Machine Learning
CPSC 340

Tutorial 12

Random Walk on Graph

Page Rank Algorithm

Label Propagation on Graph

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- Gaol: assign a label to unlabeled nodes.

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  - do majority vote among the stored labels

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Exercise

Assume we are given adjacency matrix, a labelList, a matrix where the first column contains node numbers and the second column contains class labels and starting node, write code for random walk algorithm.

RW code

```
function [y] = runRandomWalk(A,labelList,v)

while 1
    if any(labelList(:,1) == v)
        neighbours = find(A(v,:));
        nNeighbours = length(neighbours);
        ind = ceil(rand*(nNeighbours+1));
        if ind == nNeighbours+1
            ind = find(labelList(:,1)==v);
            y = labelList(ind,2);
            return
        else
            v = neighbours(ind);
        end
    else
        neighbours = find(A(v,:));
        nNeighbours = length(neighbours);
        ind = ceil(rand*nNeighbours);
        v = neighbours(ind);
    end
end

end
```

Ranking Problem

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  - Input: a large set of objects (and possibly a query object).
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- Examples:
  - Country comparisons (Global Hunger Index)
  - Academic journals (Impact factor).
  - Sports/gaming (Elo and TrueSkill)
  - Internet search engines

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- Let $Z = A^T(diag(m))^{-1}$

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- Let $Z = A^T(diag(m))^{-1}$
- $(Z)_{ij}$: probability of jumping from page j to page i via a link
- But we can go directly from page j to i by entering the address of page i in the address-bar of browser
  - add some small amount to all $(Z)_{ij}$ and normalize!

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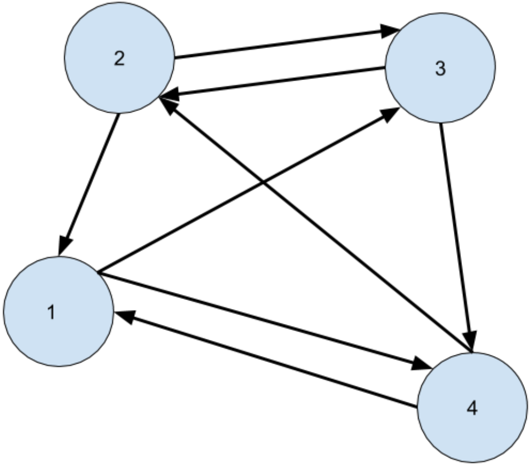
- With two above equations, we can find all p_is by solving corresponding linear system

PageRank: Exercise

Let $d = \frac{3}{4}$. For the following webpage graph, find A, m, Z and T. Then make the linear system and solve it and find P.



PageRank: Solution

$$m = \begin{pmatrix} 2 \\ 2 \\ 2 \\ 2 \end{pmatrix} A = \begin{pmatrix} 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 \end{pmatrix} Z = \begin{pmatrix} 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 \end{pmatrix}$$

$$T = \frac{1}{16} * \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix} + \frac{3}{4} * Z = \begin{pmatrix} \frac{1}{16} & \frac{7}{16} & \frac{1}{16} & \frac{7}{16} \\ \frac{1}{16} & \frac{1}{16} & \frac{7}{16} & \frac{7}{16} \\ \frac{7}{16} & \frac{7}{16} & \frac{1}{16} & \frac{1}{16} \\ \frac{7}{16} & \frac{1}{16} & \frac{7}{16} & \frac{1}{16} \end{pmatrix}$$

$P = TP \rightarrow (T - I)P = 0$ (I is Identity matrix)

$$\sum_{i=1}^{4} p_i = 1$$

PageRank: Solution

Combining two equations, we get

$$\begin{pmatrix} \frac{1}{16} & \frac{7}{16} & \frac{1}{16} & \frac{7}{16} \\ \frac{1}{16} & \frac{1}{16} & \frac{7}{16} & \frac{7}{16} \\ \frac{7}{16} & \frac{7}{16} & \frac{1}{16} & \frac{1}{16} \\ \frac{7}{16} & \frac{1}{16} & \frac{7}{16} & \frac{1}{16} \\ 1 & 1 & 1 & 1 \end{pmatrix} * \begin{pmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

$$P = \begin{pmatrix} 0.25 \\ 0.25 \\ 0.25 \\ 0.25 \end{pmatrix}$$