UBC Physics 102 Section 951
Midterm Test 2
July 14, 2003

Instructions:

1. Do not open this test until told to do so.
2. This test is closed book. You may NOT bring any material in with you. Calculators are allowed. A formula sheet is provided on the last page of this test.
3. Print your name and student number on ALL pages.
4. Show all your work to justify your answers.
5. Use the back of the page if you need more space.
6. If you use pencil there will be NO changes to your mark once your paper has been handed back.

Marks:

	Q1	Q2	Q3	Q4	TOTAL
Mark					
Max	10	10	10	10	40

Last Name: **Solution**

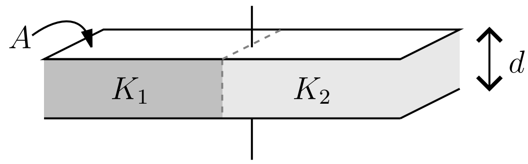
First Name: **Key**

Student Number: 00000000

Name: ______________________

Student Number: ______________________

/10 1. Two different dielectrics each fill half the space between the plates of a parallel-plate capacitor as shown below. Determine a formula for the capacitance in terms of K_1, K_2, the area A of the plates, and the separation d.



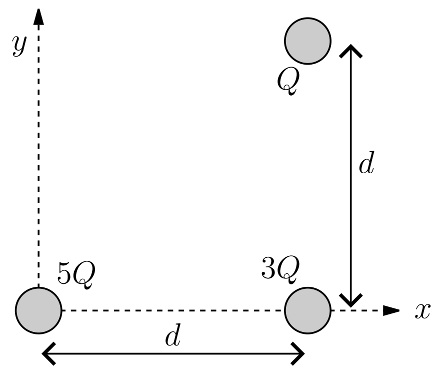
Answer: The potential difference is the same everywhere across the plates 2 so we can treat the system as two capacitors in parallel. From $\boxed{C_0 = \frac{\epsilon_0 A}{d}}$ 2 and $\boxed{C = KC_0}$ 2 the capacitance in each case is

$$C_1 = K_1 \frac{\epsilon_0 A}{2d},$$
$$C_2 = K_2 \frac{\epsilon_0 A}{2d}.$$

The equivalent capacitance of two capacitors in parallel is $\boxed{C = C_1 + C_2}$ 2 so

$$C = (K_1 + K_2)\frac{\epsilon_0 A}{2d}. \text{ 2}$$

/10 2. Consider the three positive charges in the configuration shown:



(a) (2 points) How much electrostatic potential energy does this system have?

Answer: The potential energy between any pair (q,Q) of charges is $\boxed{U = qV = \frac{kqQ}{r}}$ 1. In this case there are three pairs, so the total energy is

$$
\begin{aligned}
U &= U_{13} + U_{15} + U_{35} \text{ 1} \\
&= \frac{3kQ^2}{d} + \frac{5kQ^2}{\sqrt{2}d} + \frac{15kQ^2}{d} \\
&= \left(18 + \frac{5}{\sqrt{2}}\right)\frac{kQ^2}{d} \approx 21.5\frac{kQ^2}{d}.
\end{aligned}
$$

(b) (5 points) Where could you move the charge Q so it feels no net force (besides infinitely far away)?

Answer: The charge will feel no force if the electric fields due to the other two charges cancel out,

$$\mathbf{E} = \mathbf{E}_3 + \mathbf{E}_5 = 0. \text{ 1}$$

Since the two charges are on a line and of the same sign, the y-component of the fields will only cancel on the line itself. So we only need to look at positions on the line $y = 0$. 1

Answer, contd: The electric fields can only cancel if they are pointing in opposite directions, which only occurs if $0 < x < d$. ①
The electric fields at a point $(x,0)$ for $0 < x < d$ are

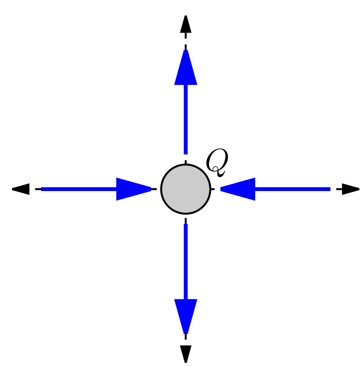
$$\mathbf{E}_3 = -\frac{3kQ}{(x-d)^2}\hat{\mathbf{i}}, \text{①}$$
$$\mathbf{E}_5 = \frac{5kQ}{x^2}\hat{\mathbf{i}}.$$

Solving $\mathbf{E}_3 + \mathbf{E}_5 = 0$ shows they will cancel when

$$x = \left(5 \pm \sqrt{15}\right)\frac{d}{2}$$

which gives the points $x \approx 0.56d$ or $x \approx 4.4d$.
But only $x \approx 0.56d$ is correct.① Outside the region $0 < x < d$ the above fields don't have the right signs so they are meaningless.
So the equilibrium position is $(x, y) = (0.56d, 0)$.

(c) (2 points) Indicate, with arrows on the diagram below, which direction Q would move if it was bumped out of the resting position found in part (b). Consider "bumps" in each of the four directions shown.

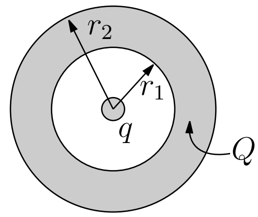


②

(d) (1 point) How would your answer to part (c) change if Q was replaced with a negative charge?

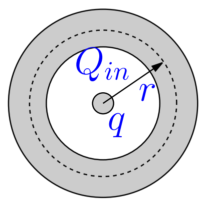
Answer: All the electric forces on the charge would be reversed so the arrows would be **reversed**.①

/10 3. Suppose a thick spherical conducting shell with inner radius r_1 and outer radius r_2 carries a total net charge Q and at its center there is a point charge q.



(a) (2 points) What total charge is found on the inner surface of the shell?

Answer: Use Gauss's law: Gaussian surface should be a sphere at radius $r_1 < r < r_2$, inside conductor.



Inside conductor electric field is always zero so $Q_{\text{encl}} = 0$. 1 So charge on inner surface, Q_{in} must be cancelled by charge at center, q:

$$Q_{in} = -q. \text{ 1}$$

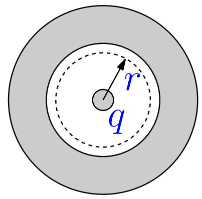
(b) (2 points) What total charge is found on the outer surface of the shell?

Answer: All charge must reside on the surface so if the total charge is $Q_{in} + Q_{out} = Q$ 1 and $Q_{in} = -q$ on the inner surface then

$$Q_{out} = Q + q. \text{ 1}$$

(c) (2 points) Determine the electric field for $0 < r < r_1$.

Answer: Gauss's law: Gaussian surface should be a sphere at radius $r < r_1$, surrounding center charge.



So $Q_{\text{encl}} = q$ 1. The E-field is perpendicular to the surface everywhere and the surface area of the sphere is $A = 4\pi r^2$. So, from $\boxed{E_\perp A = \frac{Q_{\text{encl}}}{\epsilon_0}}$ 1,

$$E = \frac{q}{4\pi\epsilon_0 r^2}.$$

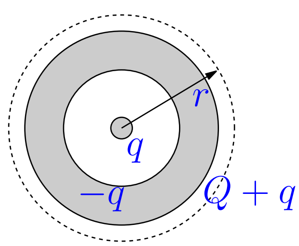
(d) (2 points) Determine the electric field for $r_1 < r < r_2$.

Answer: The electric field inside a conductor is always zero,

$$E = 0. \; 2$$

(e) (2 points) Determine the electric field for $r > r_2$.

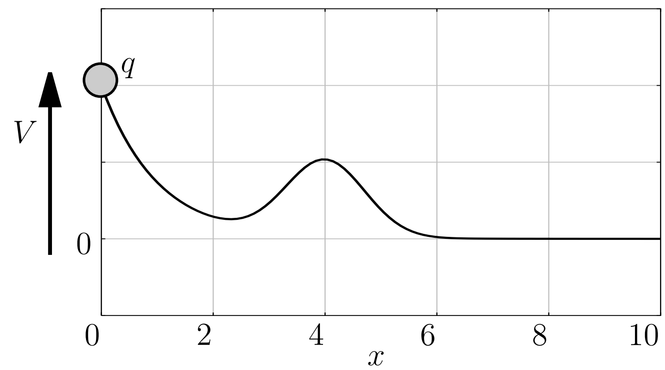
Answer: Gauss's law: Gaussian surface should be a sphere at radius $r > r_2$, surrounding outer surface.



So $Q_{\text{encl}} = Q + q$ 1. Again, from $\boxed{E_\perp A = \frac{Q_{\text{encl}}}{\epsilon_0}}$ 1,

$$E = \frac{Q + q}{4\pi\epsilon_0 r^2}.$$

/10 4. A positive charge q starts at rest at $x = 0$ in a one-dimensional potential $V(x) = 2e^{-x} + e^{-(x-4)^2}$, as shown.



(a) (3 points) Determine the speed of the charge as a function of V, if it has mass m.

Answer: The charge's potential energy is $\boxed{U = qV}$ 1. Energy is conserved so $K + U =$ constant. Initially the charge is at rest so $K(x = 0) = 0$ and

$$U(0) = qV(0) = q(2 + e^{-16}) \approx 2q.$$

So $K + U = 2q$ 1 for all x and

$$K = 2q - qV = q(2 - V).$$

Kinetic energy is given by $K = \frac{1}{2}mv^2$ where v is the speed so

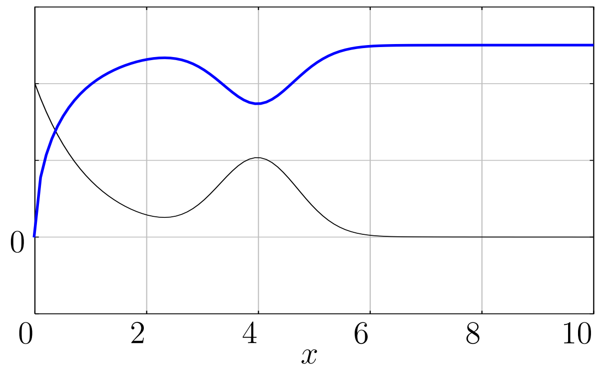
$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2q(2 - V)}{m}}. \text{ 1}$$

(b) (1 point) What will the eventual speed of the charge be?

Answer: The charge will steadily move to the right so eventually it will reach $V = 0$ and

$$v_\infty = \sqrt{\frac{4q}{m}}. \text{ 1}$$

(c) (2 points) Sketch the speed as a function of x on the graph provided.



Markers: just check the shape of the curve.② Unlabelled y-axis ok.

(d) (2 points) Determine the force on the charge as a function of x.

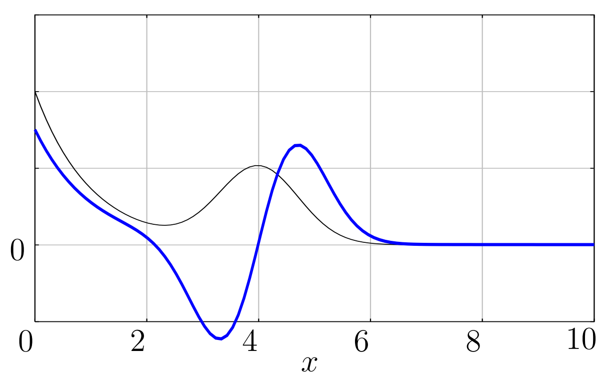
Answer: The potential $V(x)$ creates an electric field $\boxed{E_x = -\frac{dV}{dx}}$①,

$$\begin{aligned} E(x) &= -\frac{d}{dx}\left(2e^{-x} + e^{-(x-4)^2}\right) \\ &= 2e^{-x} + 2(x-4)e^{-(x-4)^2}. \end{aligned}$$

The force is $\boxed{F = qE}$ in the positive x direction,

$$F(x) = q\left(2e^{-x} + 2(x-4)e^{-(x-4)^2}\right).①$$

(e) (2 points) Sketch the magnitude of the force as a function of x.



② Unlabelled y-axis ok.

END OF EXAM

(Did you print your name and student number on every page?)