Physics 153 Section T0H - Solution to Problem 4

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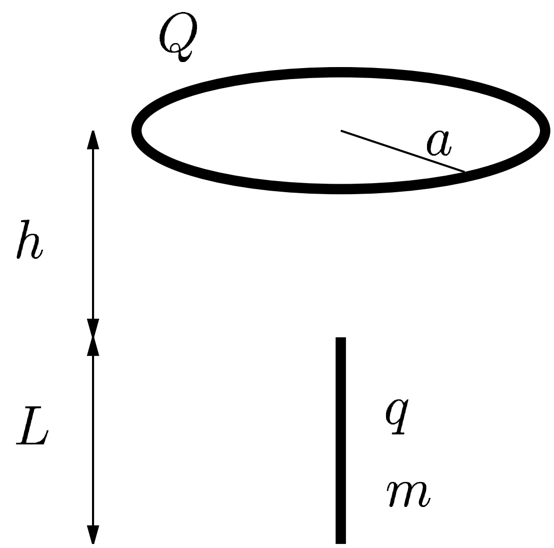
1 Assigned Problem

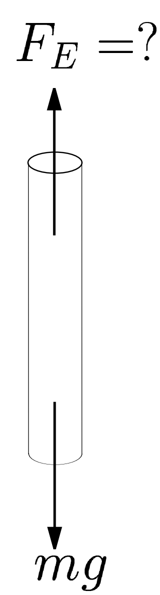


In a laboratory, a ring of charge $Q = 8\,\mu\mathrm{C}$ with radius $a = 30\,\mathrm{cm}$ levitates a charged rod of length $L = 20\,\mathrm{cm}$ and mass $m = 100\,\mathrm{g}$, as shown above. The top of the floating rod is $h = 10\,\mathrm{cm}$ below the center of the ring. (a) Draw a free body diagram for the rod in equilibrium. (b) Assuming it is uniformly distributed, what is the total charge q of the rod?

2 Solution

2.1 Part (a)

This is pretty trivial...I just wanted you to think about all the forces that are affecting the rod and how it is being held in equilibrium. Of course, all we need is an electrostatic force F_E pulling the rod up, to counterbalance the rod's weight. So the FBD simply looks as follows:



2.2 Part (b)

To solve this problem we have to calculate the electrostatic force on the rod and equate it to its weight

$$mg + F_E = 0. \tag{1}$$

Notice this defines the forces as being measured positive *downwards*, in the direction of increasing x.

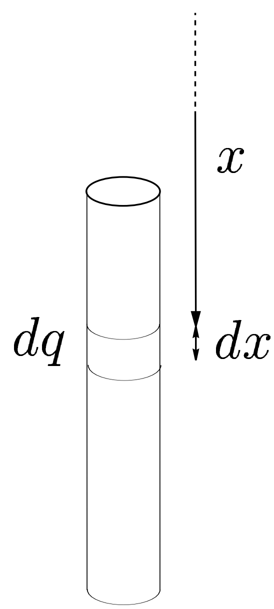
Calculating the electrostatic force requires solving an integral. We begin with the electric field at a distance x on the axis of a ring of charge Q with radius a (Tipler, Eq. (19-12))

$$E_x = \frac{kQx}{\left(x^2 + a^2\right)^{3/2}}. \tag{2}$$

In our problem x would be the distance from the center of the ring to any point x along the length of the rod so x points downward and E_x is also downward.

To find the total electrostatic force we cut the rod into tiny little charges of length dx which each have a charge

$$dq = \lambda dx \equiv \frac{q}{L} dx. \tag{3}$$



Each little charge at position x contributes dF_E to the total force

$$dF_E = E_x dq \equiv \frac{q}{L} E_x dx. \tag{4}$$

The total electrostatic force is

$$
\begin{aligned}
F_E &= \int dF_E && (5)\\
&= \int_h^{h+L} \frac{q}{L} E_x dx && (6)\\
&= \frac{q}{L}\int_h^{h+L} \frac{kQx}{\left(x^2+a^2\right)^{3/2}} dx && (7)\\
&= \frac{kQq}{L}\int_h^{h+L} \frac{x dx}{\left(x^2+a^2\right)^{3/2}}. && (8)
\end{aligned}
$$

The above integral is pretty easy and the solution is

$$
\begin{aligned}
F_E &= \frac{kQq}{L}\left[-\frac{1}{\sqrt{x^2+a^2}}\right]_h^{h+L} && (9)\\
&= \frac{kQq}{L}\left[\frac{1}{\sqrt{h^2+a^2}} - \frac{1}{\sqrt{(h+L)^2+a^2}}\right]. && (10)
\end{aligned}
$$

If we know apply our known values $Q = 8\,\mu\mathrm{C}$, $L =$

$20\,\mathrm{cm}$, $h = 10\,\mathrm{cm}$, and $a = 30\,\mathrm{cm}$ we find that

$$F_E = (290\,\mathrm{kN/C})q. \tag{11}$$

We want to find the charge q which balances the two forces

$$mg = -F_E = (-290\,\mathrm{kN/C})q \tag{12}$$

so, given $m = 100\,\mathrm{g}$,

$$q = \frac{mg}{-290\,\mathrm{kN/C}} = -3.39\,\mu\mathrm{C}. \tag{13}$$

2.3 Aside

Note that there is another, equivalent way to solve this problem. Instead of using the equation for a ring and integrating over the finite line segment, we could have used the equation for a finite line segment (Tipler, Eq. (19-5)) and integrated over the ring.