

Boolean Expressions Reduction Semantics

CPSC 509: Programming Language Principles

Ronald Garcia*

10 March 2022

(Time Stamp: 15:19, Thursday 21st March, 2024)

Program Syntax

$$t \in \text{TERM}, \quad p \in \text{PGM} = \text{TERM}$$

$$t ::= \text{true} \mid \text{false} \mid \text{if } t \text{ then } t \text{ else } t$$

Program Runtime Syntax

$$E \in \text{ECTXT}, \quad v \in \text{VALUE}, \quad r \in \text{REDEX},$$

$$v ::= \text{true} \mid \text{false}$$

$$E ::= \square \mid \text{if } E \text{ then } t \text{ else } t$$

$$r ::= \text{if } v \text{ then } t \text{ else } t$$

(Outside-In) Contexts Desugared

$$E ::= \text{hole} \mid \text{if-around}(E, t_1, t_2)$$

Context Plugging

$$\bullet[\bullet] : \text{ECTXT} \times \text{TERM} \rightarrow \text{TERM}$$

$$\square[t] = t$$

$$(\text{if } E \text{ then } t_2 \text{ else } t_3)[t_1] = \text{if } (E[t_1]) \text{ then } t_2 \text{ else } t_3$$

$\rightsquigarrow \subseteq \text{REDEX} \times \text{TERM}$

Notions of Reduction

$$\text{if true then } t_2 \text{ else } t_3 \rightsquigarrow t_2$$

$$\text{if false then } t_2 \text{ else } t_3 \rightsquigarrow t_3$$

$\longrightarrow \subseteq \text{TERM} \times \text{TERM}$

Single-step Reduction

$$\frac{r \rightsquigarrow t}{E[r] \longrightarrow E[t]}$$

$\longrightarrow^* \subseteq \text{TERM} \times \text{TERM}$

Multi-step Reduction

$$(\text{incl}) \frac{t_1 \longrightarrow t_2}{t_1 \longrightarrow^* t_2}$$

$$(\text{refl}) \frac{}{t \longrightarrow^* t}$$

$$(\text{trans}) \frac{t_1 \longrightarrow^* t_2 \quad t_2 \longrightarrow^* t_3}{t_1 \longrightarrow^* t_3}$$

$$o \in \text{OBS} = \text{VALUE}$$

$$\text{eval}_{rs} : \text{PGM} \rightarrow \text{OBS}$$

$$\text{eval}_{rs}(p) = v \text{ if } p \longrightarrow^* v$$

*© 2022 Ronald Garcia.