Tutorial 4

CPSC 340

Overview

Vector Quantization with K-means
Vector Quantization
Example

DBSCAN
DBSCAN Algorithm
DBSCAN Algorithm

Linear Algebra Review
Exercises

Vector Quantization with K-means

- Two motivations for clustering
  - Discovering object groups
  - Vector quantization
- Vector quantization:
  - Find a prototype for each cluster
  - Replace points in the cluster by their prototype
- Vector quantization with K-means:
  - Apply K-means
    - Define groups by means
    - Assign objects to nearest mean
  - Replace objects with the mean of their group

Example - Image Color Space Compression

- RGB representation
  - 24 bits per pixel (three 8-bit numbers)
- Compress color space: reduce the bits required per pixel
  - Find prototype colors, replace pixels by prototypes
  - b bits per pixel, $k = 2^b$ clusters

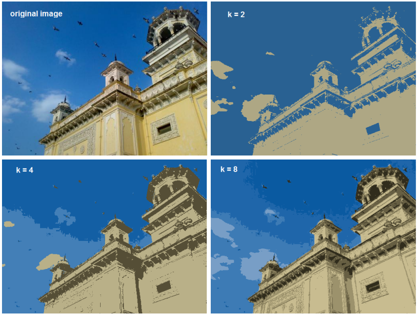

Figure: Taken from http://opencvpython.blogspot.ca/2012/12/k-means-clustering-2-working-with-scipy.html

DBSCAN

- A density-based clustering algorithm
  - Clusters are defined by all the objects in dense regions
  - Objects in non-dense regions don't get clustered.
  - Non-parametric (no fixed k)
- DBSCAN Parameters
  - Radius: maximum distance between points to be considered close
    - Points within this radius called reachable
  - MinPoints: number of reachable points needed to define a cluster
    - A point that has minPoints reachable points is called a core point
- DBSCAN Algorithm
  - Each core point defines a cluster
  - Merge clusters if core points are reachable from each other.

DBSCAN Algorithm

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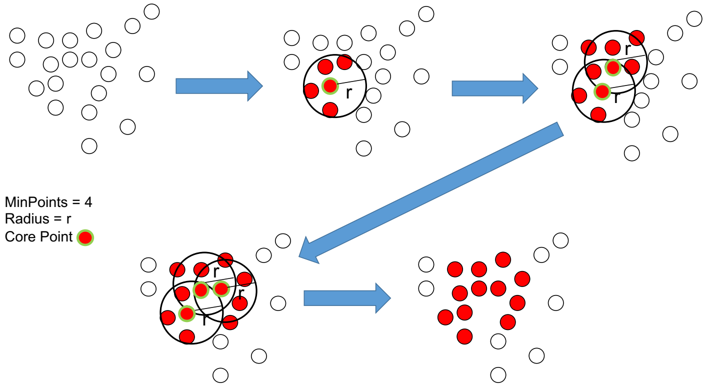

Exercise 1

Let $\alpha = 2, x = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, y = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}, z = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}, A = \begin{bmatrix} 3 & 5 & 1 \\ 2 & 3 & 4 \end{bmatrix}$

1. x^T
2. $\alpha(x + y)$ (vector addition and scalar multiplication)
3. $x^T y + ||x||^2$ (inner product and norm $||x||_2^2 = x^T x$)
4. A^T
5. Ax (matrix-vector multiplication)
6. $A^T A$ (matrix-matrix multiplication)

Exercise 2

$\{x, y, z\}$ are real-valued column vectors (of the same length) and $\{A, B, C\}$ are real-valued matrices such that the additions/multiplications below have the right dimensions, which two of the following are not true in general?

1. $x^T y = y^T x$
2. $x^T A y = y^T A^T x$
3. $x^T (y + z) = x^T y + x^T z$
4. $x^T (y^T z) = (x^T y)^T z$
5. $A + (B + C) = C + (A + B)$
6. $A(BC) = (AB)C$
7. $A(B + C) = AB + AC$
8. $AB = BA$
9. $(AB)^T = B^T A^T$

Vector notation and inner product

- Column Vector (m by 1): $w = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \end{bmatrix}$
- Row Vector (1 by n): $w^T = \begin{bmatrix} w_1 & w_2 & w_3 & w_4 \end{bmatrix}$
- The inner product between vectors of the same length is:

$$x^T w = \sum_{i=1}^{n} x_i w_i = x_1 w_1 + x_2 w_2 + \cdots + x_n w_n = \gamma$$

Sum notation to vector notation

Let $X = \begin{bmatrix} x_{11} & x_{12} & x_{13} & x_{14} \\ x_{21} & x_{22} & x_{23} & x_{24} \\ x_{31} & x_{32} & x_{33} & x_{34} \end{bmatrix}$ and $w = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \end{bmatrix}$. Compute x_1, x_1^T, and $y = Xw$.

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$$y = Xw = \begin{bmatrix} x_{11} & x_{12} & x_{13} & x_{14} \\ x_{21} & x_{22} & x_{23} & x_{24} \\ x_{31} & x_{32} & x_{33} & x_{34} \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \end{bmatrix} = \begin{bmatrix} \sum_{j=1}^4 x_{1j} w_j \\ \sum_{j=1}^4 x_{2j} w_j \\ \sum_{j=1}^4 x_{3j} w_j \end{bmatrix} = \begin{bmatrix} x_1^T w \\ x_2^T w \\ x_3^T w \end{bmatrix}$$

$y_i = x_i^T w$

Vector and matrix notation

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- In vector notation?
  - $f(x) = w^T x$ with $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, $w = \begin{bmatrix} 2 \\ 4 \\ 8 \end{bmatrix}$.
- Let

$$\begin{aligned} 4x_1 - 5x_2 &= -13 \\ -2x_1 + 3x_2 &= 9. \end{aligned}$$

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  - $Ax = b$ with $A = \begin{bmatrix} 4 & -5 \\ -2 & 3 \end{bmatrix}$, $b = \begin{bmatrix} -13 \\ 9 \end{bmatrix}$

Vector and matrix notation

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- In matrix notation?
  - $f(x) = x^T Ax$ with $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, $A = \begin{bmatrix} 4 & 3 \\ 3 & 6 \end{bmatrix}$.