

# Boolean Expressions

## Abstract Machine Semantics

### CPSC 509: Programming Language Principles

Ronald Garcia\*

10 March 2022

(Time Stamp: 15:20, Thursday 21<sup>st</sup> March, 2024)

#### Program Syntax

$$t \in \text{TERM}, \quad p \in \text{PGM} = \text{TERM}$$

$$t ::= \text{true} \mid \text{false} \mid \text{if } t \text{ then } t \text{ else } t$$

#### Program Runtime Syntax

$$E \in \text{ECTXT}, \quad v \in \text{VALUE}, \quad r \in \text{REDEX}, \quad C \in \text{CONFIG}$$

$$v ::= \text{true} \mid \text{false}$$

$$E ::= \square \mid E[\text{if } \square \text{ then } t \text{ else } t]$$

$$r ::= \text{if } v \text{ then } t \text{ else } t$$

$$C ::= \langle E, t \rangle_{\text{focus}} \mid \langle E, v \rangle_{\text{return}} \mid \langle E, r \rangle_{\text{reduce}}$$

#### (Inside-Out) Contexts Desugared

$$E ::= \text{hole} \mid \text{if-inside}(E, t_1, t_2)$$

#### Context Plugging

$$\bullet[\bullet] : \text{ECTXT} \times \text{TERM} \rightarrow \text{TERM}$$

$$\square[t] = t$$

$$(E[\text{if } \square \text{ then } t_2 \text{ else } t_3])[t_1] = E[\text{if } t_1 \text{ then } t_2 \text{ else } t_3]$$

#### $\rightarrow \subseteq \text{CONFIG} \times \text{CONFIG}$ Single-step Machine Reduction

$$\langle E, \text{if } t_1 \text{ then } t_2 \text{ else } t_3 \rangle_{\text{focus}} \rightarrow \langle E[\text{if } \square \text{ then } t_2 \text{ else } t_3], t_1 \rangle_{\text{focus}}$$

$$\langle E, v \rangle_{\text{focus}} \rightarrow \langle E, v \rangle_{\text{return}}$$

$$\langle E[\text{if } \square \text{ then } t_2 \text{ else } t_3], v \rangle_{\text{return}} \rightarrow \langle E, \text{if } v \text{ then } t_2 \text{ else } t_3 \rangle_{\text{reduce}}$$

$$\langle E, \text{if true then } t_2 \text{ else } t_3 \rangle_{\text{reduce}} \rightarrow \langle E, t_2 \rangle_{\text{focus}}$$

$$\langle E, \text{if false then } t_2 \text{ else } t_3 \rangle_{\text{reduce}} \rightarrow \langle E, t_3 \rangle_{\text{focus}}$$

#### $\rightarrow^* \subseteq \text{CONFIG} \times \text{CONFIG}$ Multi-step Reduction

$$\text{(incl)} \frac{C_1 \rightarrow C_2}{C_1 \rightarrow^* C_2} \quad \text{(refl)} \frac{}{C \rightarrow^* C} \quad \text{(trans)} \frac{C_1 \rightarrow^* C_2 \quad C_2 \rightarrow^* C_3}{C_1 \rightarrow^* C_3}$$

$$o \in \text{OBS} = \text{VALUE}$$

$$\text{eval}_B : \text{PGM} \rightarrow \text{OBS}$$

$$\text{eval}_B(p) = v \text{ if } \langle \square, p \rangle_{\text{focus}} \rightarrow^* \langle \square, v \rangle_{\text{return}}$$

\*© 2022 Ronald Garcia.