#### UBC Physics 102 Lecture 13

**Rik Blok** 

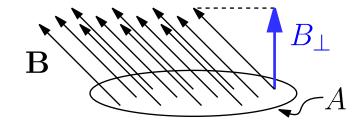


# Outline

- ▷ Faraday's law and Lenz's law
- ⊳ Emf in a moving conductor
- ▷ Transformers
- Self-inductance
- Energy storage
- ⊳ End

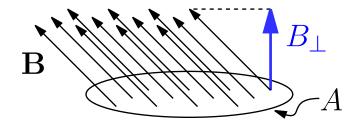


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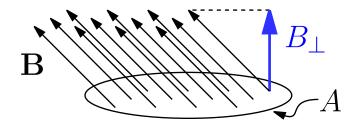


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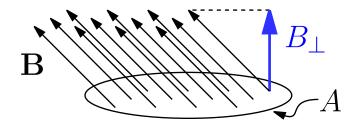
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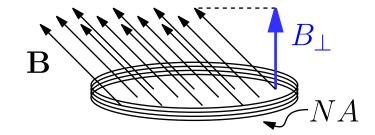
• Quantity of B-field passing through area A.

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- Like  $\Phi_E$  used in Gauss's law but don't need closed surface.
- Don't get to choose surface, is defined by a loop of wire.

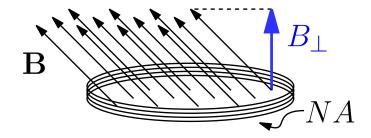


Definition: Magnetic flux, contd





**Definition:** *Magnetic flux, contd* 



 If we have a coil with N loops then same B goes through all so

$$\Phi_B = NB_{\perp}A.$$



**Junit: Weber,** Wb



**• Unit: Weber,** Wb

Unit of *B*-flux,

1 Wb = 1 T 
$$\cdot$$
 m<sup>2</sup>.



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#### Discussion: Induced emf

● Have seen current produce *B*-fields.



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- Can B-fields produce currents?
- Steady *B*-fields *cannot* but changing *B*-fields *can*.
- Definition: Induced current/emf
  - The current or voltage produced by a changing B-field.



Definition: Faraday's law



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$$\mathscr{E} = -\frac{d\Phi_B}{dt}.$$



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Changing flux induces emf equal to rate of change,

$$\mathscr{E} = -\frac{d\Phi_B}{dt}.$$

Third of Maxwell's equations.



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- Third of Maxwell's equations.
- Minus sign indicates direction of emf, or can use Lenz's law.
- Definition: Lenz's law
  - Induced emf (and current) in direction that opposes change in flux.





#### Discussion: Induced magnetic field

Changing flux induces emf.



- Changing flux induces emf.
- Emf produces current.



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- Current generates B-field.



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- Interactive Quiz: PRS 13a



Discussion: Moving conductor



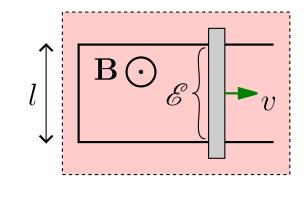
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- Consider circuit: moving rod on conducting rails.

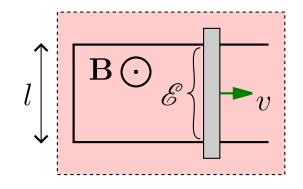


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#### Discussion: Moving conductor

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$$\frac{dA}{dt} = lv.$$

Emf is (ignoring sign)

$$\mathscr{E} = \frac{d}{dt} \Phi_B = B \frac{dA}{dt} = B l v.$$



# Emf in a moving conductor, contd

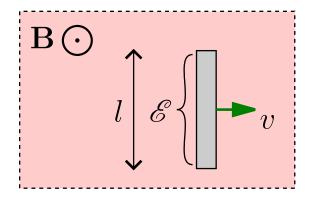
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# Emf in a moving conductor, contd

#### Discussion: Moving conductor, contd

Compare to isolated moving rod.

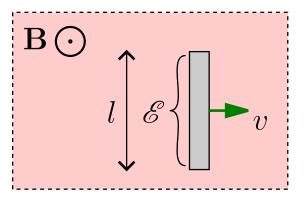




# Emf in a moving conductor, contd

#### Discussion: Moving conductor, contd

Compare to isolated moving rod.



• Force on electrons, F = qvB (up). So *E*-field in rod (down) is

$$E = \frac{F}{q} = vB.$$



Discussion: Moving conductor, contd



### Discussion: Moving conductor, contd

• E uniform so emf (voltage,  $V = -E_l l$ , ignoring sign)

$$\mathscr{E} = El = Blv.$$



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- Same as before!
- Interactive Quiz: PRS 13b



#### Derivation: Transformers



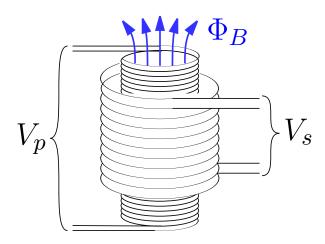
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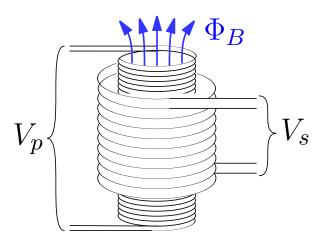
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### Derivation: Transformers

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- Can be used to "couple" 2 isolated circuits:



• Primary coil with voltage  $V_p$  generates flux:

$$V_p = N_p \frac{d\Phi_B}{dt}.$$

Derivation: Transformers, contd



### Derivation: Transformers, contd

• Flux induces voltage  $V_s$  in secondary coil:

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 Flux (per loop) same through both so voltages related by

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• If transformer efficient then power transferred from primary to secondary,  $P_p = P_s$  or  $I_pV_p = I_sV_s$ .



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- So changing current induces emf.
- Emf "impedes" change in current.



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Magnitude of voltage "response" to changing current,

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- Self-inductance is proportionality constant, L.
- Property of object: depends on shape, size, etc. For solenoid,

$$L = \frac{\mu_0 N^2 A}{l}.$$



**Definition:** Self-inductance, contd



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- A circuit component with self-inductance.
- Circuit symbol:



http://www.zoology.ubc.ca/~rikblok/phys102/lecture/

#### Interactive Quiz: PRS 13c



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  - Consider gradually ramping up current  $\hat{I}$  through inductor from zero to I.
  - Emf induced  $\mathscr{E} = -L\frac{d\hat{I}}{dt}$ . Power used by inductor,

$$P = \left| \hat{I}\mathscr{E} \right| = L\hat{I}\frac{d\hat{I}}{dt}.$$



# **Energy storage, contd**

Derivation: Energy storage, contd



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#### Derivation: Energy storage, contd

Power not lost to heat, but stored as potential energy,

$$U = \int P \, dt = \int L \widehat{I} \frac{d\widehat{I}}{dt} dt$$
$$= L \int_0^I \widehat{I} \, d\widehat{I}$$

$$U = \frac{1}{2}LI^2.$$



#### Practice Problems:

- Ch. 29: Q. 1, 3, 5, 7, 9, 11, 19.
- Ch. 29: Pr. 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 37, 39, 41, 43, 45, 55, 57, 59, 63, 65.



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