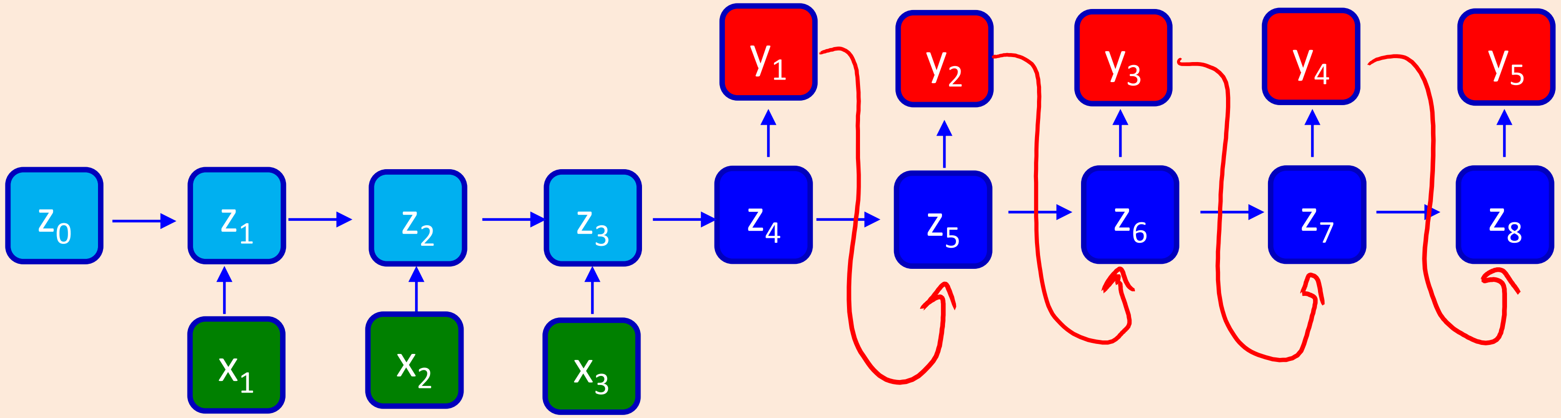
$$f(z_0, W, U_e, U_d, V) = -\sum_{i=1}^{n}\sum_{j=1}^{|y^i|} \log p(y^i_j \mid x^i_{1:t}, z_0, W, U_e, U_d, V)$$

softmax value for word at position 'j' in training data.

  - Not that this is just trying to get the label right at each "time".
    - It is not explicitly "trying to get the full sequence right".

Dependent Predictions and Beam Search

- Standard RNNs assume conditional independence of $\hat{y}_t$ values.
  - We assume they are independent given the z_t values (make inference easy).
  - This makes inference easy, but $\hat{y}_t$ "forgets" what was used for $\hat{y}_{t-1}$.
- In many applications, you want to model dependencies in the $\hat{y}_t$.
  - A common way to do this is to add edges like this:



  - This does not complicate training (where we know the y_t values).
  - But it makes decoding challenging since the y_t are dependent.
    - In this setting we typically use beam search to find a good assignment to the y_t values.
      - Stores 'k' current best decodings up to time 't' ("consider 'k' best values of y_1 when computing y_2").
      - Can be arbitrarily bad, but works if decoding is obvious as we go forwards in time.

Summary

- Autoencoders:
  - Neural network where the output is the input.
    - Non-linear generalization of PCA.
  - Encode data into a bottleneck layer, then decode predict original input.
  - Can be used for visualization, compression, outlier detection, pre-training.
- Denoising autoencoders train to uncorrupt/enhance images.
  - Useful for removing noise, adding colour, super-resolution, and so on.
- Encoding-Decoding approach to multi-label classification:
  - Have all classes shared the same hidden layer(s).
  - Reduces number of parameters.
  - Models dependencies between classes, while keeping inference easy.
- Pre-training:
  - Use parameters from model trained a on large diverse dataset, to initialize SGD for new dataset.
- Recurrent neural networks (RNNs):
  - Neural networks for sequence prediction.
  - Have connections between hidden units at adjacent times.
  - Use parameter tieing across time.
    - Allows sequences of different lengths.
    - Leads to vanishing and exploding gradients.
- Sequence-to-Sequence RNNs:
  - Encoding phase takes in one input at a time until we reach “BOS”.
  - Decoding phase outputs one output at a time until we output “EOS”.
  - Allows input and output sequences whose lengths differ.

- Next time: can machines understand language?