UBC Physics 102

Lecture 15

Rik Blok

Outline

- ▷ AC source
- ▷ R circuits
- ▷ L circuits
- ▷ C circuits
- ▷ LRC circuits
- ▷ Resonance
- ▷ End

AC source [Text: Sect. 31-1]

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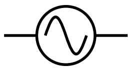
  - Circuit symbol: 

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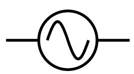
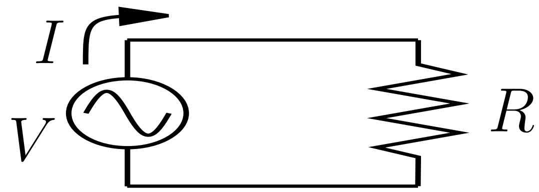
  - Circuit symbol: 
  - Note: multimeters read RMS values, not peak.

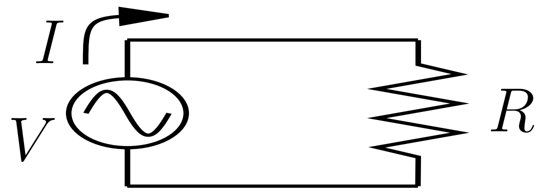
R circuits [Text: Sect. 31-2]

- **Discussion: R circuits**

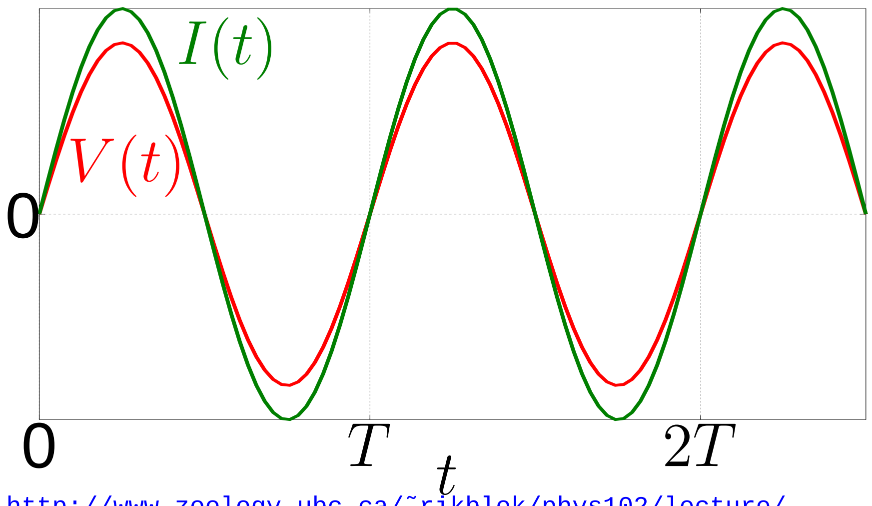


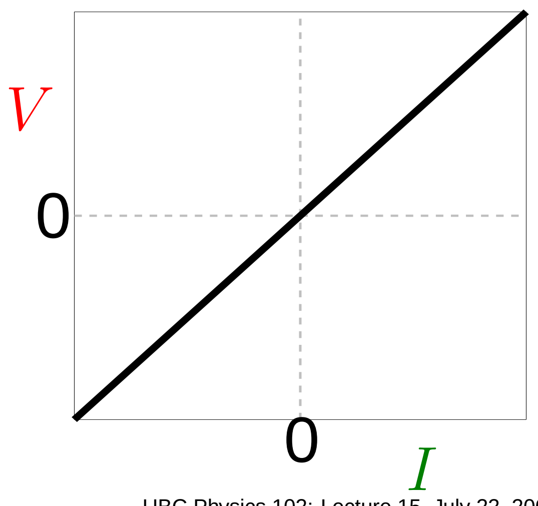
R circuits [Text: Sect. 31-2]

Discussion: R circuits



- Voltage drop across resistor, $V = IR = I_0 R \sin \omega t$ so voltage amplitude is $V_0 = I_0 R$.





L circuits [Text: Sect. 31-3]

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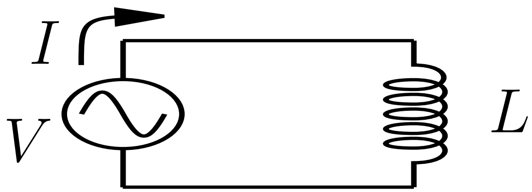

L circuits [Text: Sect. 31-3]

- **Discussion: R circuits, contd**
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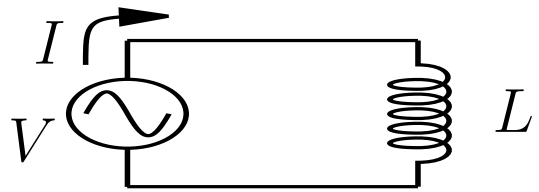
- **Discussion: L circuits**



  - By Kirchhoff's loop rule,

$$\begin{aligned} V &= L\frac{dI}{dt} = L\frac{d}{dt} I_0 \sin \omega t \\ &= \omega L I_0 \cos \omega t. \end{aligned}$$

L circuits, contd

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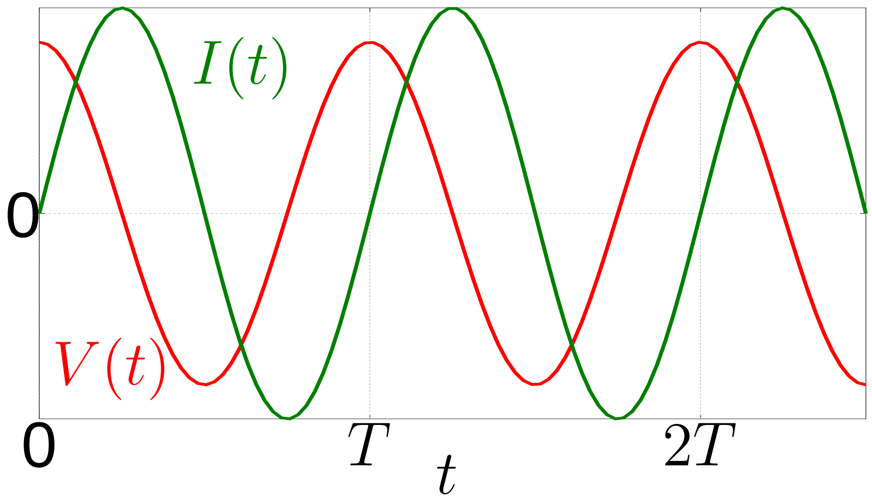
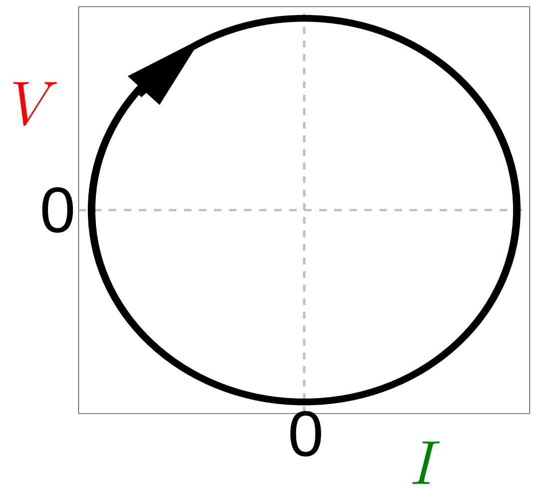
  - Current *lags* voltage by $90°$.





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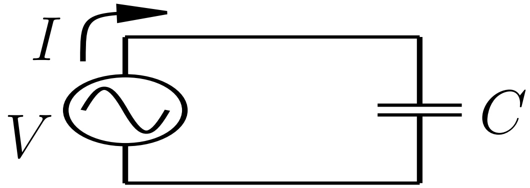
  - Like resistance except current out of phase with voltage.

C circuits [Text: Sect. 31-4]

- **Discussion: C circuits**



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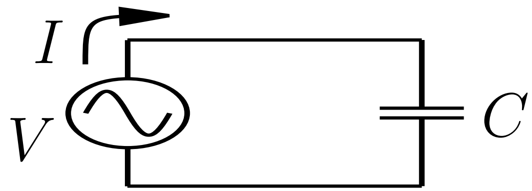

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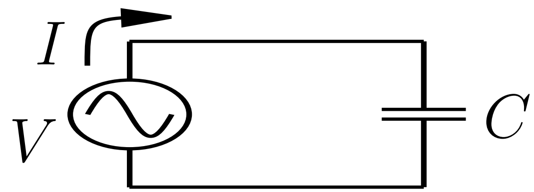

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- How does this relate to current? $I = \frac{dQ}{dt}$ so

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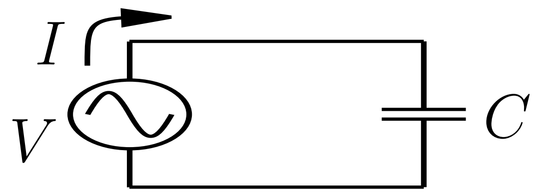

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- How does this relate to current? $I = \frac{dQ}{dt}$ so

$$\frac{dV}{dt} = \frac{I}{C} = \frac{I_0}{C}\sin\omega t.$$

- Solution is

$$V = -\frac{I_0}{\omega C}\cos\omega t.$$

C circuits, contd

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C circuits, contd

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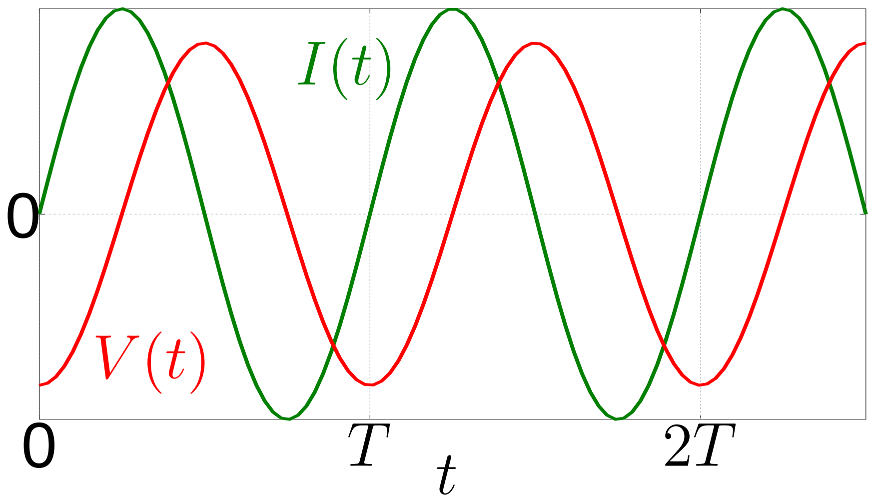
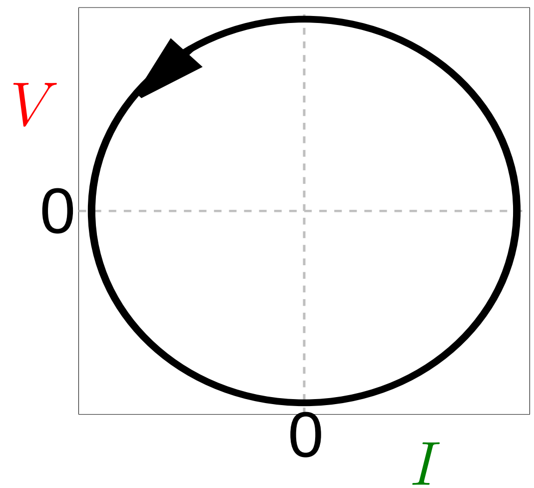
  - Current *leads* voltage by $90°$.





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  - **<u>CIV</u>**IL = in **<u>C</u>**apacitors **<u>I</u>** comes before **<u>V</u>**.

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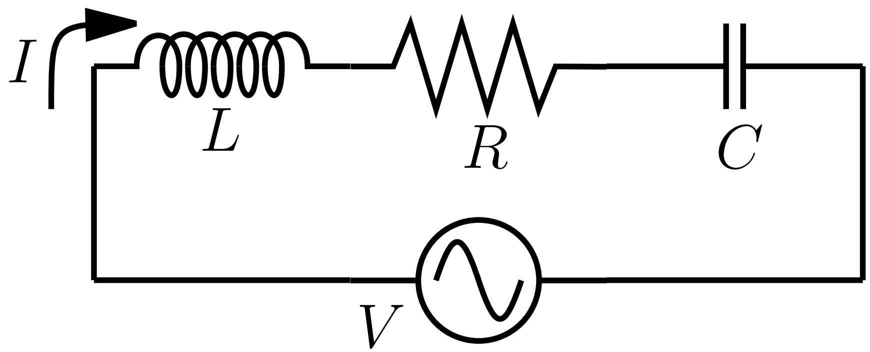
- **Discussion: CIVIL memory aid**
  - **CIV**IL = in **C**apacitors **I** comes before **V**.
  - CI**VIL** = **V** comes before **I** in inductors (**L**).

LRC circuits [Text: Sect. 31-5]

- **Discussion: LRC circuits**



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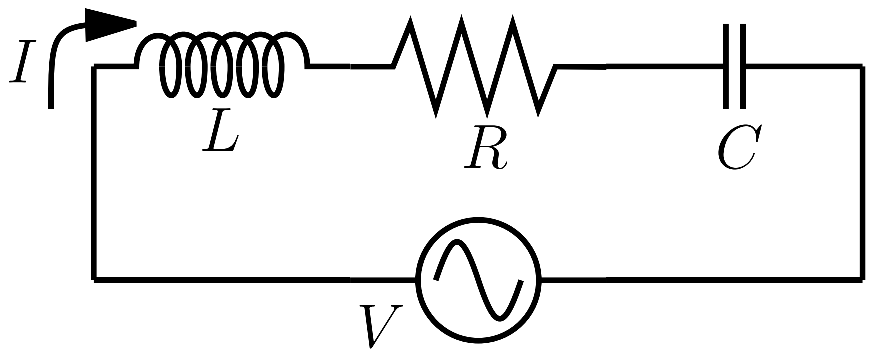

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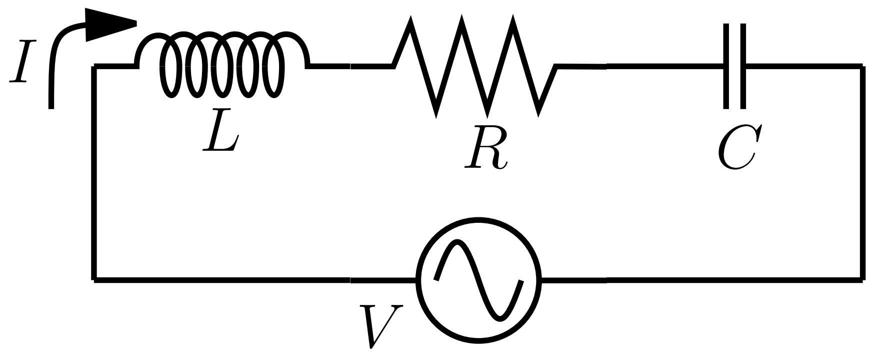

- What if circuit contains all three components in series?
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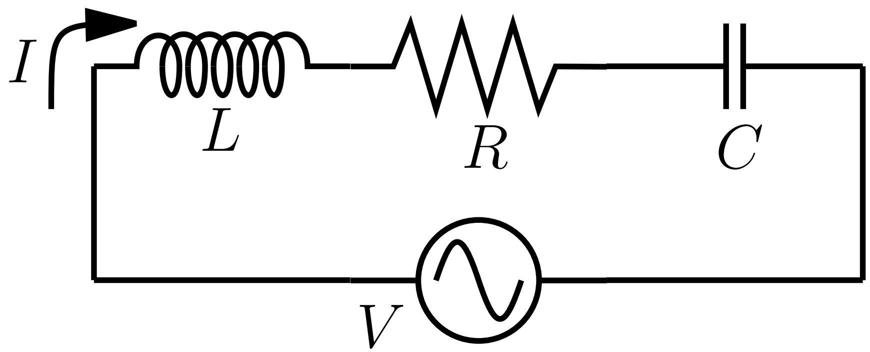

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LRC circuits [Text: Sect. 31-5]

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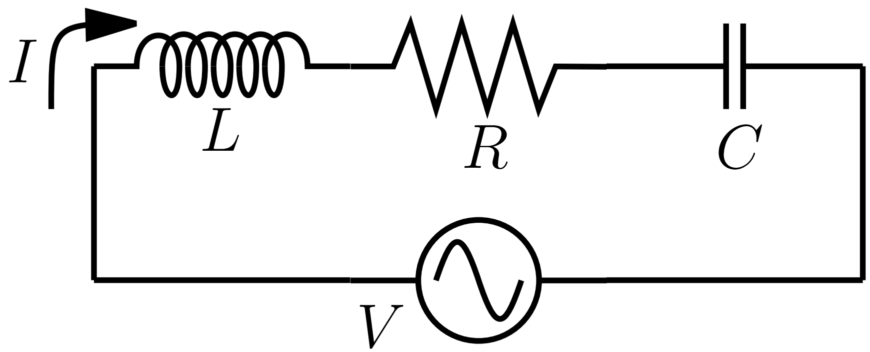

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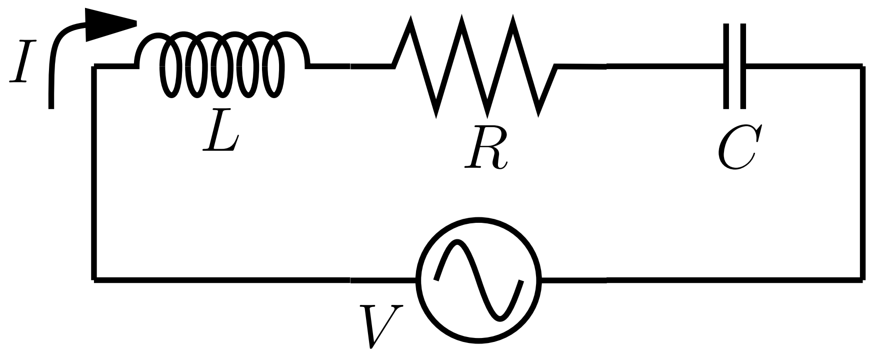

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- And what is V_0?

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- **Discussion: LRC circuits, contd**

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  - From Kirchhoff's loop rule,

$$\begin{aligned} V &= V_R + V_L + V_C \\ &= I_0 R \sin \omega t + I_0 (X_L - X_C) \cos \omega t. \end{aligned}$$

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- **Definition:** *Impedance*
  - After some geometry (hand waving) we find $V_0 = I_0 Z$.
  - Likewise, multimeters will read $V_{\mathrm{RMS}} = I_{\mathrm{RMS}} Z$ where

$$\boxed{Z = \sqrt{R^2 + (X_L - X_C)^2}.}$$

LRC circuits, contd

- **Definition:** *Impedance, contd*

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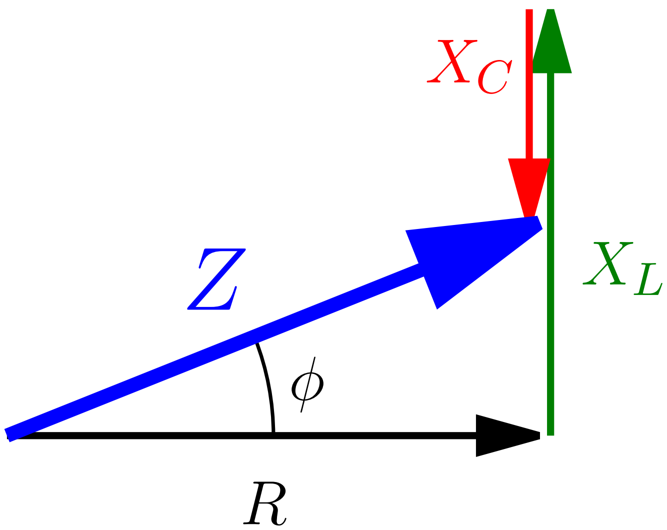
  - Vector representation:



LRC circuits, contd

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  - Summary:

Name	Symbol	Phase shift
Resistance	R	0°
Reactance	X	$\pm 90^\circ$
Impedance	Z	$\phi \quad (\|\phi\| \leq 90^\circ)$

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- **Interactive Quiz: PRS 15b**

LRC circuits, contd

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- **Interactive Quiz: PRS 15b**
- **Derivation: Power**

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- **Interactive Quiz: PRS 15b**
- **Derivation: Power**
  - L and C don't lose energy, they just swap it back and forth.

LRC circuits, contd

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  - Summary:

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- **Interactive Quiz: PRS 15b**
- **Derivation: Power**
  - L and C don't lose energy, they just swap it back and forth.
  - Only R loses energy, as heat.

LRC circuits, contd

- **Derivation: Power, contd**

LRC circuits, contd

- **Derivation: Power, contd**
  - Average rate of power consumption is $\boxed{\overline{P} = I_{\mathrm{RMS}}^2 R}$.

LRC circuits, contd

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- Average rate of power consumption is $\boxed{\overline{P} = I_{\mathrm{RMS}}^2 R}$.
- From vector representation we see $R = Z\cos\phi$ so

$$\boxed{\overline{P} = I_{\mathrm{RMS}}^2 Z \cos\phi = I_{\mathrm{RMS}} V_{\mathrm{RMS}} \cos\phi.}$$

LRC circuits, contd

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- $\cos\phi$ called **power factor**.

Resonance [Text: Sect. 31-6]

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  - Already saw a LC circuit (without any voltage source) oscillates at frequency $\boxed{\omega_0 = \frac{1}{\sqrt{LC}}}$.
  - What is so special about that frequency?
  - If we "push" an LRC circuit with an AC source at frequency ω_0,

$$
\begin{aligned}
X_L &= \omega_0 L &= \sqrt{\frac{L}{C}}, \\
X_C &= \frac{1}{\omega_0 C} &= \sqrt{\frac{L}{C}}.
\end{aligned}
$$

Resonance [Text: Sect. 31-6]

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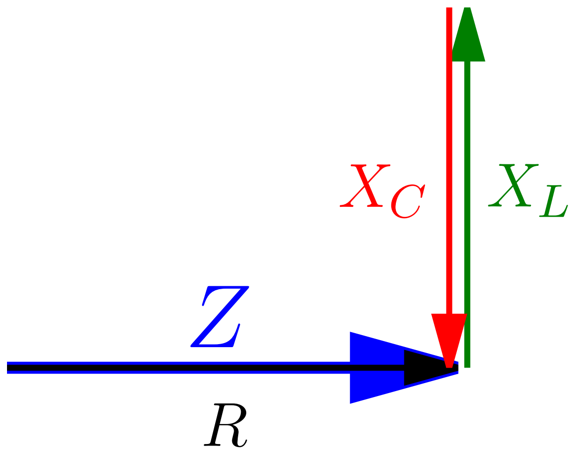
  - So $X_L = X_C$.

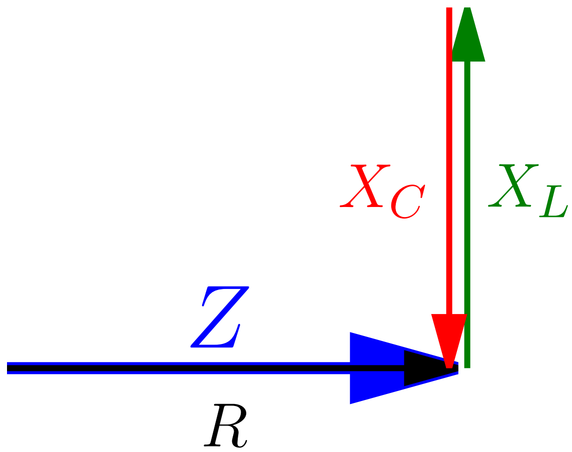
Resonance, contd

- **Discussion: Resonance, contd**



Resonance, contd

- **Discussion: Resonance, contd**



- So $\phi = 0$ and $Z = R$. Changing ω can only make Z bigger.

Resonance, contd

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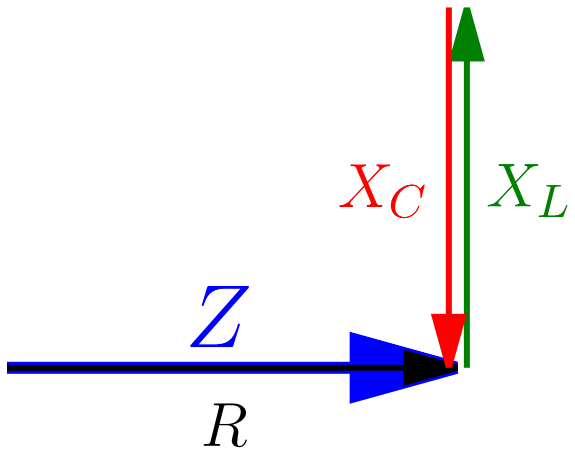

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- **Interactive Quiz: PRS 15c**

End

- **Practice Problems:**
  - Ch. 31: Q. 1, 3, 5, 7, 9, 11, 13, 15.
  - Ch. 31: Pr. 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 35, 37, 39, 41, 43, 45, 51, 53.

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- **Interactive Quiz: Feedback**

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- **Interactive Quiz: Feedback**
- **Tutorial Question: tut15**