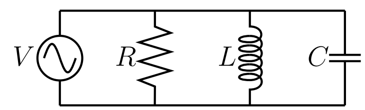
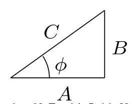
Tutorial 15 Question

- Ch 31: Pr. 48 (revised).
- A resistor R, capacitor C, and inductor L are connected in parallel across an ac generator as shown below. The source emf is $V = V_0 \sin \omega t$. Determine the current as a function of time (including amplitude and phase) (*a*) in the resistor, (*b*) in the inductor, (*c*) in the capacitor. (*d*) What is the total current leaving the source? (Give amplitude I_0 and phase.) (*e*) Determine the impedance Z defined as $Z = V_0/I_0$. (*Bonus*) What is the power factor?



- Hint: $C \sin(x + \phi) = A \sin x + B \cos x$ looks like  .

Solution

(*a*) Determine the current in the resistor.

- From Kirchhoff's loop rule the voltage across the resistor is $V = V_0 \sin \omega t$.
- The current is in phase with voltage in resistors, so $I_R = I_{R,0} \sin \omega t$.
- Ohm's law tells us $I_{R,0} = V_0/R$ so

$$I_R(t) = \frac{V_0}{R} \sin \omega t.$$

Solution, contd

(*b*) Determine the current in the inductor.

- Again, the voltage drop is V. But this time the voltage leads the current by $\phi = 90°$ (CIVIL) so

$$I_L = I_{L,0} \sin\left(\omega t - \frac{\pi}{2}\right) = -I_{L,0} \cos \omega t.$$

- The current amplitude is related to the voltage by reactance, $I_{L,0} = V_0/X_L$ so

$$I_L(t) = -\frac{V_0}{X_L} \cos \omega t.$$

- (Note: $X_L = \omega L$.)

Solution, contd

(*c*) Determine the current in the capacitor.

- Same V. Now the current leads by $\phi = 90°$ (CIVIL) so

$$I_C = I_{C,0} \sin\left(\omega t + \frac{\pi}{2}\right) = I_{C,0} \cos \omega t.$$

- Again, the amplitude is given by reactance, $I_{C,0} = V_0/X_C$ so

$$I_C(t) = \frac{V_0}{X_C} \cos \omega t.$$

- (Note: $X_C = \frac{1}{\omega C}$.)

Solution, contd

(*d*) What is the total current leaving the source?

- The total current I splits (Kirchhoff's branch rule) so

$$I = I_R + I_L + I_C = I_{R,0}\sin\omega t + (I_{C,0} - I_{L,0})\cos\omega t.$$

- To equate this to $I = I_0\sin(\omega t + \phi)$ we use the hint. The amplitude is

$$\begin{aligned} I_0 &= \sqrt{I_{R,0}^2 + (I_{C,0} - I_{L,0})^2} \\ &= V_0\sqrt{\frac{1}{R^2} + \left(\frac{1}{X_C} - \frac{1}{X_L}\right)^2}. \end{aligned}$$

- Let's wait with determining the phase ϕ until we've found the impedance Z.

Solution, contd

(*e*) Determine the impedance Z.

- If $Z = V_0/I_0$ then

$$Z = \left[\frac{1}{R^2} + \left(\frac{1}{X_C} - \frac{1}{X_L}\right)^2\right]^{-\frac{1}{2}}.$$

- Notice how this is similar (but not identical, because of the squared powers) to the formula for parallel resistors,

$$\frac{1}{Z^2} = \frac{1}{R^2} + \left(\frac{1}{X_C} - \frac{1}{X_L}\right)^2.$$

- Also notice that $Z \leq R$, always.

Solution, contd

(*d*) contd

- Now we can write down the current's phase in a more familiar form,

$$\cos\phi = \frac{I_{R,0}}{I_0} = \frac{V_0/R}{V_0/Z} = \frac{Z}{R}.$$

- Weird, that's the inverse of what we got for series LRC circuits.

(*Bonus*) What is the power factor?

- The power factor is the ratio

$$\text{Power factor} = \frac{\overline{P}}{I_{\mathrm{RMS}}V_{\mathrm{RMS}}}.$$

Solution, contd

(*Bonus*) contd

- The average power (lost through the resistor is given by

$$\overline{P} = \frac{V_{\mathrm{RMS}}^2}{R}.$$

- From part (*d*) we get $R = \frac{Z}{\cos\phi}$ so

$$\overline{P} = \frac{V_{\mathrm{RMS}}}{Z}V_{\mathrm{RMS}}\cos\phi.$$

- Since $I_{\mathrm{RMS}} = V_{\mathrm{RMS}}/Z$ we find

$$\overline{P} = I_{\mathrm{RMS}}V_{\mathrm{RMS}}\cos\phi.$$

- So the power factor is still $\cos\phi$ (same as series). □